# Arbitrage Opportunities: a Blessing or a Curse?

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This paper argues that arbitrage is limited if rational agents face uncertainty about completing their arbitrage portfolios. This "execution risk" arises in our model because of slippages in assets prices as arbitrageurs compete for the limited supply of assets needed for a profitable arbitrage portfolio. This is distinct from the existing limits of arbitrage such as noise trader risk, fundamental risk and synchronization risk. We show that execution risk is related to market illiquidity and the number of competing arbitrageurs. As a consequence, rational arbitrageurs might wait for appropriate compensation for execution risk rather than correct the mispricing immediately. This leads to the existence of arbitrage opportunities even in markets with perfect substitutes and convertibility. Economic evaluation analyses of arbitrage strategies suggest that profitable exploitation of arbitrage opportunities in such markets is rare in the presence of competition.

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## 1 Introduction

The concept of arbitrage is one of the cornerstones of financial economics. Arbitrage is widely accepted to be absent from financial markets, as exploiting any arbitrage opportunities is riskless. The simultaneity of sales and purchases of identical assets ensures that arbitrageurs require no outlay of personal endowment but only need to set up a set of simultaneous contracts such that the revenue generated from the selling contract pays off the costs of the buying contract. However, there is an increasing number of researchers challenging this basic hypothesis by demonstrating the existence of arbitrage opportunities and the associated risks because of short-selling constraints, imperfect substitutes for mispriced assets, temporary mispricing of securities in the presence of noise traders and uncertainty about the timing of price correction (De Long, Shleifer, Summers and Waldmann (1990), Shleifer and Vishny (1992), Abreu and Brunnermeier (2002) and Ofek, Richardson and Whitelaw (2004)). In this paper, we propose and study a new limit of arbitrage which exists even for assets with perfect substitutes and convertibility, in the presence of competitive arbitrageurs.

The literature on limits of arbitrage focuses on three main categories of risk: fundamental risk, noise trader risk and synchronization risk.[1] Fundamental risk exists because the value of a partially hedged portfolio changes over time as there is no perfect substitute for the mispriced asset. Arbitrageurs are subjected to this risk, as these mispricings are permanent, even if they can continue with their arbitrage strategies until the maturity of the final payoff. Noise trader risk ((De Long et al., 1990)) occurs when the existence of noise traders causes a further temporary deviation from the fundamental value of the mispriced asset. Arbitrageurs who need to liquidate their positions because of trading and wealth constraints, will incur losses. For example, an outflow of funds because of the relatively poor performance of fund managers might force arbitrageurs to liquidate their position when the arbitrage opportunity may be the greatest.[2] Abreu and Brunnermeier (2002) introduces

[1] References include Shleifer and Summers (1990), De Long et al. (1990), Shleifer and Vishny (1992), Abreu and Brunnermeier (2002), Baker and Savaşoglu (64), Daniel, Hirshleifer and Teoh (2002), Barberis and Thaler (2003), Lamont and Thaler (2003), Gagnon and Karolyi (2004), Ofek et al. (2004) and De Jong, Rosenthal and Van Dijk (2008).

[2] The near collapse of the hedge fund Long-Term Capital Management (LTCM) illustrates the importance of wealth effect and funding liquidity. For detailed analysis of the LTCM crisis see e.g. Edwards (1999), Loewenstein (2000).

a limit of arbitrage that is related to arbitrageurs' uncertainty about when other arbitrageurs will start exploiting a common arbitrage opportunity. It is known as synchronization risk and pertains to the uncertainty regarding the timing of the price correction of the mispriced asset.

More recently, there is an increasing number of works focusing on the general equilibrium analysis of risky arbitrage (e.g., Basak and Croitoru (2000), Xiong (2001), Kyle and Xiong (2001), Gromb and Vayanos (2002), Zigrand (2004), Kondor (2008) and Oehmke (2008)). These models are primarily based on convergence trading and the risks associated with the temporary divergence of mispriced assets. An example of convergence trading is the exploitation of the mispricing of dual-listed companies (DLCs). DLCs are often seen as perfect substitutes for each other in integrated and efficient financial markets, therefore their prices should move in lockstep. However, underlying shares of DLCs are not convertible into each other making any exploitation of mispricing risky as positions must be kept open until prices converge.

In this paper, we introduce a new limit of arbitrage, which we call "execution risk" and make a simple but fundamental point. We argue that there will still be risk associated with arbitrage even in the ideal condition of a complete market in the absence of arbitrage convergence trading, short-sell constraints and irrational arbitrageurs. This risk comes from the uncertainty of completing the profitable arbitrage portfolio among competitive arbitrageurs. In contrast to previous limits of arbitrage, the presented mechanism does not rely on convergence trading. Instead, it is based on uncertainty about the arbitrage return distribution due to the competition for scarce supply of the necessary assets to form a profitable arbitrage portfolio.

We provide theoretical and empirical support for this new limit of arbitrage by examining the foreign exchange (FX) market. We focus on triangular arbitrage in the FX market because, we can isolate effects of traditional impediments from execution risk.[3] Triangular arbitrage in the FX market is an example of perfect substitutes with convertibility and does not suffer from fundamental risk, noise trader risk, synchronization risk and holding costs.

We explicitly model the process by which arbitrageurs trade upon observing a violation of an arbitrage parity. Each arbitrageur maximizes her trading profits, taking into account transaction costs and the anticipated actions of other competing arbitrageurs.

