The Optimality of Interbank Liquidity Insurance

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Abstract

This paper studies banks' incentives to engage in liquidity cross-insurance. In contrast to the previous literature we do not consider a situation where banks meet centrally to determine the structure of their insurance arrangements. Instead we view interbank insurance as the outcome of *bilateral* (and non-exclusive) contracting between pairs of banks. We then ask whether this outcome is socially efficient. We present a simple model of interbank insurance and find that this is indeed the case when insurance takes place through pure transfers. This is even though liquidity support among banks sometimes breaks down. However, when insurance is only provided against some form of repayment (such as is the case, for example, with credit lines), we show that banks have a tendency to "underinsure", that is, in equilibrium they insure each other less than the socially efficient amount. We argue that this provides a role for government guarantees of interbank loans (as recently introduced in several countries) but that such guarantees should only be partial. Interestingly, even considering various forms of externalities among banks, we find that there are no cases of "overinsurance". Overinsurance, however, may arise if banks receive regulatory subsidies (explicit or implicit) in case they fail jointly.

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1 Introduction

The recent liquidity problems in the banking system have raised questions about whether the insurance banks provide to one another is efficient. This is because while some banks were clearly suffering from liquidity shortages, others seemed to have liquidity surpluses. It thus appears that liquidity insurance among banks has broken down. However, this does not suffice to conclude that there are any inefficiencies in interbank insurance and that there is hence scope for policy. It may well be that such a breakdown is part of an overall efficient outcome. In order to address the question of efficiency one needs to consider banks' private incentives to engage in insurance, and to analyze whether they lead to a socially desirable outcome. Only if this is not the case, is there a potential role for regulation in improving the outcome.

This paper provides such an analysis. The main idea is as follows. Previous literature has mostly focused on analyzing the optimal form of interbank insurance. Thus (implicitly) a situation is considered where all banks meet centrally and decide jointly about the interbank insurance structure to implement. The question we ask then is whether this insurance is also an equilibrium outcome. The reason why it may not be is that it is not be feasible for all banks to decide jointly on their insurance arrangements and, furthermore, commit to them. Instead, interbank insurance is typically implemented bilaterally (for example, bank A may decide to grant a credit line to bank B etc.). Such insurance arrangements are also non-exclusive in nature, that is, when bank A insures bank B against certain shocks, it cannot prevent (or force) bank B to also engage in bilateral insurance with other banks. As a result, even if each pair of banks in the financial system chooses their bilaterally optimal insurance arrangement, there is no guarantee that this produces the efficient outcome for the financial system as a whole. Efficiency may in particular break down when contracting between two banks poses an externality on other banks.

We develop a simple model of interbank insurance to analyze these issues. In our model there are three banks. One of the banks is randomly hit by a liquidity shock of variable size. Banks can decide ex-ante to (bilaterally) insure each other against these shocks. Insurance, however, comes at a cost. This is because when the insurance is provided to the bank hit by the shock, the liquidity holdings at the insuring banks are diminished. This puts them at risk since they may be later hit by another (aggregate) shock, which then may cause their failure.

Providing insurance thus gives rise to a trade-off. It may save the bank which has received the idiosyncratic shock, but may also result in the failure of the other banks when the aggregate shock hits. We show that the optimal form of insurance is to insure against

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small (idiosyncratic) shocks, but not to insure against large shocks. More precisely, the optimal insurance stipulates that banks fully insure each other up to a critical shock level, but do not provide any insurance beyond this level. In other words, interbank insurance optimally breaks down when banks are hit by large shocks. We also show that the critical shock level to insure against is determined by the likelihood of the aggregate shock hitting as well its expected size.

We then turn to the analysis of banks' bilateral incentives to form insurance. We first consider a simple version of the model without bank moral hazard. In this version insurance can take place in the form of a pure transfer, that is the bank which receives the shock does not have to repay the insurance payment provided by the other banks. In this version there are also no interactions among the banks beyond the insurance agreements (such interactions may arise, for example, due to spillovers from the failure of a bank).

We show that in this case equilibrium insurance is efficient, that is banks' bilateral incentives to insure each other produce the socially desirable outcome. The reason for this result is as follows. Efficiency will obtain when there are no externalities among banks, that is when the bilateral insurance decision between two banks (say, bank A and B) does not affect the third bank (bank C). Consider first a situation where bank C does not provide insurance payments to either bank A or bank B. Clearly, in this case insurance between A and B does not affect bank C since we have ruled out any spillovers from bank failures beyond the insurance itself. Consider next the situation where bank C provides insurance to bank A and/or B. When bank A and bank B decide not to insure each other, they may fail if they are hit by the idiosyncratic shock because C's insurance payment may then be insufficient to avoid failure. However, this again does not affect bank C (bank C has to provide the insurance payment in any case). For these reasons the bilateral insurance choice between the three banks can be efficient.[1]

We next consider a model where insurance payments are only provided against a repayment at a later stage. For this we introduce bank moral hazard with respect to the idiosyncratic shock. Addressing the moral hazard problem requires that receiving the idiosyncratic shock is sufficiently costly, which is what makes repayments optimal. Such arrangements can, for example, be interpreted as credit lines which banks grant to each other.

We show that in the presence of repayments banks have an incentive to "underinsure", that is, they insure each other against fewer shocks than is socially optimal. The reason for this is that now a bank may suffer from the failure of a bank it has provided insurance to,

[1] Nevertheless, we show that there can be inefficiencies because banks may coordinate on an inferior equilibrium. For our welfare analysis, however, we rule out dominated equilibria.

