The Effect of Investment and Financing Policies on Credit Risk

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ABSTRACT

We investigate the impact on credit risk of endogenous investment and capital structure decisions. To this aim, we propose a realistic dynamic structural model featuring endogenous investment, capital structure and default. We calibrate the model using accounting and market information by fitting the empirical credit risk data for different risk classes. We find that while investment and financing decisions, when made in the interest of all stakeholders, reduce credit risk, they greatly increase credit risk if they are in the best interest of shareholders, and the effect is more significant if investment and financing are jointly decided. Moreover, we find that in presence of dynamic investment/disinvestment decisions, the possibility to adapt capital structure over time in order to benefit from a positive net tax shield and to avoid distress cost is only a minor determinant of credit risk. Similarly, the effect of debt transaction costs on yield spreads is also relatively small when compared to agency costs.

JEL Classification: G12, G31, G32, E22.

Keywords: dynamic capital structure, investment, credit risk, agency costs.

Introduction

The credit risk of corporate bonds is the result of both macroeconomic conditions and firms decisions made in capital markets with frictions. Using the structural approach pioneered by Merton (1974), the literature has focused mostly on the influence of capital structure decisions and financial market imperfections on the yield spread. For example, the effects of taxation, debt covenants, transaction costs, different types of recovery, and stochastic interest rates have been considered. However, less attention has been paid to the effect of investment/disinvestment decisions on the yield spread. In the presence of market imperfections, investment decisions interact with financing decisions for at least two reasons. First, the gap between capital expenditure and internal funds is a major determinant of debt issuance decisions, and second, cash flows generated by current investments affect the future need for funds.

In this work we contend that incorporating endogenous investment and capital structure decisions, and exploring their joint effect, can advance our knowledge of the determinants of yield spreads. We propose a dynamic structural model within an infinite horizon discrete-time stochastic framework, to analyze the joint effect of dynamic investment and financing decisions on the yield spread, in a setting with market frictions, like corporate and personal taxes and debt and equity issuance costs. Investment can be financed using operating cash flows or external (debt and/or equity) funds. The debt contract is a long term callable bond, and there is a dynamic choice of capital structure and endogenous default. Likewise, the effects of costly financial distress and bankruptcy costs are included.

The joint effect of dynamic capital structure and investment policies is rather different from the case where they are separately considered. The basic idea is well summarized in the following simple example, drawn from Sundaresan and Wang (2006). Assume a firm has two compound (investment and expansion) options, and can issue a blend of debt and equity to finance the capital expenditure at the two exercise dates. The issuance is assumed to be contingent on the investment/expansion decision, with debt issued first being more senior. At the time of the first issuance, the firm is unlevered, so the capital structure decision maximizes total firm value, which in this case equals equity value. When the second portion of debt is issued, the capital structure decision together with the investment decision are made to maximize the equity value, which at this time is different from the firm value because of the presence of more senior debt. Both the capital structure *and* the investment decisions at the expansion date may be sub-optimal from senior debt holders' perspective. Hence, the price of debt at the time of the first investment/financing decision rationally incorporates agency costs related to both second best investment and second best financing.[1]

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[1]Conversely, we only have an agency issue related to investment when the debt is issued by a currently unlevered firm. In this case, the agency cost is just due to current and future equity maximizing

In this work we are interested also in exploring the effects of these agency issues on the price of corporate debt. Our model generalizes the simple idea illustrated above: we consider both expansion and contraction opportunities, and we allow capital structure decisions to be unrelated to investment decisions.[2] The model is general enough to be carried over empirical cross-sectional distributions of leverage, default rates and credit spreads.

Our work builds on existing dynamic models. The model is derived from Cooley and Quadrini (2001), although we have a richer specification for financial markets and taxes. Hennessy and Whited (2007) and Moyen (2007) describe the interaction between investment and financing policies in a firm with endogenous default risk within a one-period debt model with endogenous investment. If the bond maturity is one year, the model does not include debt transaction and refunding costs, which create a liquidity risk (from the definition given by Childs, Mauer, and Ott (2005)) and endogenously increase the default risk. Moreover, a model based on one-period debt would not account for agency costs due to a second best capital structure policy. Our model differs from theirs because we use infinite-maturity debt and consider also the effect of investment irreversibility. Our model is under many respects similar to the one by Titman and Tsyplakov (2007), but it is different from theirs because we deal also with investment reversibility and its effect on credit risk, whereas they assume that once a new asset is in place, only depreciation can reduce it. As we will see, reversibility has a significant impact on the credit risk. Our model is also similar to the one by Obreja (2006), although his purpose is to analyze the role of financial leverage to explain the distribution of expected equity returns. Lastly, Hackbarth, Miao, and Morellec (2006) take a a quite different perspective on the topic. They analyze the effect of macroeconomic conditions on credit risk, and they do so in a setting with no investment flexibility.

Given the simultaneous investment and financing decisions, the solution of the valuation problem for equity and debt is the fixed-point of a two-dimensional Bellman operator. Since our model includes perpetual (as opposed to one-period) debt, we cannot rely on the solution approach introduced by Cooley and Quadrini (2001) and extended by Moyen (2007) and Hennessy and Whited (2007) to solve the problem. Therefore, we introduce an efficient numerical algorithm for this purpose.

To calibrate the model, we compute empirical credit risk metrics using firm accounting information (Compustat), share prices (CRSP), ratings (Standard & Poor's), default rates (Moody's Investors Service (2006)), and yield spreads on industrial bonds (Reuters). Then, we match these data with simulated data from the model on a per credit merit class basis. We do no have different parameters for different risk classes (as

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investments decisions. This case has been studied by many authors, like Leland (1998), Childs, Mauer, and Ott (2005), Moyen (2007), and Titman and Tsyplakov (2007).

[2]In a similar framework, Gamba and Triantis (2008b) analyze the effect on firm value of following alternative financing policies to match the investment policies, including also the effect of a deviation from first best investment and first best financing decisions.

do Huang and Huang (2003)); rather, we have one model to fit all risk classes, while fitting also their frequencies. We think this is superior to matching the whole sample average yield spread or, alternatively, the investment bonds' and speculative bonds' average credit spreads, because we can better capture the endogenous heterogeneity of firms belonging to different credit classes.

We find that agency costs are a key determinant of the yield spreads. Investment and financing decisions made by self-interested equity holders, when separately analyzed, reduce the value of debt. This effect is much more significant when both decisions are jointly considered. In contrast, financial and investment flexibility reduce credit risk if decisions are made in the interest of all stakeholders (first best).

We also find that the possibility to dynamically change the capital structure does not significantly affect the investment policy, as long as decisions are equity value maximizing. As a consequence, at the steady state, the yield spreads in the case with both dynamic financing and dynamic investment are very similar to the ones of a firm with constant capital structure and dynamic investment. Similarly, in good states, we find that the investment policy is not affected by the possibility of issuing debt. That is, it is almost the same as the investment policy of a company constrained to remain unlevered.

We observe that the effect of the tax shield to adjust the debt level is less important in the presence of dynamic investment. That is, the main drivers of capital structure decisions are neither corporate nor personal taxes, as it would be for models based on dynamic capital structure only, but rather the interaction with investment/disinvestment choices. Also debt transaction costs have a minor effect when dynamic investment is included in the model: they are just minor determinants of capital structure decisions and of credit risk.

The outline of our work is as follows. In Section I, we present the model of the firm and the valuation framework. Next, in Section II we calibrate the model using empirical data. In Section III, we analyze the effect of financial and investment flexibility on the yield spread. We also discuss the effect of taxes and debt transaction costs. In Section IV, we offer our concluding remarks. Finally, Appendix A and Appendix B provide the details of the numerical procedure we use to solve the model and the calibration procedure, respectively.

I. The model

We introduce the valuation model for corporate securities in a setting with endogenous investment, dynamic capital structure decisions, and default.

A. Economic environment

The source of uncertainty is the productivity of firm's capital stock, denoted x. We assume that $x = e^z$, and z follows the AR(1) process

$$z_{t+1} = (1 - \rho)\bar{z} + \rho z_t + \sigma \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, 1), \quad |\rho| < 1 \tag{1}$$

where $\bar{z}$ is the long term mean, $(1 - \rho)$ is the speed of mean reversion and σ is the conditional standard deviation.[3] For valuation purposes we will consider the process under the risk-neutral measure, which differs from the one in equation (1) because $\bar{z}$ is replaced by $z^* = \bar{z} - \phi$, where ϕ is the the risk premium related to z.[4]

As a consequence, the EBITDA (operating cash flow before taxes), denoted $\pi(k, x)$, which depends on both the book value of assets, $k > 0$, and the shock, x, is

$$\pi(k, x) = xk^\alpha - f, \tag{2}$$

where $f > 0$ is a fixed cost that summarizes all expenses, and π exhibits decreasing returns to scale ($\alpha < 1$). We assume that the firm cannot change the production technology, although it can change the level of production capacity. It is worth noting that, as a consequence of operating leverage, the EBITDA rate has volatility which is higher the lower the level of k.

We assume that capital depreciates both economically and for accounting purposes at a constant rate $d > 0$. So, given a capital stock k and a debt level b, Earnings Before Taxes (EBT) are equal to the firm's EBITDA minus depreciation and the interest to be paid for the outstanding debt:

$$y(k, b, x) = \pi(k, x) - dk - rb.$$

We introduce a corporate tax function, g, defined as a convex function of EBT, which we denote y, to model a limited loss offset provision:

$$g(y) = \begin{cases} y\tau_c^+ & \text{if } y \geq 0 \\ y\tau_c^- & \text{if } y < 0, \end{cases} \tag{3}$$

where τ_c^- and τ_c^+, such that $0 \leq \tau_c^- \leq \tau_c^+ < 1$, are the marginal corporate tax rates for negative and positive earnings, respectively.[5] Investors pay taxes on the returns on the

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[3]The process in equation (1) is motivated by the fact that there is empirical evidence that earnings are persistent. See also Gomes (2001), Hennessy and Whited (2005), Hennessy and Whited (2007), and Moyen (2007).

[4]An analysis of the issues related to the existence of a martingale measure in a discrete time setting is offered by Elliott and Madan (1998).

[5]This choice for the corporate tax function is borrowed from Leland and Toft (1996). In unreported results we implemented also the more accurate convex tax function used by Hennessy and Whited (2005), with no substantial difference in our main conclusions. A more realistic tax environment is described in Liu, Qi, and Wu (2006). The one we use here is suited for the purpose of this work.

securities issued by firms. We assume that taxes on cash flows are levied at constant rates: $\tau_e \in [0, 1[$ for equity holders, and $\tau_b \in [0, 1[$ for bondholders.

We assume that the firms can raise funds by issuing equity and debt and that financial markets rationally price the cash flows paid to these securities. As for debt financing, the companies issue risky callable consol bonds with face value $b \geq 0$ and a coupon rate equal, for practical convenience, to the risk free rate r. The debt adjustment cost function is

$$q(b', b) = \begin{cases} q_0 + q_1 \, |b' - b| & \text{if } b' \neq b \\ 0 & \text{otherwise,} \end{cases} \tag{4}$$

where q_0 is a fixed component and q_1 is the issuance cost proportional to the change from current debt, b, to new debt b'.[6] The adjustment cost function, q, entails a cost in case of both an increment and a decrement of debt. This might be perceived as an oversimplification, because while issuing new debt ($b' > b$) generates underwriting costs, it is less clear what the debt retirement cost should be, as it is witnessed also by Leary and Roberts (2005), p. 2597. On the other hand, it is widely acknowledged that there are implicit costs to reduce the level of debt, due to restrictions on the possibility to pay down debt in advance or on debt repurchase or due to the illiquidity of the secondary market.

Funds can be raised also by issuing equity. In this case, a flotation cost is incurred, which is motivated by information asymmetry and underwriting fees. Hence, if the amount raised by the firm is cfe, the actual (negative) cash flow by the equity holders is $cfe \cdot (1 + \lambda_1) - \lambda_0$, where $\lambda_0 \geq 0$ is a fixed cost component and $0 < \lambda_1 < 1$ is a parameter defining the proportional flotation cost.[7]

The dynamic framework is infinite-horizon and discrete-time. We assume that the firm has two control levers: the book value of assets in place, denoted k, and the face value of outstanding debt, b.

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[6]This function for direct costs for debt restructuring is different from the one in other models. Mauer and Triantis (1994) assume $q_1 = q_1^+$ when $b' > b$, and $q_1 = q_1^-$ when $b' < b$, where q_1^- is in fact the cost for issuing equity, as in their model debt repurchase can be financed only by issuing new equity. In our model we do not have this restriction. Fischer, Heinkel, and Zechner (1989), Goldstein, Ju, and Leland (2001) and Strebulaev (2007), for analytic convenience (i.e., to preserve the scaling property), specify the adjustment cost as $q(b', b) = q_1 b'$ if $b' \neq b$ and $q(b', b) = 0$ otherwise. That is, the cost is proportional to the total amount of debt issued, and not just to the increment $b' - b$. We do not need this simplification, because we solve the valuation problem numerically.

[7]This cost function is drawn from of Gomes (2001). Instead, Hennessy and Whited (2007) use a convex flotation cost function. In unreported results we implemented it, but with no qualitative change of our main conclusions.

B. Investment, capital structure, default

At any date, the firm can decide to invest or disinvest to reach a new level of assets k'.

If there is positive investment, then the cost is represented by $\xi = k' - (1 - d)k$ and it can be financed either with internal funds, such as cash flows from operations, or with external funds, by issuing debt or equity. We assume that capital is homogeneous, so we cannot distinguish between investments made at different dates for depreciation purposes.