In equilibrium, each arbitrageur will exploit a mispricing with certainty only if the deviation exceeds the expected loss due to execution risk. The level of expected loss is dependent on the arbitrageur's expectation of the number of competing opponents and the state of market liquidity. When an arbitrage deviation is positive but below the level of the expected loss, she will only enter into the arbitrage with some probability. An arbitrageur's decision and her probability to participate under such conditions is dependent on the expected probability of participation of other competing arbitrageurs. Thus, we show that the efficiency of the market in eradicating mispricing depends on the illiquidity of the market and on the level of competition among arbitrageurs.

Empirically, we test our hypotheses using a set of reliable and detailed limit order book data from a widely traded and liquid electronic trading platform of the spot foreign exchange market. Firstly, we find that arbitrage opportunities in the FX market are not exploited instantly. This finding provides initial support for the existence of risk in arbitraging and limits of arbitrage. Arbitrageurs do not exploit arbitrage opportunities as it might not be optimal to do so immediately, or they are not being compensated appropriately for the risk they are exposed to.

[3] Studies of FX arbitrage include Branson (1969), Frenkel (1973), Frenkel and Levich (1975), Frenkel and Levich (1977), Taylor (1987), Taylor (1989), Aiba, Takayasu, Marumo and Shimizu (2002), Aiba, Takayasu, Marumo and Shimizu (2003), Akram, Rime and Sarno (2008), Fong, Valente and Fung (2008) and Marshall, Treepongkaruna and Young (2008).

Secondly, we carry out an economic evaluation of simple arbitraging strategies and find that arbitrageurs incur losses in arbitraging in the presence of other competing arbitrageurs.[4] Their losses worsen with the increasing number of competitive arbitrageurs. These results highlight the importance of how uncertainty in acquiring all necessary assets for the arbitrage portfolio at a profitable price causes limit of arbitrage. This uncertainty can also be interpreted as price slippage in arbitraging because of competition, which is comparable to market-impact costs in equity markets.[5] Put this differently, the market impact of aggregated large orders from competing arbitrageurs simultaneously (which are normally driven by computer algorithms) for multiple assets required to complete an arbitrage trade can be risky.[6] The severity of execution risk aggravates with the number of competing arbitrageurs, the limited supply of assets required for the arbitrage portfolio and the price impact of trade of these assets.

Finally, we examine the relation between the size of arbitrage deviation and the market illiquidity and find statistical significance in the relation. In particular, the deviation is positively correlated to the slope of the demand and supply schedule of the limit order book, depth of the market and the bid-ask spread. These results supports the work of Roll, Schwartz and Subrahmanyam (2007), where it is argued that market liquidity plays a key role in moving prices to eliminate arbitrage opportunities.

Taken together, this paper sheds new light on the literature of limits of arbitrage. Using assets with perfect substitutability and convertibility in the absence of traditional impediments to arbitrage, we introduce and demonstrate both theoretically and empirically the importance of execution risk in arbitraging. We provide a liquidity-based theory for impediments to arbitrage which supports the existing empirical works relating arbitrage deviations to market illiquidity. To the best of our knowledge, this paper is the first to empirically study the relation between arbitrage deviation and market liquidity using liquidity measures derived from full limit order book information.

The remainder of the paper is organized as follows. In the next section, we present the model and discuss the equilibrium of the model. In Section 3, we briefly discuss triangular arbitrage in the FX market and review the related literature. In Section 4, we describe the data and empirically test the hypotheses derived from the model. Section 5 assesses the economic value of arbitrage activities in the presence of competitive arbitrageurs. Finally, Section 6 concludes.

# 2 Model Setup

## 2.1 Markets and Assets

We consider a setup, where there are $I$ assets indexed by $i \in \{1, 2, \ldots, I\}$, which are traded in $I$ segmented markets. We assume there exists a portfolio, $RP$, consisting of all assets from the set $\{2, \ldots, I\}$ which has an identical payoff structure and a dividend stream as asset 1. For simplicity, this portfolio is assumed to include long and short positions of one unit in each asset denoted by the vector $[w_2, \ldots, w_I]$. $w_i$ takes the value of 1 if it is a long position and $-1$ if it is a short position in asset $i$. We assume that there are no short selling constraints in our market.

**Assumption 1.** There is perfect convertibility between asset 1 and portfolio $RP$.

Convertibility here is defined as the ability to convert one unit of asset 1 to one unit of portfolio

[4] Liu and Longstaff (2004) also find that an arbitrage portfolio experiences losses before the convergence date.

[5] Slippage is the transactional risk that arises from the inability of a trader to accurately foretell the price at which an order to purchase or sell will be executed, especially for large or complex trades.

[6] Algorithmic trading is the practice of automatically transacting based on a quantitative model. For work on algorithmic trading, see: Hendershott and Moulton (2007) and Hendershott, Jones and Menkveld (2007).

$RP$. An example of such a setup is the FX market where a currency can be bought directly (asset 1) or indirectly (portfolio) vis-a-vis other currencies. However, this does not apply to DLCs as these assets are not convertible into each other. Although a DLC consists of two listed companies with different sets of shareholders sharing the ownership of one set of operational businesses, a shareholder holding a share of e.g. Royal Dutch NV cannot convert it into shares in Shell Transport and Trading PLC. Inconvertibility of assets with identical payoff structures and risk exposure will imply that any exploitation of mispricings will rely on convergence trading. With Assumption 1, traditional impediments to arbitrage like fundamental risk, noise trader risk and synchronization risk will be absent in our setup.