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since in this case it may not receive the repayment. This gives rise to a positive externality from bilateral insurance. When bank A and bank B insure each other, they each reduce their likelihood of failing in response to the idiosyncratic shock. This also benefits bank C because it then becomes more likely that the latter will be able to recover any insurance payments it has made to bank A or bank B. Bank A and B will, however, ignore this effect. As a result, underinsurance among banks may arise in equilibrium. This analysis hence suggests that the breakdown of liquidity support among banks could indeed be the result of insufficient insurance arrangements (which typically require repayment).

Finally, we also consider whether there may be situations in which banks insure "too much". Aside from coordination problems, such situations may arise if two banks decide to insure each other even though this is not jointly efficient for all three banks. To analyze this possibility we consider generic externalities which arise because a joint failure of two banks causes the failure of the third bank, for example, because of spillovers of informational nature or through asset prices. In the presence of such externalities, if two banks insure each other, there will be a negative effect on the third bank. This is because, as already explained, insurance comes at the cost of a higher likelihood of joint failure.[2]

One may conjecture that banks may hence overinsure as a result. However, we show that this cannot be the case. In fact, the only situation in which overinsurance can occur in our framework is when regulators provide a subsidy to jointly failing banks. For example, when the regulator adopts a too-many-too-fail policy banks may be bailed out in a joint failure. This increases their incentives to insure themselves since the cost of insurance (which comes in the form of a higher likelihood of joint failure) is reduced. As a result, banks may overinsure in equilibrium.

Our results have interesting implications for the apparent breakdown of liquidity provision among banks in recent months. In principal, as we have shown, such a breakdown can be efficient. This is the case when the banks with liquidity needs were hit by sufficiently large shocks (that is, if they have large liquidity needs) and when the financial system overall faces the risk of substantial additional shocks. Both conditions were arguably met in recent events.

However, we have also shown that banks may have a tendency to rationally underinsure each other. This is because they do not fully internalize that saving each other has an effect on other banks in the financial system. From this perspective, the breakdown of interbank insurance may have been inefficient, suggesting that there is a role for financial regulation

[2] While we consider here interbank externalities, banking failures may also have effects outside the banking system. Kahn and Santos [13] consider a two-bank setting in which the joint failure of banks causes a failure of the payment system and incurs costs for producers in the economy. Similar to overinsurance in our paper, they show that this can cause the two banks to connect "too much".

in encouraging banks to provide more support to each other. Our analysis hence provides support for government guarantees for interbank loans that have been introduced in several countries in recent months (such as in the European Union and Canada). However, in contrast to these policies, our analysis suggests that guarantees should only be partial. The reason is that the subsidy received through the guarantee should incentivize banks to insure each other only up to the socially optimal level. By contrast, when banks are fully compensated by the government for defaults on interbank loans, they have an incentive to insure each other beyond this level, hence causing "overinsurance".

The analysis of interbank insurance has received widespread attention in the literature. In the seminal paper by Bhattacharya and Gale [6] banks under-invest in liquidity reserves when moral hazard and adverse selection problems are present. In Rochet and Tirole [16] interbank markets act as a device that banks use to monitor each other. In Freixas et al. [11] interbank markets operate in a given spacial economy and a solvency shock can cause a gridlock in the system. Allen and Gale [3] and Brusco and Castiglionesi [9] analyze the optimal form of interbank insurance in two different environments: while the former paper considers the risk coming from an (unexpected) aggregate liquidity shock, the latter considers the risk stemming from the risky behavior of banks protected by limited liability. However, both papers study interbank insurance under the (implicit) assumption that banks meet centrally and commit to a certain interbank structure.

As already said, we depart from this approach by assuming that insurance contracts are formed bilaterally. This assumption has also been the focus of the literature on non-exclusive contracts, initiated by Pauly [15]. More recent contributions include Bizer and DeMarzo [8], Bisin and Guaitoli [7], and Kahn and Mookherjee [12]. In these papers the relationship between borrowers and lenders is analyzed when borrowers can sign contracts with different lenders that are in competition with each other. The borrower can choose (either sequentially or simultaneously) more than one contract since exclusive clauses are not feasible or not enforceable. Our paper is, to our knowledge, the first to apply the notion of non-exclusivity to interbank insurance.

Another strand of literature that departs from the assumption of a fixed structure of interbank connections is the network formation approach to financial systems (see Leitner [14], Castiglionesi and Navarro [10], Babus [5]). The main interest of these papers is to analyze how financial institutions form linkages despite (or because of) the possibility of contagion. A difference to this literature is that in the present paper we are not interested in the incentives to form linkages per se, but rather in the amount of insurance provided in each connection (thus, the intensity of each link is endogenous). Another difference is that the focus in our paper is on the optimality of the equilibrium outcome, and not on the actual form of interbank connections that arise.

Our paper also connects to the literature on interbank lending, which has received a new stimulus through recent events. In Wagner [18] interbank lending may break down due to a moral hazard problem at banks and lead to an inefficient reallocation of liquidity in crisis. In Acharya, Gromb and Yorulmazer [2] inefficiencies in interbank lending arise due to monopoly power. Banks with liquidity surplus may rationally not provide liquidity to needy banks in the hope that the latter will fail, which would enable them to purchase their assets at fire-sale prices. In Allen and Carletti [4] inefficiencies in the interbank market arise because interest rates fluctuate too much in response to shocks, which precludes efficient risk sharing. Allen and Carletti also show that interbank markets may rationally break down in cases where banks have no borrowing needs (in our paper such a breakdown may even be rational when some banks have liquidity needs).

The remainder of this paper is organized as follows. Section 2 sets up the model. In Section 3 we analyze interbank insurance through pure transfers. In Section 4 we consider insurance that is provided against a repayment only. Section 5 analyzes the impact of spillovers from bank failures on the efficiency of interbank insurance. Section 6 contains the conclusions.

