Valuing Corporate Financing Strategies

Andrea Gamba* Alexander J. Triantis†

July 2008

ABSTRACT

We develop a dynamic structural model of the firm that allows us to carefully analyze the value of alternative financing strategies. We first illustrate the benefits of joint versus separate optimization of dynamic financing and investment policies. We then examine the impact on firm value of investment and financing distortions due to financial agency conflicts, and highlight the compounding of these two distortions in a fully dynamic setting. We show that simple debt contract covenants designed to restrict financing or investment behavior can decrease agency costs quite effectively. We also investigate the performance of various simple financing policy heuristics. We find that these heuristics fail to capture the full value of debt financing, even though the resulting leverage distributions may appear similar to those of optimized financing policies. Generally, our results suggest that firm values can be quite sensitive to the exact specification of financing policies in a dynamic setting.

*SAFE Center, Department of Economics, University of Verona, Verona, Italy and George Washington School of Business, George Washington University, Washington, D.C., USA

†Robert H. Smith School of Business, University of Maryland, College Park, MD

Introduction

The study of corporate financing policies has gradually evolved from a static perspective of explaining the cross-sectional variation of leverage towards a dynamic analysis of changes in debt financing over time. Various dynamic financing strategies, reflecting different levels of complexity, have been suggested to explain the patterns observed in practice. Some studies report evidence consistent with managers being fairly passive in managing capital structure, responding to changes in leverage infrequently and reversing leverage trends only in response to large shocks in profitability or other factors common to the industry or economy.[1] Other research supports the view that managers only address capital structure issues when funding a financing deficit, and appear to follow simple myopic pecking order rules.[2] More sophisticated strategies have also been analyzed, including dynamic policies designed to maximize the present value of net debt tax shields in the presence of numerous frictions, or financing policies that time the issuance of different securities based on managers' perceptions of market inefficiencies.[3]

Empirical data is being examined in increasingly more sophisticated ways in order to evaluate these various theories of dynamic capital structure. At the same time, evidence from recent surveys of CFOs reveals that executives have a tendency to follow practical, informal rules when setting capital structure, focusing on maintaining financial flexibility and a desired credit rating, rather than carefully optimizing their financing strategies to maximize the benefits of debt financing in the face of various market frictions.[4] Taken together, the evidence we have on financing behavior does not clearly point to a consistent strategy followed by all firms, nor a sense that firms are necessarily pursuing sophisticated optimized strategies. This begs the important question of whether the

[1] Welch (2004) concludes that market leverage ratios fluctuate primarily due to movements in equity values, and that debt levels are not actively managed. In contrast, Alti (2003), Fama and French (2002), Flannery and Rangan (2006), Faulkender and Smith (2008), Kayhan and Titman (2007), Kurshev and Strebulaev (2006), Leary and Roberts (2005), Lemmon, Roberts, and Zender (2007), and Strebulaev (2007) present varying evidence regarding reversion of leverage ratios back to a static or moving target.

[2] See, for example, Shyam-Sunder and Myers (1999) and Frank and Goyal (2003).

[3] Fischer, Heinkel, and Zechner (1989), Leary and Roberts (2005), Hennessy and Whited (2005), Gamba and Triantis (2008), Hennessy and Whited (2007), Moyen (2004), Strebulaev (2007), Titman and Tsyplakov (2007), Tserlukevich (2006) and Tsyplakov (2007) provide dynamic financing models based on maximizing the value of debt tax shields net of bankruptcy, tax and other transaction costs. Baker and Wurgler (2002) and Yang (2007) present evidence to support the existence of market timing strategies.

[4] Graham and Harvey (2001) survey American CFOs, while Bancel and Mittoo (2004) and Brounen, de Jong, and Koedijk (2004) survey European CFOs.

value impact of following different financing strategies, particularly ones that appear too simple or suboptimal, is all that large. This valuation issue seems central to motivating a careful study of firms' financing policies, since if values are relatively insensitive to the precise nature of the strategies employed, then recommending particular strategies may not be all that consequential. Yet, this issue appears to be relatively unexplored in the capital structure literature.

We develop a model that allows us to carefully measure the value implications of various financing strategies in a setting which is "fully" dynamic in the sense that both investment and financing can be adjusted over time. While simpler multi-period models in discrete-time are useful for gaining insights into dynamic policies, they are not as appropriate for estimating value impacts of dynamic decision making. Our model structure is similar in many ways to Hennessy and Whited (2007), but we allow for infinite horizon debt subject to issuance costs as well as for partially reversible capital, both of which impact the nature and consequences of debt and capital adjustments over time. Furthermore, we develop variants of this optimization model that allow us to examine the value effects of specific types of deviations from a fully optimized corporate financing strategy.

We first examine the magnitude of the value enhancement attributable to debt financing in our fully dynamic setting. We show that only part of this value (roughly half based on our assumptions) comes from the direct benefit of overlaying an optimally designed financing policy on top of an investment policy that is optimized for an unlevered firm. The other significant portion of the value from debt financing comes indirectly by adapting the investment policy to reflect the presence of debt financing. This result underscores the importance of jointly optimizing dynamic investment and financing policies to extract maximum benefit from debt financing.

We next carefully quantify the impact of investment and financing distortions due to financial agency problems. The current literature on measuring agency costs (e.g., Childs, Mauer, and Ott (2005) and Moyen (2007)) has focused on the value loss associated with investment distortions in the presence of debt financing. We reevaluate the loss due to investment distortions when financing is dynamic. We also examine the distortions directly attributable to the dynamic financing policy, namely that shareholders increase and decrease debt levels less frequently and are more likely to default compared to the first-best financing policy. We find that these financing distortions can decrease firm value more than the agency costs tied to investment distortions. We also show that investment and financing distortions have a compounding effect that further raises the magnitude of the total agency costs.[5]

Given that covenants are often designed into debt contracts to help mitigate these agency problems, we investigate the extent to which such covenants can be successful at reducing agency costs. Recent research on debt covenant violations by Bradley and Roberts (2004), Chava and Roberts (2007), and Roberts and Sufi (2007) indicates that these covenants allow creditors to influence the firm's investment and financing policies when these policies are most likely to cause serious agency problems. We find that simple covenants, such as an upper bound on the firm's book leverage or a restriction on asset sales, can significantly reduce agency costs. Our approach thus provides a mechanism to gauge the relative efficacy of covenants designed to mitigate agency problems.

We also examine the performance of various simple financing heuristics, such as a rule whereby the firm adjusts its leverage to a particular target only when leverage falls outside of a specified range. We find that such heuristics fail to capture much of the potential value associated with debt financing, and in some cases can even lead to a lower firm value than having no debt at all. We explore various factors that might explain the poor performance of such heuristics, and find that both the forced movements to particular preset targets, as well as the prescribed inactivity within the leverage bounds, significantly reduce the net benefits that debt financing could provide. Generally, our results suggest that firm values can be quite sensitive to the exact specification of the financing policy, and that even if the financing policy results in distributions of leverage and debt changes that are not too dissimilar to those from optimized policies, the differential impact on firm value can still be quite significant.[6]

The next section presents the general setting of our model, and then develops the different specifications of the general model required to obtain our results, including full,

[5]Sundaresan and Wang (2006) carefully delineate the different potential agency problems when investment and financing occur at multiple dates, but do not attempt to value agency costs in a fully dynamic model.

[6]Chang and Dasgupta (2006) highlight that debt financing patterns in practice might be erroneously interpreted as being driven by complex dynamic financing strategies, yet simple randomized strategies produce similar empirical results regarding leverage distributions. While we draw a similar conclusion from our analysis of heuristic rules, we take this observation a step further by showing that financing strategies that produce similar distributions of leverage may yet result in significantly different firm values.

partial, and constrained optimization models, as well as a heuristic model. Section II provides results regarding the direct and indirect value associated with debt financing, the measurement of agency costs, the effectiveness of covenants in reducing agency costs, and the performance of various heuristics. Section III concludes the paper.

I. The Model

A. Economic and Financial Setting

We model investment and financing decisions in an infinite-horizon discrete-time dynamic and stochastic framework. The control variables are the book value of assets in place, k, and the face value of outstanding debt, b.

The productivity, θ, of the firm's capital stock is uncertain. We assume that θ follows the process

$$\log \theta_{t+1} = \rho \log \theta_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, \sigma), \quad |\rho| < 1 \tag{1}$$

under the risk–neutral probability.

The operating cash flow before taxes (EBITDA), $\pi(k, \theta)$, depends upon both the book value of assets, $k > 0$, and the productivity, θ:

$$\pi(k, \theta) = \theta k^{\alpha} - F. \tag{2}$$

π exhibits decreasing returns to scale ($\alpha < 1$), and F is a fixed cost.

We assume that capital is homogeneous and that it depreciates both economically and for accounting purposes at a constant rate $\delta > 0$. The company's debt consists of risky callable consol bonds of equal priority with face value $b \geq 0$ and a coupon rate equal to the risk free rate r (coupon income to debtholders is net of personal taxes at the rate τ_b, where $0 \leq \tau_b < 1$). The Earnings Before Taxes (EBT) is equal to the firm EBITDA minus depreciation and interest:

$$y(k, b, \theta) = \pi(k, \theta) - \delta k - rb. \tag{3}$$

The corporate tax function, denoted g, is a convex function of EBT (to model a limited loss offset provision): $g(y) = y\tau_c^+$ if $y \geq 0$ and $g(y) = y\tau_c^-$ if $y < 0$, where τ_c^+ and τ_c^- are the marginal corporate tax rates for positive and negative earnings, respectively, with $0 \leq \tau_c^- \leq \tau_c^+ < 1$.

At any date, the firm can decide to invest or disinvest to reach a new level of assets k'.[7] If there is positive investment, then the cost is represented by $k' - (1-\delta)k$ and investment can be financed either with internal funds, such as current cash flows or cash balances, or with external funds, i.e., debt or equity. On the contrary, if the firm decides to disinvest, the cash inflow is $\ell((1-\delta)k - k')$, with $\ell < 1$ denoting a liquidation price. For notational convenience, for a general capital stock variation, ξ, we define the fuction $\chi(\xi, \ell)$ as $\chi(\xi, \ell) = \xi$ if $\xi \geq 0$, and $\chi(\xi, \ell) = \xi\ell$ if $\xi < 0$.

The firm may also decide at any time to increase or reduce its debt to a new level b' for the next period. Any variation of debt from b to b' entails a direct cost, $q(b', b) = q|b' - b|$, where $q \geq 0$. When the debt level is changed, all the outstanding debt is called at par and new debt is issued at the market value, in order that all debtholders retain equal seniority and preserve the rights of the current debtholders if the firm increases its debt level. Thus, there is also an indirect debt restructuring cost equal to the difference between the market value and the face value of debt.

At a given date, after observing θ and the resulting cash flow, the new level of capital k', and the new level of debt b', are chosen. Investment and financing decisions can be made with no restriction as long as the firm is not in financial distress or in a state of default.

We assume that financial distress takes place when the after-tax operating cash flow is lower than the coupon payment, $rb > \pi(k, \theta) - g(k, b, \theta)$. In this case, the firm sells the minimum amount of capital, at a discount s, to make the promised payment to debtholders. In contrast, the default occurs at a θ value which shareholders select to maximize the value of their equity given the option created by limited liability. In the event of default, assuming proportional bankruptcy costs γ $(0 \leq \gamma < 1)$, debtholders receive the recovery rate $(1 - \gamma)$ times the minimum of the unlevered market value of assets (defined below) and the face value of debt.

[7]We assume that investment instantaneously increases the productive capacity of the firm. In contrast, Tsyplakov (2007) examines the effect of the time-to-build characteristic of capital in certain industries, and shows that this can affect dynamic capital structure choices.

Based on the assumptions above, the residual cash flow to equityholders when the firm is solvent, in state (k, b, θ), and for a decision (k', b') when $b' \neq b$, is

$$
\begin{aligned}
&cfe(k, b, k', b', \theta) = \\
&\qquad \max\{\pi(k,\theta) - g(y(k,b,\theta)) - rb, 0\} + (D(k', b', \theta) - b)\,\mathcal{I}(b') - q(b', b) \\
&\qquad\qquad - \chi\left(k' - k(1-\delta) + \max\left\{\frac{rb + g(y(k,b,\theta)) - \pi(k,\theta)}{s}, 0\right\}, \ell\right), \quad (4)
\end{aligned}
$$

where $D(k', b', \theta)$ is the market price of newly issued debt, if the firm is solvent, considering that the new book value of assets is k', and the new book value of debt is b', and $\mathcal{I}(\cdot)$ is the indicator function for the event $b \neq b'$.

Taxes on cash flow to equityholders are levied at a constant rate $0 \leq \tau_e < 1$. If the optimal residual cash flow for equityholders is negative, then funds are raised by issuing new equity, and a proportional flotation cost λ $(0 < \lambda < 1)$ is incurred. Thus, when the firm is solvent, the net cash flow to equity holders is

$$
e(k, b, k', b', \theta) = \begin{cases} cfe \cdot (1 - \tau_e) & \text{if } cfe \geq 0 \\ cfe \cdot (1 + \lambda) & \text{if } cfe < 0. \end{cases} \quad (5)
$$

B. Optimization

We now set up the valuation of the firm (and its corporate securities) under different optimization conditions. In Section B.1, we model the cases where the investment and financing decisions are made to maximize either the value of equity (second best) or the total firm value (first best) in an unconstrained environment. In Section B.2, we impose constraints due to debt covenant restrictions into the second-best optimization problem. In Section B.3, we present a partial optimization model where the first-best investment or financing policy is imposed, and the other policy is determined to maximize equity value.