On the contrary, if the firm decides to disinvest, it does so at a liquidation price, and then the cash inflow is $\ell((1 - d)k - k')$, with $\ell \leq 1$. This introduces investment irreversibility in the model, and as a consequence, physical asset is not equivalent to cash.[8] For notational convenience, to describe the payoff from investment/disinvestment ξ we define the function $\chi(\xi, \ell)$ as

$$\chi(\xi, \ell) = \begin{cases} \xi & \text{if } \xi \geq 0 \\ \xi\ell & \text{if } \xi < 0. \end{cases}$$

We model also the state of financial distress (liquidity crisis): if financial distress worsens, the firm defaults. Different conditions have been used in the literature to model financial distress. We assume that distress takes place when after-tax operating cash flow is insufficient to cover the coupon payment:

$$rb > \pi(k, x) - g(y(k, b, x)).$$

In this case, the firm sells at a discount $s \leq \ell$ the minimum amount of capital, $(rb + g - \pi)/s$, to make the promised payment.[9]

Equity holders may decide to increase or reduce the debt to a new level b' for the next period. We assume that bondholders do not have the power to block any additional debt issuance. In case it is optimal to change the level of debt to b', all the outstanding debt is called at par value, b, and new debt is issued at the market value $D(k, b', x)$. While the assumption of calling at the face value preserves the rights of the current

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[8]This is different from what Hennessy and Whited (2007) and Moyen (2007) assume, because in their models the investment decision is fully reversible, or equivalently, capital is as liquid as cash. Moreover, irreversibility generates an implicit investment cost, which makes investment lumpy.

[9]While our definition of financial distress in line with Titman and Tsyplakov (2007), Strebulaev (2007), and Gamba and Triantis (2008a), the consequences of financial distress are somehow different. A reduction of cash flow is directly introduced in Titman and Tsyplakov (2007) model. In fact, in their model financial distress generates a cost that is financed either by issuing debt or equity. In ours, the asset sale implies an additional investment that is financed exactly in the same terms: either with equity or debt. This is different also from Strebulaev (2007), because asset sales are based on a discount to market value in his model, as opposed to a discount to book value in our model. Asset sales motivated by liquidity crises have been documented by Asquith, Gertner, and Scharfstein (1994).

debt holders in the event the firm increases its debt level, on the other hand, it entails a refunding cost given by the difference between the market price and the face value of debt, $D(k, b', x) - b$. Said difference will always be negative in case of a debt reduction, but it can also be positive when debt increases, and the market value of the new debt is higher than the face value of existing debt.

The firm can be taken off its steady state path in case of default on its debt obligations. From the above assumptions, all bonds have the same priority in this event. We assume endogenous default; i.e., default is decided by equity holders to preserve limited liability. In the event of default, we assume that debt holders can use debt collection laws to seize the residual value of the firm as an unlevered ongoing concern, at the depreciate capital level $k(1 - d)$, net of bankruptcy costs. Hence, the debt tax shield is lost and absolute priority rule applies. Note that the unlevered asset value includes the positive value of the option to optimally lever the firm and decide the new investment policy. Moreover, to preserve the stationarity of the infinite horizon model, we assume that in case of default, after paying the bankruptcy costs, debt holders become the new owners continuing operations henceforth.

C. Security valuation

The valuation model can be described as a dynamic program: at any date, after observing x, for given (k, b) the firm chooses a new level of capital, k', and a new level of debt, b'. If the firm is solvent, investment and financing decisions can be made with no restriction. On the other hand, in case of default, we assume that no decision is made regarding investment and capital structure, and the equity holders exercise the limited liability right by surrendering the firm to bondholders at the current level of depreciated capital.

The value of the equity, denoted $E(k, b, x)$, is the solution of a Bellman equation based on the optimization of the sum of current cash flow and the expected present value of future optimal cash flows (i.e., the continuation value).[10] The value of debt rationally incorporates the optimal policy decided by shareholders. Hence, the current value of the debt is the present value of future payoff to bondholders contingent on equity holders decisions.

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[10]We assume that managers always pursue shareholders' interest. It is beyond the focus of this work to analyze such agency issues. In addition, as mentioned in the introduction, we assume perfect symmetric information between equity holders and bondholders.

The cash flow to equity holders in case of no default, in state (k, b, x) assuming the firm is solvent, for a decision (k', b'), is

$$
\begin{aligned}
cfe(k, b, k', b', x) = {} & \max \left\{ \pi(k, x) - g(y(k, b, x)) - rb, 0 \right\} \\
& + \left(D(k', b', x) - b \right) \mathcal{I}_{b' \neq b}(b') - q(b', b) \\
& - \chi \left(k' - k(1 - d) + \max \left\{ \frac{rb + g(y(k, b, x)) - \pi(k, x)}{s}, 0 \right\}, \ell \right). \quad (5)
\end{aligned}
$$

In this equation, $\mathcal{I}_{b' \neq b}(b')$ is equal to one when $b' \neq b$ and zero otherwise, and $D(k', b', x)$ is the ex–coupon price of debt, considering that the new book value of assets is k', and the new book value of debt is b'. The first line of equation (5), at the right-hand side, captures the after tax operating cash flow, if positive; the second line presents the net flow from a capital structure change; in the third line we have the net flow from a change in the capital stock, including the effect of depreciation and of fire sales, when the after tax operating cash flow is negative.

The value of the equity at state (k, b, x) is the solution of the dynamic program

$$
E(k, b, x) = \max \left\{ \max_{(k', b')} \left\{ e(k, b, k', b', x) + \beta \mathbb{E}_{k, b, x} \left[E(k', b', x') \right] \right\}, 0 \right\}, \quad (6)
$$

where the actual cash flow to equity holders is

$$
e = \begin{cases} cfe \cdot (1 - \tau_e) & \text{if } cfe \geq 0 \\ cfe \cdot (1 + \lambda_1) - \lambda_0 & \text{if } cfe < 0. \end{cases} \quad (7)
$$

In equation (6), the discount factor is $\beta = (1 + r_z(1 - \tau_e))^{-1}$, where r_z denotes the certainty equivalent rate of return on equity flows,[11] and the expectation is computed with respect to the transition probability of the process in (1), conditional on the current state of the firm, (k, b, x).

Interpreting equation (6), at the current state, shareholders maximize their value, given by the current cash flow plus the continuation value, by selecting the new level of book value of asset and liabilities. In this case, the optimal policy is

$$
(k^*, b^*) = \arg \max_{(k', b')} \left\{ e(k, b, k', b', x) + \beta \mathbb{E}_{k, b, x} \left[E(k', b', x') \right] \right\}, \quad (8)
$$

provided that the value of equity is positive. If $e(k, b, k^*, b^*, x) + \beta \mathbb{E}_{k, b, x} \left[E(k^*, b^*, x') \right]$ is negative (i.e., the firm cannot recover from financial distress), then shareholders default

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[11]The certainty equivalent rate of return on equity flows, r_z, is determined under a tax equilibrium setting as $r_z = r(1 - \tau_b)/(1 - \tau_e)$, where τ_b is the personal tax on debt income and τ_e is the personal tax on equity income. Notice that, in the same setting, the discount factor for bond flows is $\beta_b = (1 + r(1 - \tau_b))^{-1}$. Hence, $\beta_b = \beta = \beta_e$. See Sick (1990) for details.

on servicing debt and surrender the firm to debt holders. In this case, the production capacity is kept at its current level, $k(1-d)$, and the firm becomes all-equity-financed: $(k^*, b^*) = (k(1-d), 0)$.[12] To summarize, the optimal policy is

$$\varphi(k, b, x) = \delta(k, b, x) \cdot (k(1-d), 0) + (1 - \delta(k, b, x)) \cdot (k^*, b^*), \qquad (9)$$

where

$$\delta(k, b, x) = \begin{cases} 1 & \text{in case of default} \\ 0 & \text{otherwise} \end{cases} \qquad (10)$$

is the default indicator function.

To compute $D(k, b, x)$, the ex–coupon value of debt under the assumption that the firm is solvent, we have to determine the cash flow to debt holders at (k, b, x), when firm's decisions are made:

$$cfd(k, b, x, \varphi) = (1 - \delta(k, b, x)) \left[rb(1 - \tau_b) + \mathcal{I}_{b' \neq b}(b')b + (1 - \mathcal{I}_{b' \neq b}(b'))D(k', b', x) \right]$$
$$+ \delta(k, b, x)(1 - c) \min\{E(k(1-d), 0, x), b\}, \quad (11)$$

where $(k', b') = \varphi(k, b, x)$ and $E(k(1-d), 0, x)$ is the value of the corresponding unlevered $(b = 0)$ firm from equation (6), at the depreciated capital level $k(1-d)$, and c is the proportional bankruptcy cost.[13] The first line of equation (11) displays, in the right hand side, the value to current bondholders if the firm is solvent at (k, b, x); the second line shows the value if the firm is in default.

Hence, the value of debt is

$$D(k, b, x) = \beta \mathbb{E}_{k,b,x} \left[cfd(k', b', x', \varphi) \right], \qquad (12)$$

Equations (6) and (12), determining the value of equity and debt, is a system of simultaneous non–linear equations that must be solved numerically. In Appendix A we describe the numerical method we use to solve the valuation problem.

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[12]This is different from Mauer and Triantis (1994) and Obreja (2006), who assume that an insolvent firm cannot make any decisions. Instead, in our model, if a firm is insolvent, it first tries to make up the debt payment using existing asset or issuing new equity. If still it is insolvent and shareholders do not find it profitable to refund, the firm defaults.

[13]Equation (11) sets also the debt principal as an upper bound for the cash flow to bondholders in case of bankruptcy. In unreported results, we investigated as alternative possibilities, the recovery cash flow to debt holders in case of default is either a proportion of the depreciated book value of asset, $cfd(k, b, x) = k(1 - d)(1 - c)$, or of the face value of their claim $cfd(k, b, x) = b(1 - c)$. The results presented in Section III are qualitatively not affected by this choice. We do not consider strategic debt service (see, for instance, Mella-Barral (1999)): it is beyond the scope of this paper to dissociate default from liquidation.

D. Variations on the base model

For comparison purposes, in our analysis we will consider also different versions of the baseline model described above.

The first type of variation is related to a different objective function. Instead of maximizing the value of equity, we alternatively assume that the dynamic investment and financing decisions aim at maximizing the total firm value, defined as the sum of equity and debt values. The dynamic program for the firm value, $V = E + cfd$, is

$$V(k, b, x) = \max_{(k', b')} \{e(k, b, k', b', x) + \beta \mathbb{E}_{k,b,x} [E(k', b', x')]$$
$$+ rb(1 - \tau_b) + D(k', b', x')(1 - \mathcal{I}_{b' \neq b}(b')) + b\mathcal{I}_{b' \neq b}(b')\}. \quad (13)$$

Denoting with (k^*, b^*) the maximand of the problem in (13), the corresponding value of equity is $E(k, b, x) = e(k, b, k^*, b^*, x) + \beta \mathbb{E}_{k,b,x} [E(k^*, b^*, x')]$ if it is strictly positive (and hence (k^*, b^*) is the optimal solution). Otherwise, the firm is in default and the optimal decision is $(k(1 - \delta), 0)$, the value of equity is set to zero, $E = 0$ (i.e., $V = cfd$), and the default indicator, $\delta(k, b, x)$, equals one.

The second variant is obtained by assuming that equity holders can follow the investment (or, alternatively the financing) policy, which maximizes the total firm value in equation (13), and then they choose the financing policy (alternatively, the investment policy) in their own interest, but with the investment (financing) policy constrained to be first best. This intermediate type of problem is used in a later section to isolate the effect of agency issues related to distirtions of either first best financing or first best investment.

The third type of variation is related to investment and financing flexibility. A first case is based on the assumption that there is no dynamic choice of investment and capital structure (referred to as SF-SI model).[14] Hence, we assume that debt and capital are kept constant over time. Since we are interested in a steady state solution, we assume that the physical capital of the firm is maintained at the level k by forcing the firm to expense the depreciation, dk. The second restricted specification of the model, denoted DF-SI, assumes static investment decisions (i.e., k is constant),[15] but allows for dynamic decisions on capital structure.[16] The last variation of the base model assumes static

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[14]This specification is in the same spirit of the model proposed by Leland (1994), although there are many differences as far as the cash flow process, the tax environment and the financial market are concerned.

[15]Differently from Strebulaev (2007), we do not introduce financial distress in the version with static investment and dynamic capital structure. Since in this specifications there is no investment (apart from replacement of depreciated capital), allowing for fire sales would not permit to have a steady state solution of the problem, because eventually, all the capital stock would be depleted.

[16]This specification is in the spirit of Fischer, Heinkel, and Zechner (1989) or Goldstein, Ju, and Leland (2001), although some features of the cash flow process and the tax functions are different.

capital structure (i.e., b is constant) together with dynamic investment, and will be denoted SF-DI.[17] Given the above restrictions, for convenience, we denote the baseline version (with dynamic investment and capital structure) as the DF-DI model.

II. Model calibration

This section aims at selecting the values of the model parameters for which the estimated credit risk measures reflect actual measures. Specifically, our target credit risk measures are quasi-market leverage, historical average credit spreads, and default rates per rating class. We consider seven (whole letter) risk classes, ranging from from AAA/Aaa (companies with extremely strong capacity to meet financial obligations) to C (class made of companies with rating CCC/Caa or worse).

The calibration procedure can be described as follows. We first select the quasi-market leverage (QML) as the criterion to assort companies into risk classes. The justification for this choice is provided in Appendix B.