2 The Model

The economy consists of three banks, denoted by A, B and C. There are four dates. At date 0 each bank invests in a long term asset with return R. The assets are subject to both aggregate and idiosyncratic uncertainty. They mature at the final date (date 3) with an overall return of R.

At date 1 and 2 the assets may be hit by liquidity shocks. These shocks may be either positive (=liquidity surplus) or negative (=liquidity deficit). A liquidity surplus may arise because parts of the asset mature early. A liquidity deficit may reflect unexpected liquidity needs from an asset. The shocks are pure liquidity shocks, that is they are offset by corresponding changes in the final date payoffs and hence do not change the overall return on an asset.[3] If in any period a bank's asset faces a liquidity need and the bank cannot provide the required liquidity injection, the asset cannot be continued. The return on the asset is then zero. However, any earlier injected liquidity can still be recouped in this case. There is also a storage technology which allows liquidity to be moved from one period to the next.

[3] For example, if an asset generates an unexpected surplus of l at date 1, its final date return will be $R - l$.

The date 1 liquidity shock is idiosyncratic. Nature selects at date 1 with equal probability one of the assets. This asset then has a liquidity need of $l \in [0, 2]$, while the other two assets generate a liquidity surplus of 1. The size of the shock is known at date 0 (alternatively, one may assume that the size of the liquidity shock can be contracted upon). At date 2 an aggregate liquidity need hits all banks with probability π^A ($\pi^A \in (0, 1)$). The size of the aggregate shock, denoted l^A, is assumed to be uniformly distributed on $[0, 1]$.

Before the idiosyncratic shock hits at date 1, banks can engage in bilateral contracts to insure each other against the idiosyncratic shock.[4] An insurance contract specifies a transfer $t \geq 0$ between two banks if one of them is hit by the idiosyncratic shock. The insurance contract may also specify a repayment $r \geq 0$, taking place at the final date. A (symmetric) insurance contract between two banks i and j is thus a pair $(t_{i,j}, r_{i,j})$. Note that the contract cannot be conditional on any insurance contracts with the third bank. That is, bilateral insurance between A and B, for example, cannot be conditional on the insurance A or B have agreed upon with C. This is the imperfection which is the focus of our analysis.

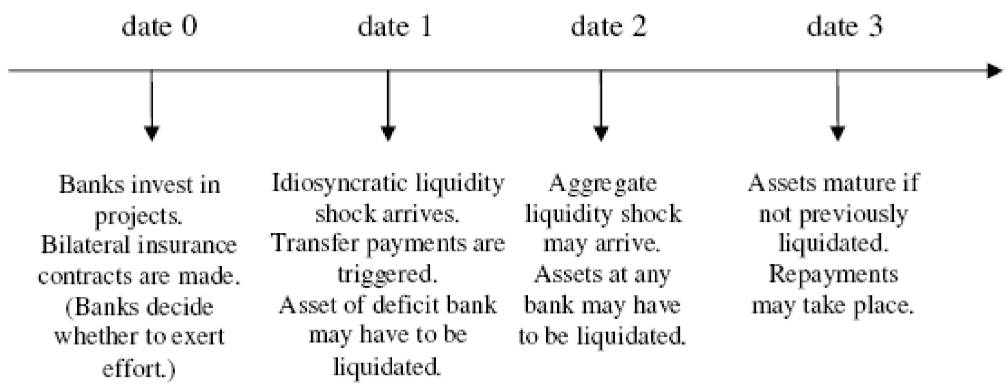
The model can be summarized as follows. At date 0 banks each invest in an asset and form bilateral insurance contracts $(t_{i,j}, r_{i,j})$. At date 1 the bank whose asset has the liquidity need l (the "deficit bank") may receive transfers from the other banks (the "surplus banks"). For example, if A is the deficit bank it collects $t_{AB} + t_{AC}$ from B and C. If $t_{AB} + t_{AC} \geq l$ the bank can meet the liquidity injection and its asset can be continued. If $t_{AB} + t_{AC} < l$ the liquidity injection cannot be provided and the asset is worthless. The bank then "fails", and its return consists only of any transfer payments $(t_{AB} + t_{AC})$ it has received. Any excess liquidity at surviving banks is stored for the next period.

At date 2, the aggregate shock may hit. If it does not hit, nothing happens and all remaining assets can be continued. If it hits, the remaining banks have new liquidity needs l^A. The assets of the banks that have a liquidity of less than l^A fail and return only any previously injected liquidity. The other assets can be continued. Again, any excess liquidity is stored. At date 3 all remaining assets mature and return R net of any previous liquidity deficit or surplus.

The timing of the model is summarized in Figure 1 (the effort choice mentioned in brackets at date 1 will be discussed in Section 4).

[4] We do not consider insurance against the aggregate shock. However, our results continue to hold if insurance can also be written against the arrival of the aggregate shock (as we discuss later, see footnote 5).

Figure 1: The Timing of the Model



3 Insurance Through Pure Transfers

We first analyze the case where there are no interactions between banks beyond their insurance payments at date 1. For this we constrain the repayment r to be zero, thus the insurance payments take the form of pure transfers.

We first solve for the (socially) efficient insurance contract. Maximizing welfare requires us here to maximize the sum of the expected returns of all banks. Recall that the liquidity shocks only affect the timing of an asset's returns but not its overall return. Therefore, the return on an asset depends only on whether it survives or not (in the latter case R is lost). Since transfers between banks cancel out in the aggregate, maximizing welfare thus simply requires minimizing the expected number of asset liquidations, that is minimizing the expected number of banking failures.