B.1. Equity and Firm Value Maximization

We first examine equity value maximization, i.e., "second-best" optimality. The value of equity at state (k, b, θ), $E(k, b, \theta)$, is the fixed point of the Bellman operator

$$E(k, b, \theta) = \max \left\{ \max_{(k', b')} \left\{ e(k, b, k', b', \theta) + \beta \mathbb{E}_{k,b,\theta} \left[E(k', b', \theta') \right] \right\}, 0 \right\}, \tag{6}$$

where θ' is the future state, and the expectation is computed with respect to the risk-neutral probability, conditional on the current state of the firm. $\beta < 1$ is the discount factor for equity flows.[8]

The optimal policy at (k, b, θ) is

$$(k^*, b^*) = \arg\max_{(k', b')} \left\{ e(k, b, k', b', \theta) + \beta \mathbb{E}_{k,b,\theta} \left[E(k', b', \theta') \right] \right\},$$

as long as the resulting equity value from this policy is not negative. Otherwise, the equityholders default on servicing debt and surrender the firm to debtholders. We define Δ^S to be the indicator function of the default event for the second best problem. When default occurs, the firm's depreciated assets are unchanged, i.e., $k^* = k(1 - \delta)$, but the firm becomes unlevered (the former bondholders take over as the new equityholders). Hence, at (k, b, θ) the optimal policy from the equityholders' program is

$$\varphi^S(k, b, \theta) = (k^*, b^*) \left(1 - \Delta^S(k, b, \theta)\right) + (k(1 - \delta), 0)\Delta^S(k, b, \theta), \tag{7}$$

with $\varphi^S(k, b, \theta) \in \mathbb{R}^2$.

[8]The discount factor for equity is $\beta = (1 + r_e(1 - \tau_e))^{-1}$, with r_e denoting the certainty equivalent rate of return on equity flows. Note that in a Miller equilibrium, $r_e = r(1 - \tau_b)/(1 - \tau_e)$, where τ_b is the personal tax on debt income. From this equilibrium condition, the discount factor for bond flows is $\beta = (1 + r(1 - \tau_b))^{-1}$, i.e., it is equal to the one used to discount equity flows. See Sick (1990) for this specification.

To determine the current (ex-coupon) market value of debt if the firm is solvent, $D(k,b,\theta)$, we first define $B(k,b,\theta,\varphi^S)$ as the value to debtholders at (k,b,θ) when the firm's decision, $(k^S,b^S)=\varphi^S(k,b,\theta)$, is made:

$$
\begin{aligned}
B(k,b,\theta,\varphi^S) = & \\
& \left(1-\Delta^S(k,b,\theta)\right)\left(rb(1-\tau_b)+D(k^S,b^S,\theta)\left(1-\mathcal{I}(b^S)\right)+b\mathcal{I}(b^S)\right) \\
& \qquad + \Delta^S(k,b,\theta)(1-\gamma)\min\left\{E(k(1-\delta),0,\theta),b\right\}, \qquad (8)
\end{aligned}
$$

where $1-\gamma$ is the recovery rate on the unlevered firm value $E(k(1-\delta),0,\theta)$, and $\mathcal{I}(\cdot)$ is the indicator function for the event $b \neq b'$. Hence,

$$
D(k,b,\theta) = \beta\mathbb{E}_{k,b,\theta}\left[B(k^S,b^S,\theta',\varphi^S)\right]. \qquad (9)
$$

We now turn to set up the first best solution, where investment and financing policies are selected to maximize total firm value rather than equity value. The value of the firm is $V=E+B$. The Bellman equation for V, when the firm is solvent, is

$$
\begin{aligned}
V(k,b,\theta) = \max_{(k',b')} & \left\{e(k,b,k',b',\theta)+\beta\mathbb{E}_{k,b,\theta}\left[E(k',b',\theta')\right]\right. \\
& \left. +rb(1-\tau_b)+D(k',b',\theta')\left(1-\mathcal{I}(b')\right)+b\mathcal{I}(b')\right\}. \qquad (10)
\end{aligned}
$$

The value of the equity given the maximand (k^*,b^*) from (10) is

$$
E(k,b,\theta) = e(k,b,k^*,b^*,\theta)+\beta\mathbb{E}_{k,b,\theta}\left[E(k^*,b^*,\theta')\right]. \qquad (11)
$$

If E is strictly positive, then (k^*,b^*) is the optimal solution. Otherwise, the firm is in default: the optimal decision is $(k(1-\delta),0)$, and the value of equity is set to zero (and the default indicator for the first best problem, $\Delta^F(k,b,\theta)$, is set equal to one).

To summarize, the optimal policy for the first best case, φ^F, can still be represented as in (7), but for possibly different (k^*,b^*) and Δ^F, given the different optimization goal. The value of debt is again found by applying equation (9) using the optimal policy φ^F in place of φ^S: $D(k,b,\theta)=\beta\mathbb{E}_{k,b,\theta}\left[B(k^F,b^F,\theta',\varphi^F)\right]$.

B.2. Constrained Optimization

To analyze the impact of covenants on firm value, we need to also consider the case of a constrained second-best optimization problem. We impose two different restrictions, one that sets a maximum book leverage ratio, and the other that does not permit disinvestment.

For the first covenant case, the firm solves the equity value maximization problem, given by equations (6) and (9), with the restriction that the (new) book leverage, b'/k', must be no higher than a specified level, L_u. Given the current state (k, b, θ), the set of feasible pairs (k', b') is denoted $\mathcal{A}^l(k, b, \theta)$, so that the optimal policy if the firm is solvent is

$$(k^*, b^*) = \arg\max_{(k',b')\in\mathcal{A}^l(k,b,\theta)} \left\{ e(k, b, k', b', \theta) + \beta \mathbb{E}_{k,b,\theta} \left[E(k', b', \theta') \right] \right\}.$$

Note that as a consequence of the constraint on leverage, the optimal policy function, denoted $\varphi^{S,l}$, and the related default indicator $\Delta^{S,l}$, are different from φ^S and Δ^S.

A special case of this specification is where $L_u = 0$, i.e., the firm is restricted to stay unlevered ($b = 0$ and $b' = 0$). Given the absence of debt, this problem is also equivalent to the first-best optimization with $b = 0$, and we denote the optimal policy for this zero-debt case as φ^Z.

For the second covenant case, we consider a firm which maximizes equity value with the restriction that the firm cannot disinvest, so that the new capital stock after each decision can not be lower than the depreciated capital, i.e. $k' \geq k(1 - \delta)$. The set of feasible decisions is denoted $\mathcal{A}^d(k, b, \theta)$, and the optimal policy if the firm is solvent is

$$(k^*, b^*) = \arg\max_{(k',b')\in\mathcal{A}^d(k,b,\theta)} \left\{ e(k, b, k', b', \theta) + \beta \mathbb{E}_{k,b,\theta} \left[E(k', b', \theta') \right] \right\}.$$

We denote the optimal policy as $\varphi^{S,d}$ and the related default indicator as $\Delta^{S,d}$.

In the above two cases, given $\varphi^{S,l}$ and $\varphi^{S,d}$, the value of debt is found by applying equation (9) using the respective optimal policy in place of φ^S.

B.3. Partial Optimization

In order to isolate several valuation effects, we also analyze partial optimization problems where either the investment or financing policy is pre-specified according to some previously determined optimal policy, and the other policy is then selected to maximize either firm or equity value. More precisely, given φ^i, with $i = F, S, Z$ as defined in Sections B.1 and B.2, we can take the investment policy,[9] $k_i = (\varphi^i)_1$, and maximize either the firm value or the equity value with respect to the financing policy. The resulting policy is defined as:

$$\varphi^{i,j(i)}(k,b,\theta) = (k(1-\delta),0)\Delta^{i,j(i)}(k,b,\theta) + (k^i, b^{j(i)})\left(1-\Delta^{i,j(i)}(k,b,\theta)\right) \tag{12}$$

where $j = F$ if financing is selected to maximize firm value, based on the Bellman operator

$$\begin{aligned} b^{j(i)} = \arg\max_{b'} \big\{ & e(k,b,k^i,b',\theta) + \beta\mathbb{E}\left[E(k^i,b',\theta')\right] \\ & + rb(1-\tau_b) + D(k^i,b',\theta')\left(1-\mathcal{I}(b')\right) + b\mathcal{I}(b')\big\}; \end{aligned}$$

or $j = S$ if financing is selected to maximize equity value, based on the Bellman operator

$$b^{j(i)} = \arg\max_{b'} \left\{ e(k,b,k^i,b',\theta) + \beta\mathbb{E}\left[E(k^i,b',\theta')\right]\right\}.$$

$\Delta^{i,j(i)}$ is the indicator function of $E \leq 0$ at $(k^i, b^{j(i)})$.

To illustrate, a possible specification of the above policy can be $\varphi^{F,S(F)}$: the first-best investment policy has been imposed, and the financing decision is then selected to maximize equity value, conditional on the first best investment (hence the notation $S(F)$ for the financing policy).

An analogous model specification arises when the financing policy $(\varphi^i)_2$ is pre-specified, and the investment policy is determined to maximize firm or equity value. An example of this case is $\varphi^{S(F),F}$: the investment policy is chosen to maximize the equity value ($j = S$), conditional on the first best financing policy ($i = F$).[10]

[9] We use $(v)_n$ to denote the n-th component of a vector v.

[10] The notation in this section also encompasses the case where either i or j is equal to Z. When $j = i$, $\varphi^{i,j(i)} = \varphi^{j(i),i} = \varphi^i$ is just either the firm ($i = F$) or equity ($i = S$) value maximization case in

C. Heuristics

We now define the valuation problem for the situation where the financing strategy is determined by a rule of thumb rather than by the maximization of either total firm value or equity value. Since we are now in a setting where simple heuristic rules are being used for the financing decision, it seems inconsistent to assume that the firm would yet be so careful as to jointly determine its investment decisions together with the financing policy. Rather, we assume that the firm optimizes its investment policy to maximize unlevered firm value since it is common in practice that the investment policy is separately determined (we also look at the case where the investment policy from the first-best case is used for a robustness check).

We assume that equityholders use a simple decision rule to control either the book leverage ($h = L$) or quasi-market leverage ($h = M$). While the capital structure and investment decisions are effectively made at the same date, we technically assume that the financing decision is made right after the investment decision if the firm is solvent. In the case where the optimal investment decision, together with the capital structure decision, does not yield a positive value for equity, the firm defaults, leaving the capital at the current level $k(1-\delta)$, and becoming unlevered after paying the bankruptcy costs.

A heuristic decision rule for capital structure can be expressed as a function of the current state, (k, b, θ), and of the investment decision, $k^j = (\varphi^j)_1$, where k^j can be either from the constrained unlevered specification ($j = Z$) or the first best ($j = F$). For convenience, we denote it $b^h = f^h(k, b, \theta, k^j)$, with $h = L, M$.

The first rule of thumb we consider is based on keeping the book leverage L, defined as debt face value over book value of assets, within the region $L_d \leq L \leq L_u$, where L_d and L_u are the lower and upper bounds of the region. Specifically, at (k, b, θ), and given the chosen new investment level k^j, $j = F, Z$, if the debt level is such that the book leverage based upon the new investment level, denoted by $L = b/k^j$, is within this region, then the debt level is left unaltered: $b^h = b$. If instead either $L > L_u$ or $L < L_d$,

Section B.1. Two additional cases (Z,L-Con.Opt and Z,L-Con.Opt-2B) introduced later in the paper also belong to the class of problems in Sections B.2 and B.3.

then b^h is chosen so that the book leverage is equal to a given target book leverage L_t ($L_d \leq L_t \leq L_u$), i.e., $b^h/k^j = L_t$. To summarize, the rule of thumb function is

$$f^L(k, b, \theta, k^j) = \begin{cases} b & \text{if } L_d \leq \frac{b}{k^j} \leq L_u \\ k^j L_t & \text{otherwise,} \end{cases} \tag{13}$$

for $j = F, Z$.[11]

The second rule of thumb is similar to the first, but based on quasi-market leverage (debt face value over the sum of debt face value and market value of equity) rather than book leverage. At (k, b, θ), given k^j, $j = F, Z$, we denote the quasi-market leverage at the new level of capital stock as $M = b/(b + E(k^j, b, \theta))$. If $M_d \leq M \leq M_u$, then the debt level must remain unchanged, i.e., $b^h = b$. Otherwise, b^h is chosen so that $b^h/(b^h + E(k^j, b^h, \theta)) = M_t$ ($M_d \leq M_t \leq M_u$). Note that $\frac{b}{b+E(k^j,b,\theta)} = M_t$ uniquely defines $\widehat{b} = \widehat{b}(k^j, \theta)$, because the value of equity is a strictly decreasing function of b, so that the left–hand–side of the above equation is strictly increasing in b. The rule of thumb function (for $j = F, Z$) is thus

$$f^M(k, b, \theta, k^j) = \begin{cases} b & \text{if } M_d \leq \frac{b}{b+E(k^j,b,\theta)} \leq M_u \\ \widehat{b}(k^j, \theta) & \text{otherwise.} \end{cases} \tag{14}$$

Given k^j, $j = F, Z$, and $b^h = f^h(k, b, \theta, k^j)$, with $h = L, M$, the value of equity and debt are determined as the fixed point of a system of non-linear equations like (9) and (11). The application of the rule of thumb may entail both a direct and an indirect transaction cost to change the capital structure and to adapt the production capacity. When the state and/or capital is low and the debt is high, equityholders may find it unprofitable to bear the costs related to the implementation of (k^j, b^h), because the equity value in (11) is zero or negative. In this case, they default and the unlevered firm is given to debtholders, net of bankruptcy costs. This means that the policy (k^j, b^h) cannot be implemented and in its place we have $(k(1-\delta), 0)$. Hence, the actual investment and financing policy when a rule of thumb is used is

$$\varphi^{j,h}(k, b, \theta) = (k(1-\delta), 0)\Delta^{j,h}(k, b, \theta) + (k^j, b^h)\left(1 - \Delta^{j,h}(k, b, \theta)\right), \tag{15}$$

[11] We also later introduce a slightly different heuristic, denoted Z,L-2B, where the target is equal to the closest boundary of the inactivity region, i.e., where $L_t = L_d$ if $b/k^j < L_d$ and $L_t = L_u$ if $b/k^j > L_u$.

where $\Delta^{j,h}$ is the default indicator function related to $\varphi^{j,h}$.

