High Water Marks in Competitive Capital Markets[*]

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Abstract

We model the effect of the standard high-water-mark provision of hedge funds when the supply of capital is competitive and managerial ability is uncertain.. We find that confidence in a manager's ability is crucial to the provision's effect, and this effect is to boost the initial fund size, and to depress initial expected returns while increasing subsequent expected returns. We also find that expected returns can be non-monotonic in past returns, higher for somewhat poor returns than above or below, that flows can be unresponsive to performance, and that fund size decreases with the manager's effort cost.

1   Introduction

Much of the world's investment in securities passes through hedge funds. What distinguishes these funds is not the securities they trade, or how they trade them, but how their managers are paid; typically, they get a base fee and an incentive fee with a high-water-mark (HWM). The incentive fee gives managers a fraction of profits, while the HWM limits this fee to profits above an investment's

historical maximum. Intuitively, this provision benefits investors by focusing the fee on net value creation. However, as Berk and Green (2004) observe, competitive investors compete their benefits away. This does not mean that the effect of the HWM is competed away, but rather that its implications are something other than making investors wealthier. The goal of this paper is to identify these implications by introducing a HWM into a model of money management in competitive capital markets.

The analysis of Berk and Green (2004) develops two main premises about the value added by fund management - that it is uncertain, and that it exhibits decreasing returns to scale. In competitive capital markets, they show, the result is that the response of flow to performance equalizes mutual funds' expected returns. Since hedge fund managers are also likely to combine uncertain value-added with decreasing returns to scale, and hedge-fund investors are also competitive and have opportunities to rebalance, we situate our analysis in the same enivironment.

The potential for a different result for hedge funds, compared to mutual funds, lies in the HWM's asymmetric effect on new and old investment. In the mutual-fund context, new investment enters on the same terms as old investment, so when the expected return on new investment is competed down to a reservation level, so is the expected return on old investment. But with a HWM, old investment invests on better terms after a loss, because it does not pay the incentive until the loss is made up, whereas new investment has no loss to make up, and therefore pays higher expected fees. So if expected profits after a loss are zero for new investment they must be positive for old investment.

Besides the HWM, another distinctive feature of hedge funds, especially recently, is their propensity to fold. Because this tends to follow poor performance, and because poor performance is when the HWM can boost the old investors' expected returns by reducing what they pay the manager, the risk of folding is likely to be important to the HWM's effect.

We analyze the effect of the HWM with a two-period model that embeds learning, decreasing returns to scale and competitive capital in the context of hedge funds. There is a simple and an expanded version of this model, where the simple version has binary returns and costless effort and is solved analytically, and the expanded version has continuous returns and costly effort, and is solved numerically. So the simple version gives many of the key results in closed form, while the expanded version gives the shape of the relations between past performance on the one hand, and flows or future performance on the other, and it also captures the manager's participation constraint.

Our main finding is that the HWM alters both the cross section and time series of expected returns, and this alteration depends most of all on how well the public knows the manager's ability. If two managers have the same expected ability but different precisions regarding ability, then the higher-precision manager has lower expected returns initially, and higher expected returns later on. And when funds reopen for new investment, expected returns are zero for those that did well, but positive for those in a range of poor performance. The HWM also makes fund flows unresponsive to past performance for a range of outcomes

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that predict positive returns, and increases the initial fund size. These effects decrease as the manager's effort cost grows.

The paper is in four sections. Section 2 provides some background and covers the relevant literature, Section 3 presents the model, and Section 4 summarizes and concludes.

2   Background and Literature

A foundational paper is Goetzmann, Ingersoll and Ross (2003), which is concerned with valuing hedge-fund compensation contracts, and posits a continuous-time model where the adaptive response of investment to performance is reduced-form. A number of papers, such as Hodder and Jackwerth (2005) and Basak, Pavlova and Shapiro (2004), are concerned with managers' risk choices as a response to their convex incentive pay, and show an economically significant encouragement to alter risk. The HWM is incorporated into this risk choice by Panageas and Westerfield (2008), which shows that the combination of the HWM and a long-horizon for the manager can undo this encouragement. In Aragon and Qian (2007), the HWM is found to add value when investors are uncertain about managerial quality, and withdrawing is costly. In Rouah (2005) the probability that the fund folds is found empirically to be higher for funds below the HWM, and the implications of the distance from the HWM for both investor and managerial walkaway, as well as the manager's risk choice, are addressed theoretically and empirically in Ray and Chakraborty (2007). The Berk and Green (2004) hypothesis that the response of flow to performance equalizes future returns was recently tested on hedge funds by Naik, Ramadorai and Stromqvist (2007) and Fung, Hsieh, Naik and Ramadorai (2008), who conclude that the effect is strong in recent years, and weaker in earlier years.

This paper focuses on the implications of optimal investor flows given the prevailing contract design, rather than optimal contract design or risk choice. The key to the contribution of this paper is the conditional nature of the HWM. If all investors in a hedge fund have the same HWM, regardless of their investment date, then there is a well-known free-rider problem: investors can avoid paying incentive fees by entering after losses borne by others (see, e.g., Anson, 2001, and Lee, Lwi and Phoon, 2004). The usual remedy for this problem[1] is "share equalization," which amounts to creating a separate share class for each investment date, and then charging incentive fees on each share depending on its own HWM (see Lee, Lwi and Phoon, 2004, Aragon and Qian, 2007, Huetl, Loistl and Zellner, 2008). Our modeling of the HWM takes this approach.

3   Model

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[1] Though not the only remedy; see Das, Kish, Muething and Taylor (2002) and Technical Committee of ther International Organization of Securities Commissions (2004).

3.1 Overview

The two versions of the model are the same to a point. Both have two periods and a manager who can manage money in both, and arbitrarily many investors with arbitrarily much money to invest. Everybody is risk-neutral, discounts at zero and has reservation utility of zero. In both versions we capture decreasing returns to scale by assuming that the manager adds a fixed amount of expected value, so that extra investment spreads this expectation across more dollars, and we capture learning about the manager by assuming that this fixed amount is imperfectly known, so that investors update their priors on the manager using the first-period return. The incentive contract is the standard base fee / incentive fee with HWM hedge fund contract, and for simplicity we assume a zero hurdle rate (i.e. the incentive fee applies to any net profits on an investment, not the net profits in excess of a hurdle rate such as LIBOR). Also for simplicity we abstract from limited liability of the investors, so they experience all net profits or losses of the fund, and we assume that the manager does not invest in the fund.

The differences between the two versions are that 1) the first models the manager's value-added with a binary distribution, whereas the second models it with a normal distribution, and 2) the first implicitly assumes no effort cost, whereas the second assumes a positive effort cost and endogenizes the choice to make a second-period effort. We refer to the former as the binary version, and the latter as the continuous version. To make the results of the versions more comparable, the second largely reuses the notation and terminology of the first.

3.2 Binary Version

3.2.1 Model Setup

There are three dates, 0, 1 and 2, with the first period running from 0 to 1 and the second from 1 to 2. A money manager can manage in both periods, and in each period his dollar return is either σ or $-\sigma$, which we refer to as a high or low return, respectively. Everybody, including the manager, agrees at date 0 that the manager's probability of achieving a high return is π_0. Everybody also agrees that if the manager achieves a high first-perod return then his probability of a high second-period return is π_H, and if his first-period return is low then his probability of a high second-period return is π_L, where $\frac{1}{2} < \pi_L < \pi_0 < \pi_H < 1$, and to be consistent, $\pi_0 \pi_H + (1 - \pi_0)\pi_L = \pi_0$.[2] Everybody is risk-neutral, discounts at 0 and has reservation utility of 0.