Proposition 1 *An efficient contract $t^* = t_{AB} = t_{AC} = t_{BC}$ is given by*

$$t^* = \begin{cases} \frac{l}{2} \text{ if } l \leq \frac{1}{\pi^A} - 1 \\ 0 \text{ if } l > \frac{1}{\pi^A} - 1 \end{cases} . \tag{1}$$

Proof. *Without loss of generality focus on the case where bank A is the deficit bank (that is the bank whose asset has a liquidity deficit at date 1). For a given liquidity deficit l there can be in principle two different situations: either none insures bank A against this deficit ($t_{AB} + t_{AC} < l$) or one does ($t_{AB} + t_{AC} \geq l$). Note that insurance is always possible since $l \leq 2$ and hence never exceeds the combined liquidity holdings of the surplus banks (= 2).*

Consider first the case of no insurance ($t_{AB} + t_{AC} < l$). If bank A is not fully insured against the liquidity deficit, it fails at date 1. Bank B and C survive date 1 and store their remaining liquidity (net of any transfers to A) of $1 - t_{AB}$ and $1 - t_{AC}$, respectively. At date 2 they will thus survive the aggregate shock if the latter does not exceed $1 - t_{AB}$ and $1 - t_{AC}$, *respectively. Given that the aggregate liquidity shock is uniformly distributed on* $[0,1]$*, the likelihood that it exceeds the liquidity holdings of these banks is* t_{AB} *and* t_{AC}*, respectively. Recalling that the aggregate shock hits with probability* π^A*, the expected number of bank failures at date 2 is hence* $\pi^A(t_{AB}+t_{AC})$*. It follows that it is optimal to set transfers equal to zero (*$t^*_{AB}=t^*_{AC}=t^*=0$*). The total number of banking failures is hence 1 (the deficit bank is certain to fail, while the surplus banks survive).*

Consider next the case of insurance ($t_{AB}+t_{AC}\geq l$*). The deficit bank is now insured against the idiosyncratic shock. Hence no bank fails at date 1 and any excess liquidity is stored. If at date 2 the aggregate shock (*l^A*) does not hit, no bank will fail at all. If it hits, bank A fails if* $l^A > t_{AB}+t_{AC}-l$*, while bank B and C fail if* $l^A > 1-t_{AB}$ *and* $l^A > 1-t_{AC}$*, respectively. The total expected number of banks failing when the aggregate shock arrives is hence*

$$(1-(t_{AB}+t_{AC}-l))+(1-(1-t_{AB}))+(1-(1-t_{AC}))=1+l \tag{2}$$

and is independent of t_{AB} *and* t_{AC}*.*[5] *Thus, without loss of generality we can set for the optimal contract* $t^*_{AB}=t^*_{AC}=\frac{l}{2}$ *(*$=: t^*$*), that is a deficit bank gets just enough transfers to survive at date 1 and the surplus banks contribute equally to its insurance. Given a probability of the aggregate shock arriving of* π^A*, the total expected number of banking failures is then*

$$\pi^A(1+l). \tag{3}$$

From comparing this to the expected number of banking failures when there is no insurance (= 1), it follows that it is optimal to insure the deficit bank iff $1\geq\pi^A(1+l)$*, or, rearranging, iff* $l\leq\frac{1}{\pi^A}-1$*.* ■

Proposition 1 shows that it is optimal to insure against small liquidity deficits ($l\leq\frac{1}{\pi^A}-1$) but not against large ones ($l>\frac{1}{\pi^A}-1$). What is the intuition for this result? Providing insurance creates a trade-off here. On the one hand, sufficient transfers can always save the deficit bank at date 1. On the other hand, transfers reduce the available liquidity at surplus banks. This may cause their failure when they get into troubles as well due to the aggregate shock.[6] If the liquidity deficit is small, small transfers are sufficient to save the deficit banks. Then only a large aggregate shock can cause the failure of the surplus banks. It is hence less likely that they fail as a result of insuring the deficit bank.

[5]Since the expected losses from the aggregate shock are independent of the distribution of the liquidity holdings, contracts which are conditional on the *arrival* of the aggregate state cannot improve upon the allocation. However, there are situations where the allocation could be improved if, additionally, the size of the aggregate liquidity shock were contractible.

[6]There is hence "contagion" in the sense that a liquidity problem at one bank can cause the failure of other banks.

Conversely, if the liquidity deficit is large, large transfers are required to save the deficit bank. Already relatively small aggregate shocks then cause the failure of the surplus banks. Thus, it is (socially) more costly to insure the deficit bank against larger liquidity shocks.

Moreover, Proposition 1 also shows that the condition for insurance being optimal becomes tighter when the likelihood of the aggregate shock π^A increases. This is because it is then more likely that the surplus banks themselves have (future) liquidity needs, which makes it (socially) more costly to insure the deficit bank.

Note that, while we focus here on (ex-ante) insurance among banks, an alternative way to deal with liquidity shocks is for a deficit bank to borrow at date 1 against the date 3 returns on its asset. In some situations this can indeed replace interbank insurance but in many situations it cannot. The reason is that when the liquidity deficit is significant, the required repayment for the bank becomes large. This is, among others, because the repayment also has to compensate the lenders for their higher likelihood of failure. Such repayments may not be feasible because they may exceed the date 3 return of the deficit bank. We analyze the case for borrowing and lending (as an alternative to ex-ante insurance) in the Appendix.

3.1 Equilibrium Insurance

We now consider banks' individual incentives to form insurance. For this we study whether banks have an incentive to deviate from the efficient contract. In other words, we ask whether there exists an equilibrium that is efficient. Since banks can form insurance only bilaterally, it is a priori unclear whether an equilibrium can also be efficient for the banking system as a whole. In particular, bilateral insurance between two banks may impose externalities on the third bank, which may preclude an efficient outcome.