So, to find the value of the corporate securities under the application of a heuristic capital structure decision, we have to find the fixed point of the system

$$\begin{aligned} E(k,b,\theta) &= e(k,b,k^{j,h},b^{j,h},\theta) + \beta\mathbb{E}_{k,b,\theta}\left[E(k^{j,h},b^{j,h},\theta')\right] \\ D(k,b,,\theta) &= \beta\mathbb{E}_{k,b,\theta}\left[B(k^{j,h},b^{j,h},\theta',\varphi^{j,h})\right], \end{aligned} \tag{16}$$

where the function B is defined in (8), and $(k^{j,h},b^{j,h}) = \varphi^{j,h}(k,b,\theta)$ from the policy function in (15).

Table I summarizes the specific cases of the different optimizations described above which we solve and discuss in the next section.

II. Results

A. Direct and Indirect Effects of Financing on Firm Value

We first analyze the value contribution attributable to debt financing by examining three different cases: 1) the zero-debt case, Z, where the firm optimizes its investment policy to maximize firm value; 2) the Z,F(Z) case where the firm uses the optimal unlevered investment policy found in Z, and then optimizes its financing policy conditional on this investment policy, i.e., F(Z), to maximize firm value; and 3) the first-best case, F, where both investment and financing policies are jointly optimized to maximize firm value.

Figure 1 shows the firm values in the first two cases (Z and Z,F(Z)) as a percentage of the first-best (F) value, across a range of values for the underlying stochastic profitability variable, θ, assuming the firm starts with an intermediate value of capital and an initial debt level of zero.[12] The firm valuations are based on the parameter values shown in Table II, which are similar (or equivalent) to those used in other dynamic financing and investment papers (e.g. Gamba and Triantis (2008)).

[12] Using the intermediate value of capital allows us to capture a steady-state representation of the firm in terms of capital in place. The zero debt level enables us to measure the value of debt financing relative to the zero debt case.

By benchmarking against the first-best value, we highlight the value that may be lost by not following the best policy that jointly optimizes investment and financing. Comparing Z to the first-best case, the value gained from having debt financing is approximately 3.5-6% of the value of the unlevered firm, depending on the level of current profitability as captured by θ.[13] This reflects the present value of the corporate interest tax shields and lower issuance costs of debt financing relative to equity, net of the effects of personal taxes, bankruptcy costs and the direct and indirect costs associated with changing debt over time. Given the number of factors affecting the value of debt financing and their complex interaction, the value attributable to debt financing is difficult to accurately quantify absent a dynamic model such as the one we develop. Note that debt financing has a larger percentage contribution to value when θ is lower. This effect is due to the fact that the unlevered firm value declines significantly when current profitability is very low, while the present value attributable to the net benefits of future debt financing drops by a relatively lower percentage.[14]

The intermediate case Z,F(Z) shows that if the firm follows the optimal unlevered investment policy, and then selects a dynamic debt policy to maximize firm value, only part (between one third to two-thirds for most θ) of the value of debt financing is captured. The residual value attributable to debt financing comes from the firm altering its investment policy given that it can finance investment with debt rather than just equity, and can take further advantage of the net benefits of debt. This highlights that there is both a direct and indirect benefit to debt financing, and to realize the full value potential of debt financing, the firm must jointly optimize its investment and financing policies.

B. Agency Costs due to Investment and Financing Distortions

We now examine the loss in firm value due to investment and financing distortions that emerge from shareholder-bondholder agency conflicts. We calculate the values for three second-best cases where either investment, financing, or both are selected to maximize

[13]For values of θ at or above the median, the net value of debt financing is approximately 4% of asset value, which is consistent with the recent empirical finding in van Binsbergen, Graham, and Yang (2007).

[14]The dips in percentage value for the Z and Z,F(Z) cases relative to F also reflects that while value increases monotonically with θ for all cases, it does so at different rates for the different cases.

shareholder value, rather than firm value. The second-best case (denoted by S) captures the total agency costs due to shareholder-bondholder conflicts since both investment and financing policies are jointly optimized to maximize shareholder value. We isolate the part of the agency costs due solely to investment distortions by designing an intermediate second-best case where the first-best financing policy is used, but the investment policy is selected to optimize shareholder value. This case is denoted by S(F),F, signifying that the investment policy is second best conditional on the first best financing, i.e. S(F), while the financing policy is equivalent to that in the first-best case (F). Similarly, in the case F,S(F), we isolate the effects of financing distortions by using the first-best investment policy (from F), but selecting the financing policy to optimize shareholder value conditional on following the first-best investment policy.

For each of these three cases, as well as the zero-debt case which we provide again as a baseline, Figure 2 shows the resulting firm value as a percentage of the first-best firm value benchmark as in Figure 1, again looking across a range of θ values, and assuming the firm starts with an intermediate value of capital and a debt level of zero.[15] While agency costs are larger when there is a pre-existing higher debt level, the zero initial debt level seems to be particularly appropriate when benchmarking against the zero-debt case. We will later examine the entire distribution of firm value levels to ensure that our results are not specific to particular nodes in the state space.

As discussed previously in the context of Figure 1, the curve for the zero debt case (Z) relative to the first-best case indicates that having access to debt financing, and dynamically optimizing it together with the investment policy in order to maximize firm value, increases firm value around 3.5%-6% relative to an all-equity financed firm. However, if the financing and investment policies are selected to maximize shareholder value rather than firm value, a significant portion of the value potential of debt financing will not be captured. For instance, at the median θ value of one, the second-best (S) value is at approximately 97% of the first-best value, indicating that agency costs are about 3% of first-best value and thus erase almost three-quarters of the potential value from debt financing.

[15]In all figures (except Figure 4, where we plot distributions), we assume an intermediate value of capital of 8.53, which is the median capital level for the S, Z, and Z,F(Z) benchmark cases (based on the simulations we perform).

Agency costs result from distortions to both investment and financing policies in our fully dynamic framework. Debt overhang causes underinvestment in new capital relative to the first-best case, as well as excessive disinvestment in low states. This agency problem has been studied by Childs, Mauer, and Ott (2005), Moyen (2007) and others by assuming a first-best financing policy (or the simpler case of a static optimal capital structure) and examining the difference between employing a first-best versus second-best investment policy. Our S(F),F case captures exactly this agency problem of investment, and we find a value loss of approximately .5%-1.5%.

Our framework also allows us to capture in a similar way the effects of distortions to the financing policy due to agency conflicts. Such agency costs stem from shareholders avoiding debt repurchases in low profitability states, thus being more prone to allow the firm to slip into default, and also from being reticent to issue additional debt in high profitability states. Analogous to the way in which we captured the agency costs associated with investment distortions, we examine the case F,S(F) where the first-best investment policy (from F) is followed, while the financing policy is optimized by shareholders conditional on the pre-selected first-best investment policy. The corresponding curve in Figure 2 shows that the cost associated with financing distortions is 50-100% larger than the magnitude of the cost associated with investment distortions based on our input assumptions. This suggests that agency costs due to financing distortions may be even more important to consider than those associated with investment distortions, despite the fact that they have been relatively overlooked in the literature, largely since most agency models do not capture the dynamic evolution of financing.

Note that the loss in value in the second-best case is not a simple sum of the value losses from each of the two constrained second-best cases where only either investment or financing is distorted. Rather, there appears to be a considerable compounding effect of these agency problems. Both investment and financing agency problems are anticipated by creditors and are priced into debt contracts, and in turn reduce the incentive to issue additional debt over time and make it quite costly to buy back debt at par. In fact, as we shall see later, the firm takes on much less debt in the second-best case relative to the first-best case. The resulting effect on firm value is that the contribution from debt financing is quite low.

C. The Impact of Covenants on Firm Value

Covenants are typically included in debt agreements to prevent the types of agency problems that lead to the value loss examined above. We study the impact on firm value of imposing two common covenants. The first covenant requires that the firm maintains a book leverage ratio below a specified upper bound. In practice, this might be alternatively framed as a minimum net worth constraint.[16] Roberts and Sufi (2007) provide evidence that firms decrease net debt issuing activity following a financing covenant violation, indicating that there is a mechanism in place to prevent increases in debt, or to counter the unwillingness to decrease debt, when the firm is highly leveraged.

The second covenant we examine constrains the firm from disinvesting. In other words, the maximum rate of decrease in the capital level is the depreciation rate. This prevents shareholders from liquidating the firm's assets, particularly as the firm approaches default, in order to pay themselves large dividends. In practice, it is common to find covenants restricting the ability to divest assets without bondholder approval, as well as covenants that prevent the payment of liquidating dividends. This absolute restriction on disinvestment might be unnecessarily strict in that it restricts the firm from reducing the capital level in bad times even if there is little debt and bondholders might support the decision to sell capital. However, we frame the covenant in this simple way to show that even though there may be some value loss attributable to the restriction, there is nonetheless a very significant net increase in value (i.e., we are understating the reduction in agency costs from preventing investment distortions).

Figure 3 shows the firm values corresponding to the second-best case (S), as well as to the second-best case constrained by each of the two covenants, as a percentage of the first-best firm value across a range of θ values, again assuming an initial intermediate value of capital and a zero debt level. The presence of either covenant increases firm value. The covenant that places an upper bound of .55 on book leverage eliminates at least two-thirds of the value loss attributable to agency costs.[17] It does so not only by preventing the firm from following a financing policy that allows the firm to drift

[16] We have also examined a minimum coverage ratio (EBITDA/interest) covenant, which yields similar results since this metric is highly correlated with book leverage in our model.

[17] We use an upper bound of .55 for illustrative purposes; it does not necessarily represent an optimized covenant. This upper bound, though, is empirically plausible if investors want to ensure that the debt remains within the investment-grade range of debt ratings.

into default in low states, but also by indirectly restricting shareholders from selling off capital, which would otherwise result in too high of a book leverage ratio. Thus, this financing covenant is particularly effective in our setting. The covenant which directly prevents disinvestment also decreases agency costs, though rather less effectively than the covenant that bounds the leverage ratio. As mentioned above, this could be due to restricting asset sales that might increase firm value in some circumstances.

While Figure 3 shows the firm value results for a particular starting point of capital and leverage, we also find similar results by examining the distribution of firm values based on a Monte Carlo simulation with 10,000 replications along 50-year time paths of state variables, where the firm visits different capital-leverage nodes through time. The percentiles of these distributions are shown in Table III, based on using the same state variable paths in each of the cases examined. The constrained maximum leverage covenant case delivers values that are close to the first-best case, while the no disinvestment covenant case leads to values that are only somewhat above the second-best case, consistent with our findings in Figure 3.

The impact of covenants on the firm's financing decisions can also be seen more directly in Figure 4 by examining the distribution of the firm's book leverage for the different cases shown in Figure 3 (based on running the same 10,000 sample paths for 50 time steps in each case). In the first-best case, an aggressive debt policy can be pursued since debt holders know that when there is a negative shock to profitability, leverage will usually be reduced by the firm in order to prevent costly default. In contrast, under equity value maximization, investors will value debt at a significant discount endogenizing shareholders' incentives to distort investment and financing policies, and thus shareholders will only be willing to issue a very moderate amount of debt. However, when the covenant that constrains leverage is introduced, the optimal leverage increases, and despite the upper bound of .55 imposed on the book leverage distribution, the average leverage is very close to the average for the first-best case. In contrast, the leverage for the case where the covenant constrains the ability to sell capital is significantly lower, closer to the second-best case. Since agency problems in the restricted disinvestment case are only moderately abated, the agency costs still result in significantly reduced incentives to increase debt financing, as in the second-best case.

Finally, we examine the distribution of debt level *changes* (as a percentage of the current firm value) in Table IV. The last column shows the probability of positive or negative debt changes. These probabilities indicate that the firm manages its debt level most actively (about 60% of the time) in the first-best case, and much less actively in the second-best case, as expected. Note that under the maximum leverage covenant, the firm decreases its debt more frequently than under the second-best case (19% of the time versus 12%), and with larger magnitude of changes (the percentile changes shown are conditional on the change in debt being either positive or negative). This is consistent with the ability of this covenant to control agency costs and to allow the firm to dynamically manage its capital structure in order to capture greater value from debt financing.[18]

D. The Performance of Financing Heuristics

We now examine the value impact of using heuristic, rather than optimized, financing policies. Figures 5 and 6 illustrate the performance of a simple heuristic rule based on managing the firm's book leverage. We focus on book leverage given that this is a key metric that rating agencies – and thus firms – appear to focus on when managing their leverage (another important metric is interest coverage, but in our model this is closely correlated to book leverage).[19] The rule specifies that the firm maintains its current debt level if the book leverage is between .35 and .55, and otherwise rebalances the debt level to a target book leverage of .45. The upper and lower bounds for the band of acceptable leverage are chosen consistent with a range of empirical book leverage ratios for firms with lower investment grade ratings, which tends to be a sweet spot that many firms target.[20] We will later examine the effect of using heuristics that are based on alternative leverage bounds and target policies.