Then, we use Monte Carlo simulation to create a sample of companies and to compute the quasi-market leverage ratio together with the target credit risk measures for each company in the sample. Note that, since in the DF-DI model the leverage at a given date is path dependent, the only way to generate a mapping between quasi-market leverage and the credit risk measures is by explicitly considering the pattern of past decisions; i.e., by using simulation. With this mapping, we generate the estimated distribution of risk classes and compute the average of the credit risk measures per rating class. The simulation procedure, the classification of simulated companies into credit classes, and the computation of estimated metrics, all needed for the calibration process, are explained in detail in Appendix B.

Table V displays a summary of the empirical metrics for our dataset, which consists of an unbalanced panel of 9048 US firms (excluding financial, insurance, and real state firms) from years 1997 to 2005, with between 887 and 1110 companies per year. Appendix B also expounds on the dataset.

Table I reports the values of model parameters. We take some of these values from previous research and we modify some other with the purpose of (i) reasonably improving the fitting of the simulated credit metrics with respect to the empirical ones, and (ii) making unlikely to have optimally unlevered firms in the sample. This process produces the "best fit" values or calibrated values.

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[17]A model with static debt and dynamic investment has been studied by many authors. For instance, Leland (1998), Childs, Mauer, and Ott (2005), Titman and Tsyplakov (2007), and Moyen (2007).

Specifically, the value of the parameters of the flotation cost function, $\lambda_0 = 0.08$ and $\lambda_1 = 0.09$, are derived from Gomes (2001) and are greater than the ones in Strebulaev (2007). Proportional debt adjustment costs are provided in Fischer, Heinkel, and Zechner (1989) with $q_1 = 0.01$. Our constant and proportional transaction costs are higher: $q_0 = 0.04 = q_1$. We use a return to scale of $\alpha = 0.476$, higher than the one in Gomes (2001), who proposes $\alpha = 0.30$ but with a different hypothesis on productive technology. We include also a fixed cost, to increase both the operating and (as a consequence) the financial leverage. Gomes (2001) also provides $\rho = 0.62$ and $\sigma = 0.15$, that are lower than ours. In Moyen (2004), $\alpha = 0.45$, $\rho = 0.6$ and $\sigma = 0.2$ and the fixed cost is $f = 1.3$. So, our parameters are similar to hers, with the exception of a higher ρ. Note that Hennessy and Whited (2005) provide a (structural) estimate of ρ equal to 0.74, not far from our choice. The depreciation rate, d, and the salvage value, s, are the same as in Hennessy and Whited (2005). The selling price of capital stock in case of disinvestment, $\ell = 0.9$, represents the degree of investment irreversibility, and is greater than the one in Gamba and Triantis (2008a). The personal tax rates $\tau_e = 0.15$ and $\tau_b = 0.20$ are in line with the ones in Hennessy and Whited (2007) and in many other papers. The corporate tax rate is the same as in Hennessy and Whited (2007) ($\tau_c^+ = 0.40$). With our selection of parameters, the rate of net tax advantage to debt in case of positive earnings is 34.12%. The tax rate for losses is 15%, which means a limited possibility of loss offsetting. Finally, the risk-free rate is 5% on a annual basis and the risk-premium is 6%, in line with many other models.

Table V collects also the simulated statistics for the three credit risk measures. While the fitting of QML to empirical data is excellent for all credit classes, and is good for credit spreads and default rates in classes from AAA through B, the fitting for class C is rather poor. We consider this a satisfactory result for the purpose of our work.

Finally, we conduct a sensitivity analysis of some model parameters to make sure that our results are not driven by any single feature of the model. In our experiments, we noticed that the parameters related to the production function, α, f, the corporate tax on positive earnings, τ_c^+, are the ones that have the largest impact on the average yield spread and the average default rate. To a lesser extent, the average yield spread and the average default rate depend also on the parameters related to the exogenous state variable, ρ and σ, on the tax rate on corporate bonds, τ_b, on the liquidation price, ℓ, and on the salvage value, s. Finally, the other parameters, like the tax rate on negative earning, τ_c^-, the equity flotation costs, λ_0 and λ_1, the debt adjustment costs, q_0 and q_1, and the bankruptcy cost, c, have an even smaller impact on the distributions, and are used mostly to improve the fitting of simulated credit spreads.

III. Results

A. Financial flexibility and investment flexibility

In this section, we analyze how the price of debt depends on dynamic investment and capital structure choices. We compare the spread between the yield on corporate debt and the risk-free rate, $YS = rb/D - r$, where D is the ex–coupon value of debt in equation (12), for the base case model with dynamic investment and dynamic financing decisions (DF-DI), to the yield spread for three restrictions of the same model: DF-SI (dynamic financing and static investment), SF-DI (static financing and dynamic investment), and SF-SI (static financing and static investment), using the values of model parameters from Table I.

The upper panel of Figure 1 plots the yield spread against the productivity of the firm's capital stock, x at a debt level (in nominal terms) $b = 2$ and at a capital stock $k = 7.68$. Unless stated differently, we will always base our subsequent discussion on this choice of k and b.[18] The yield is obtained considering that equity holders exploit flexibility (whether only financial (DF-SI), or only investment (SF-DI) or both (DF-DI)) in their own interest (second best).

In general, the negative slope of the yield spread as a function of the productivity is as expected: highly profitable companies generate enough operating cash flows and have no problems to meet current debt obligations. More importantly, we observe that SF-DI and DF-DI have almost the same yield spread,[19] and that the spreads of DF-SI and SF-SI are very close to each other. In addition, the cases with dynamic investment have the highest spreads in all states. Lastly, financial flexibility, even if decided in the sole interest of shareholders, reduces credit risk.

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[18] $k = 7.68$ and $b = 2$ are the modes of the simulated distribution of capital stock and nominal debt, respectively. The qualitative results presented below do not depend of the specific choice of k and b. In unreported analysis we found the same qualitative results for different levels of k and b. These are available from the authors on request.

[19] The result that introducing financing flexibility slightly reduces the yield spread differs from that of Dangl and Zechner (2004). They show that, comparing a (continuous time, infinite horizon) model with dynamic capital structure choice to a model with static capital structure, the yield spread of the former is (for many, but not all parameter choices) higher that the yield spread of the latter. The explanation they offer is that, in a dynamic capital structure setting, the firm will increases the debt exposure in good states (high cash flow from operations). We will later show this is not the case in our framework: the firm is not willing to increase the debt only to to exploit the tax shield, when the capital stock is held constant (static investment). The debt level is only increased, in case of dynamic investment, to fund investments.

The bottom panel of Figure 1 depicts the yield spread in case the current debt and the capital stock cannot be changed over time. Specifically, this is derived from the value of the firm's debt by considering only the default option,

$$D^0(k, b, x) = \lim_{T \to \infty} \left(\sum_{t=1}^{T} \beta^t (1 - \tau_b) rb(1 - CDP_{k,b,x}(t)) + \beta^T b(1 - CDP_{k,b,x}(T)) \right.$$

$$\left. + \sum_{t=1}^{T} \beta^t \mathbb{E}_{k,b,x} \left[R(k, b, x') \delta(k, b, x') \right] \right). \quad (14)$$

In the above equation, $CDP_{k,b,x}(t)$ is the cumulative default probability up to year t at the current state;[20] $R(k, b, x) = (1 - c) \min\{E(k(1 - d), 0, x), b\}$ is the recovery in case of default, and $\delta(k, b, x)$ is the default indicator function. Equation (14) reads as follows: the first line is the present value of coupon payments in case the firm does not default; the second line is the present value of the expected recovery in case of default. Notice that the expectation of recovery from default at year t requires the computation of the marginal default probability at t assuming the firm is not in default at $t - 1$. The yield spread in the bottom panel of Figure 1 is then computed as $YS^0 = rb/D^0 - r$.

First, we notice that for the DF-SI case, the possibility to adapt capital structure over time reduces credit risk. Second and more important, YS^0 is zero for the two cases of dynamic investment (SF-DI and DF-DI); i.e. the default option alone contributes nothing to credit risk in these cases and the yield spread is entirely due to current and future investment and financing decisions together with the fact that these decisions are made in the sole interest of equity holders. In the above comparison, it is impossible to disentangle theses two effects and to provide an assessment of the importance of this opportunistic behavior on credit risk.

In Figure 2 we plot the spread of the same four versions of the model when the decisions are made in the best interest of all stakeholders (first best), as opposed to just equity holders (second best). First, we notice that with dynamic investment and financing decisions, debt is almost riskless in the first best case, whereas in the second best case (Figure 1) we had the highest yield spread. Keeping investment constant, while dynamic financing is first best, slightly increases credit risk in bad states, although the yield spread is lower than the one under the corresponding second best case. This reveals a somehow surprising result that there can be an agency issue related to capital structure decisions. We will explore this in a later section. On the other hand, holding capital structure constant, while dynamic investment is first best, increases the yield spread

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[20]The marginal default probability at t, is defined as $DP(t) = \Pi_{nd}^{t-1} \Pi_d \delta$, where Π is the Markov transition probability matrix (see Appendix A), Π_{nd} is the transition matrix restricted to non-default states, according to the indicator function δ, and Π_d is the matrix restricted to transitions from non-default states to default states, based on δ. The cumulative default probability is defined as $CDP(T) = \sum_{t=1}^{T} DP(t)$. The expected recovery from default is given by $\Pi_{nd}^{t-1} \Pi_d R$.

by a bigger extent. It is noteworthy that, although investment decisions are made to maximize the firm value and so the spread is smaller than in the corresponding second best case, the credit risk is much bigger than in the static case (SF-SI).

The big differences between first best (Figure 2) and second best (Figure 1) suggest that not only investment and financing decisions are interrelated, but also they jointly encourage an opportunistic behavior from shareholders. Therefore, it is important to analyze the mechanism in which these agency issues operate and interact as far as credit risk is concerned. This is the goal of the next subsection.

B. Agency costs

Figure 3 shows the yield spread vs. the state variable, x, when both financing and investment decisions are endogenous (DF-DI), using the values of model parameters from Table I. Both decisions can be made in the best interest of shareholders (second best policy, hereafter (S)) or of all stakeholders (first best policy, hereafter (F)). These cases are the same as reported in Figures 1 (upper panel) and 2, respectively.

As previously remarked, we find that second best investment and financing decisions together reduce significantly the price of bonds. It is important to stress that these two distortions from a first best policy are inherently interconnected. By construction, financing decisions affect and are affected by investment decisions. Nevertheless, for clarity sake, we analyze each type of deviation from a first best policy in isolation.

To correctly measure the contribution to credit risk of either a second best investment policy or of a second best financing policy, we compute the spread for two intermediate cases, denoted DF-DI(F,S) and DF-DI(S,F) respectively. In the first one, DF-DI(F,S), at any date, after observing x, a new level of debt is chosen by picking b^* from the solution of the firm value maximization problem in (13). Conditional on this choice, equity holders select the level of capital stock in their best interest. Thus, by comparing the yield spread for this intermediate case to the one for DF-DI(F), we isolate the impact of a second best investment policy. In the second intermediate case, DF-DI(S,F), the decision process is reversed: equity holders choose their best financing policy conditional on a first best investment policy (i.e., conditional on a k^* coming from the solution of problem (13)). This case permits to isolate the effect of a second best debt policy on the yield spread. Both these intermediate cases are included in Figure 3. We notice that following a second best financing, while investment is first best, increases the spread but to a lower extent than following also a second best investment. On the other hand, when the debt policy is first best and investment is second best, default risk is increased to a

higher level than in the DF-DI(S) case.[21] To explain this behavior we have to analyze also the investment and the financing policies.

Table II shows the average investment policy in case of dynamic financing for different values of the state variable. Both financing and investment decisions can be first best or, alternatively, second best. For each level of the state variable, x, the average investment policy is computed as the average of the ratio $k^*/(k(1-d))$ in non-default states, where the average is taken with respect to current k, and k^* is the optimal investment. Notice that k^*, under second best, is the optimal investment policy from equation (6), whereas, under first best, it is derived from equation (13). The current nominal debt is $b = 2$. An average ratio above/below one implies the firm is investing/disinvesting. The comparison of DF-DI(F,S) to DF-DI(F), which are the cases of interest here, shows the distortion on the investment policy.

In good states (x higher than 1), equity holders acting in their best interest under-invest relative to first-best.[22] A lower investment with exactly the same debt policy (by construction) increases the payout to equity holders. In bad states ($x < 1$), we observe a great deal of disinvestment; i.e., equity holders have strong incentive to sell part of the asset. This behavior increases the severity of default (i.e., it increases the probability of default and reduces the recovery value). In general, shareholders disinvest in bad states essentially because they try to avoid financial distress costs.[23] Actually, while equity holders do not suffer from default costs, in a liquidity crisis they bear the cost of selling assets at a discount with a lower proceed than the liquidation price in case of voluntary disinvestment, because $s < \ell$. This cost can be partly avoided by generating enough cash to pay the coupon through disinvestment. Hence, in bad states, we see that in the F,S case, they disinvest more readily than in the F case. Finally, for intermediate states (x equal to 1) equity holders invest more in their own interest than they would do to maximize total firm value. If things go badly, they are better prepared to service a first-best debt policy payments, because they have more capital to sell and to collect cash from. If things go well, they will use the debt funds to generate higher payouts; that is, they will choose a lower level of investment, as described above.

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[21]This happens for the DF-DI(F,S) case at states around one. This is because, when firms are forced to adopt a first best debt policy, they assume more debt and since second best investment is lower, they default at higher states, when x is just below one.