The manager has no money, and there are arbitrarily many competitive investors who have money they can invest with the manager at both dates 0

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[2] This would result from, for example, the skilled managers having a higher probability of high returns, and investors updating the probability that the manager is skilled (as in Gervais, Lynch and Musto, 2005)

and 1. We denote the amount invested with the manager at date 0 as I_0. The money management contract is exogenously specified as a base fee of $x > 0$ paid at the beginning of a period and an incentive fee $y > 0$ of net profits paid at the end of the period, where the incentive fee has a HWM provision. Thus at date 0 the manager gets a base fee of xI_0, and at date 1 the manager gets $y(\sigma - xI_0)$ if the first-period return was high, and nothing if the first-period return was low. For the second period, the HWM provision dictates that any first-period losses suffered by an account must be made up in the second before the incentive fee can be charged to that account. This does not apply to any new date 1 investment in the fund, which will have a new HWM struck at the money. If investment is added to the fund at date 1, the new investment and old investment share in the fund's pre-incentive-fee return in proportion to their invested amounts, and then pay incentive fees out of their take. If investors remove a fraction of their investment before rolling over, the HWM on the remaining investment goes down by the same fraction. All second-period investment pays the base fee x at date 1.

3.2.2 Model Solution

There are two decisions in this version: the amount I_0 that is invested at date 0, and the adjustment, either adding or subtracting investment, at date 1. In equilibrium, the I_0 chosen at date 0 is the one at which investors' expected profits are zero, where this expectation is over both first-period profits and any second-period expected profits that depend on I_0. At date 1, if the expected return on new investment is positive, investors will add until the expected return on new investment is zero, and if investors face a negative expected return on reinvestment they will subtract until the expected return on reinvestment is zero.

For the solution below it is handy to have notation for break-even investment levels. The investors' expected first-period return, given their choice of I_0, is $-xI_0 + \pi_0(\sigma - y(\sigma - xI_0)) + (1 - \pi_0)(-\sigma)$. Thus, the I_0 which sets the first-period expected return to zero, which we denote $I_0^{B/E}$, is

$$I_0^{B/E} = \frac{\sigma[\pi_0(2 - y) - 1]}{x(1 - \pi_0 y)}.$$

Similarly, we take $I_1^{B/E}$ to be the investment in the fund at date 1 after a low first-period return that would set the second-period expected return to zero if the HWM for *all* second-period investment were at the money (that is, the HWM applied to new investment):

$$I_1^{B/E} = \frac{\sigma[\pi_L(2 - y) - 1]}{x(1 - \pi_L y)}.$$

To ensure that investment occurs even after a low return, we impose a parameter restriction that ensures this break-even quantity is positive:

$$\pi_L > \frac{1}{2 - y}.$$

The solution is greatly simplified by several observations, which we state as lemmas:

Lemma 1 *The first-period expected return is not positive, so the initial investment is at least $I_0^{B/E}$, and the second-period expected return is not negative.*

Proof. If the first-period expected return were positive, investors would have a profitable strategy of adding investment to the fund at date 0 and removing it at date 1. The arbitrarily large number of investors ensures that this entry would not be deterred by the effect of additional investment on the profitability of existing investment. The base fee grows proportionately with investment but the value added by the manager does not change, so this entry would wipe out the expected return. By construction of $I_0^{B/E}$ this means the initial investment is at least $I_0^{B/E}$. The second-period expected return cannot be negative because investors would make expected profits by removing their money from the fund with no future consequences because the second period is the last. ■

Lemma 2 *The second-period expected return is always 0 after a high first-period return.*

Proof. The first-period profit means that any rolled-over investment is at its HWM, which means it has the same HWM as any added investment. Therefore, if it is profitable to add investment, it will be added until its expected return is zero, which means the expected return of rolled-over investment is also zero. If it is not profitable to add investment then rolled-over investment is also not profitable, and investors will remove money from the fund until the expected return is zero. ■

Lemma 3 *If investors roll over all their investment after a low first-period return, they pay no incentive fee on this rolled-over amount at date 2, but they would pay an incentive fee on any added investment.*

Proof. A low return means the initial investment lost $xI_0 + \sigma$. The best the fund can do in the second period is make σ, so the first-period loss cannot be made up entirely, so there can be no incentive fee on the rolled-over investment. On the other hand, any new investment has no losses to make up, so it would pay an incentive fee after a high return. ■

Lemma 4 *If the expected second-period return from rolling over all first-period investment is positive, then either 1) additional investment is profitable, in which case additional investment enters until its expected return is zero, reducing the expected return on rolled-over investment, but not to zero, or 2) additional investment is not profitable, in which case no money is added or removed. If the expected second-period return from rolling over all first-period investment is negative, then investors will remove investment until the expected return is zero.*

Proof. If additional investment is profitable, then competition will force its expected return to zero. This will not force the expected return on rolled-over investment to zero, since its HWM is strictly higher, making its expected return strictly higher. If rolled-over investment is profitable but additional investment is not, then investors make losses either adding or removing investment but profit from staying put, so they stay put. If rolled-over investment is not profitable then investors profit from removing investment until its expected return reaches zero. ∎

These lemmas imply that, in equilibrium, investors either have zero expected returns in both periods, or they have negative expected returns in the first period and positive after a low first-period return. The former we refer to as the *Reduce* case, because investors reduce their investment after a low first-period return. The latter admits two possible cases: the *Add* case where investors add investment after a low return, and the *Hold* case where no money goes in or out.

One more piece of notation makes the proofs go smoothly. We take I_0^{HOLD} to be the initial investment such that the investors' overall expected return is zero, assuming they roll over all investment at date 1 after a low return, without adding or removing, and without constraining the sign of either expected return. This is the solution to the equation

$$
\begin{aligned}
&-x I_0^{HOLD} + \pi_0(\sigma - y(\sigma - x I_0^{HOLD})) + (1 - \pi_0)(-\sigma) \\
&+(1 - \pi_0)[-x(I_0^{HOLD}(1 - x) - \sigma) + \pi_L(\sigma) + (1 - \pi_L)(-\sigma)] = 0
\end{aligned}
$$

which works out to

$$
I_0^{HOLD} = \frac{\sigma[\pi_0(2 - y) + (1 - \pi_0)(2\pi_L + x - 1) - 1]}{x[(1 - \pi_0 y) + (1 - \pi_0)(1 - x)]}
$$

We can now start identifying the equilibrium value of I_0, which we call I_0^*.

Proposition 1 *We are in the Reduce case, where expected returns are* 0 *in both periods and* $I_0^* = I_0^{B/E}$, *if and only if* $I_0^{HOLD} \leq I_0^{B/E}$.