[18]In unreported results, we find that debt reduction in the second best case happens for θ values around .75. When $\theta > 1$, debt reduction doesn't occur because the firm is doing well. When θ is significantly below 1, debt reduction doesn't occur because the firm is doing very poorly and the indirect cost to restructuring debt (due to buying it back at par) is too high. For intermediate θ values, the equityholders find it optimal to reduce debt to avoid triggering financial distress. Note that in the absence of a financial distress condition, there will never be any debt reduction in the second best case, as pointed out in Dangl and Zechner (2004).

[19]We have replicated our results using heuristics based on Debt/EBITDA and interest coverage ratios, and they yield virtually identical results. We also have examined heuristics based on market leverage, which we report later in this section.

[20]This range is roughly in line with the 25th and 75th percentiles of the distribution of book leverage for BBB rated firms during 1997-2006, where book leverage is calculated as long-term and short-term debt divided by long-term and short-term debt plus book shareholders' equity. Using the statistics in Table I of Kisgen (2006) (for 1986-2001), the upper bound of 55% is approximately the median book

The graphs shown in Figures 5 and 6 differ based on the assumptions made regarding the investment policy. It seems conceptually inconsistent to assume that a firm would follow a simple heuristic rule to determine its financing policy, and yet would carefully optimize its investment policy conditional on this financing policy. Instead, we assume that the firm follows a prespecified (exogenous) investment policy.[21] In Figure 5, we assume that the firm uses the unlevered investment policy, i.e., it optimizes its investment decisions assuming equity financing, and then uses a simple financing rule to obtain benefits from debt financing.[22] In Figure 6, we assume alternatively that the first-best investment policy is used, in order to provide a robustness check.

Examining Figure 5 more carefully, the three curves represent the percentage of first-best value (conditional on using the optimal unlevered investment policy, i.e., the case Z,F(Z)) for the following three cases: the all-equity financing (Z) case; the case where the financing policy is selected to optimize shareholder value (Z, S(Z)); and the case where the financing heuristic described above is used (Z,L). The zero-debt case is again a useful benchmark in that it shows the percentage value loss from having no access to debt financing, and thus the potential gain from introducing debt financing given a particular investment policy. The first-best financing policy adds 2-3% to the value of the firm, consistent with what we saw in Figure 1. The second-best financing policy captures only about half of this potential additional value from debt financing, and much less when the state variable is low and little or no debt is issued by shareholders. The heuristic financing policy performs rather poorly, contributing only a fraction of the value enhancement from debt when firm profitability is high. For values of θ below 1.5, the heuristic actually begins to destroy value. In other words, the firm is better off having no debt whatsoever, rather than having a naively managed debt policy.

Before analyzing the shortcomings of the heuristic policy more carefully, we note that Figure 6 shows that when the first-best investment policy is used, the heuristic financing policy (F,L) seems to provide some value relative to the all-equity case (F,Z),

leverage for BB rated debt, while a debt level of 35% would be more consistent with a highly rated AA bond, which is a less populated region for corporate bonds.

[21]It is quite possible that firms use heuristic rules when making investment decisions as well. McDonald (2000) investigates investment policy heuristics, and finds that they may lead to a reasonable approximation to first-best value (in an unlevered setting). We do not simultaneously pursue the issue of investment heuristics here, since our objective is to focus solely on the effect of employing financing heuristics.

[22]Most real options models that have been developed to guide normative investment decision making in a dynamic setting explicitly or implicitly assume all-equity financing.

but yet is still far inferior to second-best (F,S(F)) or first-best financing strategies (F). The value gap for the zero-debt case (F,Z) ranges from 10% for high θ levels to 25% for low levels of θ. This is much larger than the gap between Z and F shown in Figure 1 since the zero-debt case here is based on the first-best investment policy rather than the investment policy that is optimized for no debt financing. The higher capital levels of the first-best case require the firm to access costly equity financing since no debt financing is available.

The heuristic financing policy is able to capture only some of the potential value gain from debt financing in the higher θ cases, and as we saw in Figure 5, it might even destroy value relative to an unlevered firm. There are various suboptimalities associated with the heuristic financing policy, which combined are apparently quite costly. First, the heuristic may be leading the firm to releverage at low and high debt levels more often than it should, leading to the frequent payment of direct issuance costs of debt and equity, as well as bearing the indirect cost of covering the discount on debt when the firm repurchases debt at par. Second, the heuristic might lead to a higher incidence of default (despite the presence of an upper bound on leverage) and thus debt would sell at a greater discount, which in turn increases the indirect cost associated with forced financial restructurings, since debt is repurchased at its par value. Third, it is possible that the firm should be rebalancing its capital structure more often *within* the leverage bounds, rather than simply following a policy of inertia in this region as the heuristic specifies. Fourth, the firm always releverages back to a single target leverage ratio, which is unlikely to be optimal given that it does not take into account the existing leverage ratio (e.g. whether the leverage is above the upper bound or below the lower bound). We investigate these four possible explanations in turn.

The first possible explanation for the poor performance of our heuristic policy is that it forces the firm to rebalance its debt either too frequently or too much, thus generating significant losses due to transaction costs. To see whether this is indeed true, we examine the changes in debt that result from the heuristic financing policy relative to changes in the first-best and second-best dynamic financing policies. The last column in Table V shows that the heuristic (either F,L or Z,L) results in much less frequent debt changes (for the same underlying simulation paths), which is consistent with the relatively large region of inertia between the upper and lower leverage bounds. However, when a debt change does takes place, the magnitude of the change is much larger than in the first-best case (particularly for extreme movements).

To see whether the effect of less frequent, but larger, movements in debt could lead to larger transaction costs for heuristics relative to optimal financing strategies, and thus poorer performance, we compare the performance of the heuristic with and without issuance costs. Figure 7 shows the value of the Z,L heuristic relative to the first-best financing strategy Z,F(Z) (this percentage curve denoted by Z,L), as well as the same ratio when there are no debt or equity issuance costs (the percentage curve when there are zero costs is denoted by Z,L-ZC). The curves are very similar to each other for higher θ values, but there is a slight improvement for lower θ values when there are no issuance costs. This implies that the issuance costs do not appear to explain much of the poor performance of the heuristic, particularly for higher θ values. We also find later that a heuristic which nudges the leverage ratio back to the closest boundary (rather than a single internal target) when it exits the bounds does not perform much better, even though the magnitude of debt changes are significantly reduced.

In addition to direct financing issuance costs, the firm also bears the indirect cost associated with the discount on debt when the firm restructures its debt. Despite the upper bound imposed by the heuristic, we find that the default incidence is higher in the case where the heuristic is imposed than in either the first-best or second-best cases. Presumably there are cases where shareholders would find it preferable to default on the debt rather than to repurchase debt (at par) in order to bring the leverage ratio back into the acceptable range. Put differently, given that we don't allow the firm the option of having speculative grade debt, the firm may occasionally simply default on its debt obligations. If this were to explain the poor performance of the heuristic (particularly when θ is low), then eliminating the deadweight costs associated with default, and thus reducing the significant discount on the value of debt, might markedly improve the performance of the heuristic relative to the first-best case. The curve labeled Z,L-ZBC in Figure 7 examines this possibility. For low θ values there does appear to be some improvement, but overall default costs appear to have a limited ability to explain the poor performance of the heuristic.

The remaining explanations for the poor performance of the heuristic concern the ways in which the heuristic limits the ability of the firm to fine tune its financing policy. The simple heuristic we have presented does not allow the firm to optimize its financing policy when the leverage lies within the acceptable leverage bounds. Furthermore, if the leverage exits these bounds, it forces the leverage back to a specific interior target rather than allowing the firm to pick the best target within the bounds.

In Figure 8, we examine the effects of relaxing these restrictions. The curve labeled Z,L-Con.Opt.-2B allows the firm to change its debt level if it is within the acceptable leverage bounds (in order to maximize firm value), but if the current debt level would result in a leverage ratio outside of the acceptable bounds, it forces the leverage to the closest boundary (.35 on the low side and .55 on the high side). This constrained optimization results in a much higher firm value for θ values higher than the median level of one, and mitigates a substantial portion of the loss even for lower θ values. This implies that a key reason why the heuristic we have posed does not perform well is that fine tuning the capital structure even close to the optimal value can create significant value. The Z,L-Con.Opt. case further releases the restrictions of the heuristic by allowing the firm to select an optimal target within the bounds if it were to otherwise exit the bounds, rather than being forced to a specific target. This additional flexibility also somewhat improves firm value. Thus, the simplistic nature of the heuristic which restricts the firm from refining its leverage, and forces it to a specific leverage ratio regardless of other conditions, significantly reduces the value which could be gained from debt financing. This is particularly true when the state variable is low and the firm is forced to either stay within the acceptable leverage band or to default.

Since it is hard to precisely specify the heuristic rules that firms may use (and presumably they vary widely across firms), we examine a few alternative heuristic specifications that all share the characteristic of being simple to implement (i.e., they should not reflect the type of complex constrained optimization strategies examined in Figure 8). There are several different dimensions in which the heuristic could be modified, but we focus on three: selecting different bounds for the leverage region, choosing alternative targets, and using alternative metrics. In Figure 9, we examine the following four specific alternatives to the standard heuristic (Z,L), which is also shown for purposes of comparison: i) Z,L-L uses the base case bounds (.35,.55), but has a target leverage equal to the lower bound of .35, rather than the mid-point of the leverage range; ii) Z,L-2B has the same base case bounds, but if the leverage were to end up below the lower bound, the debt level is adjusted so as to go back to the lower bound, while if the leverage were to end up above the upper bound, the debt level is reset to get to the leverage ratio of .55 (i.e.

the target is the closest boundary of the acceptable leverage range); iii) Z,M, where the quasi-market leverage must be maintained between a lower bound of .10 and an upper bound of .30, and the target quasi-market leverage is .20; and iv) Z,L-MM, where the bounds are selected based on the upper and lower five percentile points of the first-best book leverage distribution (lower bound = .22 and upper bound = .61), and the target (.55) was determined based on an optimization that matches the first and second moments of the resulting book leverage distribution most closely to the moments of the first-best book leverage distribution (the Z,F(Z) case). In all cases, the values are shown as a percentage of the first-best value under the optimal unlevered investment policy (Z,F(Z)).

There is some variation in the performance of the heuristics. In terms of picking a better target than the middle of the range, it appears that moving back to the closest bound (Z,L-2B) does not do as well as targeting the middle of the leverage range. While the issuance cost for debt and equity is variable, there is an implicit fixed cost due to repurchasing all debt at par value when a restructuring occurs, and thus just pushing leverage back to the edge of the bounds results in an increase in the frequency of recapitalizations and the associated deadweight costs. The heuristic performs better if the firm always targets the lower bound of the leverage range (Z,L-L). This is particularly true when θ is low, which is precisely where a more conservative debt policy may help to avoid default and thus decrease the discount on debt that creates the large indirect cost to issuing debt and recapitalizing over time. Likewise, it appears that the specific heuristic we examine based on quasi-market leverage (Z,M) may also lead to a somewhat more conservative debt policy and thus higher firm value. Yet, even these improved heuristics still result in significant value loss, and perform worse than the second-best financing policy (Z,S(Z)) shown in Figure 5.

Finally, note that the heuristic whose bounds are based on the top and bottom 5 percentile points of the first-best (Z,F(Z)) book leverage distribution, and whose target is selected to match the first two moments of this distribution most closely, leads to lower firm values than most other heuristics. This indicates that financing policies that produce similar distributions of leverage ratios may still result in quite different firm values (even with identical investment policies). The first-best financing policy is much more subtle than a simple heuristic, and, specifically, is less likely to lead to default. More generally, this raises the issue of whether empirical studies that focus on leverage distributions can tell us much about the value impact of different financing strategies, particularly since the financing policy as the firm approaches default is rarely observed.

III. Conclusions

In this paper, we provide a framework to measure the effect of different financing strategies on firm value. We first examine direct and indirect effects of debt financing on firm value, and show that jointly optimizing financing and investment policies significantly increases firm value relative to separately optimizing these policies. We then measure the impact of agency problems on firm value. Our results extend the current literature on quantifying agency costs in that we separate the effects of both financing and investment distortions in a framework that allows for dynamic rebalancing of debt levels. We find that shareholders increase and decrease debt levels less frequently, and are more likely to default, as compared to the first-best optimal financing policy. We show that the impact of financing distortions is quite significant, and that investment and financing distortions tend to have a compounding effect on reducing firm value. Relatively simple covenants included in debt agreements can mitigate these agency problems, and we measure the reduction in agency costs due to the presence of such covenants.

We also examine the performance of various financing heuristics. Despite the fact that these simple rules can provide leverage distributions that are similar to those from optimized financing strategies, their performance from a value perspective varies significantly and can be quite poor in many cases. Our intention overall, however, is not to provide a conclusive verdict on whether rules of thumb can capture most of the potential value associated with financing, but rather to provide an approach to more rigorously assess alternative financing strategies, and to illustrate that the design of financial strategies can in fact have a significant impact on firm value. Further research along these lines needs to push along two dimensions. First, more sophisticated heuristics should be developed that are likely to be descriptively more accurate in terms of what managers are doing in practice, or, more importantly, superior from a normative perspective. Second, it is important that the optimal model that serves as the benchmark incorporates as many realistic features as possible in order to properly evaluate the performance of alternative strategies.