However:

Proposition 2 *The efficient outcome t^* also constitutes an equilibrium.*

Proof. *We have to show that banks have no (individual) incentives to deviate from the efficient outcome. For this we have to distinguish between the case where the efficient outcome is insurance ($l \leq \frac{1}{\pi^A} - 1$) and where it is not ($l \geq \frac{1}{\pi^A} - 1$). We start with the second case.*

1. Insurance is inefficient ($l \geq \frac{1}{\pi^A} - 1$). The optimal contract is then $t^* = 0$. In order to analyze banks' incentives to deviate, we consider without loss of generality bank A and B's incentives to change their bilateral insurance contract t_{AB} to an amount larger than 0. In doing this, both banks take as given their other insurance agreements, that is between A and C and B and C: $t_{AC} = t_{BC} = 0$. A deviation will only affect bank A and B if either *of them is the deficit bank at date* 1*. This is because the bilateral insurance between A and B is not invoked if bank C is the deficit bank. Without loss of generality presume again that bank A is the deficit bank. If bank A and B decide to remain uninsured (that is, if they leave* t_{AB} *at* 0*), bank A fails at date* 1*. Bank B always survives since it can store one unit of liquidity and thus withstand any aggregate shock. Thus, there is only one banking failure among the two banks A and B. Now consider the case where they both choose to insure each other by agreeing on an* $t_{AB} \geq l$ *(note that this is only feasible if* $l \leq 1$ *because bank B has only one unit of liquidity). The banks may then fail when the aggregate shock arrives. Similar to the expressions in Proposition 1, their total expected failures from this are* $\pi^A(1 + l - t_{AB}) + \pi^A t_{AB} = \pi^A(1 + l)$*. Comparing the expected failures under deviation and no deviation, we see that banks only deviate if* $\pi^A(1 + l) < 1$*, which contradicts the assumed (social) optimality of no insurance (*$l \geq 1/\pi^A - 1$ *from above).*

2. Insurance is efficient ($l < \frac{1}{\pi^A} - 1$*). The efficient contract now stipulates to save the deficit bank, with each of the surplus banks contributing* $\frac{l}{2}$*. Consider again the incentives of bank A and B to deviate, and presume that bank A is the deficit bank. If both banks do not deviate, bank A is saved at date* 1 *but bank A and B may fail when the aggregate shock arrives. Their total expected number of failures from this is* $\pi^A(1 + \frac{l}{2})$ *(observe that we now have the term* $\frac{l}{2}$ *rather than* l *as under 1.; this is because in the case above B had to insure bank A alone, making insurance more costly). If both banks deviate from insurance, there is one bank which certaintly fails (bank A). The two banks also receive a transfer* $\frac{l}{2}$ *from bank C whenever bank A has the deficit. However, this does not affect their deviation incentives since these transfers occur regardless of whether or not they deviate. Thus, the banks will deviate iff* $\pi^A(1 + \frac{l}{2}) > 1$*, which contradicts the above condition that insurance is optimal (*$l < \frac{1}{\pi^A} - 1$*).* ■

What is the reason why banks do not deviate from the efficient outcome? Loosely speaking, the bilateral insurance choice between two banks will be efficient if it does not pose any externalities to the third bank. In the case of no insurance the third bank does not interact at all with the first two banks and hence there cannot be any externality. In the case of insurance the third bank does interact with the other banks because it has to make transfers to the deficit bank. But these transfers only depend on the idiosyncratic liquidity shock and are not affected by the bilateral insurance between the other two banks. Hence, there is again no externality and the equilibrium can be efficient.

Notice, however, that a potential inefficiency may arise because banks may coordinate on an inefficient equilibrium. Recall from the proof of Proposition 2 that the condition for banks not finding it optimal to deviate from an insurance outcome is $l < 2(\frac{1}{\pi^A} - 1)$, while the condition for the optimality of insurance is $l < \frac{1}{\pi^A} - 1$. Thus there are parameter constellations for l ($2(\frac{1}{\pi^A}-1) > l > \frac{1}{\pi^A}-1$) where insurance is inefficient but banks would not deviate from an insurance outcome. Intuitively, this is because when the third bank already provides insurance, lower insurance payments are required by the other two banks in order to save a deficit bank. As a result, these two banks perceive lower costs of insuring.

4 Credit Lines

So far interbank insurance took place in the form of pure transfers. There was hence no interaction between banks beyond the risk sharing at date 1. In this section we introduce moral hazard at the bank level. The presence of moral hazard makes it no longer efficient to provide insurance for free. A certain repayment on the insurance transfers is now needed in order to (at least partially) force a bank to internalize the costs of ending up in a liquidity deficit. Such insurance contracts with repayments are essentially credit lines granted by banks to each other. The repayment adds another interaction among banks which, as we will see, provides a potential source of inefficiency.

We now assume that at date 0, after insurance contracts have been signed, each bank can decide whether or not to exert effort. If each bank undertakes effort, the date 1 (idiosyncratic) liquidity shock arrives only with probability π^I ($\pi^I \in (0,1)$). When it arrives it hits with equal probability one of the banks, as before. A bank's probability of receiving the shock is thus $\frac{\pi^I}{3}$. If one bank does not exert effort, the liquidity shock arrives with probability 1. It then hits this bank with probability $1-\frac{2}{3}\pi^I$ ($> \frac{\pi^I}{3}$) and the other two banks with probability $\frac{\pi^I}{3}$ each. Thus, if a bank does not exert effort it increases its risk of receiving the liquidity shock by $1-\pi^I$, while the risk for all other banks stays the same. If more than one bank does not exert effort, we assume for simplicity that each of the assets without effort become worthless. Effort is assumed to incur private (non-monetary) costs of e ($e > 0$) per bank. The timing of actions can be seen from the previous Figure 1, now including the effort choice at date 1.