Proof. From above we know that $I_0^* \geq I_0^{B/E}$, so if $I_0^{HOLD} \leq I_0^{B/E}$ then we must have $I_0^* \geq I_0^{HOLD}$. Since $I_0^{HOLD} \leq I_0^{B/E}$ implies nonnegative expected first-period returns, it also implies nonpositive expected second-period returns. If expected second-period returns are nonpositive for $I_0^* = I_0^{HOLD}$, they must be nonpositive for $I_0^* \geq I_0^{HOLD}$, so the expected first-period return must be zero, so we must have $I_0^* = I_0^{B/E}$. Conversely, if we are in the *Reduce* case we know that expected second-period returns are zero, so the expected first-period returns are zero, so we must have $I_0^* = I_0^{B/E}$. This is incompatible with $I_0^{HOLD} > I_0^{B/E}$, because that would imply positive expected second-period returns from rolling over investment with $I_0^* = I_0^{HOLD}$, and therefore with any lower I_0^*. ∎

Proposition 2 *We are in the Hold case, and* $I_0^* = I_0^{HOLD}$ *if and only if* $I_0^{HOLD} > \max\{I_0^{B/E}, \frac{I_1^{B/E} + \sigma}{1 - x}\}$.

Proof. The Hold case requires positive expected second-period expected returns from rolling over, which is true if and only if $I_0^{HOLD} > I_0^{B/E}$. It also requires negative expected second-period returns on additional investment. This is true if and only if $I_0^{HOLD}(1-x) - \sigma > I_1^{B/E}$, because it states that the rolled-over amount, $I_0^{HOLD}(1-x) - \sigma$, is greater than the break-even investment level at the HWM of new investment. ∎

Corollary 1 *In the Hold case, the expected first-period return is $-(1-\pi_0)R^{HOLD}$ and the expected second-period return after a low return is R^{HOLD}, where*

$$R^{HOLD} = \frac{2\sigma[\pi_L(1-\pi_0 y) - \pi_0(1-y)(1-x)]}{(1-\pi_0 y) + (1-\pi_0)(1-x)}$$

Proof. To calculate the expected second-period return after a low return, plug I_0^{HOLD} into the expected second-period return, i.e. $-x[(1-x)I_0^{HOLD} - \sigma] + (2\pi_L - 1)\sigma$, which gives R^{HOLD}. For the overall expected return to be zero, the expected first-period return must be $-(1-\pi_0)$ times this number. ∎

Proposition 3 *We are in the Add case if and only if $I_0^{B/E} < I_0^{HOLD} < \frac{I_1^{B/E}+\sigma}{1-x}$, and in that case, $I_0^* = I_0^{ADD}$ where*

$$I_0^{ADD} = \frac{\sigma[(\pi_0(2-y)-1)(\pi_L(2-y)-1) - x(1-\pi_0)2\pi_L y(1-\pi_L)]}{x[(1-\pi_0 y)(\pi_L(2-y)-1) - (1-x)(1-\pi_0)2\pi_L y(1-\pi_L)]}$$

Proof. If $I_0^* = I_0^{HOLD} > I_0^{B/E}$ then second-period expected returns are positive, and if $I_0^{HOLD} < \frac{I_1^{B/E}+\sigma}{1-x}$ then additional investment will occur, bringing down the profitability of rolled-over investment, thereby pushing the overall expected return as of date 0 below zero. To bring the expected return back to zero, investors must invest less, but investment will not fall all the way to $I_0^{B/E}$, because in that case they would be breaking even in the first period and making expected profits in the second. So there is an I_0^{ADD} between I_0^{HOLD} and $I_0^{B/E}$ at which they break even overall. To identify I_0^{ADD}, we use the fact that rolled-over investment will share the fund's pre-incentive-fee return with new investment in proportion to their invested amounts, and we use our result that the total of rolled-over and additional investment is $I_1^{B/E}$. Thus, I_0^{ADD} solves

$$-xI_0^{ADD} + \pi_0(\sigma - y(\sigma - xI_0^{ADD})) + (1-\pi_0)(-\sigma) + (1-\pi_0)[-x(I_0^{ADD}(1-x)-\sigma)$$

$$+\frac{I_0^{ADD}(1-x) - \sigma}{I_1^{B/E}}(\pi_L(\sigma) + (1-\pi_L)(-\sigma))] = 0$$

The solution is the quantity in the proposition. ∎

Corollary 2 *In the Add case, the expected first-period return is $-(1-\pi_0)R^{ADD}$ and the expected second-period return after a low return is R^{ADD}, where*

$$R^{ADD} = \frac{2\sigma\pi_L y(1-\pi_L)(\pi_0[2(1+xy-x)-y]-1)}{(1-\pi_0 y)(\pi_L(2-y)-1) - (1-x)(1-\pi_0)2\pi_L y(1-\pi_L)}$$

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Proof. To calculate the expected second-period return after a low return, plug I_0^{ADD} into the expected second-period return, i.e. $-x[(1-x)I_0^{ADD}-\sigma]+(2\pi_L-1)\sigma$, which gives R^{ADD}. For the overall expected return to be zero, the expected first-period return must be $-(1-\pi_0)$ times this number. ∎

3.2.3 Comparative Statics

To help interpret the solution, it is useful to formalize a concept that proves crucial: the precision of investors' date 0 beliefs regarding the manager's value-added. Since higher date 0 precision means less weight on first-period returns in date 1 beliefs, i.e. π_L and π_H closer to π_0, we say that

Definition 1 *In the binary version, Investors have **higher precision** if $\frac{\pi_L}{\pi_0}$ is higher.*

This is not the only way to define precision, but it is intuitive and it is the form in which the concept arises in the solution.

Looking at the boundary between the *Reduce* case and the *Add* and *Hold* cases, we see that precision determines whether expected returns are zero:

Proposition 4 *If precision is sufficiently high then expected profits are negative in the first period and positive after a low return. Otherwise, expected profits are always zero.*

Proof. Investors make negative expected first-period returns and positive expected returns after low first-period returns if and only if we are in either the *Hold* or the *Add* case, which is true if and only if $I_0^{HOLD} > I_0^{B/E}$, which is true if and only if

$$\frac{\pi_L}{\pi_0} > \frac{(1-y)(1-x)}{1-\pi_0 y}.$$

Since y, x and π_0 are all between 0 and 1, the expression on the RHS is too, so if π_L is sufficiently close to π_0 then we are in either the *Hold* or the *Add* case. ∎

It follows immediately that non-zero expected returns arise with lower precision if fees are higher:

Corollary 3 *The level of precision at which expected first-period returns become negative and expected returns after low returns become positive decreases as either the base fee or the incentive fee goes up.*

Proof. The derivative of the RHS with respect to both x and y is negative, so given π_0, the threshold value of π_L for the *Hold* and *Add* cases goes down as x or y goes up. ∎

A higher base fee means rolled-over investment is smaller relative to the initial investment, so it pays that much less fee going forward. A higher incentive fee means rolled-over investment has a greater advantage after a low return, since it does not pay an incentive fee going forward.