## 2.2 Traders

There are $I$ groups of local traders, who operate only within their own corresponding markets. For example, local trader group 1 operates only in market 1 and local trader group 2 operates only in market 2. We assume each group of traders can only trade assets in their own market. There are groups of liquidity traders who trade the asset for exogenous reasons to the model. Liquidity is offered by these traders in the limit order book (LOB) through quotes posting. Asymmetric demands and income shocks to these local traders may cause transient differences in the demand for assets in each market. This captures the idea that similar assets can be traded at different prices until arbitrageurs eliminate the mispricing.

In addition to the local traders, we also assume the existence of $k$ competitive risk-neutral arbitrageurs. These arbitrageurs can trade across all markets and exploit any existing mispricings. We assume all exploitations are conducted via simultaneous sales and purchases of identical assets with no requirement of any outlay of personal endowment. Arbitrageurs will use market orders to ensure the simultaneity of sales and purchases of mispriced assets. For simplicity, we assume:

**Assumption 2.** All arbitrageurs can only buy one unit of each asset needed to form an arbitrage portfolio.

Violation of this assumption will not change the implications of the model.

We denote the set of all arbitrageurs in the market by $\mathcal{K} = \{1, \ldots, k\}$ and the set of all opponents of arbitrager $j$ for $j \in \{1, \ldots, k\}$ by $\mathcal{K}_{-j} = \mathcal{K} \setminus \{j\}$ .

## 2.3 Limit Order Book

We assume that all participants in our setup have access to a publicly visible electronic screen, which specifies a price and quantity available at that price. Liquidity traders compete for prices as in Glosten (1994). There is no cost in posting, retracting or altering any limit orders at any time except in the middle of a trade execution. All participants are able to see details (all quoted prices and depths) of the demand and supply schedules of the LOB. All prices are assumed to be placed in a discrete grid.

We assume there are only two layers in our discrete demand and supply schedules. The first layer consists of the best bid and ask prices and the quantities available at this prices. The best bid and ask prices of asset $i$ are denoted by $p_i^b$ and $p_i^a$ respectively. The corresponding quantities available at the best bid and ask prices of asset $i$ are denoted by $n_i^b$ and $n_i^a$. The next best available bid price of the asset is $p_i^b - \Delta_i^b$ and the next best ask price is $p_i^a + \Delta_i^a$ at the second layer. $\Delta_i^b$ and $\Delta_i^a$ are the price differences between the best and second best price for demand and supply schedules respectively. As a simplifying assumption, prices of all assets at the second layer are assumed to be available with infinite supply. The modeled structure of the LOB can be visualized in Figure 1.

Insert Figure 1 here

### 2.4 Arbitrage Deviation

As defined earlier, the portfolio $RP$ consists of all assets from the set $\{2, \ldots I\}$. The best price at which one unit of portfolio $RP$ can then be bought is $P^a$; where

$$P^a = \sum_{i=2}^{I} w_i p_i (w_i),$$

, $p_i(w_i) = p_i^b$ if $w_i = -1$ and $p_i(w_i) = p_i^a$ if $w_i = 1$. The best price at which one unit of portfolio $RP$ can be sold is $P^b$. Since portfolio $RP$ and asset 1 have identical payoff structure, dividend streams and risk exposure, they should have the same price. Taking the transaction costs into account, a mispricing occurs if:

$$\begin{aligned} P^a &< p_1^b \\ P^b &> p_1^a. \end{aligned}$$

and it will be exploited by arbitrageurs.[7] We define the magnitude of the mispricing then as

$$A = \max\left\{0, P^b - p_1^a, p_1^b - P^a\right\}$$

where either $P^b - p_1^a > 0$ or $p_1^b - P^a > 0$.[8]

### 2.5 Arbitraging Strategies

Competition is the notion of individuals and firms striving for a greater share of a market to sell or buy goods and services. Professional arbitrageurs frequently compete against each other in exploiting any observable arbitrage opportunities in financial markets. With limited and scarce supply of required asset available to form an arbitrage portfolio, we assume that there exists an excess demand for these assets among competitive arbitrageurs such that:

**Assumption 3.** $\max\left\{n_i^a, n_i^b\right\} < k$ for each $i = 1, \ldots I$.

With arbitrageurs only permitted to purchase one unit of each required assets, we assume that there are always more arbitrageurs than the maximum number of available assets. The assumption is made for exposition purposes; execution risk exists as long as there are shortages of supply in at least one of the required assets. Market orders are preferred by arbitrageurs over limit orders because of the advantage of immediacy. With the enormous technological advances in trading tools over recent years, algorithmic trading is widely used in exploiting arbitrage opportunities. These algorithmic trades lead to almost simultaneous exploitation of arbitrage opportunities by large numbers of professional arbitrageurs in the financial market. Thus, arbitrageurs who want to trade upon observing any

[7] The introduction of transaction costs affects the no-arbitrage condition by creating a band within which arbitrage is not profitable. The conditions state that a mispricing exists only if what is bought can be sold at a more expensive price, where $P^b$ is the selling price and $P^a$ is the buying price.

[8] This statement is true under the assumption of positive bid-ask spreads: $p_1^a > p_1^b$ and $P^a > P^b$. Let us also assume that there exists a mispricing such that $P^b > p_1^a$. With these assumptions, $P^a > P^b > p_1^a > p_1^b$ implying that $p_1^b - P^a < 0$. On the other hand, if $p_1^b - P^a > 0$, we have $p_1^a > p_1^b > P^a > P^b$, i.e., $P^b - p_1^a < 0$.

mispricing are assumed to submit their market orders simultaneously. In this paper, we also assume that

**Assumption 4.** All arbitrageurs have the same probability of executing their market orders at the best available price when they submit market orders simultaneously.