[22] The underinvestment distortion gives rise to the classical overhang problem, in the sense of Myers (1977), if the debt level is not changed. On the contrary, if the debt level is changed, current debt holders are reimbursed at par, and play no role in the distortion. New bond holders are the ones who anticipate the failure to invest at the first-best, and incorporate it into a lower price of newly issued debt. This is precisely the situation we are describing here. Hennessy and Whited (2007) have already documented underinvestment with a source different from debt overhang.

[23]Later in this section we provide also other motivations for disinvestment in bad states.

From the above analysis, we can state that there is an agency issue related to invest-ment policy.[24] In our dynamic setting, debt holders rationally incorporate this incentive to implement an investment policy of self-interest into a lower price of debt. This is so independently of the current level of debt, b, and of the optimal face value of debt, b^*. Ac-tually, if $b = 0$ and $b^* > 0$, then the new debt is issued at $D(k^*, b^*, x)$ and the shareholders bear the cost of the sub-optimal investment policy because they collect less cash from debt issuance. If instead $b > 0$, then equity holders can decide either to change or to keep the current debt level. In the first case ($b^* > 0$ and $b^* \neq b$), the new bondholders ratio-nally anticipate the equity-maximizing investment policy, and they transfer this agency cost to equity holders through the price $D(k^*, b^*, x)$, and consequently, through the yield spread. Hence, shareholders immediately bear the consequences of their (second best) investment decision through the net cash flow from debt change, $D(k^*, b^*, x) - b$. Also in the second case ($b^* = b$), the equity maximizing investment policy is incorporated into the debt price although, differently from the previous case, bond holders cannot readily transfer this cost to equity holders because the debt contract is long-term. Yet, they an-ticipated this distortion from the first best decision when, in a previous year, the equity holders decided to issue debt for a face value b. So the price of debt reflects not only current, but also future sub-optimal investment decisions. Precisely, in our discussion of the investment policy distortion (case DF-DI(F,S) of Table III), we see that in good states there is also a change in the level of debt. Current debt holders do not suffer the consequence of sub-optimality, for they receive more than the fair value of their claim, $b \geq D(k^*, b, x)$. On the contrary, shareholders are the ones who immediately bear the consequences of the distortion.

As shown by Gamba and Triantis (2008b), agency issues related to financing decisions can be as important as the ones for the investment decisions. This agency cost is the debt value shortfall deriving from an equity maximizing, as opposed to the firm value maximizing, financing decision. To isolate the effect of this type of agency issues we compare the first best case F to S,F; i.e., a case with second best financing conditional on first best investment.

Table III provides the average debt policy in several cases. For each level of the state variable, x, the average debt policy is computed as the average of the ratio b^*/b in non-default states, where the average is taken with respect to current b, and b^* is the optimal face value of debt from equation (8) under second best and from (13) under first best. The book value of assets is $k = 7.68$. An average ratio above/below one implies the issuance/retirement of debt. The comparison of DF-DI(S,F) to DF-DI(F) shows the distortion on the debt policy: in good states, with the exception of very high values for x, equity holders acting in their best interest issue more debt than under the first best case. With this policy, the current bondholders receive the par value, and since $b \geq D(k^*, b, x)$,

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[24]This analysis is in line with the approach proposed by other authors like Leland (1998), Childs, Mauer, and Ott (2005), Moyen (2007), where the capital structure decision is made to maximize the total firm value.

they do not suffer from a second best capital structure decision. On the other hand, in bad states shareholders no longer pay down the debt, as it would be optimal from a firm value maximizing perspective. In this case, the current bondholders bear the cost of sub-optimality of capital structure decision (i.e., no change of the debt level). While they cannot readily transfer this cost to equity holders, they rationally anticipate this in previous years when the debt was issued. So the price of debt is lowered (or the yield is increased) by future second best capital structure decisions. Titman and Tsyplakov (2007) also reported that value-maximizing firms have a more symmetric incentive to increase and decrease debt. On the contrary, they also find that equity-maximizing firms are not inclined to reduce their debt.

This distortion is absent when debt maturity is prespecified, either optimally, as in Childs, Mauer, and Ott (2005),[25] or not, as in Hennessy and Whited (2007) and Moyen (2007). In all these cases, the debt decision is always made after the debt expiry, when the firm is unlevered, and so the price of newly issued debt fully reflects all the agency costs.

Lastly, for the current choice of parameters, we do not see any optimal debt reduction from the perspective of equity holder. This seems to be in line with the argument presented in Dangl and Zechner (2004) that shareholders have a put option to default and they never reduce the debt level because this would decrease the value of the option. In our model, this argument is no longer valid because we have financial distress costs, which are borne by equity holders. Actually, for a different choice of parameters, such that the cost of financial distress is particularly high relative to debt transaction costs, we have that if debt level is low, for some particular unfavorable states, it is in the best interest of equity holders to reduce the debt exposure to avoid potential distress costs.

We can now analyze the joint effects on credit risk of distortions on first best investment and financing decisions. Comparing the average investment policy of DF-DI(F) to that of DF-DI(S) in Table II, we observe underinvestment in good states, which is (by construction) rationally incorporated by bondholders into a lower debt price. The key to understand this distortion of the investment policy lies precisely in the second best financing policy. When the debt level is decided under the second best case, equity holders choose a much lower one because debt financing is relatively more expensive than under the first best case. Less debt funds are raised, and as a consequence, should equity holders keep investment at the first best level, they would necessarily finance it using external equity, which is not optimal from their viewpoint. As a result, they react by reducing investment.[26] In the end, in DF-DI(S) we see a lower investment ratio and a lower level of debt, compare to first best case, so that shareholders have on average

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[25]In Childs, Mauer, and Ott (2005) the debt maturity is chosen at the initial date to maximize the total firm value.

[26]Titman and Tsyplakov (2007) also report an incentive to underinvest when investment and financing decisions are made in the shareholders' interest.

about the same level of cash flow as the one they have under the first best policy. Note also that, in good states, the investment policy for DF-DI(S) is similar to that of a firm constrained to stay unlevered; i.e. the additional funds raised trough debt are not invested, and are used to increase the payout to shareholders.

From Table II, in bad states for the DF-DI(S) case, we observe a stronger incentive to sell part of the asset than in the DF-DI(F) case, tough to a lower extent compared to DF-DI(F,S). Disinvestment goes together with a constant debt policy; i.e., instead of using the cash to pay down the debt, equity holders are using the proceeds to complement the low cash flow from operations in an attempt to keep their cash flows at a reasonably constant level. It is this joint opportunistic effect that creates credit risk and increases the yield for DF-DI(S), as we see in Figure 3. A second motivation for the liquidation of assets in bad states comes from the fact that the volatility of the cash flow rate is increased when k is low, due to the operating leverage induced by fixed costs. To clarify this point, in Table IV we compute the cash flow rate volatility, σ_{CF}, for three different capital levels, $k = 4, 7, 10$, assuming that the current productivity is $x = 1$ and the current face value of debt is $b = 2$. The cash flow for a given k and x, is

$$CF(k, x) = \pi(k, x) - g(\pi(k, x) - dk - rb) - rb,$$

where $\pi(k, x)$ is the EBITDA from equation (2) and g is the corporate tax function from (3). The growth rate of cash flow is defined as $CF(k, x')/CF(k, 1) - 1$, and μ_{CF} and σ_{CF} are its expected value and standard deviation, respectively. From the last row of the table, we can see that the lower k the higher σ_{CF}. Hence, operating leverage permits equity holders to better exploit the convexity of their value function.[27] This is why, in Table II, we have that also equity holders managing a firm constrained to stay unlevered, disinvest, though only in very bad states.

Notice that the distortion of keeping the level of debt constant only shows in states where the firm disinvests. Differently from Titman and Tsyplakov (2007) (but similarly to Moyen (2007)), in our model shareholders can partially reverse the investment decisions made in previous steps. This is an important feature of the model that is worth exploring further. Figure 4 and Table II show, respectively, the yield spread and the investment policy for the two extreme cases of no-disinvestment and fully reversible investment. By no-disinvestment we precisely mean that $\xi = k' - (1 - d)k \geq 0$ in cases of no liquidity crisis, so that voluntary sales of capital stock are ruled out.[28] The fully reversible case is when disinvestment does not entail costs, or $\ell = 1$.[29] From Table II,

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[27]From unreported results, we obtain that the higher the current face value of debt, the bigger is the incentive to disinvest.

[28]Yet, fire sales can occur in case of financial distress. For this reason, the average investment ratio lies slightly below one in bad states.

[29]This makes the capital stock equivalent to a highly profitable form of cash balance. Yet, in our model (see equation (9)) shareholders cannot sell assets to the point of causing the default, because in that case their policy is undone and the firm asset is restored.

we learn that the average investment ratio is the same in good states for the two cases at hand and for the base case DF-DI(S). However, this is not true in bad states. As expected, equity holders are more actively disinvesting if the asset is reversible. The consequence is a reduction of the collateral (i.e., the value of unlevered equity) in case of default and a slightly higher yield spread than in the base case. The opposite is true in the case of no disinvestment. The collateral is rarely reduced (only in cases of fire sales, when the reduction is forced), and debt holders ask for a lower yield on their claim.

Financial flexibility does not affect much credit risk when decisions are second best. This can be seen from Table II: joint investment and financing policies in the sole interest of equity holders, DF-DI(S), generate an investment policy identical in all states to that of a static capital structure, but with dynamic investment, SF-DI(S). This is due to the fact that, in good states, the distortion on the financing policy under second best, which consists of issuing less debt, pushes the investment ratio down to, on average, the same level as the one in the SF-DI(S). For low values, there is disinvestment independently from a static or dynamic capital structure. The same investment together with the same financing in bad states produce a yield spread in the fully dynamic case almost identical to the one in the SF-DI(S) case (see Figure 1).

When investment is static and debt is the only lever, DF-SI,[30] equity holders still make decisions of self-interest, but the effect on the yield spread is not as important as in the fully dynamic case. Figure 5 plots the yield spread for the variation on the base model with only dynamic capital structure. As shown, the spread derived from the second best financing policy is greater than the one from the first best policy. From the corresponding average debt policy in Table III, we can see that in good and intermediate states the firm tends to reduce debt when decisions are firm value maximizing. This reduction makes debt almost riskless in those states. This is not the case if equity holders act in their own interest, as in DF-SI(S). No matter how good the state is, debt is almost always kept constant, even if the net tax shield is positive. The consequence is that, although reduced in good states, the yield spread never reaches zero.

From Table III, we have that, when the capital structure is the only lever (DF-SI), in bad states the firm issues debt, no matter if following a first or second best goal. In the base model (DF-DI), in bad states voluntary disinvestment is preferred to fire sales since the cost is lower. Likewise, disinvestment is preferred to the issuance of new securities, because of direct and indirect (e.g., the refunding cost) transaction costs. In the latter case, debt issuance is preferred to equity issuance. This is so because, although shareholders would pay the refunding cost coming from a lower market value of debt in case new debt is issued, they would never inflate new equity cash if the state is poor. In the case at hand (DF-SI), as disinvestment and fire sales are ruled out, there are only two channels to raise cash to service the coupon: either debt or equity issuance. If the current debt level is low, the refunding cost is either small or positive when increasing the debt

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[30]This case is similar, but not identical, to the one presented by Dangl and Zechner (2004).

(i.e., the cash inflow from new debt issuance, $D(k, b', x)$, is higher than current debt, b), and so there is a net inflow from the debt policy. Moreover, if new debt is issued, current debt holders receive more than the fair value of their claim ($b \geq D(k, b, x)$). Therefore, issuing debt is the best policy for all stakeholders. This is the reason why we observe a debt policy ratio higher than one in bad states under first best. In addition, comparing the debt policy of DF-SI(F) and DF-SI(S), we see a stronger incentive to issue debt in the latter case. As it would happen with disinvestment in the fully dynamic case, shareholders issue more debt with the aim of using part of the proceeds to complement their low cash flow generated by the operations.

C. Corporate and personal taxes

In the previous analysis, we have noticed that many of the observed effects on yield spreads are motivated by how a specific feature affects the debt policy. Since Modigliani and Miller (1958) and (1963), the net tax shield has been considered a key determinant of capital structure decisions and hence of credit risk. We investigate here the influence of taxes (and specifically of τ_c^+ and τ_e) on credit risk, in a framework where the capital structure choice is motivated also by investment/disinvestment and is affected by agency issues.[31]

To this aim we compare three cases. In the first, we use the baseline parameters ($\tau_c^+ = 40\%$ and $\tau_e = 15\%$). In the second case, we reduce τ_c^+ to 35% while keeping τ_e at the base value. On the contrary, in the third case, we reduce τ_e to 10.23%, with τ_c^+ at the base value. The net tax advantage to debt, when earnings are positive, in our setting is computed as

$$\tau_c^+ - \left(1 - \frac{1 - \tau_b}{1 - \tau_e}\right),$$

and it is approximately 34.12% in the base case, and 29.12% in the other two cases, where either the corporate tax rate or the personal tax rate on equity is reduced.

The effect of a change on taxes is almost the same for all three specifications of the model – fully dynamic, only dynamic financing and static: a reduction on corporate taxes always has a larger impact on the yield spread than an equivalent reduction on personal taxes on equity flows. Figure 6 plots the yield spread against the state variable for the SF-SI, and the DF-DI model, and for the three cases at hand. We see that a lower corporate tax rate significantly reduces the spread, whereas an almost similar reduction of the personal tax rate does not change the spread.

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[31]We exclude τ_b from our analysis because our valuation setting is neutral (i.e., the price of debt, and also the yield spread, does not change) with respect to personal taxes on bond flows (see Gamba, Sick, and Aranda León (2008)), except in case of default, because we do not model a tax credit for bondholders on bankruptcy losses.