Precision is also key to the boundary between the *Hold* and *Add* cases:

Proposition 5 *There is no Add case if precision is too low or the base fee is too high. If precision is sufficiently high and the base fee is sufficiently low, then we are in the Add case if the incentive fee is sufficiently low.*

Proof. We have from above that we are in the Hold or Add case if precision is sufficiently high. And the condition $I_0^{HOLD} < \frac{I_1^{B/E} + \sigma}{1-x}$ simplifies to

$$y[\pi_0 \pi_L x + \frac{1}{1-y}(1 - \pi_L)(1 - \pi_0)(1 - x)] < \pi_L - (1 - x)\pi_0.$$

Since the LHS of this expression is always positive for positive y and goes to zero as $y \to 0$, it is satisfied by some $y > 0$ if and only if the RHS is positive, which is true if and only if $\frac{\pi_L}{\pi_0} > 1 - x$. Thus, there exists a sufficiently low y to put us in the Add case if x is sufficiently low and $\frac{\pi_L}{\pi_0}$ is sufficiently high. ■

Corollary 4 *The initial size of a fund is invariant with respect to precision about the manager's ability in the Reduce case, increases with precision in the Hold case, and decreases with precision in the Add case.*

Proof. In the Reduce case, $I_0^* = I_0^{B/E}$, which does not depend on π_L. In the Hold case, $I_0^* = I_0^{HOLD}$ which is always increasing in π_L. In the Add case, $I_0^* = I_0^{ADD}$, and to see that this decreases in π_L, note that the derivative of R^{ADD} (a long expression, available on request) is decreasing in π_L if and only if $\pi_0[2(1 + xy - x) - y] > 1$, which is true if and only if $R^{ADD} > 0$, which has to be true in the Add case. And if the expected-return after a low return is decreasing, the first-period expected return must be increasing, which can only happen if the initial investment is decreasing. ■

Corollary 5 *The expected first-period return decreases with precision in the Hold case, and increases with precision in the Add case. The opposite is true for the expected return after a low first-period return.*

Proof. When initial fund size increases with precision in the *Hold* region, this directly decreases the first-period expected return, and when it decreases in the *Add* case, this increases the first-period expected return. The second-period expected return must move in the opposite direction to keep the sum at zero. ■

Since the source of expected returns after a loss is the wedge between rolled-over and new investment driven by the incentive fee, intuition suggests that the expected return increases with the incentive fee. We can see that, as long as the base fee is not too high, this intuition holds:

Proposition 6 *In the Hold case the initial size of the fund and the initial expected return go down, and the expected return after a loss goes up as the incentive fee goes up.*

Proof. The derivative of R_{HOLD} w/r/t y is

$$\frac{2\sigma\pi_0(1-\pi_0)(1-x)(2-x-\pi_L)}{[(1-\pi_0 y)+(1-\pi_0)(1-x)]^2}$$

which is always positive and the derivative of I_0^{HOLD} w/r/t y is

$$\frac{-2\sigma\pi_0(1-\pi_0)(2-x-\pi_L)}{x[(1-\pi_0 y)+(1-\pi_0)(1-x)]^2}$$

which is always negative. Since the expected return after a loss is $R_{HOLD}/(I_0^{HOLD}(1-x)-\sigma)$, it follows that the expected return goes up as y goes up. And since the expected return after a loss goes up and the probability of a loss stays the same, the initial expected return must go down. ∎

The simplicity of the two-return space delivers analytical results but inhibits the analysis of the cross section of returns. For this purpose, we enrich the model with a continuous distribution, and also with a more realistic participation constraint for the manager represented by an effort cost.

3.3 Continuous Version

3.3.1 Model Setup

There are again three dates, 0, 1 and 2, defining a first and second period which we will index with t, and there are again arbitrarily many investors with money and one manager with no money, and everybody is risk-neutral with 0 reservation utility and discounts at 0. The money-management contract is again a base fee of x and an incentive fee of y, both exogenously specified, with a HWM provision. The differences are that the manager must make an effort to add value, and his returns are normally distributed. Specifically, if the manager makes an effort that costs him $c > 0$, then the value his management adds in the first period is $V_1 = A + \varepsilon_1$, and if he does not make the effort then $V_1 = \varepsilon_1$. The first summand A is constant over time, but known imprecisely: at date 0, everybody, including the manager agrees that $A \sim N(\mu_A, \sigma_A^2)$, where $\mu_A > 0$. The second summand is period-specific noise, i.i.d. across time, and everyone agrees that $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ for $t = 1, 2$. At date 1 the manager can again make an effort costing c, and if he does then $V_2 = A + \varepsilon_2$ and if he does not then $V_2 = \varepsilon_2$. The effort is observable but not contractible, and investors make their investment decision after observing whether the effort was made.

The order of events is as follows. At date 0, the manager decides whether to make an effort, and then investors choose the amount I_0 to invest. The manager removes his base fee of xI_0, and then adds V_1, so that the investors' net profit before the incentive fee is $V_1 - xI_0$, and the incentive fee is $y\max\{0, V_1 - xI_0\}$. Thus, if we let $R_1(I_0, V_1)$ be the first-period return (we denote the realized i^{th}-period return R_i and the expected return $\overline{R}_i$), we have

$$R_1(I_0, V_1) = V_1 - xI_0 - y\max\{0, V_1 - xI_0\}$$

and the value of the date 0 investment after the first period, which we denote $I_0^1(V_1, I_0)$, is $I_0 + R_1(I_0, V_1)$. Note that if $V_1 \geq xI_0$ then the HWM on this existing investment is the same as new investment for the second period, whereas if $V_1 < xI_0$ then rolled-over investment pays no incentive fee on the first $xI_0 - V_1$ of second-period net profit. If only a fraction of investment is rolled over, then it pays no incentive fee on this fraction of $xI_0 - V_1$. At date 1 the manager decides whether to make an effort, then investors choose their investment for the second period, then the manager removes his base fee, V_2 is realized, and incentive fees, if any, are removed.

3.3.2 Model Solution

In contrast to the binary version, where we solve analytically for the date 0 investment I_0^*, here we solve for it numerically. We do this by positing a value for I_0 and then calculating analytically the expected profit it implies for the investors in the first period, and the expected profit in the second period, conditional on the first-period outcome and imputing the investors' optimal date 1 action. We then integrate numerically over the distribution of first-period outcomes to calculate the overall expected profit implied by I_0. We then numerically identify I_0^* as the I_0 such that this overall expectation is zero.

We start by simplifying the strategy space with a parameter restriction. There are four choices in this version - the manager's two effort choices, and the investors' two investment choices. We narrow this down to the three choices we want to focus on with a restriction that guarantees a date 0 effort. To motivate this restriction, note that, with investors breaking even overall, the manager receives all surplus expected as of date 0. Also, note that if the manager makes a date 0 effort then this expectation is the first-period expected surplus $\mu_A - c$ plus any second-period expected surplus, whereas if he makes only a date 1 effort then his expected surplus is just $\mu_A - c$, and if he makes no effort then it is 0. Thus, we can guarantee a date 0 effort with this parameter restriction:

$$\mu_A > c.$$

Note that this is sufficient but not necessary.

Before commencing the solution, some preliminary observations and some new notation are useful. From our Normal distribution assumption we get the familiar result that the posterior distribution of A, given the observation of A plus Normal noise, is linear in the observation, and also Normal:

$$A \mid V_1 \sim N\left(\mu_1(V_1), \left(\frac{\sigma_\varepsilon^2}{\sigma_A^2 + \sigma_\varepsilon^2}\right)\sigma_A^2\right),$$

where

$$\mu_1(V_1) = \mu_A + \left(\frac{\sigma_A^2}{\sigma_A^2 + \sigma_\varepsilon^2}\right)(V_1 - \mu_A)$$

so that $V_2 = A + \varepsilon_2$ is distributed $N\left(\mu_1(V_1), \left(\frac{\sigma_\varepsilon^2}{\sigma_A^2 + \sigma_\varepsilon^2}\right)\sigma_A^2 + \sigma_\varepsilon^2\right)$. Because it comes

up repeatedly, we use the shorthand

$$\sigma_1^2 = (\frac{\sigma_\varepsilon^2}{\sigma_A^2 + \sigma_\varepsilon^2})\sigma_A^2 + \sigma_\varepsilon^2.$$

It is also convenient to have a shorthand representation of the portion of the expectation of a Normal that is above a constant, as this comes up in the calculation of expected incentive fees. So, letting f be the pdf of the Normal distribution with mean μ and variance σ^2, we have

$$T(\mu, \sigma^2, k) = \int_k^\infty x f(x) dx = \sigma\phi(\frac{\mu - k}{\sigma}) + \mu\Phi(\frac{\mu - k}{\sigma})$$

where ϕ and Φ are the pdf and cdf, respectively, of the Standard Normal. Using this notation we can identify $I_1^{B/E}(V_1)$, which, analogously to the binary version, is the break-even level of investment at date 1 following a return of V_1, assuming all investment has the HWM of new investment. It is the solution to

$$\mu(V_1) = x I_1^{B/E}(V_1) + y T(\mu_1(V_1), \sigma_1^2, x I_1^{B/E}(V_1)).$$

This equation states that the expected value added by management equals the expected fees paid by investors, assuming that investors pay an incentive fee for all profits in excess of their base fee.