A. Numerical solution

In Section I, we introduced the valuation problem for equity and debt, given all the firm policies introduced in Sections I.B, and I.C. Here we describe how we solve the problem using a numerical approach.

The general structure of the problem is as follows. Given a stationary policy function φ, the state transition function is $(k', b', \theta') = \psi(k, b, \theta) = (\varphi(k, b, \theta), \theta')$.

From equations (8) and (9), the value of debt is

$$\begin{aligned} D(k,b,\theta) = \beta\mathbb{E}_{k,b,\theta}&\left[\left(1-\Delta^S(k,b,\theta)\right)\right] rb(1-\tau_b)+ \\ &\beta\mathbb{E}_{k,b,\theta}\left[\left(1-\Delta^S(k,b,\theta)\right)\left(1-\mathcal{I}(b^S)\right)D(k^S,b^S,\theta)\right]+ \\ &\beta\mathbb{E}_{k,b,\theta}\left[\left(1-\Delta^S(k,b,\theta)\right)\mathcal{I}(b^S)\right]b+ \\ &\beta\mathbb{E}_{k,b,\theta}\left[\Delta^S(k,b,\theta)\min\left\{(1-\gamma)E(k(1-\delta),0,\theta),b\right\}\right] \end{aligned} \tag{17}$$

where the first line is the value of the future after-tax coupon payment, the second line is the value of debt at the future date if the debt level is not changed, the third line is the value of the debt repayment when debt is changed, and the fourth line is the value of the recovery payoff in the case of default.

By denoting the payment $d(k,b,\theta) = rb(1-\tau_b) + \mathcal{I}(b^S)b$ and the recovery payoff $R(k,b,\theta) = \min\{(1-\gamma)E(k(1-\delta),0,\theta),b\}$, the value of the securities is the fixed point of the system of equations

$$\begin{aligned} &E = e_\psi + \beta\mathbb{E}_\psi E \\ &D = \beta\mathbb{E}_\psi\left(\left(1-\Delta^S\right)d + \Delta^S R\right) + \beta\mathbb{E}_\psi\left(1-\Delta^S\right)(1-\mathcal{I})\,D(\varphi), \end{aligned} \tag{18}$$

where $\mathbb{E}_\psi[\cdot]$ denotes the Markov transition operator of ψ, and $D(\varphi)$ is the value of debt after the new policy is adopted at the end of next year.

It should be noted that, given ψ, while the first equation in (18) is linear in the unknown function E, the second equation is generally non-linear because the unknown function D must be determined at φ. This makes the problem of finding the solution for the second equation much more difficult than for the first equation. If the debt contract matures at the end of each period, as in Cooley and Quadrini (2001), Hennessy and

Whited (2007) and Moyen (2007), the last term would vanish because $\mathcal{I} \equiv 1$, and the second equation would become

$$D = \beta \mathbb{E}_{\psi} \left(\left(1 - \Delta^S \right) d + \Delta^S R \right),$$

which is computed in closed form once we know ψ. As a consequence, the solution is easily found when debt has a one-period maturity.

In the long-term debt case of the system of equations in (18), the solution is found numerically by a discrete state-space method based on modified policy iteration, as proposed by Puterman and Shin (1978). The Gauss-Hermite quadrature method is used to approximate the dynamics of θ with a finite state Markov chain. We discretize the state variable k and b by setting the upper and lower bound for capital, k_u and k_d respectively, and the upper and lower bounds for debt, b_u and b_d respectively, in a way that they are never binding for the optimization problem. We solve the model using 27 points for θ, 27 points for k, and 37 points for b.

References

Alti, A., 2003, How sensitive is investment to cash flow when financing is frictionless?, *Journal of Finance* 58, 707–722.

Baker, M. P., and J. Wurgler, 2002, Market timing and capital structure, *Journal of Finance* 57, 1–32.

Bancel, F., and U. R. Mittoo, 2004, Cross-country determinants of capital structure choice: A survey of European firms, *Financial Management* 33, 103–132.

Bradley, M., and M. Roberts, 2004, The structure and pricing of corporate debt contracts, Working paper, Duke and Wharton.

Brounen, D., A. de Jong, and K. Koedijk, 2004, Corporate Finance in Europe: Confronting Theory with Practice, *Financial Management* 33, 71–101.

Chang, X., and S. Dasgupta, 2006, Target Behavior and Financing: How Conclusive is the Evidence?, Working paper, Hong Kong University of Science and Technology.

Chava, S., and M. Roberts, 2007, How Does Financing Impact Investment? The Role of Debt Covenant Violations, *Journal of Finance* forthcoming.

Childs, P. D., D. C. Mauer, and S. H. Ott, 2005, Interactions of Corporate Financing and Investment Decisions: The Effects of agency conflicts, *Journal of Financial Economics* 76, 667–690.

Cooley, T.F., and V. Quadrini, 2001, Financial Markets and Firm Dynamics, *American Economic Review* 91, 1286–1310.

Dangl, T., and J. Zechner, 2004, Credit Risk and Dynamic Capital Structure Choice, *Journal of Financial Intermediation* 13, 183–204.

Fama, E. F., and K. R. French, 2002, Testing tradeoff and pecking order predictions about dividends and debt, *Review of Financial Studies* 15, 1–33.

Faulkender, M., and J. Smith, 2008, Are adjustment costs impeding realization of target capital structure?, Working paper, University of Maryland.

Fischer, E.O., R. Heinkel, and J. Zechner, 1989, Dynamic Capital Structure Choice: Theory and Tests, *Journal of Finance* 49, 19–40.

Flannery, M. J., and K. P. Rangan, 2006, Partial adjustment toward target capital structures, *Journal of Financial Economics* 79, 469–506.

Frank, M. Z., and V. K. Goyal, 2003, Testing the pecking order theory of capital structure, *Journal of Financial Economics* 67, 217–248.

Gamba, A., and A. J. Triantis, 2008, The Value of Financial Flexibility, *Journal of Finance* 63.

Graham, J. R., and C. R. Harvey, 2001, The Theory and Practice of Corporate Finance: Evidence from the Field, *Journal of Financial Economics* 60, 187–243.

Hennessy, C. A., and T. M. Whited, 2005, Debt Dynamics, *Journal of Finance* 60, 1129–1165.

Hennessy, C. A., and T. M. Whited, 2007, How Costly is External Financing? Evidence from a Structural Estimation, *Journal of Finance* 62, 1705–1745.

Kayhan, A., and S. Titman, 2007, Firms' histories and their capital structure, *Journal of Financial Economics* 83, 1–32.

Kisgen, D., 2006, Credit Ratings and Capital Structure, *Journal of Finance* 61, 1035–1072.

Kurshev, A., and I. A. Strebulaev, 2006, Firm Size and Capital Structure, Working paper, Stanford University GSB.

Leary, Mark. T., and M. R. Roberts, 2005, Do Firms Rebalance their Capital Structure?, *Journal of Finance* 60, 2575–2619.

Lemmon, M. L., M. R. Roberts, and J. F. Zender, 2007, Back to the beginning: persistence and the cross-section of corporate capital structure, *Journal of Finance* forthcoming.

McDonald, R. L., 2000, Real Options and Rules of Thumb in Capital Budgeting, in M. J. Brennan, and L. Trigeorgis, eds.: *Project Flexibility, Agency, and Competition:*

New Developments in the Theory and Application of Real Options (Oxford University Press, New York).

Moyen, N., 2004, Investment-Cash Flow Sensitivities: Constrained versus Unconstrained Firms, *Journal of Finance* 49, 2061–2092.

Moyen, N., 2007, How Big Is the Debt Overhang Problem?, *Journal of Economic Dynamics and Control* 31, 433–472.

Puterman, M. L., and M. C. Shin, 1978, Modified Policy Iteration Algorithms for Discounted Markov Decision Problems, *Management Science* 24, 1127–1137.

Roberts, M., and A. Sufi, 2007, Control Rights and Capital Structure: An Empirical Investigation, Working paper, Wharton and Chicago.

Shyam-Sunder, L., and S. C. Myers, 1999, Testing static tradeoff against pecking order models of capital structure, *Journal of Financial Economics* 51, 219–244.

Sick, G., 1990, Tax-Adjusted Discount Rates, *Management Science* 36, 1432–1450.

Strebulaev, I. A., 2007, Do Tests of Capital Structure Theory Mean What They Say?, *Journal of Finance* 62, 1747–1787.

Sundaresan, S., and N. Wang, 2006, Dynamic Investment, Capital Structure, and Debt Overhang, Working paper, SSRN.

Titman, S., and S. Tsyplakov, 2007, A Dynamic Model of Optimal Capital Structure, *Review of Finance* 11, 401–451.

Tserlukevich, Y., 2006, Can Real Options Explain Financing Behavior?, *Journal of Financial Economics* forthcoming.

Tsyplakov, S., 2007, Investment Frictions and Leverage Dynamics, *Journal of Financial Economics* forthcoming.

van Binsbergen, J., J. R. Graham, and J. Yang, 2007, The cost of debt, Working paper, Fuqua School of Business.

Welch, I., 2004, Capital structure and stock returns, *Journal of Political Economy* 112, 106–131.

Yang, Baozhong, 2007, A Dynamic Model of Corporate Financing with Market Timing, Working paper, Stanford University GSB.

Symbol	*Description*
F	firm value maximizing (first best) investment and financing
S	equity value maximizing (second best) investment and financing
Z	optimal unlevered investment, zero debt
S(F),F	second best investment conditional on first best financing
F,S(F)	second best financing conditional on first best investment
S,l	second best investment and financing, with constrained leverage
S,d	second best investment and financing, with no disinvestment
Z,F(Z)	first best financing conditional on optimal unlevered investment
Z,S(Z)	second best financing conditional on optimal unlevered investment
F,Z	first best investment, zero debt
F,L	first best investment, book leverage heuristic
Z,L	optimal unlevered investment, book leverage heuristic
Z,L-ZC	as in "Z,L", with zero transaction costs
Z,L-BZC	as in "Z,L", with zero bankruptcy costs
Z,L-Con.Opt.	unlevered investment, optimized leverage lies between L_d and L_u
Z,L-Con.Opt.-2B	as in Z,L-Con.Opt, but $L_t = L_d$ if below L_d, and $L_t = L_u$ if above L_u
Z,L-L	as in "Z,L", with $L_t = L_d$
Z,L-2B	as in "Z,L", with $L_t = L_d$ if below L_d, and $L_t = L_u$ if above L_u
Z,L-MM	as in "Z,L", with $L_d = .23$, $L_u = .65$ and $L_t = .55$
Z,M	optimal unlevered investment, quasi-market leverage heuristic

Table I: **Notation for the Different Investment and Financing Policy Cases**

ρ	persistence of state variable	0.80
σ	annual volatility of state variable	0.20
r	annual risk-free borrowing rate	5%
τ_e	personal tax rate on equity cash flows	15%
τ_b	personal tax rate on bond coupons	22%
τ_c^+	corporate tax rate for positive earnings	40%
τ_c^-	corporate tax rate for negative earnings	20%
α	production return-to-scale parameter	0.48
δ	annual depreciation rate	0.10
F	fixed production cost	1.34
s	fire-sale discount for asset sales	0.45
ℓ	liquidation price for disinvestment	0.65
γ	proportional bankruptcy costs	0.60
λ	variable flotation cost for equity	0.05
q	variable issuance cost for debt	0.01
L_d	lower bound for book leverage heuristic	0.35
L_u	upper bound for book leverage heuristic	0.55
M_d	lower bound for quasi-market leverage heuristic	0.10
M_u	upper bound for quasi-market leverage heuristic	0.30

Table II: **Base Case Parameter Values**

percentile	5	25	50	75	95
F	5.52	8.97	12.11	15.55	20.20
S,l	5.80	8.89	11.55	15.29	20.09
S,d	4.73	8.17	10.91	14.08	18.81
S	4.80	8.05	10.67	13.91	18.44
Z	4.55	7.69	10.06	13.07	17.97

Table III: **Percentile Points for Firm Value Distributions**. The table presents the firm value percentiles for the first-best (F), second-best (S), maximum book leverage covenant (S,l), no disinvestment covenant (S,d), and unlevered (Z) cases. The distributions are obtained from using in each case the same simulated panel of 50 years' observations of $10,000$ Monte Carlo replications of the base case firm. The base case parameters shown in Table II are used. The sample is obtained by a Monte Carlo simulation of θ and the application of the optimal policy φ, for the five cases listed above. The optimal policy is obtained from the numerical solution of the model using 27 points for θ, 27 points for k, and 37 points for b.

percentile	5	25	50	75	95	Prob.
F (>0)	0.02	0.03	0.07	0.13	0.26	0.24
F (<0)	0.09	0.08	0.06	0.03	0.02	0.35
S (>0)	0.02	0.03	0.04	0.09	0.20	0.13
S (<0)	0.08	0.06	0.04	0.03	0.02	0.12
S,l (>0)	0.02	0.03	0.08	0.16	0.24	0.14
S,l (<0)	0.10	0.08	0.06	0.04	0.02	0.19
S,d (>0)	0.01	0.02	0.05	0.10	0.23	0.12
S,d (<0)	0.09	0.07	0.06	0.04	0.02	0.12