For example, if there were three arbitrageurs vying for one available unit of asset at the best available price, the probability of an arbitrageur successfully acquiring this asset will be one-third. Arbitrageurs who are unsuccessful in acquiring the required asset at the best available price will execute their market orders at the next best available price. These prices are then $p_i^b - \Delta_i^b$ and $p_i^a + \Delta_i^a$ for sell and buy trades respectively for asset $i$. Thus, the penalty for missing a buy trade at the best price or the price slippage in one of required asset $i$ is $\Delta_i^a$. In this circumstance, the arbitrageur will be left with a payoff of $A - \Delta_i^a$.[9] The worst situation an arbitrageur could faced is one in which she fails to acquire all the required assets at the best available price. Her payoff at this instant will be $\Delta_i(w_i) = \Delta_i^b$ if $w_i = -1$ and $\Delta_i(w_i) = \Delta_i^a$ if $w_i = 1$. We assume that her payoff in the worst scenario is negative,

$$A - \sum_{i=1}^{I} \Delta_i(w_i) < 0.$$

All arbitrageurs have two possible strategies upon observing an arbitrage opportunity, " to trade" or "not to trade". An arbitrageur who chooses not to trade will have a payoff of zero. We also assume that all information, arbitrageurs' strategies, preferences and beliefs are common knowledge.

## 2.6 Equilibrium Arbitrage

Given the model described above, arbitrageurs will choose whether to participate in exploiting arbitrage opportunities of a particular deviation size, $A$. Arbitrageurs seek to maximize their expected payoffs and will only trade if there is a positive payoff. The equilibrium payoff of arbitrageurs is given by the following theorem

**Theorem 1.** *If the probability of getting the best price for asset $i$ for arbitrageur $j$ is $\mathbf{P}_i^j$, then her expected payoff $E(U^j)$ is given by*

$$E\left(U^j\right) = A - \sum_{i=1}^{I} \Delta_i(w_i)\left(1 - \mathbf{P}_i^j\right). \tag{1}$$

*Proof.* See appendix.

In equilibrium, Equation 1 shows that an arbitrageur's expected payoff is dependent on the number of competing arbitrageurs and the price slippage $\Delta(w)$. The $\left(1 - \mathbf{P}_i^j\right)$ term on the right hand side of Equation 1 captures the probability of trader $j$ not getting the best price in market $i$. In this case, the arbitrageur faces loss due to price slippage of $\Delta_i(w_i)$. Thus, the term $\Delta_i(w_i)\left(1 - \mathbf{P}_i^j\right)$ represents the expected loss for asset $i$. This loss arises from the execution risk of competing against other arbitrageurs for the observed mispricing between the two identical assets. Due to the independence among losses across $I$ different markets, the total expected loss $E\left(L^j\right)$ of arbitrageur $j$ can be written as follows:

[9] Let there be a mispricing, such that $A = p_1^b - P^a > 0$, and an arbitrageur failing to get the best price in market $i$. If $w_i = 1$, then the profit of the arbitrageur will be: $p_1^b - \sum_{\iota=2}^{i-1} w_\iota p_\iota(w_\iota) - p_i^a - \Delta_i^a - \sum_{\iota=i+1}^{I} w_\iota p_\iota(w_\iota) = p_1^b - P^a - \Delta_i^a = A - \Delta_i^a$.

$$E\left(L^j\right)=\sum_{i=1}^{I}\Delta_i\left(w_i\right)\left(1-\mathbf{P}_i^j\right).$$

Hence the expected payoff is the difference between the observed mispricing $A$ and the expected loss due to execution risk.

In the case of full participation by all the arbitrageurs in exploiting the arbitrage opportunity, the probability of trader $j$ executing a market order at the best price for asset $i$ is

$$\mathbf{P}_{i|n_i,k}^j=\frac{n_i}{k},$$

where $n_i$ denotes the quantity available at the best price and $k$ denotes the number of competing arbitrageurs. The subscripts on $P_{i|n_i,k}^j$ highlight the role of $n_i$ and $k$ in affecting the success of executing a best price market order. As the number of competing arbitrageurs increases, the probability of success converges to zero. As the breadth of asset $i$ increases, an arbitrageur is more likely to execute her best price market order.[10] This expression is obtained with assumptions of simultaneous market orders submissions by arbitrageurs upon observing a mispricing and equal probability of arbitrageurs in acquiring an asset at the best available price. In this case, Equation 1 can be rewritten as

$$E\left(U^j\right)=A-\sum_{i=1}^{I}\Delta_i\left(w_i\right)\left(1-\frac{n_i}{k}\right). \tag{2}$$

If $E\left(U^j\right)\geq 0$, it is Pareto optimal for the trader to use the strategy "trade" and to receive a positive payoff. As the number of arbitrageurs increases, the expected payoff $E\left(U^j\right)$ converges to $A-\sum_{i=1}^{I}\Delta_i\left(w_i\right)$. With the assumption made earlier that the expected payoff for failing to acquire all assets in the arbitrage portfolio at the best available price is negative, $A-\sum_{i=1}^{I}\Delta_i\left(w_i\right)<0$, arbitrageurs are expected to suffer losses with increasing competition. The severity of these losses or the cost of execution failure increases with $\Delta_i\left(w_i\right)$. As $\Delta_i\left(w_i\right)$ increases with market illiquidity, the cost of execution failure increases with market illiquidity. This demonstrates that competition for scarce supply of assets and market illiquidity exacerbate execution risk when exploiting arbitrage opportunities.