We start by identifying the cases that can arise at date 1, and the expected returns and date 1 investment levels they imply. These cases are conceptually different from the cases in the binary version in that they refer to the date 1 outcome conditional on a given choice of I_0 and outcome V_1, as opposed to the date 1 outcome conditional on the equilibrium choice I_0^* and a loss. In the binary version there are three cases - the *Hold*, *Add* and *Reduce* cases - that can arise after losses, plus another case that arises after profits, which we can call the *Profit* case. These are also possible date 1 outcomes in the continuous version, as is a fifth case where the manager makes no effort, which we call the *Fold* case. As before, the investor makes second-period expected profits in the Hold and Add cases and otherwise breaks even, but now there is the additional question whether the manager's expected fee income is sufficient to warrant making an effort, i.e. whether it exceeds c. We run through the cases, and in each we calculate the investors' expected second-period net return $\overline{R}_2(V_1, I_0)$ and total date 1 investment $I_1(V_1, I_0)$. First, a simple result:

Lemma 5 *If the manager does not make an effort at date 1, then investment in the fund goes to zero.*

Proof. If the manager does not make an effort, then $V_2 = \varepsilon_2$, which has mean zero. Since the fund charges fees, it is immediate that investors make negative profits at any positive investment level. So there is no investment. ∎

Then the five cases:

Lemma 6 *We are in the Profit case if and only if*

$$
\begin{aligned}
V_1 &\geq x I_0, \\
I_1^{B/E}(V_1) &> 0
\end{aligned}
$$

and

$$\mu_1(V_1) \geq c.$$

In this case, $\overline{R}_2(V_1, I_0) = 0$ and $I_1(V_1, I_0) = I_1^{B/E}(V_1)$.

Proof. The first inequality establishes that the fund made first-period profits. The second inequality establishes that investors break even at some positive investment level. Since there is an incentive fee even at zero investment, this is not automatically true. The third inequality establishes that expected surplus is not negative. If these conditions all hold it is immediate that investors will invest $I_1(V_1, I_0) = I_1^{B/E}(V_1)$ and $\overline{R}_2(V_1, I_0) = 0$. If the first condition does not hold then by definition this is not the *Profit* case, if the first holds but the second does not then investors will not invest and the fund will fold, and if the first holds but the third does not then the manager will not make an effort and the fund will fold. $\blacksquare$

Lemma 7 *We are in the Hold case if and only if*

$$
\begin{aligned}
V_1 &< x I_0 \\
I_1^{B/E}(V_1) &\leq I_0^1(V_1, I_0)
\end{aligned}
$$

and

$$\mu_1(V_1) \geq x I_0^1 + y T(\mu_1(V_1), \sigma_1^2, x I_0^1(V_1, I_0) + x I_0 - V_1) \geq c.$$

In this case, $\overline{R}_2(V_1, I_0) = \mu_1(V_1) - (x I_0^1 + y T(\mu_1(V_1), \sigma_1^2, x I_0^1(V_1, I_0) + x I_0 - V_1))$ and $I_1(V_1, I_0) = I_0^1(V_1, I_0)$.

Proof. The first condition establishes that $R_1(V_1, I_0) < 0$, so that the HWM of rolled-over investment is higher than that of new investment. The second condition establishes that if all investment is rolled over, new investment is unprofitable. The third condition establishes that the manager's expected value-added exceeds the expected fees on rolled-over investment, and these expected fees exceed the manager's effort cost. So, the manager will choose to make an effort, because if he does, then investors will find it profitable to roll over all their investment, and will pay fees in excess of c. Investors will not find it profitable to add investment, and no investor will benefit from removing investment. The expected return is simply the difference between the manager's expected value-added and the investors' expected fees. $\blacksquare$

Lemma 8 *We are in the Add case if and only if*

$$
\begin{aligned}
V_1 &< x I_0 \\
I_1^{B/E}(V_1) &> I_0^1(V_1, I_0)
\end{aligned}
$$

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and

$$\mu_1(V_1) - \overline{R}_2(V_1, I_0) \geq c.$$

where

$$\overline{R}_2(V_1, I_0) = (\frac{I_0^1(V_1, I_0)}{I_1^{B/E}(V_1)})y(T(\mu_1(V_1), \sigma_1^2, xI_1^{B/E}(V_1))$$

$$-T(\mu_1(V_1), \sigma_1^2, xI_1^{B/E}(V_1) + [\frac{I_1^{B/E}(V_1)}{I_0^1(V_1, I_0)}](xI_0 - V_1)))$$

and $I_1(V_1, I_0) = I_1^{B/E}(V_1)$.

Proof. The first condition establishes that $R_1(V_1, I_0) < 0$, and the second establishes that investors will increase investment from the rolled-over amount up to $I_1^{B/E}(V_1)$, provided an effort is made. The third condition establishes that the manager will make an effort if the investors' expected return is $\overline{R}_2(V_1, I_0)$, since it ensures that the manager's value added minus the investors' expected return exceeds the manager's effort cost. To see that the fourth equation gives the investors' expected return, note that the expected return is entirely due to the the HWM on $I_0^1(V_1, I_0)$ of the $I_1^{B/E}(V_1)$ invested dollars being higher by $xI_0 - V_1$. Since these $I_0^1(V_1, I_0)$ dollars get $\frac{I_0^1(V_1, I_0)}{I_1^{B/E}(V_1)}$ of the fund's return, this return must be $\frac{I_1^{B/E}(V_1)}{I_0^1(V_1, I_0)}(xI_0 - V_1)$ higher to reach the point where rolled-over investment starts paying incentive fees. ∎

Lemma 9 *We are in the Reduce case if and only if*

$$
\begin{aligned}
V_1 &< xI_0 \\
c &\leq \mu_1(V_1) < xI_0^1 + yT(\mu_1(V_1), \sigma_1^2, xI_0^1(V_1, I_0) + xI_0 - V_1)
\end{aligned}
$$

and there exists an $I > 0$ such that

$$xI + yT(\mu_1(V_1), \sigma_1^2, xI + [\frac{I}{I_0^1(V_1, I_0)}](xI_0 - V_1) = \mu_1(V_1)$$

in which case $\overline{R}_2(V_1, I_0) = 0$ and $I_1(V_1, I_0) = I$.