Table IV: **Debt Change Distributions With and Without Covenants**. The table presents the probabilities of increases and decreases in debt, as well as the percentiles for the distribution of debt changes (as a percentage of current firm value), conditional on either an increase (>0) or a decrease (<0) in the debt level, for the following four cases: first best (F); second best (S); maximum book leverage covenant (S,l); and no disinvestment covenant (S,d). The distributions are obtained from using in each case the same simulated panel of 50 years observations of $10,000$ Monte Carlo replication of the base case firm. Base case parameters are in Table II. The sample is obtained by a Monte Carlo simulation of θ and the application of the optimal policy φ, for the four cases listed above. The optimal policy is obtained from the numerical solution of the model using 27 points for θ, 27 points for k, and 37 points for b.

percentile	5	25	50	75	95	Prob.
F (> 0)	0.02	0.03	0.07	0.13	0.26	0.24
F (< 0)	0.09	0.08	0.06	0.03	0.02	0.35
S (> 0)	0.02	0.03	0.04	0.09	0.20	0.13
S (< 0)	0.08	0.06	0.04	0.03	0.02	0.12
F,L (> 0)	0.09	0.11	0.13	0.17	1.04	0.08
F,L (< 0)	0.14	0.12	0.11	0.09	0.09	0.03
Z,L (> 0)	0.07	0.08	0.18	0.76	2.76	0.03
Z,L (< 0)	0.18	0.11	0.09	0.08	0.08	0.02
Z,L-MM (> 0)	0.23	0.70	0.94	1.54	5.77	0.02
Z,L-MM (< 0)	0.16	0.12	0.10	0.09	0.08	0.00

Table V: **Debt Change Distributions for Heuristics**. The table presents the probabilities of increases and decreases in debt, as well as the percentiles for the distribution of debt changes (as a percentage of current firm value), conditional on either an increase (> 0) or a decrease (< 0) in the debt level, for the following five cases: first best (F); second best (S); heuristic case (F,L) using the first best investment policy together with a book leverage policy with a lower bound of .35, an upper bound of .55, and a target of .45; heuristic case (Z,L) using the unlevered investment policy together with a book leverage policy with a lower bound of .35, an upper bound of .55, and a target of .45; and the heuristic (Z,L-MM) where the bounds are selected based on the upper and lower five percentile points of the first-best book leverage distribution assuming unlevered investment (the lower bound is .22, the upper bound is .61), and the target (.55) was determined based on an optimization matching the first and second moments of the resulting book leverage distribution of the heuristic most closely to the moments of the first-best book leverage distribution (the F,L case). The distributions are obtained from using in each case the same simulated panel of 50 years observations of $10,000$ Monte Carlo replication of the base case firm. Base case parameters are in Table II. The sample is obtained by a Monte Carlo simulation of θ and the application of the optimal policy φ, for the five cases listed above. The optimal policy is obtained from the numerical solution of the model using 27 points for θ, 27 points for k, and 37 points for b.

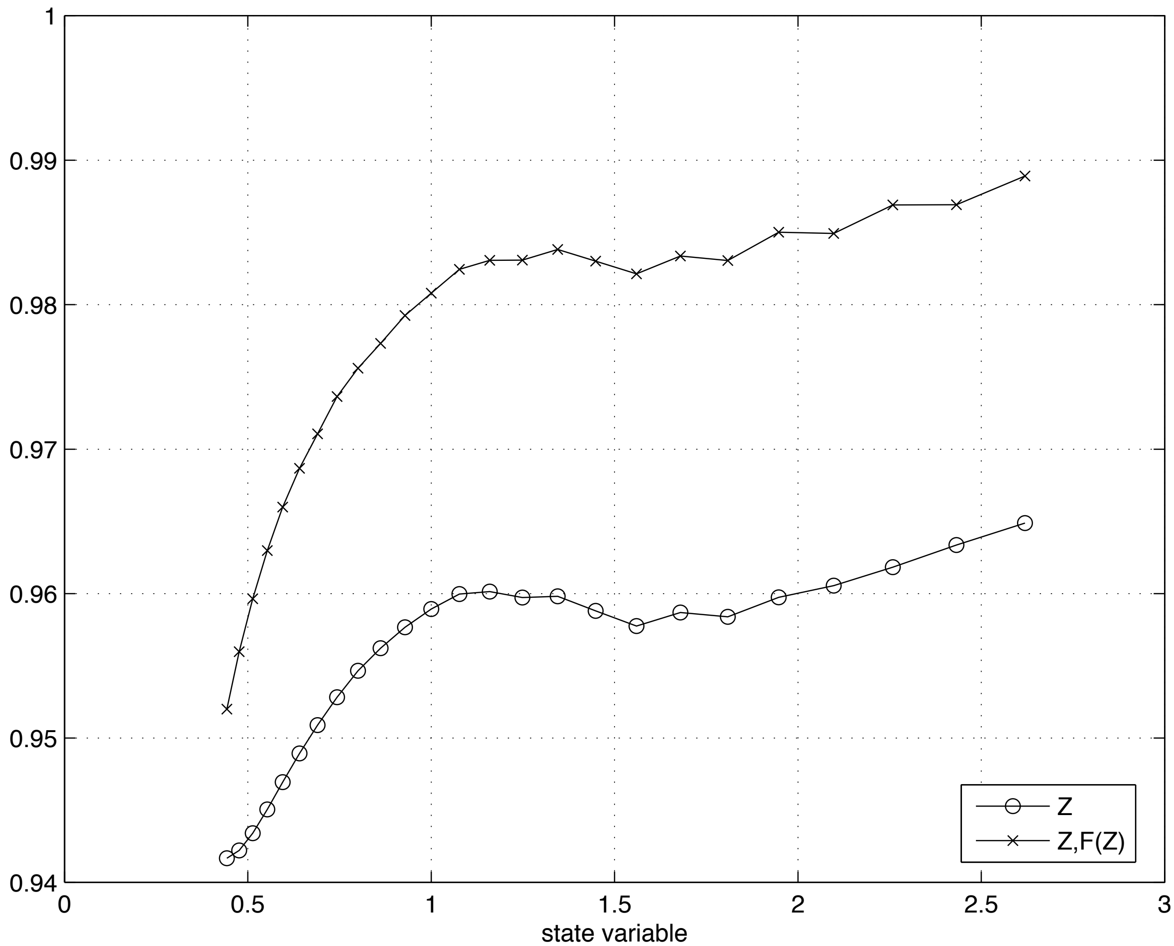

Figure 1: **Direct and Indirect Effects of Financing on Firm Value**. The figure shows the value of an unlevered firm (Z), and the value of a firm that follows the unlevered investment policy but takes on leverage to maximize firm value (Z,F(Z)), as a percentage of the first best firm value (F), for different values of θ (the state variable). The values are from the numerical solution of the model using 27 points for θ, 27 points for k, 37 points for b, based on the parameter values in Table II. The plots are based on initial values of $k = 8.53$, the median of the capital stock distribution for the unlevered firm, and $b = 0$.

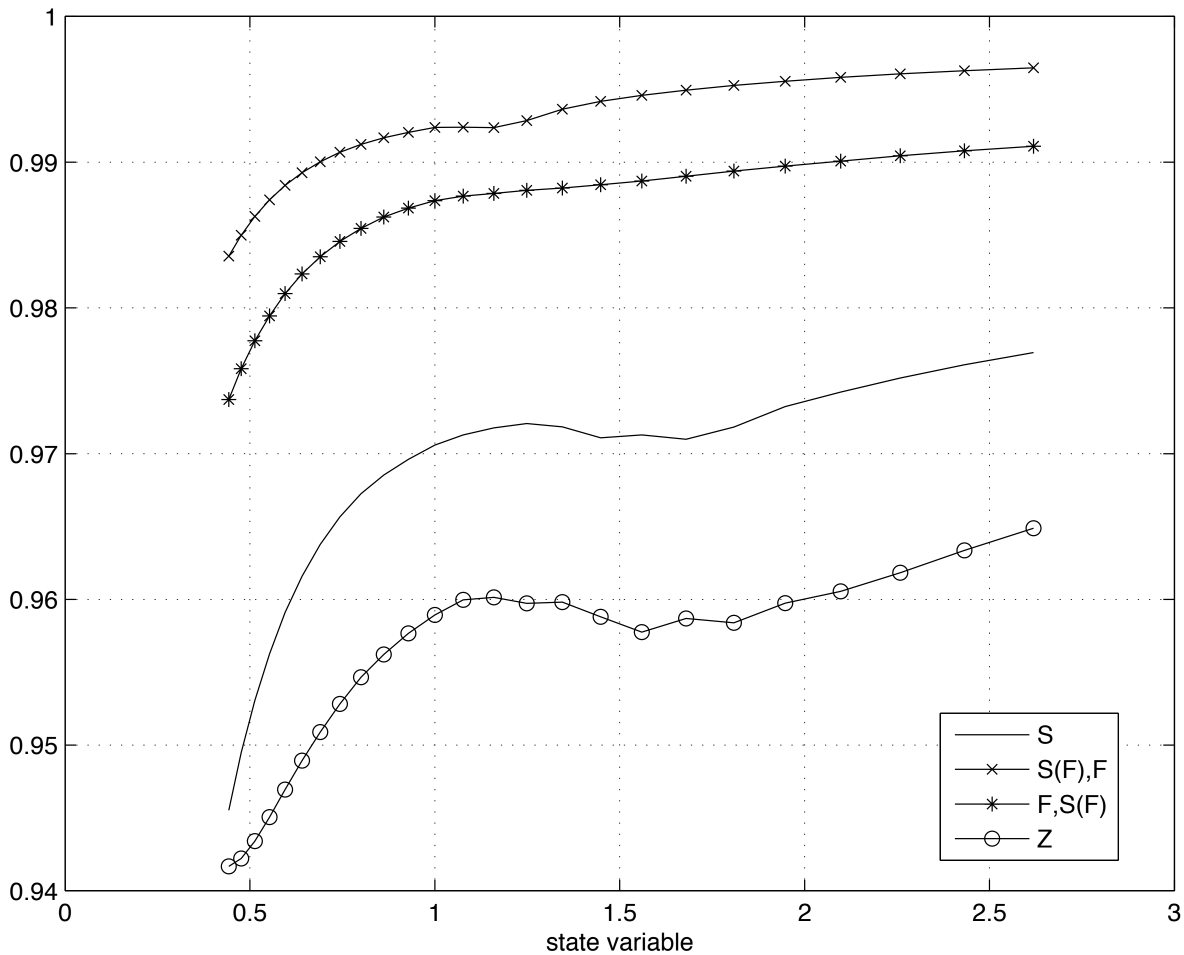

Figure 2: **Impact of Investment and Financing Distortions**. The figure plots firm values in four different cases as a percentage of the first best (F) firm value, for different values of θ (the state variable). The four cases are: no debt financing (Z); equity value maximization (S); first-best financing policy, and shareholder optimization of investment conditional on first best financing (S(F),F); and first-best investment and shareholder optimization of financing conditional on first best investment (F,S(F)). The values are from the numerical solution of the model using 27 points for θ, 27 points for k, 37 points for b, based on the parameter values in Table II. The plots are based on initial values of $k = 8.53$, the median of the capital stock distribution for the second best case (S), and $b = 0$.

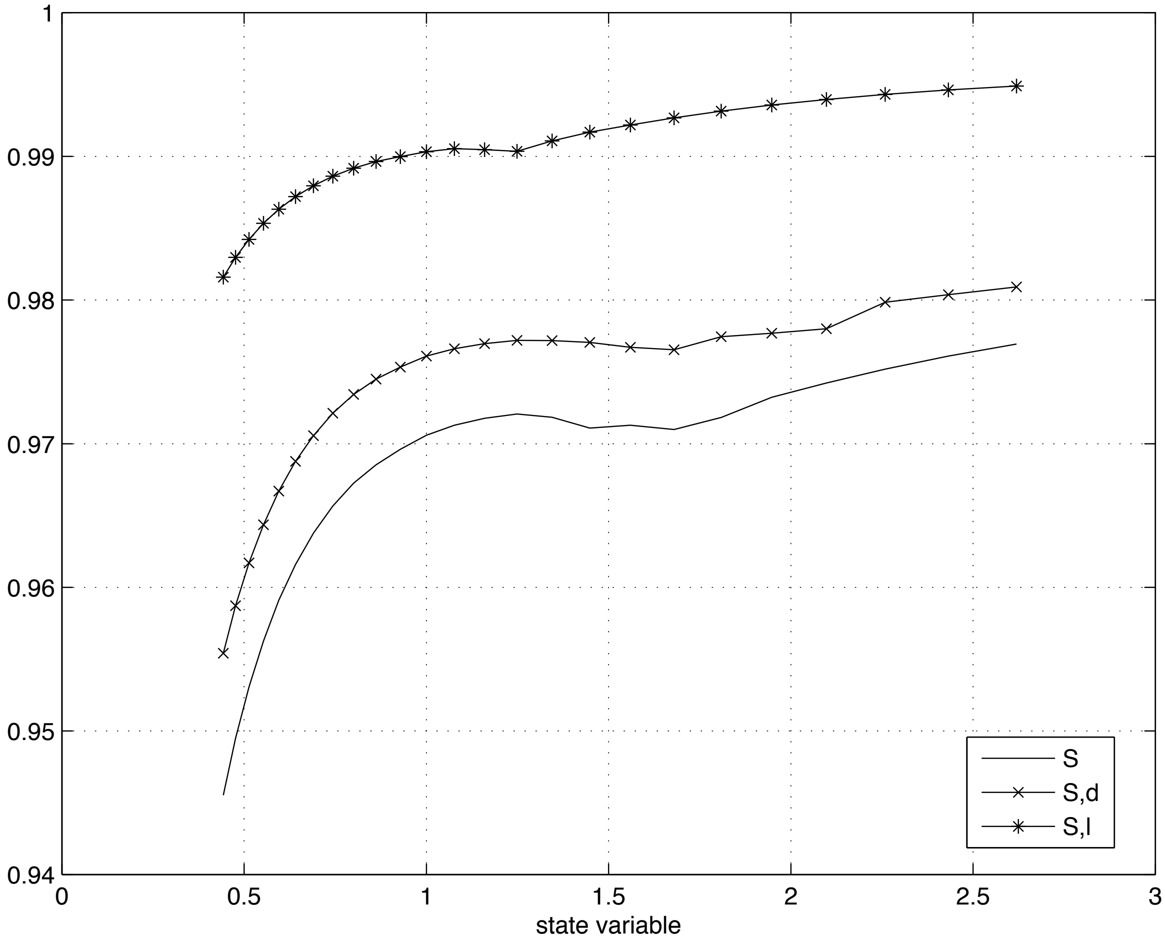

Figure 3: **Effect of Covenants on Firm Value**. The figure plots the firm values under three second-best cases as a percentage of first-best firm value, for different values of θ (the state variable). The bottom curve (S) represents the unconstrained shareholder optimization case. The other two curves depict cases where either financing or investment behavior is constrained by a covenant: S,l imposes an upper bound of .55 on the book value of leverage; and S,d restricts the firm from selling off its capital. The values are from the numerical solution of the model using 27 points for θ, 27 points for k, 37 points for b, based on the parameter values in Table II. The plots are based on $k = 8.53$, the median of the capital stock distribution for the second best case (S), and $b = 0$.