If the observed positive arbitrage deviation is smaller than the total expected loss due to execution risk, then it is not optimal for arbitrageurs to "trade" with probability one as $E\left(U^j\right)<0$. Under these circumstances, we assume that arbitrageurs adopt mixed strategies in their arbitrage strategies, where they participate in the market but with only a positive probability of exploiting any mispricing. We will denote the probability of participation of arbitrageur $j\in\{1,\ldots,k\}$ by $\pi_j\in[0,1]$. For a mixed strategy profile $\Pi=(\pi_1,\ldots,\pi_k)$, we use a standard notation $\Pi_{-j}=(\pi_1,\ldots,\pi_{j-1},\pi_{j+1},\ldots,\pi_k)$ to denote a strategy profile of all arbitrageurs other than $j$. We add the subscript, $\Pi_{-j}$, in the notation $\mathbf{P}_{i|n_i,k,\Pi_{-j}}^j$ to underline its dependence on strategies of trader $j$'s opponents.

Let $2^{\mathcal{K}_{-j}}$ denotes a family of all subsets of the set $\mathcal{K}_{-j}$ of all opponents of arbitrager $j$. $S$ is a subset of this family, such that $S\in 2^{\mathcal{K}_{-j}}$, where $|S|$ denotes the number of elements in $S$. The following theorem provides an expression for the probability of failing to execute a best price market order for those arbitrageurs in market $i$.

**Theorem 2.** *Let the opponents of trader $j$ play the strategy profile $\Pi_{-j}=(\pi_1,\ldots,\pi_{j-1},\pi_{j+1},\ldots,\pi_k)$. Then:*

[10] Breadth of an asset is defined as the quantity available at the best price.

(i) *the probability of failing to execute a best price market order in market $i$ is given by*

$$\bar{\mathbf{P}}^j_{i|n_i,k,\Pi_{-j}} = \sum_{S\in 2^{\mathcal{K}-j}} \prod_{s\in S} \pi_s \prod_{s\in \bar{S}} (1-\pi_s) \bar{\mathbf{P}}^j_{i|n_i,|S|}, \tag{3}$$

*where* $\bar{\mathbf{P}}^j_{i|n_i,|S|} = \begin{cases} 0 & \text{if} \quad |S| \leq n_i - 1 \\ 1 - \frac{n_i}{|S|+1} & \text{if} \quad |S| > n_i - 1. \end{cases}$

(ii) *the probability* $\bar{\mathbf{P}}^j_{i|n_i,k,\Pi_{-j}}$ *decreases monotonically with the number of existing arbitrageurs $k$.*

*Proof.* See appendix.

By Nash's theorem, there exists a mixed strategy profile $\Pi$ that forms a Nash equilibrium for the above game. According to Theorem 1, the expected payoff of arbitrageur $j$ in the case of mixed strategies is $\pi_j E\left(U^j|\Pi_{-j}\right)$, where

$$\begin{aligned} E\left(U^j|\Pi_{-j}\right) &= A - \sum_{i=1}^{I} \Delta_i\left(w_i\right)\left(1 - \mathbf{P}^j_{i|n_i,k,\Pi_{-j}}\right) \\ &= A - \sum_{i=1}^{I} \Delta_i\left(w_i\right) \bar{\mathbf{P}}^j_{i|n_i,k,\Pi_{-j}} \end{aligned}$$

is the expected payoff of arbitrageur $j$ playing pure strategy "trade" while her opponents use mixed strategies $\Pi_{-j}$. In equilibrium, the expected payoff of arbitrageur $j$ is dependent on the mixed strategies of other participating arbitrageurs, the number of existing arbitrageurs and the breadth of the market.

The following theorem characterizes the mixed strategy equilibria of the game.

**Theorem 3.** *If a mixed strategy profile $\Pi = (\pi_1, ..., \pi_k)$ with $\pi_j \in (0,1)$ is a Nash equilibrium of the game, then*

(i) $E(U^j|\Pi_{-j}) = 0$

(ii) $\pi_j = \pi_{j'} = \pi$ *for each $j, j'$ in $\mathcal{K}$.*

*Proof.* See appendix.

The above theorem states that risk neutral arbitrageurs demand an expected payoff of zero and have an identical probability of participation, $\pi$, in a mixed strategy equilibrium. Since strategies of all arbitrageurs are identical, we will drop all superscript $j$ to simplify the notation. As a consequence of Theorems 1 and 3, we obtain the following:

**Corollary 4.** *If $\pi \in (0,1)$ is an equilibrium probability of participation of the arbitrageurs, then:*

(i) *the observed arbitrage deviation is a linear function of the differences between the best and the next best prices on the corresponding markets*

$$A = \sum_{i=1}^{I} \Delta_i\left(w_i\right) \bar{\mathbf{P}}_{i|n_i,k,\pi}; \tag{4}$$

*(ii) the observed arbitrage deviation is a linear function of the slopes of the demand and supply schedules in the corresponding markets*

$$A = \sum_{i=1}^{I} \lambda_i (w_i) \, n_i (w_i) \, \bar{\mathbf{P}}_{i|n_i,k,\pi}, \tag{5}$$

*where* $\lambda_i (w_i) = \frac{\Delta_i^a}{n_i^a}$ *if* $w_i = 1$ *and* $\lambda_i (w_i) = \frac{\Delta_i^b}{n_i^b}$ *if* $w_i = 0$.