This can be motivated as follows. Corporate taxes are levied on earnings before taxes (EBT). As a consequence, a reduction on corporate taxes increases the net operating cash flow. On the one hand, this higher cash flow can be used to better service the debt, i.e, to avoid financial distress; on the other hand, the residual after the coupon payment augments the cash flow to equity holders. Both destinations reduce the default probability, and consequently, the yield spread.[32] On the contrary, if we reduce personal taxes on equity flows by the same percentage, first of all it does not increase the net operating cash flow. Second, the effect is smaller, because the tax rate is applied to equity cash flow, which is always lower than the EBT due to the presence of investment.[33] The conclusion is that personal taxes have a lower impact on the value of equity, the default probability, and consequently, the yield spread. Alternatively, to see a reduction on the yield spread of a similar magnitude to the corporate tax case, we would need a bigger reduction on τ_e.

D. Transaction costs

In this section, we want to see if the effects of a debt policy motivated also by investment/disinvestment decisions are due also to debt transaction costs.

Figure 7 plots the yield spread vs the state variable, for the DF-SI and the DF-DI versions of the model with and without direct transaction cost of debt ($q_0 = q_1 = 0$), when financing and investment decisions are made to maximize equity value. We compare these cases to the corresponding baseline cases. Table III compares the average debt policy of DF-SI and DF-DI with and without debt transaction costs.

As for the DF-SI case, zero transaction costs on debt reduces the yield spread.[34] When compared to the debt policy of the base case case with transaction costs, DF-SI(S), we can see that more debt is issued in bad states, practically no debt is retired at the intermediate states, and more debt is issued in good states, because debt issuances and retirements are less expensive. In addition, in very good states, equity holders are free to issue more debt to exploit the net tax shield, as, in case there is a subsequent downturn of x, they can conveniently reduce the debt exposure, fully exploiting financing flexibility. In the end, this policy reduces the risk and, consequently, the yield spread. Actually, when there are no transaction costs, debt can become riskless in good states,

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[32]In unreported analysis, we observe that a change of τ_c^+ (e.g., 1-2%) does not influence the yield spread until it increases the cash flow to a given threshold such that the default probability is reduced. In addition, we also obtain that a higher corporate tax rate on negative earnings, τ_c^-, (i.e., a significant rebate in case of losses) also, and for the same reason, reduces the yield spread.

[33]This is maintenance investment in the SF-SI and DF-Si cases. For the DF-DI, disinvestment only takes place to avoid financial distress.

[34]In unreported analysis, we noticed that lower flotation cost on equity also reduces the yield spread in the case of DF-SI. Since the results for zero equity transaction costs are very similar to the ones we are presenting here, we report only the analysis of the effect of debt transaction costs.

due to the positive role of a dynamic capital structure policy, as in the first best case (see Figure 5).

As for the DF-DI model, transaction costs have almost no effect on the yield spread. Compared to the previous case, the effect on the debt policy is less significant (and in unreported results we have that there is almost no affect on the investment policy). The fact that transaction costs only impacts the DF-SI case reinforces the idea that model parameters directly affecting the debt policy (for instance, transaction costs discussed here or corporate taxes discussed in the previous section) have only a small effect on the yield on corporate debt.[35]

IV. Concluding remarks

We have developed a discrete-time and infinite-horizon structural model for credit risk of corporate debt featuring endogenous investment, endogenous capital structure, and default. Investment and leverage policies are simultaneously decided to maximize equity value. Corporate investments are partially irreversible and are financed with internal and external equity and with debt. The debt contract is a defaultable consol bond, with a call provision. The firm faces both constant and proportional debt adjustment costs and both constant and proportional equity flotation costs. The financial distress cost is represented by a fire sale discount. In the case of bankruptcy, bondholders pay bankruptcy costs and receive the unlevered ongoing concern. The corporation has a convex tax function, thus including a limited loss offset provision, and investors face personal taxes, with higher rates for bond flows than for equity flows. We assume that equity holders and bondholders have the same information.

Our goal was to analyze how the price of debt, or alternatively the yield spread, depends on dynamic investment and capital structure choices, in a setting with market frictions. To this end, we calibrated the model using firm accounting information from Compustat North America Industrial Annual and market data from the CRSP, from Moody's Investor Service for the cumulative default rates, and Reuters for the credit spread paid by industrial bonds. We calibrated the model by matching default risk statistics (leverage, yield spread, default rates) on a per credit class basis.

Our results show that, when policies are firm value maximizing, dynamic investment and dynamic capital structure decisions reduce the yield on corporate debt with respect to the case where both capital structure and investment are static. However, exactly the opposite is true if dynamic decisions are made in the best interest of equity holders: debt becomes riskier and yield spreads are higher than in the static case. The spread

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[35]In unreported analysis we computed YS^0 for both DF-SI and DF-DI. We observe that zero transaction costs have a positive effect (i.e., a reduction in the yield spread) only in the first case.

when investment and financing decisions are second best is similar to the one of a firm with constant capital structure such that the investment/disinvestment policy is decided in the sole interest of shareholders. We also find that the possibility to adapt the capital structure over time does not modify the investment policy, as long as shareholders' make self-interested decisions.

We observe that the effect of the tax shield on the yield spread is of second order in the presence of dynamic investment. That is, the main driver of capital structure decisions is not the tax shield as in models based on dynamic capital structure only. Rather, the main driver is the interaction of capital structure decisions with investment/disinvestment decisions. The same can be said about debt transaction costs. In the end, apart from the fluctuations of economic conditions, it is the financing policies influenced by - and influencing - the investment policies that really affects credit risk of corporate debt.

A. Numerical solution of the fixed point problem

In this section we describe the numerical procedure we use to solve the dynamic programs introduced in Section I. For the sake of brevity, here we will refer only to the more general model with dynamic investment and dynamic capital structure, assuming equity value maximization.

The general valuation model for equity and debt presented in this paper belongs to the class of continuous decision infinite horizon Markov Decision Processes. The solution method is based on successive approximations of the fixed point solution of the valuation operator using a dynamic programming approach. For numerical purposes we apply this method to an approximate discrete state-space and discrete decision valuation operator.[36]

Gauss-Hermite quadrature method (see Tauchen (1986)) is used to approximate the dynamics of the AR(1) process of $z = \log(x)$ with a finite state Markov chain. We take S discrete, equally spaced, abscissae in an interval of semi-width $I_p = 3.5\sigma/\sqrt{1-\rho^2}$, and centered on the long term mean of process, z^*. The set of the discretized state variable is $\widetilde{Z} = \{\widetilde{z}(s) \mid s = 1, \ldots, S\}$. Let w be the distance between two successive elements of $\widetilde{Z}$ and $\eta = z^*(1-\rho)$. The transition probability matrix from $\widetilde{z}(i)$ to $\widetilde{z}(j)$, for all $i = 1, \ldots, S$ is

$$\Pi(i,1) = \mathcal{N}\left(\frac{\widetilde{z}(1) - \eta - \rho\widetilde{z}(i) + w/2}{\sigma}\right);$$

$$\Pi(i,j) = \mathcal{N}\left(\frac{\widetilde{z}(j) - \eta - \rho\widetilde{z}(i) + w/2}{\sigma}\right) - \mathcal{N}\left(\frac{\widetilde{z}(j) - \eta - \rho\widetilde{z}(i) - w/2}{\sigma}\right),$$

for $i = 2, \ldots, S-1$, and

$$\Pi(i,S) = 1 - \mathcal{N}\left(\frac{\widetilde{z}(S) - \eta - \rho\widetilde{z}(j) - w/2}{\sigma}\right).$$

Then we define the actual state space $\widetilde{X} = \left\{e^{\widetilde{z}(s)} \mid s = 1, \ldots, S\right\}$.

We set the upper bound for capital stock, k_u, and for the face value of the debt, b_u respectively, in a way that they are never binding for the optimization problem. We discretize $[0, k_u]$, to obtain $\widetilde{K} = \left\{\widetilde{k}_j = k_u(1-d)^j \mid j = 1, \ldots, N_k\right\}$. The interval $[0, b_u]$ is discretized into N_b equally spaced values, gathered into the set $\widetilde{B}$. We denote (k, b, x) as the discretized control variable.

We solve the problem

$$E(k, b, x) = \max\left\{\max_{(k', b')}\left\{e(k, b, k', b', x) + \beta\mathbb{E}_{k,b,x}\left[E(k', b', x')\right]\right\}, 0\right\},$$

---

[36]See Rust (1996) or Burnside (1999) for a survey on numerical methods for continuous decision infinite horizon Markov Decision Processes.

where function $e(k, b, k', b', x)$, defined in equation (7), depends also on $D(k, b, x) = \beta \mathbb{E}_{k,b,x} [cfd(k', b', x', \varphi)]$ and

$$
cfd(k, b, x, \varphi) = \begin{cases} rb(1 - \tau_b) + \beta \mathbb{E}_{k,b,x} [D(k^*, b, x')] & \text{if } b = b^* \text{ and } E(k, b, x) > 0 \\ rb(1 - \tau_b) + b & \text{if } b \neq b^* \text{ and } E(k, b, x) > 0 \\ (1 - c) \min\{\max\{E(k(1 - d), 0, x), 0\}, b\} & \text{if } E(k, b, x) = 0 \end{cases}
$$

in all points of the discrete state space. More succinctly, the fixed point problem is

$$
\begin{aligned} E &= \Gamma(E, D) \\ D &= \Psi(E, D) \end{aligned} \tag{15}
$$

where Γ and Ψ denote the approximate Bellman operators.

The fixed point solution of the system of non-linear equations (15) is found by successive approximations. This means that, given the guesses E_0 and D_0, we iterate the following

$$
\begin{aligned} E_j &= \Gamma(E_{j-1}, D_{j-1}) \\ D_j &= \Psi(E_{j-1}, D_{j-1}) \end{aligned}
$$

until convergence. For the optimal set of parameters in Table I, we use $k_u = 13$ and $b_u = 6$. We solve the model using $S = 27$, $N_k = 27$ $N_b = 37$. Tolerance is set at 10^{-14}.

Given the optimal solution, we can determine the optimal policy $\varphi(k, b, x)$ by looking for the arg-max of equity at the discrete states (k, b, x).

B. Calibration

In this section we first describe the dataset and next explain in more detail the calibration method. Specifically, we elaborate on the selection of the criterion to assort companies into credit classes, the simulation procedure, the classification of simulated firms into credit classes, and the computation of the estimated metrics.

Empirical data on US firm are obtained from Compustat. We exclude financial, insurance and real estate firms (SIC code 6000-6999) and also regulated utility firms (SIC code 4900-4999). Second, we drop any firm-year observation with (i) non listed share price in the Center for Research on Security Prices (CRSP)/Compustat Data Merged files, (ii) non available Standard & Poor's Long Term Issuer Credit Rating or (iii) any missing data for the variables considered. We end up with an unbalanced panel of firms from years 1997 to 2005 with between 887 and 1110 companies per year (in total the data set has 9048 observations).

Data variables used in the computation of company metrics are: book value of common equity (Compustat item 60),[37] book value of long-term debt (item 9), book value of short-term debt (item 44), earnings before interest and taxes or EBIT (item 178), interest expenses (item 15), fiscal year end share price (CRSP item PRCC12) and, finally, number of outstanding shares (CRSP item CSHOQ12). The firms' metrics built upon these variables and their definitions are: (1) quasi-market leverage, defined as total debt (long-term plus short-term) over total debt plus the product of market price of share and number of outstanding shares; (2) total debt over EBIT; (3) interest coverage, defined as EBIT over interest expenses; and (4) return on assets (ROA) computed as EBIT over total debt plus the product of market price of share and number of outstanding shares.[38]

Next, based on the S&P's Long Term Issuer Credit Rating, we sort firms in seven (whole letter) risk classes, ranging from AAA/Aaa (companies with extremely strong capacity to meet financial obligations, code 2) to C (class made of companies with rating CCC/Caa or worse; i.e., code 17 and higher).[39] Within each risk class, from the panel data we compute the mean of each metric relevant for our analysis.

The average annual default rates are obtained as $1 - \exp\left(1/10 \cdot \log(1 - DR_{10})\right)$, where DR_{10} is the Moody's 10 year default rate (Average issuer-weighted corporate percentage default rates by whole letter rating, 1983-2005. Moody's Investors Service (2006)). In particular, the 10 year default rate we used are (in %): 0.208 for AAA, 0.415 for AA, 1.248 for A, 4.721 for BBB, 21.038 for BB, 46.931 for B, and 78.673 for C. The average credit spreads paid by industrial bonds in each specific rating class are obtained from Reuters and are referred to year 2004. All the above results are summarized in Table V.

As for the selection of the best credit class assorting criterion, we run several ordered probit models on our Compustat sample. In all of them, the dependent variable is the S&P's seven (whole letter) risk classes discussed previously. It takes values from 1 (class AAA) to 7 (class C). As potential independent variables (i.e., as sorting criteria), we have selected the following metrics: book-leverage (debt over debt plus book value of equity), quasi-market-leverage (debt over debt plus the product of market share price and number of outstanding shares), debt over EBITDA, debt over EBIT, EBITDA over interest expenses, EBIT over interest expenses, and finally ROA (EBIT over total debt plus the book value of equity).

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[37]Alternatively, it is total assets (item 6) less total liabilities (item 181) less preferred stocks (item 10).

[38]Total debt over EBIT, the interest rate coverage, and ROA are used in the classification procedure described below.