Proof. The first condition establishes that $R_1(V_1, I_0) < 0$, and the second that the manager recovers his effort cost if the investors break even, but if the investors roll over all their investment, they do not break even. The third condition establishes that there is a reduced investment level at which investors do break even. So if the manager makes an effort, investors reduce their investment to this break-even level at which the manager gets at least his effort cost, so the manager makes an effort and this is what happens. ∎

Lemma 10 *If none of the other four cases apply, we are in the Fold region where the manager makes no effort, $I_1(V_1, I_0) = 0$ and $\overline{R}_2(V_1, I_0) = 0$.*

Proof. The other four cases exactly characterize where the investors either stay put or adjust their investment to a different positive amount. The only alternative is for investors to remove all investment, in which case the manager will not make an effort. ■

Now that we have calculated $R_1(V_1, I_0)$ and $\overline{R}_2(V_1, I_0)$, and we know that the distribution of V_1 is $N(\mu_A, \sigma_A^2 + \sigma_\varepsilon^2)$, we can identify the equilibrium choice I_0^*:

Proposition 7 *At date* 0*, investors choose the I_0^* that solves*

$$\int_{-\infty}^{\infty} (R_1(V_1, I_0^*) + \overline{R}_2(V_1, I_0^*)) f(V_1) dV_1 = 0$$

where f is the pdf of the Normal distribution with mean μ_A and variance $\sigma_A^2 + \sigma_\varepsilon^2$.

Proof. This is by construction the break-even investment level, so it is the equilibrium result of competition between arbitrarily many investors. ■

Before we solve this numerically, we can make a few observations. As with the binary version, expected returns are non-positive in the first period and non-negative in the second. Also, it is immediate from the functional form of R_1 that $\overline{R}_1$ decreases in I_0. The effect of I_0 on $\overline{R}_2$ is less clear, because there are effects in both directions. On the one hand, higher I_0 means a higher value of rolled-over investment I_0^1, which increases the fee investors pay when they roll over in the *Hold* and *Add* regions, and thereby decreases $\overline{R}_2$. On the other hand, because higher I_0 reduces R_1, it increases $\overline{R}_2$ by decreasing fees because it increases the loss that must be made up before the incentive fee is charged. In other words, it widens the gap between the fees paid by rolled-over vs. new investment.

3.3.3 Comparative Statics: Precision

In the binary version, more precision regarding the manager is captured by less updating after a bad return. In this version there is a continuum of bad returns, so the analog is the slope of the posterior $\mu_1(V_1)$ on the return V_1:

Definition 2 *In the continuous version, Investors have **higher precision** if $\frac{\sigma_A^2}{\sigma_A^2 + \sigma_\varepsilon^2}$ is lower.*

We can illustrate the equilibrium choice graphically by choosing a vector of parameters and then plotting the expected second-period return and the negative of the expected first period return as a function of I_0, so that the equilibrium I_0^* and expected returns are identified by their intersection. To do this, and also to simultaneously illustrate the effect of date 0 precision about the manager's ability, we choose two vectors, the difference being the variance σ_A^2 of the prior distribution on A. So we choose 2% and 20% for x and y, 100

for σ_ε^2, 5 for c, 10 for μ_A and then alternately 1 and 5 for σ_A^2. The result is in Figure 1.

Figure 1 shows the first-period expected return decreasing as I_0 increases (note that the two lines for $\overline{R}_1(V_1, I_0)$ are almost the same, so we do not label them separately), and it shows the equilibria as the intersections with the the lines representing the second period. The effect of precision is analogous to the finding with the binary version:as it increases, initial investment goes up, first-period expected returns grow more negative, and second-period expected returns grow more positive. The intuition is the same, too: more precision means less learning, and therefore better prospects after losses, when rolled-over investment has an advantage over new investment. It also means worse prospects after gains, but that is irrelevant since there is no advantage after gains.

To illustrate the equilibrium relation between first-period returns and second-period regions, we run the following exercise. First, we plot the expected second-period return from rolling over first-period investment as a function of the first-period gross return V_1, which we call the Rollover line. Then on the same graph we plot the same line except that we assume that the HWM in the second period is the HWM for new investment, and we call this the Add line. With this setup, the Profit region is to the right of $V_1 = xI_0$, and we can identify the other regions, except the Fold region, in the loss region on the left: when the Rollover line is above zero and the Add line is below, then rolling over is profitable but adding is not, so this is the Hold region; when both are above zero then both are profitable so we are in the Add region; and when both are below zero then investors need to withdraw to break even, so we are in the Reduce region. We generate this plot for the two parameter vectors from Figure 1; the results are in Figures 2 (low precision) and 3 (high precision).[3]

In Figure 2, both lines are generally below zero in the loss region, but there is a range where the rollover line is above zero. Thus, with these parameter values expected second-period returns are generally zero, and money generally flows out after losses, but there is a range of moderate first-period losses that predicts second-period profits and zero flows. To understand the shapes of these lines, note first that when precision is low, investors learn more about the manager from his realized returns, so the positive effect of past performance on updating is relatively stronger compared to its negative effect through the decreasing returns to scale. Thus, the lines slope upward more. But as we move right to left into the loss region, the HWM effect kicks in for the Rollover line. This effect is strong at first because the return required for a second-period incentive fee is increasing through the thick part of the second-period return distribution, but the increase asymptotes to zero as we move to the tail of this distribution. Thus, as the first-period loss grows we get the observed shape of the Rollover line, reaching upward and then bending back to its downward trend.

---

[3]The plots do not identify the *Fold* regions because they do not indicate when expected fees will fall short of c. This constraint is never binding in the ranges plotted in Figures 2 and 3, so there is no *Fold* region.

In Figure 3, the higher precision implies less learning from realized returns, so the decreasing returns to scale overwhelm the updating, sloping the lines down rather than up. Thus, as first-period losses grow, the Add line eventually crosses zero and adding becomes optimal (though at almost -10 standard deviations of V_1). So with these parameter values, there are inflows and positive expected returns after extreme losses, no flows and positive expected returns after moderate losses, and outflows and zero expected returns after gains.

To represent the effect of precision on expected returns on a larger scale we plot the regions in (σ_A^2, V_1) space. That is, we start with the same parameter vector as before, and then for each of a range of values of σ_A^2 we calculate the ranges of V_1 that produce the different second-period regions, and then plot the result with the regions shaded different colors. This graph is Figure 4, and it shows tthat the Hold and Add regions predominate when precision is high, and then the Add region drops away and the Hold region shrinks, with the Reduce region on either side. For high enough σ_A^2 there is no Hold region, so expected returns are always 0. So, as we found in the binary version, the effect of HWMs on expected returns requires sufficiently high precision.

3.3.4 Comparative Statics: Effort Cost

For the manager to participate in the second period, his expected value-added minus the investors' expected profits must exceed his effort cost. So as his effort cost rises, the investors' latitude to make expected profits after losses, which lower the expected profits, is likely to narrow. And if second-period expected profits decrease, then so must the investors initial investment, to bring the overall expected profit back to zero by increasing the first-period expected profit. To explore this dynamic we plot the equilibrium initial fund size against the effort cost for the two parameter vectors from the figures above, letting c range from the minimum 0 to the maximum $\mu_A = 10$, and present the result as Figure 5. We see that as effort cost rises from zero it initially has no perceptible effect, reflecting the extreme unlikelihood that the posterior on A falls that far, but investment levels eventually start dropping and converging, reflecting the shrinking significance of second-period expected profits. Thus, fund size shrinks as the manager's costs rise, but this is not because the fund becomes more expensive but rather that its participation constraint is more likely to bind in future scenarios where investors would otherwise have benefitted.