We first write the condition that exerting effort is productive, which requires that a bank would also undertake effort in the absence of any insurance. A bank fails in the absence of insurance whenever it receives the idiosyncratic liquidity shock. The probability of this is $\frac{\pi^I}{3}$ if effort is undertaken and $1-\frac{2}{3}\pi^I$ if not. Given effort costs e, effort is hence exerted when $\frac{1}{3}\pi^I R + e \leq (1-\frac{2}{3}\pi^I)R$. Rearranging this gives

$$(1-\pi^I)R \geq e. \tag{4}$$

Quite intuitively, this condition states that the expected reduction in liquidation costs induced by exerting effort, $(1-\pi^I)R$, has to at least offset the effort costs e. We assume this condition in the following to be fulfilled.

The efficient contract, as defined in Proposition 1, remains unchanged. There is only the added requirement that repayments have to be chosen such that effort is induced. Proposition 3 shows next that the equilibrium may now display underinsurance:

Proposition 3 *When insurance takes the form of credit lines, banks may deviate from an efficient insurance outcome (equilibrium underinsurance).*

Proof. *We show that when insurance is optimal there are parameter values for which banks deviate from the insurance outcome. Assume thus that $l < \frac{1}{\pi^A} - 1$, that is insurance is optimal (from Proposition 1). We first derive the required repayment r which induces effort under insurance for the optimal contract ($t^* = \frac{l}{2}$). For this we analyze how a bank's pay-off changes when it decides to exercise effort. If it does so, it has to incur the effort costs e but it also becomes less likely to receive the idiosyncratic shock. Receiving this shock is now costly for two reasons. First, because it may lead to the failure of the bank, and second, because the bank has to make a repayment when it survives. More specifically, when the bank receives this liquidity shock it fails with probability π^A (inducing costs of R) but survives with probability $1-\pi^A$ (inducing costs r due to repayment). However, receiving the liquidity shock also has a benefit because the bank gets a total transfer of l from the other banks. Given that effort reduces the probability of receiving the liquidity shock by $1 - \pi^I$, effort is hence exerted iff*

$$(1 - \pi^I)(\pi^A R + (1 - \pi^A)r - l) \geq e. \tag{5}$$

Rearranging for r gives $r \geq \frac{1}{1-\pi^A}(\frac{e}{1-\pi^I} - \pi^A R + l)$. From this we can define with $\overline{r}$ the critical repayment that just induces effort:

$$\overline{r} = \frac{1}{1-\pi^A}(\frac{e}{1-\pi^I} - \pi^A R + l). \tag{6}$$

This critical repayment is larger than zero, for example, for sufficiently large effort costs e. Note that $\overline{r}$ refers to the total repayment a bank has to make (that is, the sum of the repayments to the two other banks).

We consider next the incentives for banks to deviate from an efficient insurance outcome with $t^* = \frac{l}{2}$ and a total repayment that induces effort. As we will see later the scope for an inefficient deviation is minimized when repayments are low. We thus set the total repayment to its minimum possible value $r = \overline{r}$. Furthermore we assume that repayments are equally split among banks: $r_{AB} = r_{AC} = r_{BC} = \overline{r}/2$. The deviation we consider is one where banks A and B deviate from the insurance outcome by no longer insuring A through transfers from B (that is $t_{AB} = 0$ if A gets the liquidity shock). Note that any asymmetry in the transfers between A and B does not affect their joint pay-off and hence *not their deviation incentives. We focus on situations where bank A may be the deficit bank (otherwise the deviation does not matter).*

The two bank's payoffs if they do not deviate are as follows. The probability of bank A being the deficit bank is $\frac{\pi^I}{3}$. *With probability* $1-\pi^A$ *the aggregate shock does not arrive subsequently and both banks survive. Their joint costs are then* $\frac{\overline{r}}{2}$ *because this is the repayment bank A has to make to bank C at date 3. With probability* $\pi^A(1-\frac{l}{2})$ *the aggregate shock arrives but is not so high that bank B fails (recall that its liquidity holdings at date 2 are* $1-\frac{l}{2}$*). The costs are then R because of the failure of bank A. With probability* $\pi^A\frac{l}{2}$ *the aggregate shock arrives and is high enough to make bank B fail as well. The joint costs are then 2R. In addition to the above effects, whenever bank A is the deficit bank, the two bank's payoffs are enhanced by a transfer from bank C of* $\frac{l}{2}$. *Finally, there are also the effort costs for both banks: 2e. The total costs for bank A and bank B are thus*

$$\frac{\pi^I}{3}\left((1-\pi^A)\frac{\overline{r}}{2}+\pi^A(1-\frac{l}{2})R+\pi^A\frac{l}{2}\cdot 2R-\frac{l}{2}\right)+2e. \tag{7}$$

If the banks decide to deviate as described above, their costs are as follows. Since now the critical repayment is no longer reached, bank A does not undertake effort. The probability of it getting the liquidity deficit is then $1-\frac{2}{3}\pi^I$. *When it receives the liquidity shock it now always fails, while bank B survives. The costs from this are* $R-\frac{l}{2}$ *in this case (the costs are reduced by* $\frac{l}{2}$ *through the transfer A receives from C). Since effort costs are e (effort for bank B), the total costs for banks A and B are*

$$(1-\frac{2}{3}\pi^I)(R-\frac{l}{2})+e. \tag{8}$$

Comparing both costs we can see that a deviation takes place iff

$$\frac{1}{3}\pi^I\left((1-\pi^A)\frac{\overline{r}}{2}+\pi^A(1-\frac{l}{2})R+\pi^A\frac{l}{2}\cdot 2R-\frac{l}{2}\right)+2e>(1-\frac{2}{3}\pi^I)(R-\frac{l}{2})+e. \tag{9}$$