*Proof.* See appendix.

Equation 4 shows that the magnitude of the arbitrage deviation is associated with the execution risk for each of the $I$ number of asset in an arbitrage portfolio. The total execution risk compensation or the arbitrage deviation can be seen as the sum of individual compensation for execution risk for each individual asset. Each of these individual components will depend on the cost of execution failure, $\Delta (w_i)$, and the failure probability of executing the best price market orders, $\bar{\mathbf{P}}_{i|n_i,k,\pi}$. Thus, the arbitrage deviation is also a function of the breadth of the asset supply, the number of existing arbitrageurs and the number of participating arbitrageurs.

## 2.7 Main Implications

The equilibrium of the model illustrates three main observations. First, an arbitrageur faces execution risk in acquiring her arbitrage portfolio at the best price in the presence of competitive arbitrageurs. This risk stems from the uncertainty in acquiring the arbitrage portfolio at a profitable price because of competition for the scarce supply of profitable arbitrage portfolios. From Corollary 4, arbitrageurs demand a compensation for the execution risk and will participate in arbitrage activities with certainty only if the observed mispricing exceeds the equilibrium payoff. Arbitrage deviations below the equilibrium payoff will not be exploited by arbitrageurs adopting a pure strategy. If arbitrageurs were to adopt mixed strategies with some positive probability of participation when the deviation is below the equilibrium payoff, mispricings might not be exploited immediately. This is consistent with the existing literature on limits of arbitrage, which suggests that arbitrage opportunities exist because exploiting them can be risky. However, the nature of risk arbitrage in the current literature relies on the existence of convergence trading while the driver of our risk is arbitrage competition. The first result also sheds new light to the existing literature on existence of triangular arbitrage opportunities in the FX market (Aiba et al. (2002), Aiba et al. (2003) and Marshall et al. (2008). As triangular arbitrage violations are mispricings of assets with perfect substitutability and convertibility and are free from fundamental, noise trader and synchronization risk, we argue that they exist simply because of execution risk.

Secondly, execution risk in arbitraging worsen with increasing number of competitive arbitrageurs. This is because the failure probability of acquiring the arbitrage portfolio at a profitable price increases with the number of competing arbitrageurs. Thus, arbitrageurs incur more losses with increasing competition. This highlights the problem of infinite arbitrageurs' demands in a world of finite resources. The relevance of the number of competing arbitrageurs is analogous to the economic problem of scarcity, where not all the goals of society can be fulfilled at the same time with limited supply of goods. An increasing competition for limited number of exploitable arbitrage opportunities brings upon execution risk that prevents efficient elimination of asset mispricings.

Finally, the demanded compensation for execution risk in equilibrium increases with the price impact of trades and market illiquidity. Our model provides a theoretical framework for recent empirical evidence of the relation between the deviation from the law of one price and market illiquidity (e.g. Roll et al. (2007), Deville and Riva (2007), Akram et al. (2008), Fong et al. (2008) and Marshall et al. (2008)). In equilibrium, we have shown that the cost of failure is related to the slope of the demand and supply schedules. The steeper the slope, the more illiquid is the market and the higher is the cost of failure in acquiring an arbitrage portfolio at the best price.

Given these findings, we establish the following testable hypotheses:

1. **Arbitrage is not eliminated instantly in the market.**

2. **The existence of competitive arbitrageurs induces potential losses in arbitraging.**

3. **These losses increase with the number of competing arbitrageurs**

4. **The size of arbitrage deviation is proportional to market illiquidity.**

We will test these hypotheses by examining the triangular arbitrage parity in FX market. As triangular parity condition establishes a relation between two assets of perfect substitutability and convertibility, this controls for the existence of other impediments to arbitrages. In the next section, we will discuss about triangular arbitrage in the FX market and the relevant literature.

# 3 Triangular Arbitrage in the FX Market

In the foreign exchange market, price consistency of economically equivalent assets implies that exchange rates are in parity. They should be aligned so that no persistent risk-free-profits can be made by arbitraging among currencies. Triangular arbitrage involves one exchange rate traded at two different prices, a direct price and an indirect price (vis-a-vis other currencies). Arbitrage profits could potentially be made by buying the lower of the two and selling the higher of the two simultaneously. Triangular arbitrage conditions ensure price consistency by arbitraging among the three markets. Let us denote $S\left(A/B\right)$ is the number of units of currency $A$ per unit of currency $B$ in the spot foreign exchange market. Arbitrageurs are often assumed to eliminate any price discrepancy if the inferred cross-rate between currency A and B is known through the two currencies' quotes vis-a-vis the third currency $C$. The triangular no-arbitrage conditions are then expressed as

$$S\left(A/B\right) = S\left(A/C\right).S\left(C/B\right),$$

$$S\left(B/A\right) = S\left(B/C\right).S\left(C/A\right),$$

in the absence of transaction costs. It is important to consider transaction costs while investigating the presence of exploitable arbitrage. Transaction costs in this case can be seen as a per-unit charge which is captured in the bid-ask spread. The bid-ask spread covers the adverse selection, inventory and the order processing costs that a liquidity provider charges. $S\left(A/B^{ask}\right)$ is defined as the price that must be paid to buy one unit of currency $B$ with currency $A$ and $S\left(A/B^{bid}\right)$ is the number of units of currency $A$ received for the sale of one unit of currency $B$ , where $S\left(A/B^{ask}\right) > S\left(A/B^{bid}\right)$. Taking the transaction costs into account, the triangular no-arbitrage conditions are