[39]Originally, our data time period was 1995-2005. As a stability check, we split the sample into two subsamples (1995-1999, and 2000- 2005) and computed the median for each subsample. We also carried out the analysis on a yearly basis. On this last analysis, we noticed that for years 1995 and 1996 companies in class C were very few, and more importantly, a great number of them had a zero figure for long-term debt in the data base. To avoid distortions, we decided not to include these two years in the sample.

In unreported analysis, we estimated the model using each of the potential criteria as the sole independent variable. We observed that the independent variable is always statistically significant except for EBITDA and EBIT over interest expenses. However, when book leverage together with quasi-market-leverage is considered, the first variable is no longer significant. Quasi-market-leverage then carries the same information on the credit worthiness of a firm as book leverage. As a result, we dropped book leverage from the final model specification. Similarly, when both debt over EBITDA (or, alternatively, debt over EBIT) and quasi-market-leverage are introduced, the first variable becomes not significant. Again, the same conclusion follows.

Our final specification (Table VI) only includes quasi-market leverage and ROA as independent variables. The large t-ratio on quasi-market leverage lead us to conclude that this metric constitutes the primary credit risk sorting criterion. As such, we classify a firm by minimizing the distance between the simulated quasi-market leverage, $b/(E(k, b, x) + b)$, and the same measure, specific for each rating class, from Table V.

As for the simulation procedure, we simulate $20,000$ paths for the state variable x using the stochastic model in equation (1) for 150 steps (years). Next, we apply the optimal policy φ from equation (9). At every step, along each path, the realization of the exogenous state variable, combined with the current debt level and stock of capital provides the endogenous values of equity, debt and yield spread as determined by the optimization problem. To get rid of the influence of the initial condition, we drop the first 50 steps.

In case of default, the new equity holders carry on operations for the current depreciated capital $k(1 - d)$, pay bankruptcy costs and, in case, optimally issue new debt and invest in new capital stock. For a particularly low level of capital stock, the firm would remain unlevered and would never find optimal to restart operations. To keep the size of the sample of active firms constant, we assume that a lower bound for capital, denoted k_d, is set in our simulation so that the condition $\Pr\{\pi(k_d, x) > dk_d\} > 0$ holds true. This means that there is a positive probability that the EBIT will be positive in the next step and the firm is restarted.[40]

At every step from 50 to 149 we classify firms into the seven credit classes (from AAA to C) by minimizing the distance between the simulated quasi-market leverage of the firm and the mean for the empirical quasi-market leverage for the specific class. Firms in default and firms which are optimally unlevered are not considered.

Once companies are classified into credit classes, we compute the other two target measures of credit risk in the following manner. First, at every t for the ω-th firm, we compute the yield spread as $rb_t/D(\omega, t) - r$, where $D(\omega, t)$ is the ex coupon price and b_t is the current par value of debt. Next, we take the average of the simulated yield

---

[40]Alternatively, we can say that the state (k, b, x) with $k = k_d$ is not absorbing. In our simulations, we set k_d such that $\Pr\{\pi(k_d, x)dk_d\} = 0.5$.

spreads within each class, and compute the time average of this for years from year 50 to 149. Second, we compute the annual default rate for each class as the default relative frequency on the number of non-default firms at the beginning of the year. Then, we take the time average of the simulated frequencies for each class from year 50 to 149.

At this point we compare the simulated values of the selected metrics to the target values. If the fitting is not good, we change the parameters to improve it. The procedure is repeated until a satisfactory fitting is obtained. Table V shows the simulated values of the metrics for the "best fit" parameters.

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ρ	AR(1) persistence	0.80
σ	annual volatility of the state variable	0.20
$\bar{x}$	long term value of the state variable	1
r	annual risk-free borrowing rate	5%
ϕ	premium on cash flows risk	6%
τ_e	personal tax rate on equity flows	15%
τ_b	personal tax rate on coupon	20%
τ_c^+	corporate tax rate for positive earnings	40%
τ_c^-	corporate tax rate for negative earnings	15%
α	production return-to-scale parameter	0.476
d	annual depreciation rate	10%
f	fixed production cost	1.35
s	fire-sale discount for asset sales	0.45
ℓ	liquidation price for disinvestment	0.90
c	proportional bankruptcy costs	0.70
λ_0	fixed flotation cost for equity	0.08
λ_1	variable flotation cost for equity	0.09
q_0	fixed issuance cost for debt	0.04
q_1	variable issuance cost for debt	0.04

Table I: **Model parameters**

x	0.53	0.64	0.76	1.00	1.31	1.57	1.87
DF-DI(F)	0.92	1.11	1.15	1.29	1.63	1.77	1.88
DF-DI(F,S)	0.49	0.83	1.02	1.67	1.43	1.50	1.64
DF-DI(S,F)	0.88	1.09	1.15	1.29	1.63	1.77	1.88
DF-DI(S)	0.74	0.93	1.02	1.29	1.38	1.42	1.49
DF-DI(zero debt)	0.94	1.04	1.29	1.33	1.35	1.41	1.48
DF-DI(no disinv.)	0.95	0.99	1.02	1.25	1.38	1.42	1.49
DF-DI(reversible)	0.41	0.89	0.98	1.26	1.39	1.42	1.49
SF-DI(S)	0.74	0.93	1.02	1.29	1.38	1.42	1.49

Table II: **Investment policy**. The table provides the investment ratio $k^*/(k(1-d))$ at $b = 2$, averaged with respect to k, vs the productivity parameter, x. k^* is the optimal investment. We consider the baseline model, DF-DI (dynamic financing and dynamic investment), for the base case parameters and for the first best case (F), second best case (S), second best investment conditional on first best financing (F,S), second best financing conditional on first best investment (S,F). Moreover, we consider the second best DF-DI for for zero debt transaction costs ($q_0 = q_1 = 0$). We consider also the second best DF-DI either with fully reversible asset $\ell = 1$, or with no disinvestment ($k' - (1-d)k \geq 0$). Lastly, we compute the investment policy of a firm constrained to stay unlevered (*zero debt* or $b = 0 = b'$). The plots are obtained using $S = 27$, $N_b = 27$ and $N_k = 37$ points for the approximate solution.

x	0.53	0.64	0.76	1.00	1.31	1.57	1.87
DS-DI(F)	0.90	0.90	0.91	1.00	1.29	1.91	2.64
DS-DI(F,S)	0.90	0.94	0.99	1.00	1.29	1.91	2.64
DS-DI(S,F)	1.00	1.00	1.00	1.00	1.40	2.13	2.63
DF-DI(S)	1.00	1.00	1.00	1.00	1.00	1.28	1.49
DF-DI(reversible)	1.00	1.00	1.00	1.00	1.00	1.28	1.33
DF-SI(F)	0.95	1.13	1.02	0.92	0.89	0.84	0.81
DS-SI(S)	1.04	1.23	1.10	1.00	0.98	1.00	1.00
DF-SI(no debt trans. cost)	1.36	1.25	1.09	0.91	0.90	1.25	1.69
DF-DI(no debt trans. cost)	1.00	1.00	0.91	1.00	1.05	1.05	1.89

Table III: **Debt policy**. The table provides the debt ratio b^*/b at $k = 7.68$, averaged with respect to b, vs x. b^* is the optimal debt level. We consider the baseline model, DF-DI (dynamic financing and dynamic investment), and DF-SI model (dynamic financing and static investment), for the base case parameters and for the first best case (F), second best case (S), second best investment conditional on first best financing (F,S), second best financing conditional on first best investment (S,F). Moreover, we consider both the second best DF-DI and DF-SI cases for zero debt transaction costs ($q_0 = q_1 = 0$) and for fully reversible investment ($\ell = 1$). The plots are obtained using $S = 27$, $N_b = 27$ and $N_k = 37$ points for the approximate solution.

k	4	7	10
$CF(k)$	0.4507	0.9250	1.3254
μ_{CF}	-0.0025	0.0205	0.0203
σ_{CF}	0.6005	0.3577	0.2912