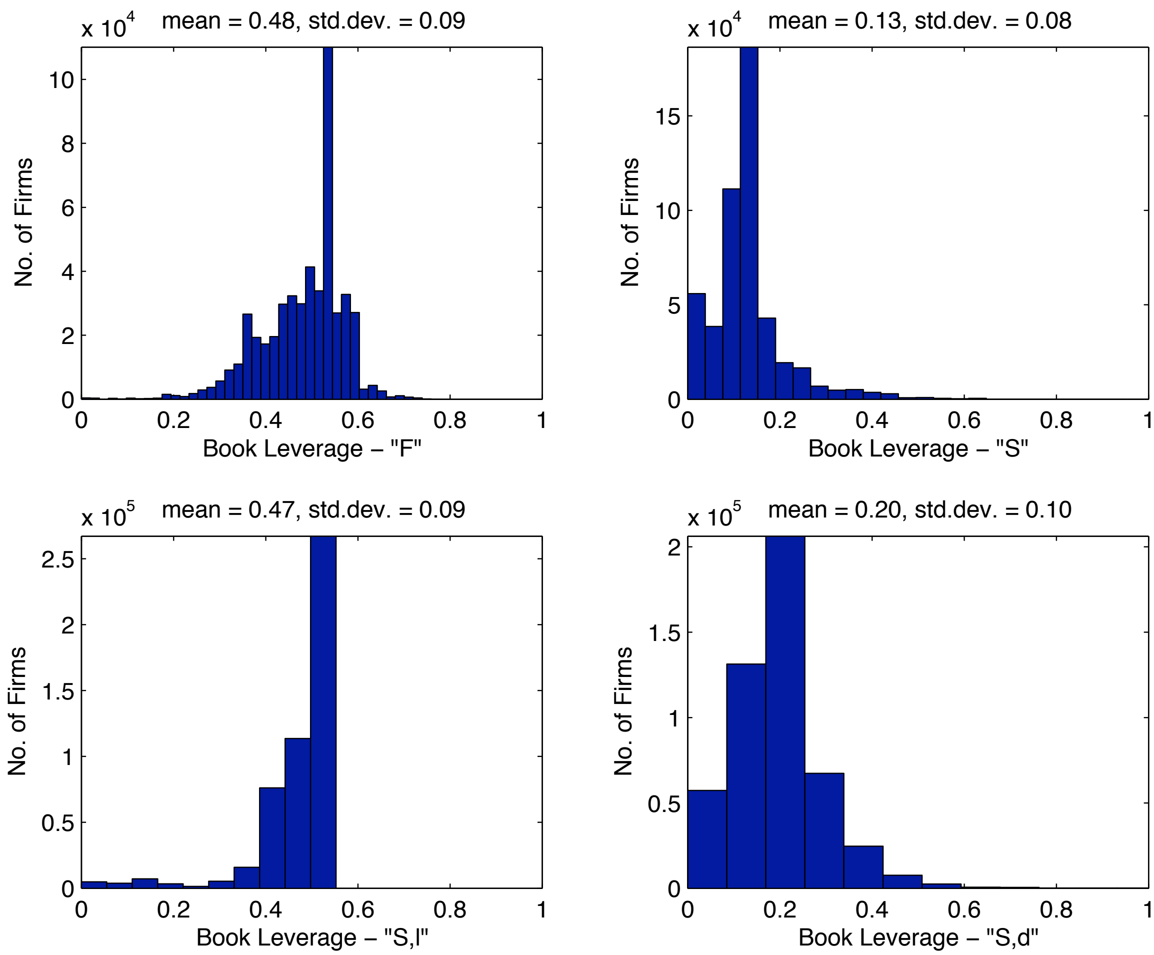

Figure 4: **Distribution of Book Leverage for Different Financing Strategies**. The figure plots the histograms of book leverage distribution for four cases using the same simulated panel of 50 years observations of $10,000$ Monte Carlo replications of the base case firm. Base case parameters are shown in Table II. The sample is obtained by a Monte Carlo simulation of θ and the application of the optimal policy φ, for the first best (F), second best (S), and the two covenant cases – book leverage constrained to be less than .55 (S,l), and no asset sales (S,d). The optimal policy is obtained from the numerical solution of the model using 27 points for θ, 27 points for k, 37 points for b, based on the parameter values in Table II.

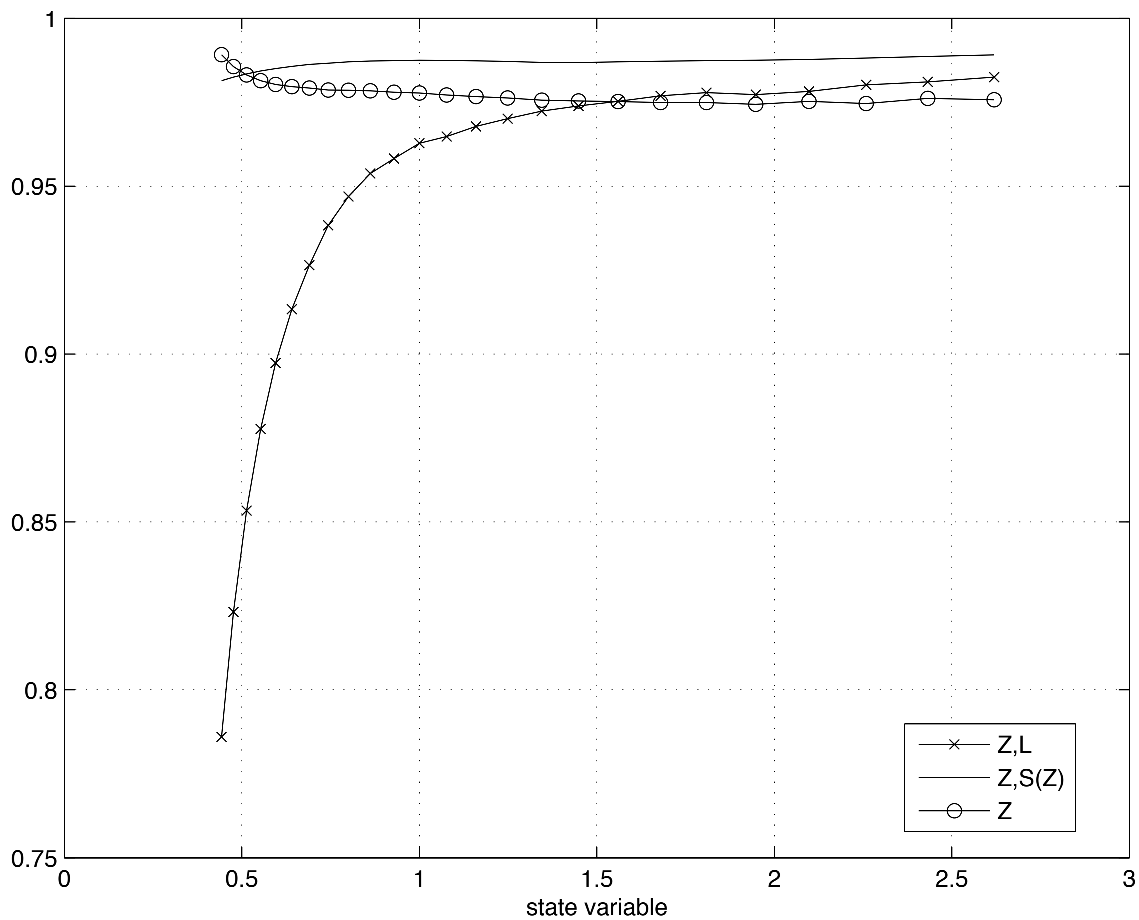

Figure 5: **Performance of the Financing Heuristic Assuming an Unlevered Investment Policy**. The figure plots firm values for three financing cases: the heuristic rule (Z,L); second best financing (Z,S(Z)), and zero debt financing (Z), as a percentage of the firm value using the same optimal unlevered investment policy, but with first-best financing (Z, F(Z)). The values are from the numerical solution of the model using 27 points for θ, 27 points for k, 37 points for b, based on the parameter values in Table II. The plots are based on $k = 8.53$, the median of the capital stock distribution assuming an unlevered investment policy, and $b = 0$.

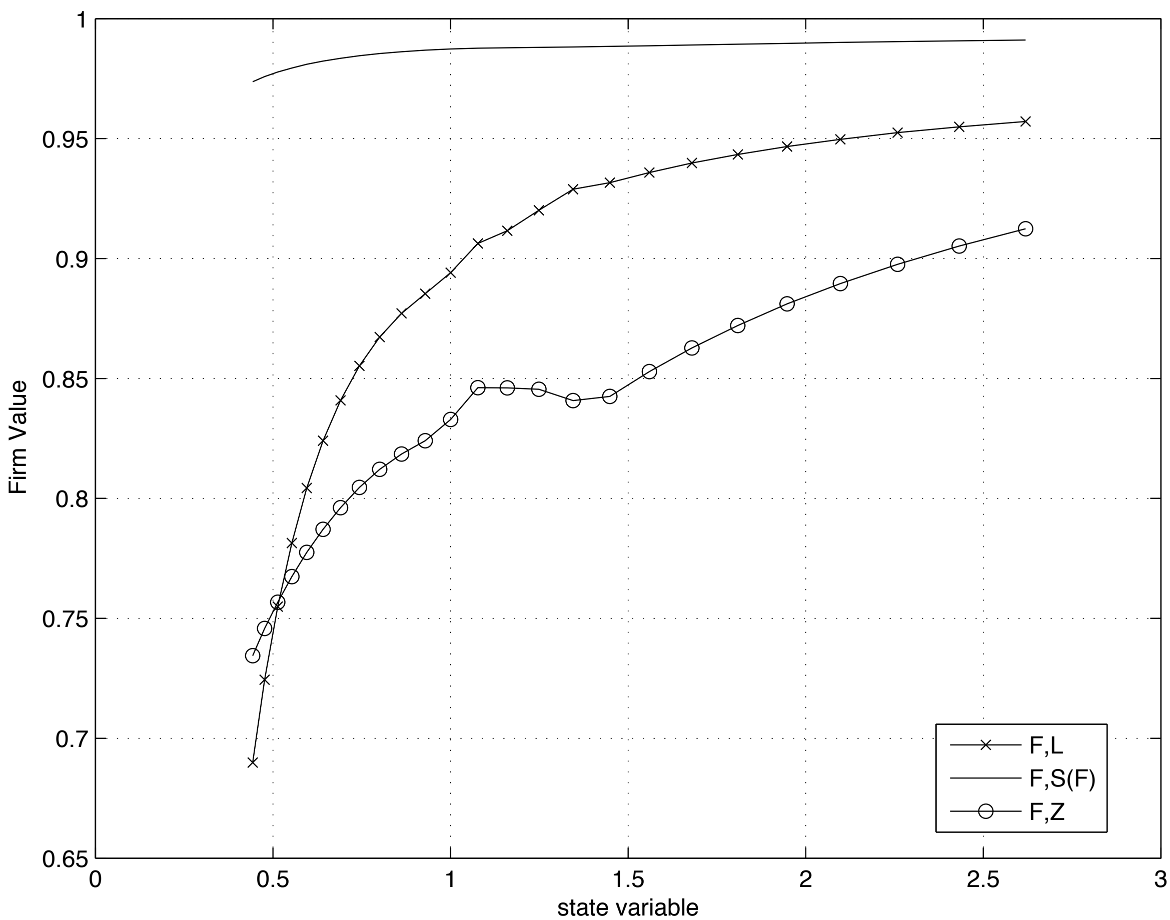

Figure 6: **Performance of the Financing Heuristic Assuming a First Best Investment Policy**. The figure plots firm values for three financing cases: the heuristic rule (F,L); second best financing (F,S(F)), and zero debt financing (F,Z), as a percentage of the first-best firm value. The values are from the numerical solution of the model using 27 points for θ, 27 points for k, 37 points for b, based on the parameter values in Table II. The plots are based on $k = 8.53$ and $b = 0$.