Note that this condition is more easily fulfilled when $\overline{r}$ *is higher. The reason is that when there is deviation from the insurance outcome, the deficit bank fails more often and there are hence less occasions where it has to do the repayment.*

In the following we show that there are parameter values for which there are deviations from an (efficient) insurance. For this we show that there is a deviation from insurance for the idiosyncratic shock at which insurance is marginally efficient: $l=\frac{1}{\pi^A}-1$. *Furthermore, we also assume that effort is just worthwhile:* $e=(1-\pi^I)R$ *(from equation 4). Using this and the equation for* $\overline{r}$ *to substitute* l, e *and* $\overline{r}$ *into equation (9) one obtains after rearranging*

$$R+\frac{1}{\pi^A}>R-\frac{3(1-\pi^I)}{\pi^A\pi^I}. \tag{10}$$

Since the left-hand side of (10) is larger than R but its right-hand side is smaller than R, this condition is fulfilled for all feasible π^A, π^I *and R.* ■

The intuition for why there can now be underinsurance is the following. When two banks move away from insurance, they become more likely to fail. This induces a negative effect on the third bank because in this case the bank is not repaid. This effect is not internalized by the two deviating banks, creating an incentive to insure "too little".

5 Overinsurance

We have just shown that underinsurance may occur when insurance transfers are provided against a repayment, such as is the case with credit lines. Can there also be overinsurance? That is, is it possible that two banks may decide to deviate from an efficient no-insurance outcome?[7]

Generally, when two banks deviate from a no insurance outcome by insuring each other they create a trade-off. They reduce the likelihood that a single bank fails (since the deficit bank is now insured) but they increase the likelihood that both banks fail jointly (since now also the surplus bank may fail when the aggregate shock arrives). If either effect poses externalities on the other bank, their deviation decision may become inefficient.

We typically associate the joint failure of banks with external effects. For example, when two banks fail the ensuing liquidation of assets may cause fire-sale prices and thus also lead to the failure of the third bank (e.g., Wagner [17]). Alternatively, there may be negative informational spillovers, causing depositors' runs. Or, the third bank may suffer from a loss of future risk sharing opportunities since there is now no longer a bank with which it can share its idiosyncratic risk.[8] Therefore, a deviation from no insurance may conceivably pose negative externalities that appear to give banks an incentive to insure too much.

However, there cannot be overinsurance as a result of such externalities. To see this, suppose that the joint failure of two banks creates negative costs $K > 0$ for the third bank (if it would otherwise survive). For overinsurance to occur we need the two banks to have an incentive to deviate from an efficient no-insurance-outcome. As shown earlier, in the absence of K social and private incentives are completely aligned in this case. That is, two banks only have an incentive to deviate from no insurance if insurance were socially preferable. Since the costs K are external to the two banks, they obviously do not affect their incentives to deviate from a no insurance outcome. Moreover, the costs K also do not affect the optimality of no insurance relative to a situation where all three banks insure

[7] Note that, as already pointed out earlier, there can be overinsurance in the sense that banks may coordinate on an equilibrium in which there is "too much" insurance. However, this coordination failure is not our focus here.

[8] We have modelled such an effect in an earlier version (calculations are available on request).

each other. This is because in either case there are no situations where two banks fail and one survives and hence externalities from a joint failure of two banks are absent. The externalities only arise if two banks deviate from a no insurance outcome to a situation where two banks insure each other; but as just pointed out they only do this if no insurance is in fact not optimal.

The only way in which overinsurance can arise in our setting is if the deviating banks directly gain from their joint failure. This may be, for example, because of a too-many-too-fail policy (e.g., Acharya and Yorulmazer [1]). When several banks fail at the same time, the functioning of the financial system may be no longer guaranteed. The regulator may then decide to bail-out the jointly failing banks, for example by giving them (implicit) subsidies. This increases their incentives to deviate from a no insurance outcome and may lead to inefficiently high insurance.

6 Conclusion

This paper emphasizes the importance of considering banks' bilateral incentives in the analysis of interbank insurance. While previous literature has analyzed the optimal form of interbank linkages, these linkages may in practice not be implemented. This is because interbank insurance may be formed bilaterally, and not jointly among all banks.

Understanding any potential deviation of the interbank insurance which arises from banks' bilateral contracting from the optimal insurance is of paramount importance for financial regulation. This is because a breakdown of interbank insurance itself does not necessarily indicate inefficiency. Indeed, as we have shown, it may be desirable for banks to not insure each other against certain outcomes. Only when the equilibrium deviates from the efficient outcome, is there a role for regulation.

Our analysis has shown, that in principle, such a deviation can go either way. The failure of an insured bank imposes negative externalities on the insuring banks because it can then no longer repay the insurance payment. This creates a tendency for banks to underinsure. That is, there are shocks against which it is optimal to insure, but in equilibrium banks fail to do so. In such situations there is a role for regulators to encourage banks to cross-insure more. This tendency for underinsurance should be particularly pronounced when there are strong negative externalities of bank failures (which in our model happen to arise from a failure to repay).

We have shown, however, that banks may also have an incentive to insure "too much" in order to exploit the possibility of receiving regulatory subsidies in a systemic crises. The reason is that interbank insurance trades-off a lower probability of individual failures with a higher likelihood of joint failures. Therefore, if banks perceive lower costs from the latter, their incentives to insure increase beyond the efficient level. This effect will be important when public bail-outs of banks are likely. As a result of the experience of the current crisis, the perceived likelihood of such bail-outs has certainly increased. In the future, there may hence be a potential role for regulators to discourage insurance among banks.