Table IV: **Volatility of the Cash Flow Rate**. We compute the volatility of the cash flow rate, σ_{CF}, for three different capital levels, $k = 4, 7, 10$, assuming that the productivity is at its long-term average value, $x = 1$ and the current face value of debt is $b = 2$. The cash flow for a given k and x, is $CF(k, x) = \pi(k, x) - g(\pi(k, x) - dk - rb) - rb$, where the EBITDA, $\pi(k, x)$, is defined in equation (2) and the corporate tax function, g, is from (3). The growth rate of cash flow is defined as $CF(k, x')/CF(k, 1) - 1$. μ_{CF} is the expected value of the growth rate and σ_{CF} is the standard deviation of the cash flow rate.
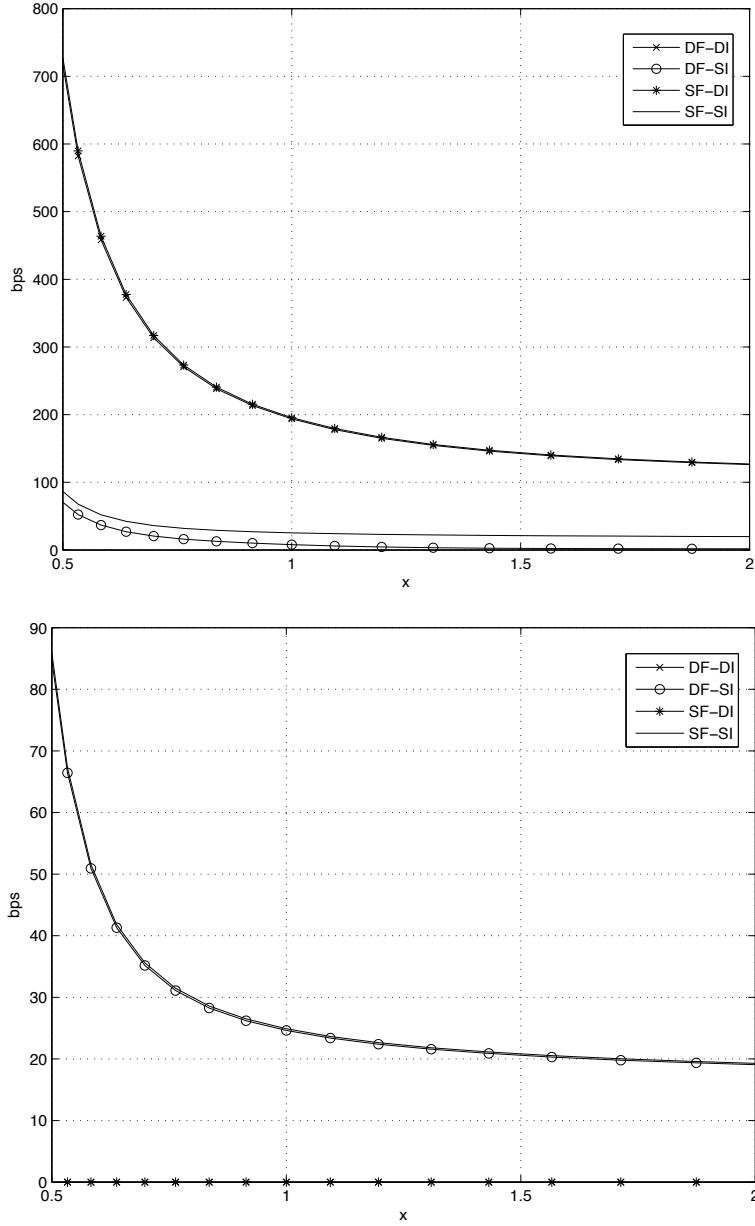
Figure 1: **Yield spread**. The upper panel plots the spread between the yield on corporate debt and the risk-free rate, $rb/D(k, b, x) - r$, with respect to the productivity parameter, x. $D(k, b, x)$ is the ex–coupon price of debt from equation (12). The bottom panel plots yield spread assuming the current capital, k, and debt, b, cannot be changed. The debt price is D^0 from equation (14). The yield spreads (in basis points/year) are determined for a current face value of debt $b = 2$ and for a current capital stock $k = 7.68$. We consider the baseline model, DF-DI (dynamic financing and dynamic investment), and three restrictions of the model: DF-SI (dynamic financing and static investment), SF-DI (static financing and dynamic investment), and SF-SI (static financing and static investment). All policies maximize shareholders' value. The plots are obtained using $S = 27$, $N_b = 27$ and $N_k = 37$ points for the approximate solution.
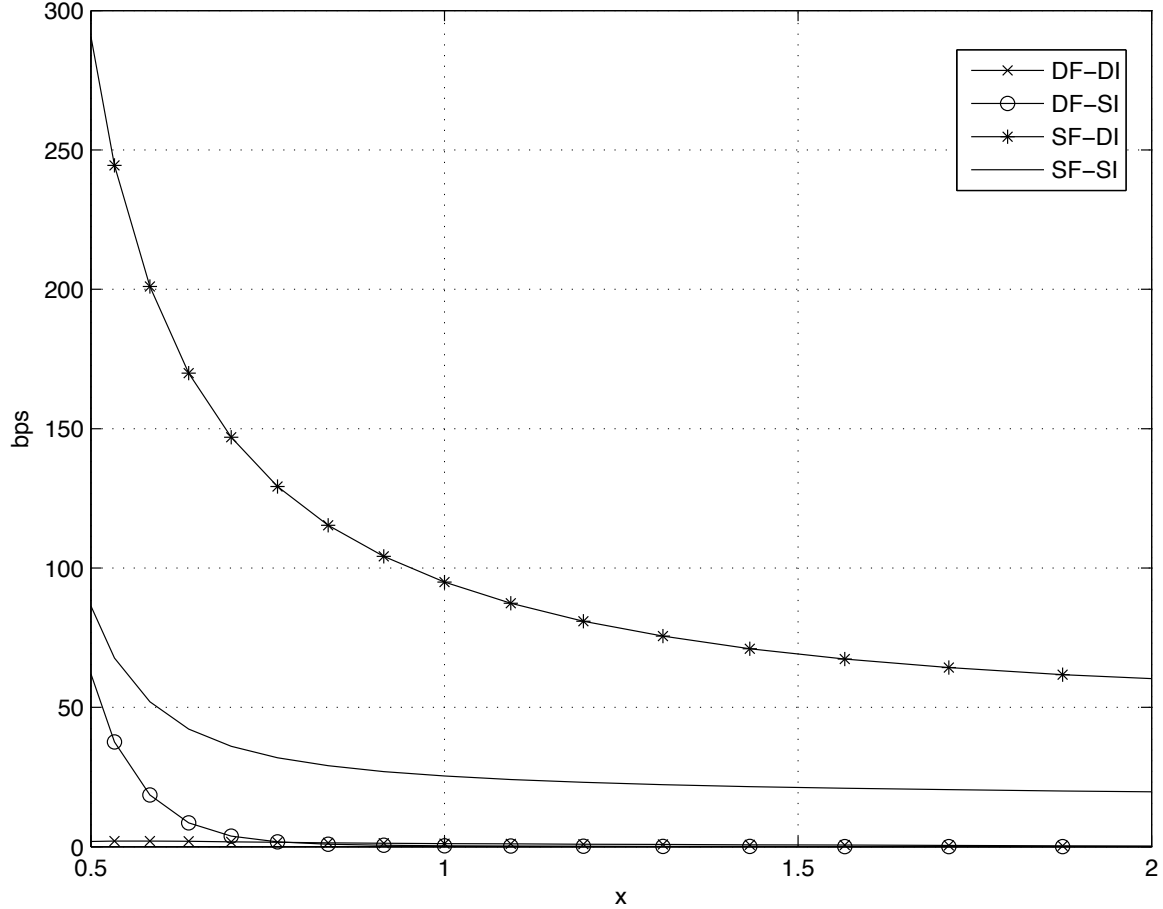
Figure 2: **First best**. The graph plots the spread between the yield on corporate debt and the risk-free rate, $rb/D(k,b,x) - r$, with respect to the productivity parameter, x. $D(k,b,x)$ is the ex–coupon price of debt from equation (12). The yield spread (in basis points/year) is determined for a current face value of debt $b = 2$ and for a current capital stock $k = 7.68$. We consider the baseline model, DF-DI (dynamic financing and dynamic investment), and three restrictions of the model: DF-SI (dynamic financing and static investment), SF-DI (static financing and dynamic investment), and SF-SI (static financing and static investment). All these cases are considered under first best. The plots are obtained using $S = 27$, $N_b = 27$ and $N_k = 37$ points for the approximate solution.
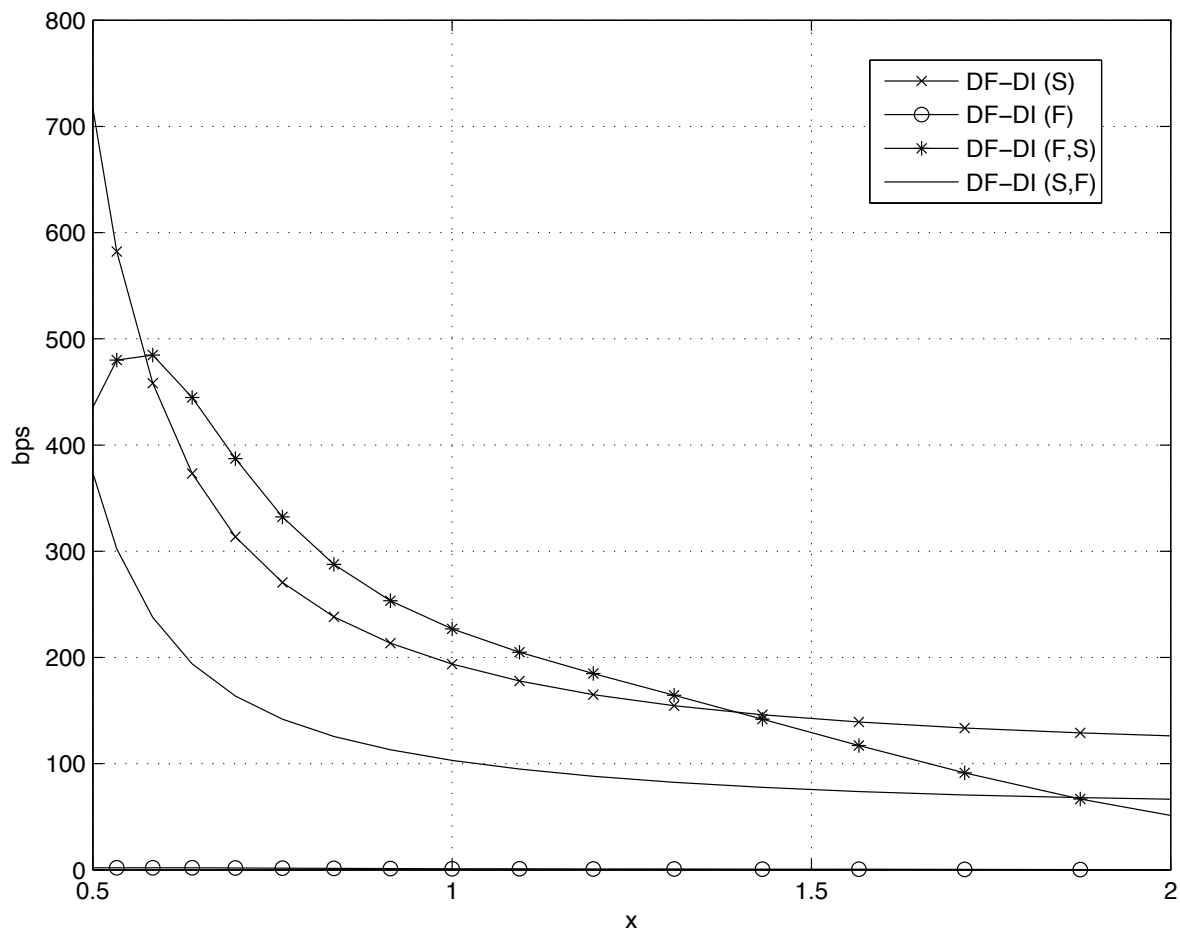
Figure 3: **First best vs second best policy**. The graph plots the spread between the yield on corporate debt and the risk-free rate, $rb/D(k, b, x) - r$, with respect to the productivity parameter, x. $D(k, b, x)$ is the ex–coupon price of debt from equation (12). The yield spread (in basis points/year) is determined for a current face value of debt $b = 2$ and for a current capital stock $k = 7.68$. We consider the baseline model, DF-DI (dynamic financing and dynamic investment), with four different possible goals of firm's policy: second best investment and financing (S); first best investment and financing (F); second best investment conditional on first best financing (F,S); second best financing conditional on first best investment (S,F). The plots are obtained using $S = 27$, $N_b = 27$ and $N_k = 37$ points for the approximate solution.
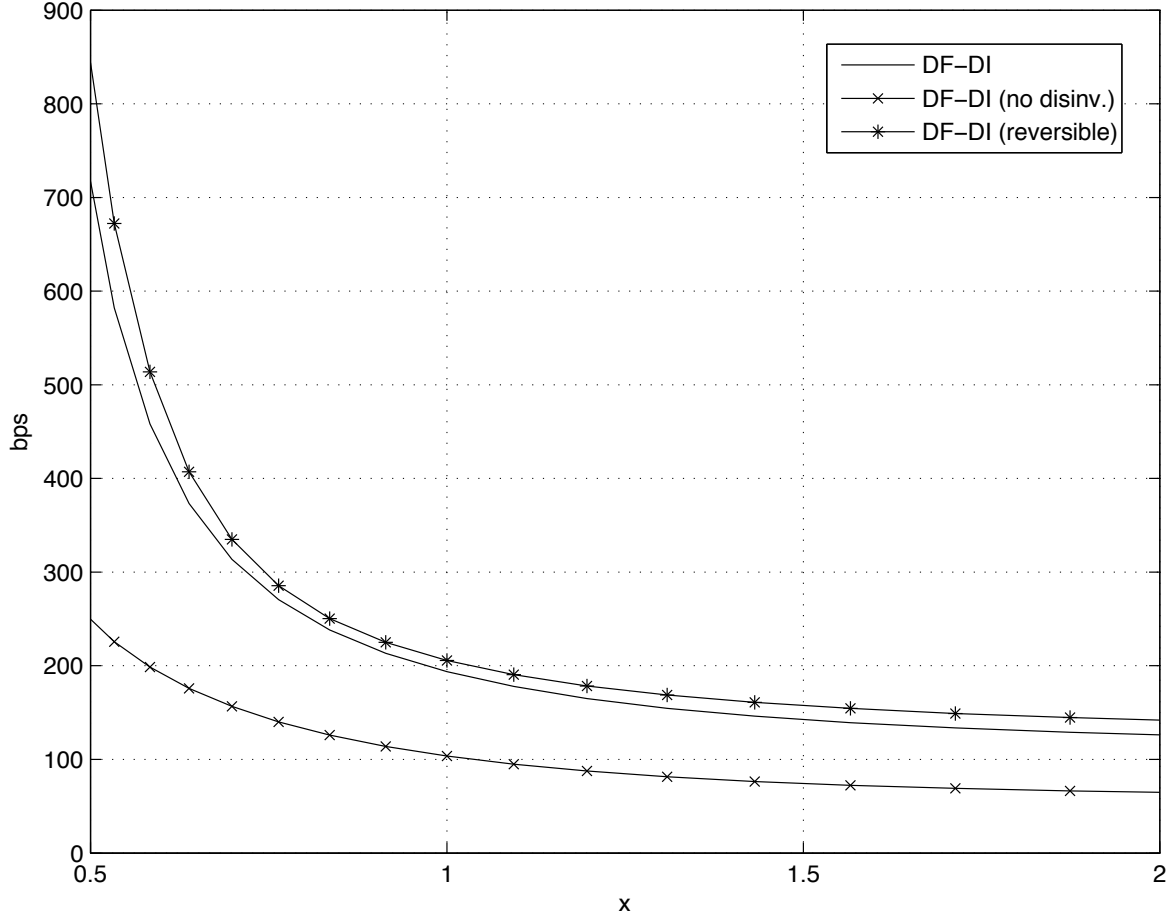
Figure 4: **Asset flexibility**. The figure plots the spread between the yield on corporate debt and the risk-free rate, $rb/D(k, b, x) - r$, with respect to the productivity parameter, x. $D(k, b, x)$ is the ex coupon price of debt from equation (12). The yield spread (in basis points/year) is determined for a current face value of debt $b = 2$ and for a current capital stock $k = 7.68$. We consider the DF-DI model (dynamic financing and dynamic investment) for the baseline parameters and for either *no-disinvestment* ($k' - (1 - d)k \geq 0$, when there is no financial distress), or fully *reversible* asset ($\ell = 1$). All cases are under second best. The plots are obtained using $S = 27$, $N_b = 27$ and $N_k = 37$ points for the approximate solution.
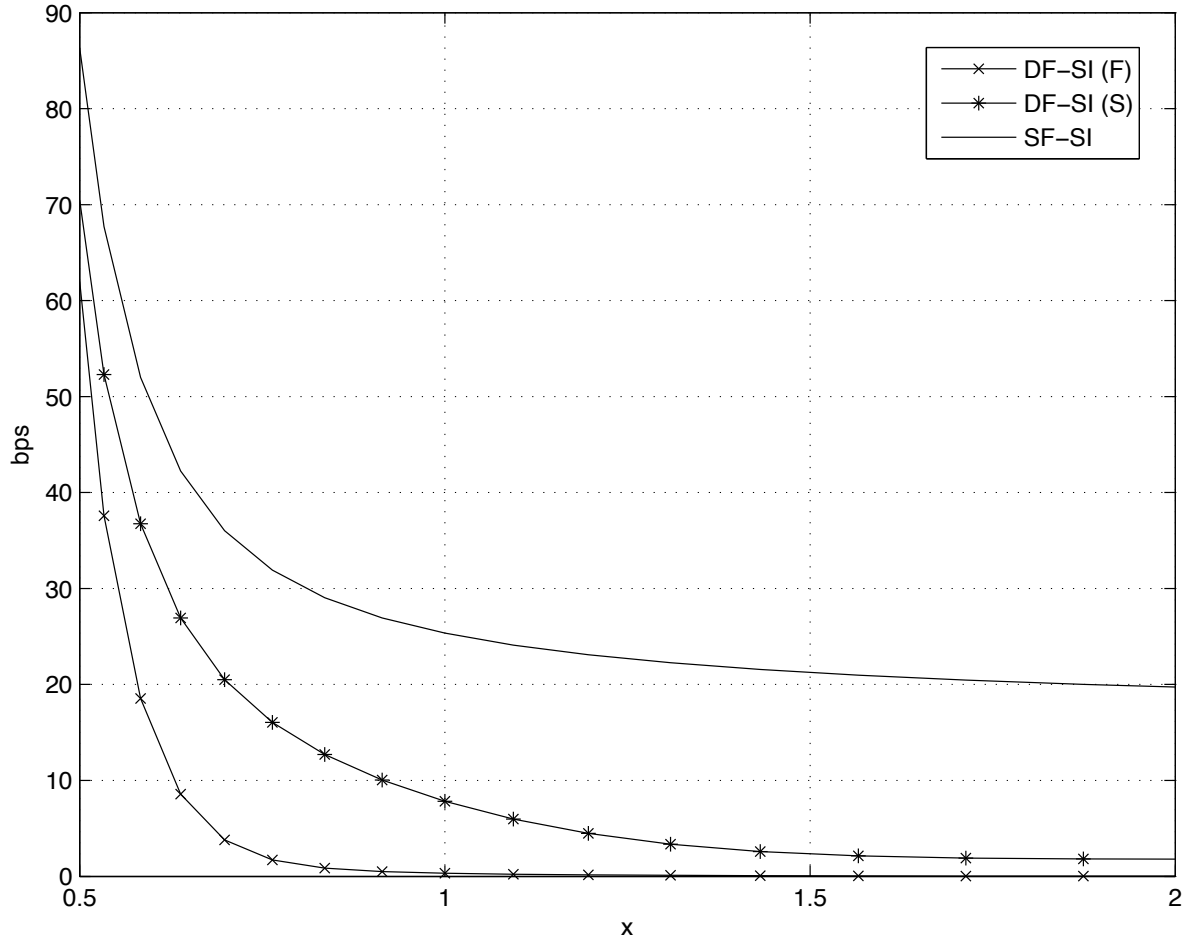
Figure 5: **Dynamic capital structure only**. The graph plots the spread between the yield on corporate debt and the risk-free rate, $rb/D(k, b, x) - r$, with respect to the productivity parameter, x. $D(k, b, x)$ is the ex–coupon price of debt from equation (12). The yield spread (in basis points/year) is determined for a current face value of debt $b = 2$ and for a current capital stock $k = 7.68$. We consider the DF-SI model (dynamic financing and static investment) either under first or second best, and the static model SF-SI. The plots are obtained using $S = 27$, $N_b = 27$ and $N_k = 37$ points for the approximate solution.
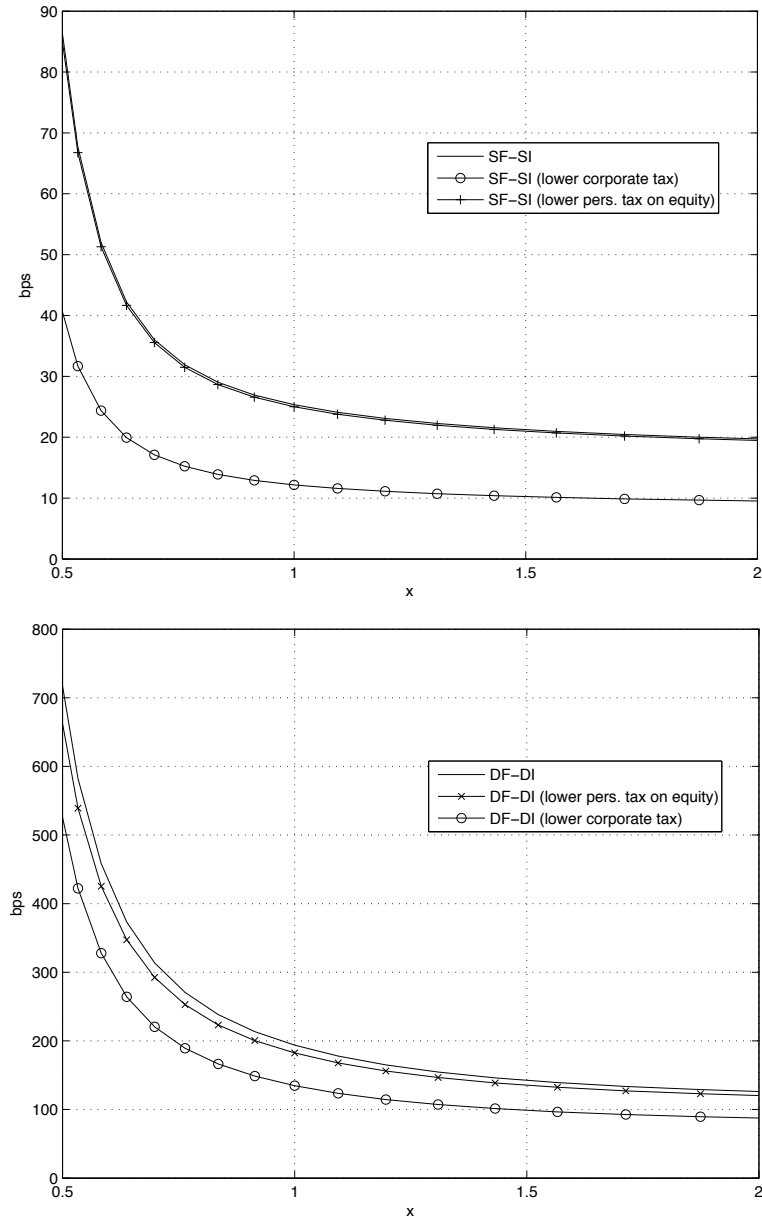
Figure 6: **Personal and corporate taxes**. The upper panel plots the spread between the yield on corporate debt and the risk-free rate, $rb/D(k, b, x) - r$, with respect to the productivity parameter, x. $D(k, b, x)$ is the ex–coupon price of debt from equation (12). The yield spread (in basis points/year) is determined for a current face value of debt $b = 2$ and for a current capital stock $k = 7.68$. We consider the SF-SI model (static financing and static investment) with the baseline tax parameters, and with either lower personal taxes on equity flows ($\tau_e = 10.23\%$), or lower corporate taxes on positive earnings ($\tau_c^+ = 0.35$). The bottom panel plots the spread, computed in exactly the same manner, for the DF-DI model. The plots are obtained using $S = 27$, $N_b = 27$ and $N_k = 37$ points for the approximate solution.
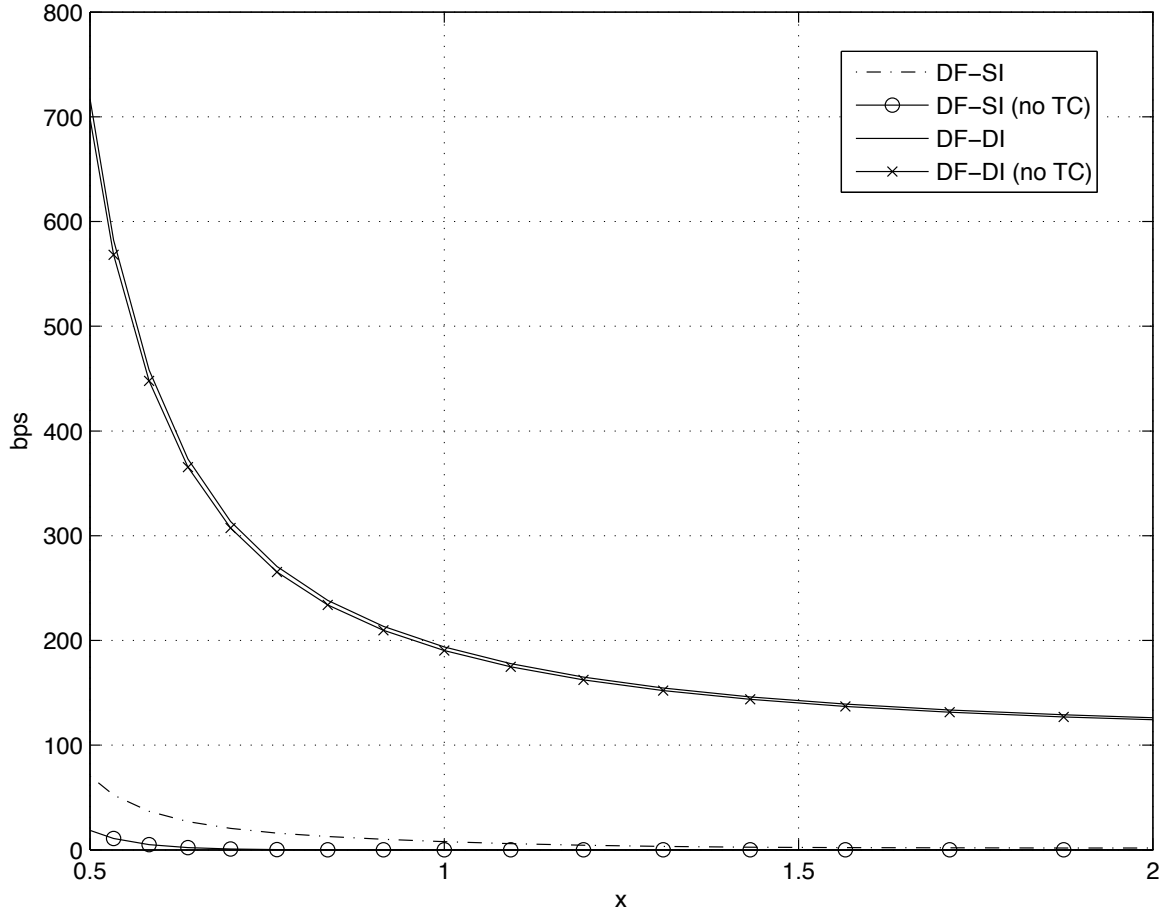
Figure 7: **Transaction costs**. The upper panel plots the spread between the yield on corporate debt and the risk-free rate, $rb/D(k, b, x) - r$, with respect to the productivity parameter, x. $D(k, b, x)$ is the ex coupon price of debt from equation (12). The yield spread (in basis points/year) is determined for a current face value of debt $b = 2$ and for a current capital stock $k = 7.68$. We consider the DF-SI model (dynamic financing and static investment) and the DF-DI model (dynamic financing and dynamic investment) under second best with the baseline parameters, and with zero transaction costs (no TC) on debt changes ($q_0 = q_1 = 0$). The bottom panel plots the debt ratio b^*/b at $k = 7.68$, averaged with respect to b, vs x. We consider the DF-DI and DF-SI models with the baseline tax parameters, and with zero transaction costs on debt changes. The plots are obtained using $S = 27$, $N_b = 27$ and $N_k = 37$ points for the approximate solution.