3.3.5 Comparative Statics: Incentive Fee

The incentive fee has opposing implications for the first and second periods. For the first period, a higher incentive fee is purely a drag on the investors' profits, and therefore reduces the initial investment required for a given expected return. For the second period, the incentive fee is the source of expected profits, because it is the wedge between rolled-over and new investment. To gauge the net effect, we repeat the exercise from Figure 1, except now we leave σ_A^2 at 1 and instead vary the incentive fee from $y = 20$ to $y = 15$. The result, in Figure 6, shows

the incentive-fee decrease moving the equilibrium to a higher initial fund size, and lower expected first-period losses and second-period profits. Or to put it another way, by raising his incentive fee, the manager shrinks his fund and he also moves some expected compensation from the future to the present.

3.4 Discussion

Embedding the HWM in its context of learning, decreasing returns to scale and competitive cash flows leads to predictions for fund size and the time series and cross section of expected returns:

- In the cross section of managers, controlling for expected value-added, those with longer track records (e.g. more management experience at other hedge funds or mutual funds) are likely to be the ones about whom the market has more certainty as to the value they add. So the prediction is that the HWM has a greater effect in their case, depressing initial returns, increasing later returns and boosting initial fund size. We should see new investment after bad returns only if precision is high and the incentive fee is low.

- In the cross section of management styles, those with higher costs (e.g. investing in overseas markets, obscure or otherwise research-intensive asset classes) should exhibit lower initial fund size and higher initial expected returns.

- In the cross section of incentive fees, initial expected returns and fund size should go down, and expected returns after losses should increase as the incentive fee goes up.

- In the cross section of returns, if precision is high then expected returns should be higher after intermediate-size losses than after large or small losses. Net flows should be less responsive to returns in this region of intermediate losses. As precision goes to zero, these effects should go away.

Our analysis is not an attempt to explain the existence of HWMs, but from the predictions we get at least one rationale. Since a HWM pushes the investor to a negative expected initial net return, it follows that the manager's expected initial fee revenue is higher than it would be without the HWM. If the manager needs more expected revenue to get started, or alternatively if the manager has a higher intertemporal discount rate than his investors do, then this effect of the HWM may be the desired effect. However, this works only to the extent that the HWM is valuable, so it works better for better-known managers than for rookies, and more for management styles with lower effort costs.

The analysis of Aragon and Qian (2007) *is* an attempt to explain the existence of HWMs, and the finding is that the HWM reduces the deadweight cost of liquidating after bad performance, by encouraging investors to stay. What

our analysis can add to this finding is that the encouragement is stronger when ex ante uncertainty about the manager is less, and when the manager's effort cost is less.

Along those same lines, the HWM is worth more when investors have some assurance that the fund will not try to renegotiate the contract when its performance lands it in a region where their expected profits were going to be positive. We do not model this explicitly, but our results suggest that factors that increase the probability of such renegotiation will tend to undo the HWM effects we derive. Since it is presumably at least somewhat embarrassing to do this to investors, proxies for the manager's reputation capital should predict greater HWM effects. The damaging renegotiation does not have to be with respect to the fees for reinvestment; it could be the fees for new investment instead. Since the expected profits on reinvestment arise from its better terms relative to that of new investment, a renegotiation that leads to new investment breaking even at a lower base or incentive fee would reduce the profits on reinvestment, unless reinvestment wasn't going to be profitable anyhow.

4   Conclusion

High-water marks are fundamental to money-management, and have therefore attracted considerable analysis. However, this analysis has abstracted from the endogeneity, as theorized by Berk and Green (2004) and documented by Fung et al. (2008), of investment and expected returns to past returns. We show that this endogenity is key to the testable implications of high-water marks for the cross section of managers, management styles, returns and fee levels. The analysis also suggests a rationale for the usage of high-water marks.

Another perspective on these results is that the high-water mark forges an unusual connection between returns and the investment opportunity set. The asset-pricing literature notes the importance to investors of the link between their performance and their future investment opportunity set (e.g., Brennan, 1998, Xia, 2001). What the high-water mark delivers in equilibrium is a relation opposite to the intuition – the investment opportunity set presented to an incumbent investor is better after a bad return than after a good one because the high-water mark defends his expected return from competition. But as the manager's track record shrinks, his expected return heads to zero, at which point this defense is no longer helpful.

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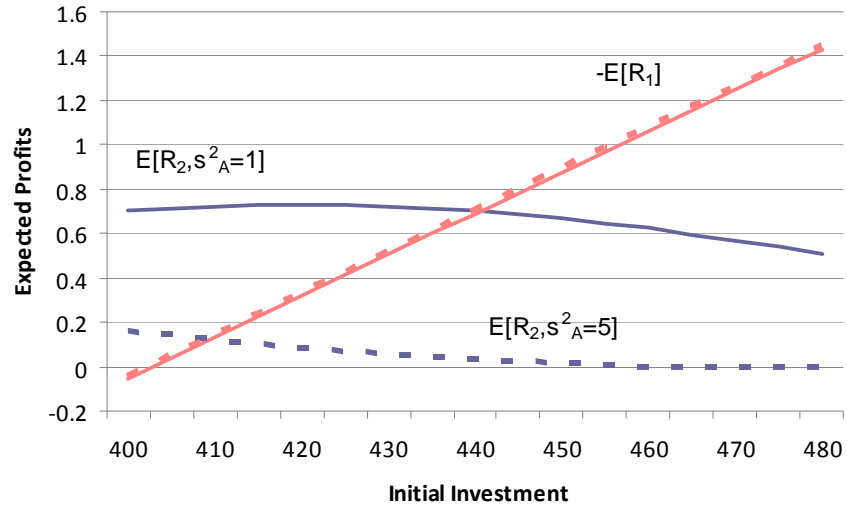
6 Figures