Appendix: Borrowing versus Insurance

We show in the following that there are situations where borrowing at date 1 cannot replace (efficient) insurance. The problem with borrowing is that if a deficit bank borrows to withstand a liquidity deficit, this borrowing has to be worthwhile for the other two banks at date 1 (or at least not make them worse off), while insurance only has to be worthwhile from an ex-ante perspective.

Consider a situation where the two surplus banks lend $\frac{l}{2}$ each to save the deficit bank. A surplus bank's pay-off if it does not lend is simply R since it never fails when it does not give away liquidity at date 1. Its pay-off with lending is as follows. With probability $1-\pi^A$ the surplus bank will survive together with the deficit bank. In this case it gets R plus the agreed repayment per bank, denoted $\frac{r_L}{2}$, minus the amount lent out, $\frac{l}{2}$. With probability $\pi^A \frac{l}{2}$ the surplus bank fails and does not receive anything. Finally, with probability $\pi^A(1-\frac{l}{2})$ the surplus bank survives but the deficit bank fails. In this case there is no repayment and the pay-off is $R - \frac{l}{2}$.

The break-even repayment for the surplus banks is determined implicitly by setting equal the payoffs under lending and no lending:

$$R = (1-\pi^A)(R + \frac{r_L}{2} - \frac{l}{2}) + \pi^A \frac{l}{2} \cdot 0 + \pi^A(1-\frac{l}{2})(R - \frac{l}{2}). \tag{11}$$

From this we can derive the inequality

$$R < (1-\pi^A)(R + \frac{r_L}{2} - \frac{l}{2}) + \pi^A(1-\frac{l}{2})R. \tag{12}$$

Rearranging for r_L gives

$$r_L > l(\frac{\pi^A R}{1-\pi^A} + 1).$$

The maximum feasible repayment at date 3 is R, thus we must have that $r_L \leq R$. Using this to substitute r_L, and rearranging for l, gives us the condition that determines when lending is feasible

$$l < R\frac{1-\pi^A}{\pi^A(R-1)+1}.$$

Contrast the last expression with the condition for the optimality of saving the deficit bank (Proposition 1), that is $l \leq \frac{1}{\pi^A} - 1$. The condition for the feasibility of lending is more binding when $R\frac{1-\pi^A}{\pi^A(R-1)+1} < \frac{1}{\pi^A} - 1$, or, after rearranging, if $\pi^A \leq 1$. Thus, the lending condition is always more binding. It follows that there are always shocks l for which saving the deficit bank is desirable but which cannot be achieved through lending.

References

[1] Acharya, V. and T. Yorulmazer (2006), 'Cash-in-the-Market Pricing and Optimal Resolution of Bank Failures', *Review of Financial Studies*, forthcoming.

[2] Acharya, V., D. Gromb and T. Yorulmazer (2008), 'Imperfect Competition in the Inter-Bank Market for Liquidity as a Rationale for Central Banking', mimeo, London Business School.

[3] Allen, F. and D. Gale (2000), 'Financial Contagion', *Journal of Political Economy*, **108**: 1-33.

[4] Allen, F. and E. Carletti (2008), 'Interbank Market Liquidity and Central Bank Intervention.', mimeo, Wharton School

[5] Babus, A. (2007), 'The Formation of Financial Networks', Manuscript, Erasmus University of Rotterdam.

[6] Bhattacharya, S. and D. Gale (1987), 'Preference Shocks, Liquidity and Central Bank Policy', in W. Barnett and K. Singleton (eds), '*New Approaches to Monetary Economics*', Cambridge University Press, Cambridge, pp. 69-88.

[7] Bisin, A. and D. Guaitoli (2004), 'Moral Hazard with Non-exclusive Contracts', *RAND Journal of Economics*, **35** (2): 306-328.

[8] Bizer, D. and P. DeMarzo (1992), 'Sequential Banking', *Journal of Political Economy*, **100** (1): 41-61.

[9] Brusco, S. and F. Castiglionesi (2007), 'Liquidity Coinsurance, Moral Hazard and Financial Contagion', *Journal of Finance*, **62** (5): 2275 - 2302.

[10] Castiglionesi, F. and N. Navarro (2008), 'Optimal Fragile Financial Networks', Center Discussion Paper n. 2007-100, Tilburg University.

[11] Freixas, X., B. Parigi and J. C. Rochet (2000), 'Systemic Risk, Interbank Relations and Liquidity Provision by the Central Bank', *Journal of Money, Credit and Banking*, **32**: 611-638.

[12] Kahn, C. and D. Mookherjee (1998), 'Competition and Incentives with Non-exclusive Contracts', *RAND Journal of Economics*, **29** (3): 443-465.

[13] Kahn, C. and J. Santos (2007), 'Endogenous Financial Fragility and Prudential Regulation', working paper, Federal Reserve Bank of New York.

[14] Leitner, Y. (2005), 'Financial Networks: Contagion, Commitment, and Private Sector Bailouts', *Journal of Finance*, **6**: 2925-2953.

[15] Pauly, M. (1974), 'Overinsurance and Public Provision of Insurance: The Roles of Moral Hazard and Adverse Selection', *Quarterly Journal of Economics*, **88**: 44-62.

[16] Rochet, J. C. and J. Tirole (1996), 'Interbank Lending and Systemic Risk', *Journal of Money, Credit and Banking*, **28**: 733-762.

[17] Wagner, W. (2006), 'Diversification at Financial Institutions and Systemic Crises', Center Discussion Paper n. 2006-71, Tilburg University.

[18] Wagner, W. (2008), 'The Homogenization of the Financial System and Liquidity Crisis', *Journal of Financial Intermediation*, **17**: 330-356.