$$S\left(A/B^{ask}\right) \geq S\left(C/B^{bid}\right).S\left(A/C^{bid}\right)$$

$$S\left(A/B^{ask}\right) - S\left(C/B^{bid}\right).S\left(A/C^{bid}\right) \geq 0, \tag{6}$$

$$S\left(B/A^{ask}\right) \geq S\left(C/A^{bid}\right).S\left(B/C^{bid}\right)$$

$$S\left(B/A^{ask}\right) - S\left(C/A^{bid}\right).S\left(B/C^{bid}\right) \geq 0. \qquad (7)$$

Any deviation from equation (6) or (7) would represent a textbook riskless arbitrage opportunity.[11]

Aiba et al. (2002) find the presence of exploitable arbitrage opportunities that last about 90 minutes a day in the FX market using transaction data between the yen-dollar, dollar-euro and yen-euro from the period January 25, 1999 to March 12, 1999. However, this study uses a relatively short sample of transaction data from the early phase of the electronic FX market (before 2000) and major developments in the electronic market have since taken place. Marshall et al. (2008) finds the existence of exploitable arbitrage opportunities using 1 year binding quote data from EBS and argues that these opportunities are monies left on the table to compensate arbitrageurs for their service in relieving market maker's order imbalance. Marshall et al. (2008) also establishes a relation between arbitrage deviations and the bid-ask spread. However, the theoretical relation between arbitrage deviations and market illiquidity remains unclear in their paper. Moreover, data limitation has restricted their study of cost of immediacy and arbitrage profits in using bid-ask spread as their only measure of liquidity.

We extend the triangular arbitrage literature in the FX market with an alternative hypothesis for the existence of triangular arbitrage opportunities. We supplement and strengthen the existing literature with a liquidity-based theoretical model for execution risk. We extend the current empirical analysis with a more detailed data set that allows us to investigate the relation between arbitrage and market illiquidity using the limit order book.

## 4 Data Sources and Preliminary Analysis

While most previous research used data during the early rise of the electronic platform before the 2000, this paper uses tick by tick data from Reuters trading system Dealing 3000 for three currency pairs. US dollar-euro (dollars per euro), US dollar-pound sterling (dollars per pound) and pound sterling-euro (pounds per euro) (hereafter USD/EUR, USD/GBP and GBP/EUR respectively). The sample period runs from January 2, 2003 to December 30, 2004. The Bank for International Settlement (BIS, 2004) estimates that trades in these currencies constitute up to 60 percent of the FX spot transactions, 53 percent of which are interdealer trades which indicates that our data represent a substantial part of the FX market.[12]

The data analyzed consists of continuously recorded transactions and quotations between 07:00-17:00 GMT. All weekends and holidays are excluded. The advantage of this dataset is the availability of volume in all quotes as well as all trades and hidden orders, which allows one to reconstruct the full limit orderbook, without making any ad-hoc assumptions. For each quote, the dataset reports the currency pair, unique order identifier, quoted price, order quantity, hidden quantity (D3000 function), quantity traded, order type, transaction identifier of order entered and removed, status of market order, entry type of orders, removal reason, time of orders entered and removed. The time stamp of the data has an accuracy of one-thousandth of a second. This extremely detailed dataset facilitates the easy reconstruction of the limit order book. To reconstruct the limit order

[11] The conditions state that what is bought cannot be sold at a higher price immediately.

[12] See Osler (2008), Lyons (2001) and Rime (2003) for a more detailed survey about the institutional features of the foreign exchange market.

book, we start at the beginning of the trading day tracking all types of orders submitted throughout the day and updating the order book accordingly. Thus all entries, removals, amendments and trade executions are accounted for when the book is updated.

### 4.1 Summary Statistics

In this section, we report the preliminary statistics of arbitrage deviation and clusters (sequences) of profitable triangular arbitrage deviations. A cluster is defined as consisting of at least one consecutive profitable triangular arbitrage deviation. There are a total of 139,548 arbitrage opportunities and 2,583 blocks of arbitrage clusters. A round-trip arbitrage opportunity is identified by the following way:

1. Record the latest quoted best bid and best ask prices for the three currency pairs in our portfolio.

2. Identify if an arbitrage opportunity exists. Check this by selling one unit of currency 1 (e.g. USD/GBP) and buying currency 2 (e.g. EUR/GBP). This is equivalent to selling USD for GBP and using the GBP from sales to purchase EUR. Thus our net position will be short USD/ long EUR, which we will compare against the quoted rate for currency 3 (e.g. USD/EUR). We will sell currency 3 to obtain an arbitrage profit if the quoted rate is lower than our current position. If the rate is higher than our current position, we will rerun this exercise by buying currency 1 (e.g. USD/GBP) and selling currency 2 (e.g EUR/GBP). We then purchase currency 3 and check if we have a positive profit. All purchases and sales are carried out at the relevant ask and bid prices respectively.