Figure 1: Expected second-period return and negative of expected first-period return with x=2%, y=20%, c=5, μ_A=10, σ_ε^2=100 and σ_A^2=1 and 5.
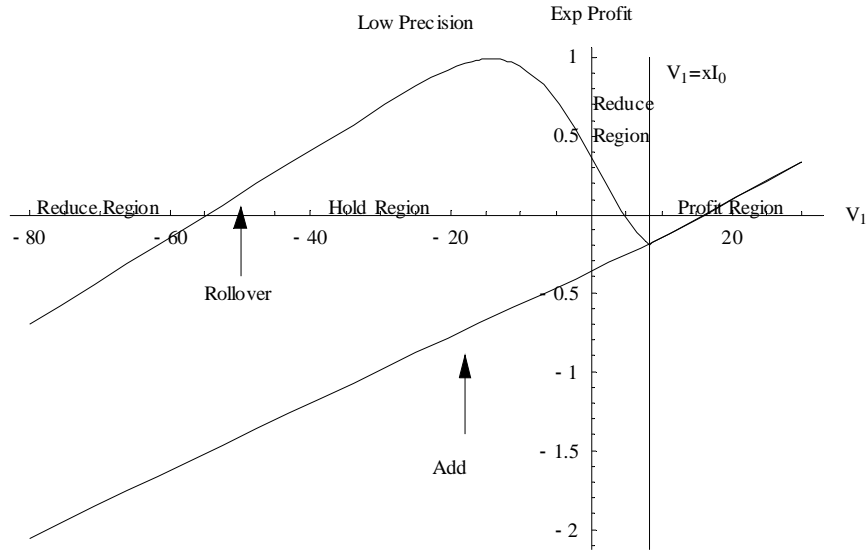
Figure 2: The Add (bottom) and Rollover (top) lines for $x=2\%$, $y=20\%$, $c=5$, $\mu_A=10$, $\sigma_\varepsilon^2=100$, $\sigma_A^2=5$ and $I_0^*=408.852$, and the regions they imply. There is no Fold region for this range of V_1.
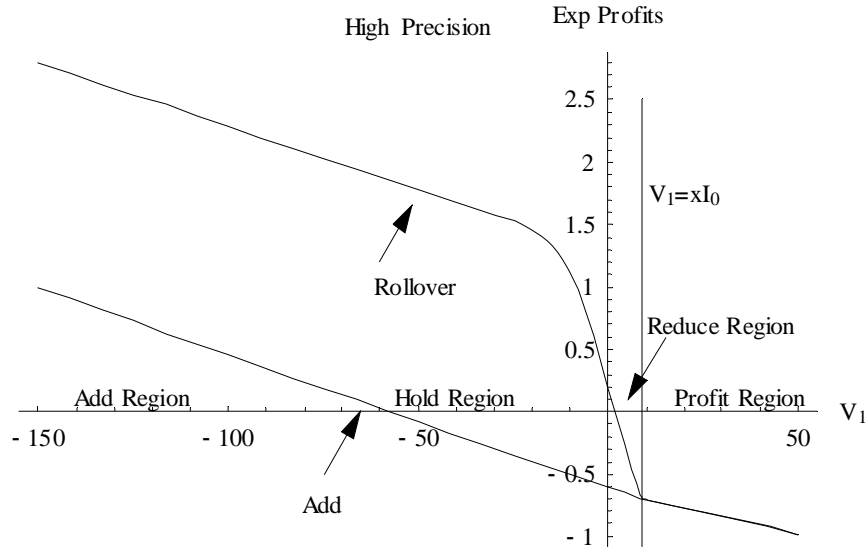

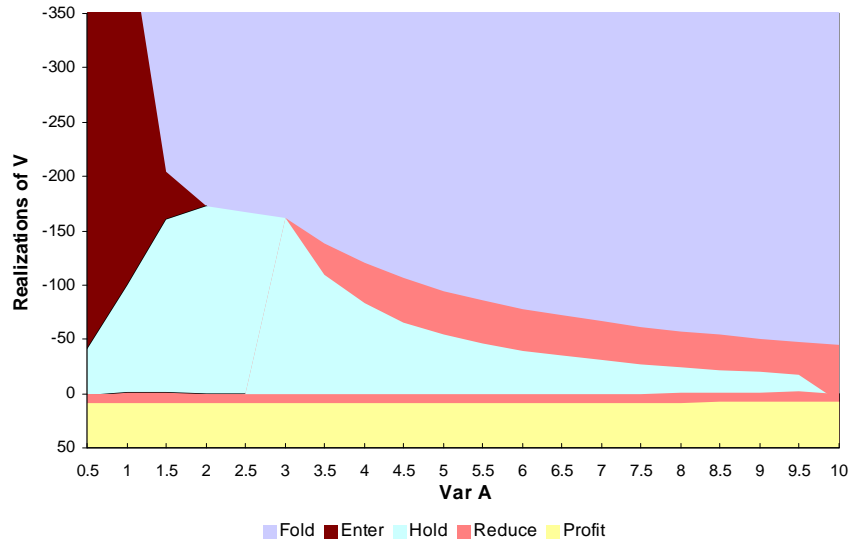Figure 3: The Add (bottom) and Rollover (top) lines for $x=2\%$, $y=20\%$, $c=5$, $\mu_A=10$, $\sigma_\varepsilon^2=100$, $\sigma_A^2=1$ and $I_0^*=439.974$, and the regions they imply. There is no Fold region for this range of V_1.

Figure 4: The various regions as functions of the prior precision about the manager and the realized first-period return.
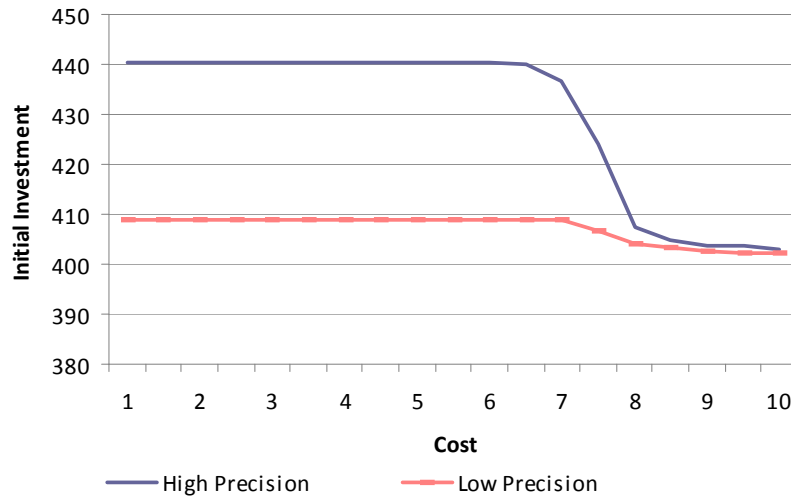

Figure 5: Initial investment as a function of the manager's effort cost. Parameter values are x=2%, y=20%, c=5, μ_A=10, σ_ε^2=100 and σ_A^2=1 (high precision) and 5 (low).
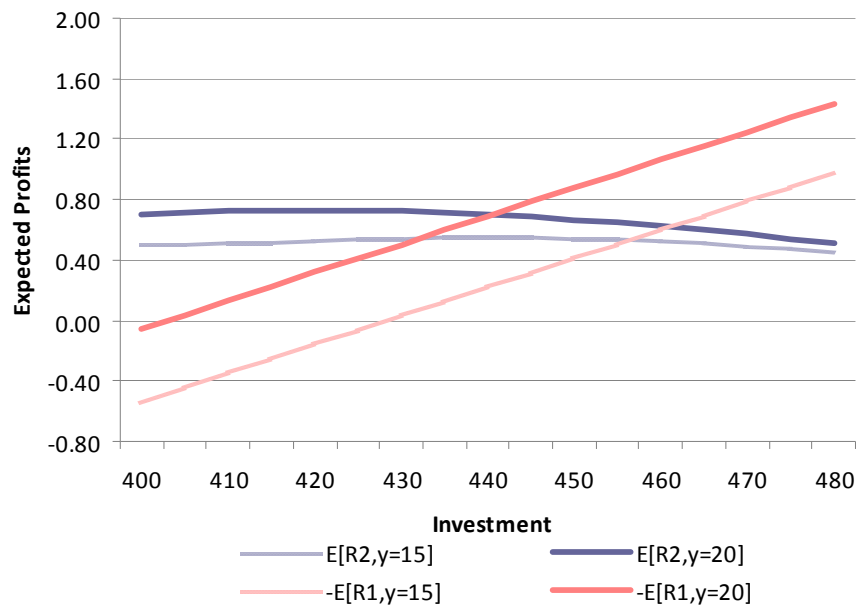
Figure 6: Expected second-period return and negative of expected first-period return with x=2%, c=5, μ_A=10, σ_ε^2=100 σ_A^2=1 and y=15% and 20%.