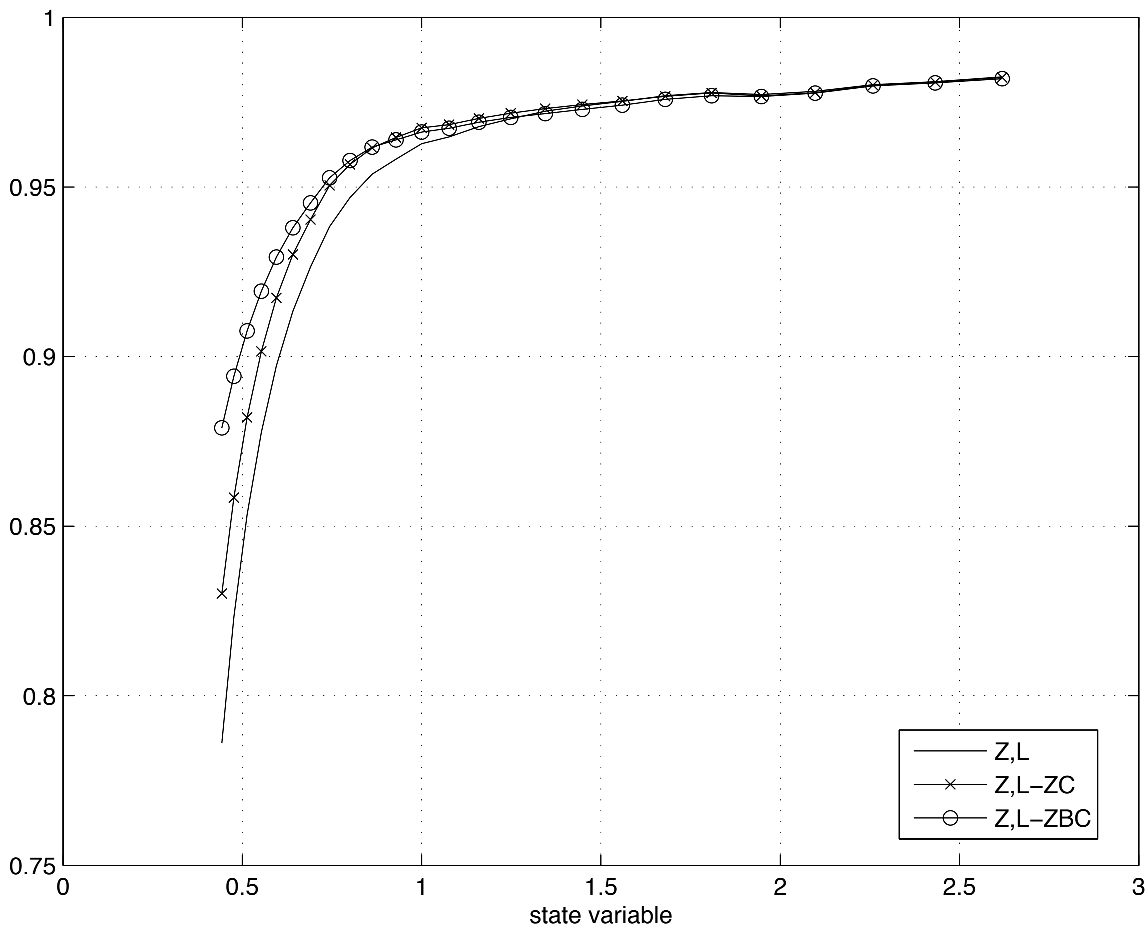

Figure 7: **The Effect of Issuance and Bankruptcy Costs on Heuristic Performance**. The figure plots firm values under the book leverage financing heuristic with unlevered investment, as a percentage of firm values under first-best financing, for three separate cases: i) base case costs (Z,L); ii) zero issuance costs (Z,L-ZC); and iii) zero bankruptcy costs (Z,L-ZBC). In each case, the optimal unlevered investment policy is used. The values are from the numerical solution of the model using 27 points for θ, 27 points for k, 37 points for b, based on the parameter values in Table II. The plots are based on $k = 8.53$, the median of the capital stock distribution assuming an unlevered investment policy, and $b = 0$.

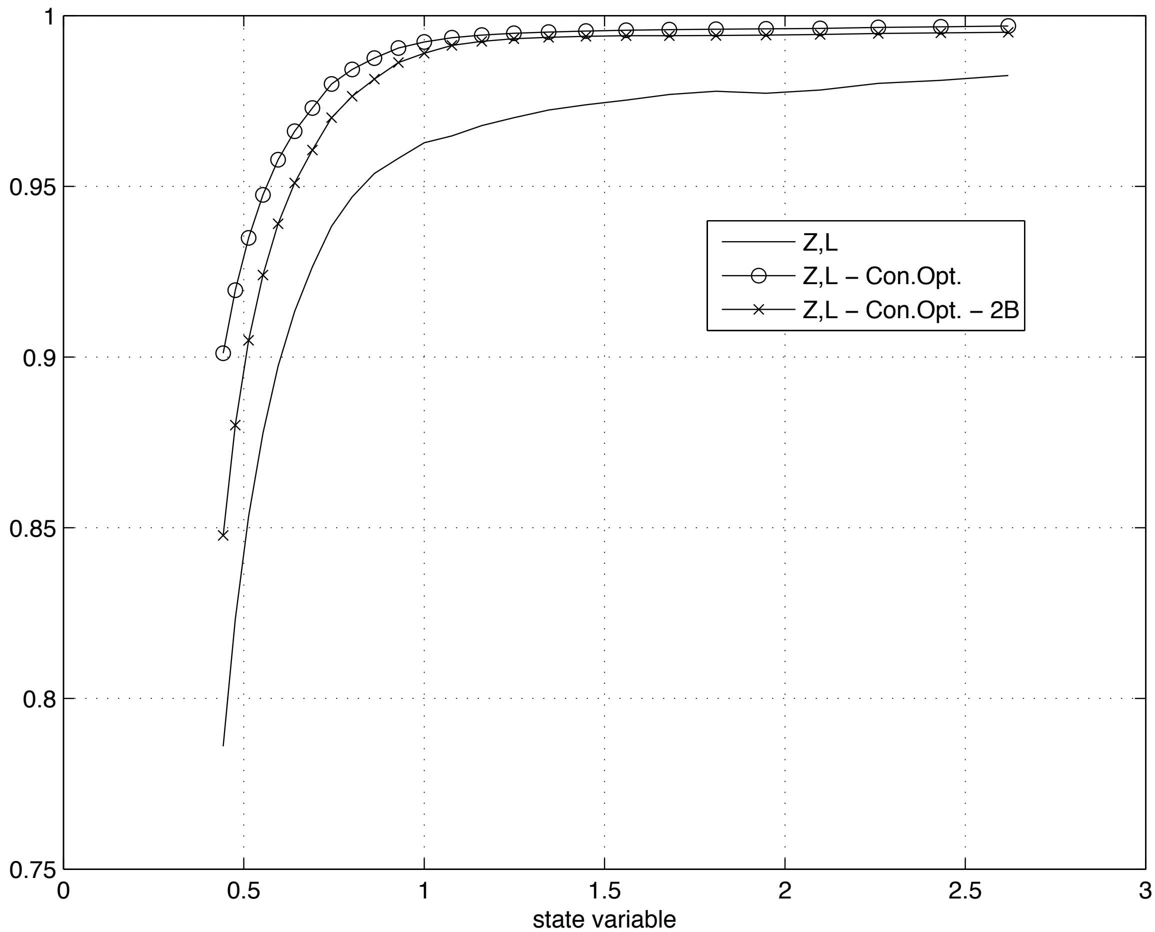

Figure 8: **The Effect of Restrictions Imposed by the Heuristic**. The figure examines two constrained optimizations that highlight key factors explaining the value loss associated with the heuristic financing rule. The curve labeled Z,L-Con.Opt.-2B represents the value of a firm where leverage is constrained to fall within the bounds [.35, .55], and the firm must return to the closest boundary if the leverage ratio were to otherwise exit the bounds if the debt level were not changed; however, the firm may adjust its leverage if the current debt level (and the new debt level) maintain a leverage ratio within the bounds. The case Z,L-Con.Opt. represents the value of a firm where the leverage can be changed at any time as long as the leverage falls within the bounds [.35, .55]. The values in all cases (including the basic Z,L heuristic case) are shown as a percentage of the first-best Z,F(Z) case (i.e. the optimal unlevered investment policy is assumed). The values are from the numerical solution of the model using 27 points for θ, 27 points for k, 37 points for b, based on the parameter values in Table II. The plots are based on $k = 8.53$ and $b = 0$.

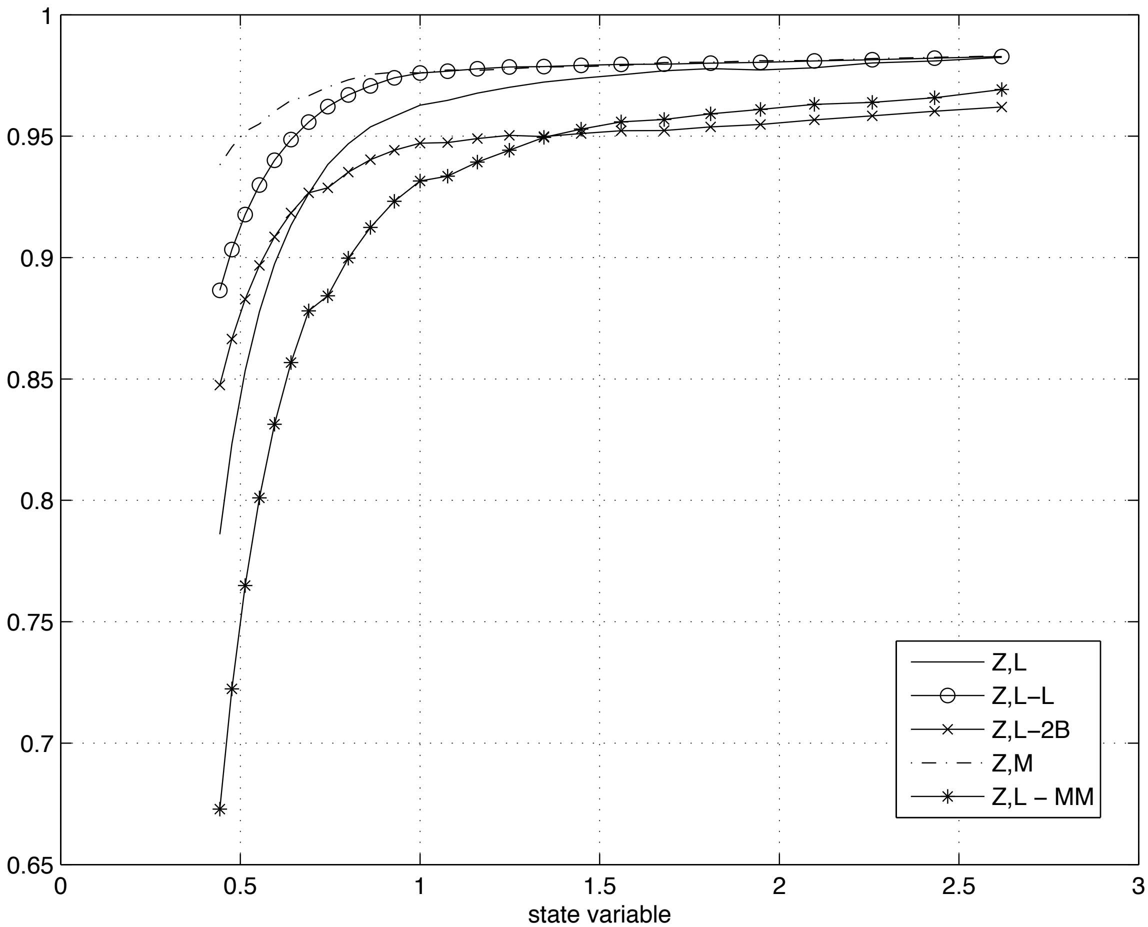

Figure 9: **The Performance of Alternative Heuristics**. The figure plots firm values for the standard heuristic (Z,L), as well as the following four alternative heuristics: i) Z,L-L has the base case bounds (.35,.55), and a target leverage equal to the lower bound of .35; ii) Z,L-2B has the same base case bounds, but the target is the closest boundary of the acceptable leverage range; iii) Z,M, where the quasi-market leverage must be maintained between a lower bound of .10 and an upper bound of .30, and the target market leverage is .20; and iv) Z,L-MM, where the bounds are selected based on the upper and lower five percentile points of the first-best book leverage distribution (lower bound = .22 and upper bound = .61), and the target (.55) was determined based on an optimization matching the first and second moments of the resulting book leverage distribution of the heuristic most closely to the moments of the first-best book leverage distribution (the Z,F(Z) case). In all cases, the values are shown as a percentage of the first-best value under the optimal unlevered investment policy (Z,F(Z)). The values are from the numerical solution of the model using 27 points for θ, 27 points for k, 37 points for b, based on the parameter values in Table II. The plots are based on $k = 8.53$ and $b = 0$.