| | QML (%) | | | YS (bps) | | | DR (%) | |
| | Empirical | Simulated | | Empirical | Simulated | | Empirical | Simulated |
		mean	*s.e.*		*mean*	*s.e.*		*mean*
AAA	6.29	5.66	0.01	55	77	0.14	0.02	0.02
AA	8.50	9.86	0.00	65	111	0.18	0.04	0.13
A	16.12	15.84	0.01	92	173	0.29	0.13	0.22
BBB	26.01	25.16	0.01	149	311	0.59	0.48	1.54
BB	34.74	34.17	0.01	234	438	1.27	2.33	3.76
B	43.38	43.71	0.02	675	603	1.53	6.14	6.42
C	57.19	57.07	0.04	1500	1122	3.27	14.32	23.17

Table V: **Simulated vs empirical credit risk statistics.** Source of empirical statistics: Compustat North America Industrial Annual files. Financial, insurance and real state firms (SIC code 6000-6999) and regulated utility firms (SIC code 4900-4999) are excluded), Reuters (2004) and our computations from Moody's Investor Service (2006). YS is the average yield spread by whole letter class, in basis points (*bps*), measured as the difference between the yield on 10 years maturity corporate bonds and the rate of return on treasuries. QML is the average percentage of total debt over total debt plus the product of market price of share and number of outstanding shares. DR is the average annual default rates obtained as $1 - \exp(1/10 \cdot \log(1 - DR_{10}))$, where DR_{10} is the Moody's 10 year default rate (Average issuer-weighted corporate percentage default rates by whole letter rating, 1983-2005. Source: Moody's Investor Service (2006)). In particular, the 10 year default rates are: 0.208 for AAA, 0.415 for AA, 1.248 for A, 4.721 for BBB, 21.038 for BB, 46.931 for B, and 78.673 for C. Estimates (and the related standard error, s.e.) from a simulated sample of 20,000 firms for 150 years, following the procedure described in Appendix B. We drop the first 50 years to reduce the influence of the initial conditions. We used the optimal policy and the related value of firm's securities of the model. The numerical solution is obtained using the algorithm presented in Appendix A, with $S = 27$, $N_b = 27$ and $N_k = 37$. At every step t from 50 to 149 we classify firms into seven whole letter credit classes from AAA to C by minimizing the distance between the simulated quasi-market leverage of the firm and the empirical quasi-market leverage for the specific class. Firms in default and firms which are optimally unlevered are not considered. Next, we compute the yield spread for the ω-th firm at t as $rb_t/D(\omega, t) - r$, where $D(\omega, t)$ is the ex-coupon price and b_t is the current par value of debt. We compute also the annual default rate for each class as the default relative frequency on the number of non-default firms at the beginning of the year. Finally, we compute the average of the simulated credit spreads and default frequencies within each class, and then we take the time average for each class from year 50 to 149.

Ind. Var.	coefficient	std. error	t	p-value	[95% Conf. Interval]
QML	.02433	.0005	45.27	0.000	[.0233, .0254]
ROA	-.2439	.0283	-8.61	0.000	[-.2994, -.1884]

Table VI: **Ordered Probit Model for the S&P's credit risk classes**. Results of the ordered probit analysis of firm classification criterion. Data are from Compustat North America Industrial Annual files. Financial, insurance and real state firms (SIC code 6000- 6999) and regulated utility firms (SIC code 4900-4999) are excluded. The dependent variable is the S&P's seven (whole letter) risk classes discussed previously. It takes values from 1 (class AAA) to 7 (class C). QML (*quasi-market leverage*) is given by total debt over total debt plus the product of market price of share and number of outstanding shares. ROA (*return on assets*) is given by EBIT over total debt plus the book value of equity. t is the t-ratio. p-value is the probability of getting a value of the test statistic as or more extreme than that observed by chance alone, if the null hypothesis H_0, is true.