Summary statistics for transaction and firm quotes data are reported in Table 1. The table reports information on the average inter-quote duration (in seconds), average bid-ask spread (in pips), the average of slope (in basis points per billion of the base currency), depth and breadth (in million of base currency) of the demand and supply schedule across the sample. On average a quote arrives every 1.05, 1.71 and 1.31 seconds for USD/EUR, USD/GBP and GBP/EUR respectively. This is much lower than the quote arrival rate of 15-20 seconds reported by Engle and Russell (1998) and Bollerslev and Domowitz (1993). The increase in trading activities in the FX market is attributed to the recent propagation of electronic trading platforms, which enables large financial institutions to set up more comprehensive trading facilities for the increasing numbers of retail investors. The average bid-ask spreads during the arbitrage cluster are found 2.126, 2.065 and 1.026 pips for USD/EUR, USD/GBP and GBP/EUR, respectively, indicating that at first glance, D3000-2 is a very tight market as highlighted in Tham (2008).

The average slopes of the demand schedules are 31.37, 85.73 and 68.37 basis point per billion of currency trade for GBP/EUR, USD/EUR and USD/GBP, respectively. The average slopes of the supply schedules are 36.41, 99.07 and 74.60 basis point per billion of currency trade for GBP/EUR, USD/EUR and USD/GBP, respectively. The average depth across the market for the three currency pairs ranges from 29 millions to 50 millions indicating that the FX market is a very liquid market. The breadth is about 3 million which is just enough to satisfy one average size of the market order. Hence, the summary statistics indicates that the currency pairs of interest are traded on a highly liquid market with high price sensitivity.

Insert Table 1 here

Table 1 presents the preliminary statistics on the deviations from triangular arbitrage parity. The panel reports information on the arbitrage deviations (in pips), average arbitrage duration (in seconds) and the numbers of arbitrage opportunities in an arbitrage cluster.[13] Panel A of Table 2 shows that the mean of the average arbitrage profit within a block is about 1.53 pips with a standard deviation of 1.78. The positive average deviation value implies that, *on average*, triangular arbitrage is profit making after accounting for transaction costs (i.e. net bid and ask spread and 0.2 pip trade fees).[14] Furthermore, the associated $t$-statistics in Panel B suggest that the deviations are statistically significant.

Insert Table 2 here

There are occasions where the deviation is as high as 53 pips. Table 3 presents the number of arbitrage deviation across our sample. The majority of the deviations are of 1 pip but there is a significant number of deviations between 3 pips to 19 pips.

The average duration of a block of arbitrage opportunities is 1.37 seconds indicating that profitable deviations are eliminated from the market rather quickly. The standard deviation of the duration is about 6.56 seconds. The sizable difference between the mean and standard deviation of the duration indicates that the durations are not exponentially distributed. This suggests that there are market conditions (i.e. low market liquidity) where the duration of the arbitrage clusters is persistently high. The average number of quotes and trades within a cluster is 4.00. However, there are occasions where it takes up to 45 correction of the quotes, cancelations and orders for the deviation to disappear.

Insert Table 3 here

Overall, the preliminary evidence reveals the existence of potential profitable arbitrage opportunities which are small in number relative to the total number of quotes and observations in our data, but they are sizeable and relatively long-lived.

# 5 Empirical Results

We first test the validity of a common textbook arbitrage assumption that arbitrage opportunities are eliminated instantly from the market. Next, we carry out an economic evaluation of arbitrage strategies with competitive arbitrageurs to study the potential profit and loss for arbitrageurs. Finally, we test for the relation between market illiquidity and triangular arbitrage deviations.

## 5.1 Instant Elimination of Arbitrage Opportunities

The preliminary analysis has identified the existence of apparent triangular arbitrage opportunities, which confirms findings by Aiba et al. (2002), Aiba et al. (2003) and Marshall et al. (2008). However,

[13]A pip, which stands for "price interest point", represents the smallest fluctuation in the price of a currency. Depending on the context, normally one basis point 0.0001 in the case of EUR/USD and GBD/USD. GBP/EUR is displayed in a slightly different way from most other currency pairs in that although one pip is worth 0.0001, the rate is often displayed to five decimal places. The fifth decimal place can only be 0 or 5 and is used to display half pips.

[14]There are costs involved in obtaining a Reuters trading system, but given that market participants are bank dealers who participate in the foreign exchange market for purposes other than arbitrage these costs are sunk costs to a bank who wishes to also pursue arbitrage.

the finance literature often assumes that these opportunities will be eliminated instantly by arbitrageurs in the market. We first revisit the hypothesis that arbitrage opportunities are eliminated instantly given the implications of our model.

**Hypothesis 1: Triangular arbitrage is not eliminated instantly in the FX market.**

To investigate this hypothesis, we examine the observed arbitrage opportunities and group them into clusters. A cluster consists of at least one profitable triangular arbitrage deviations. The duration of a cluster will simply be the elapsed time required for exchange rates to revert to no arbitrage values, after a deviation has been identified. We test Hypothesis 1 by investigating if the duration of the deviations is statistically difference from zero. The associated $t$-statistics in Panel B of Table 2 suggests that the durations of the arbitrage clusters are statistically different from zero. Although the statistical result rejects the null hypothesis of immediate elimination of arbitrage opportunities, the null hypothesis of a zero duration arbitrage cluster is in fact unrealistic. Arbitrage opportunities will probably be eliminated in an efficient market by the next incoming trade or quote, which very often takes more than a fraction of a second.

To account for this, we test the null hypothesis by splitting the arbitrage clusters into two groups. The first group consists of arbitrage clusters that are consistent with a textbook arbitrage example in that arbitrage opportunities in this group are eliminated by any next incoming order (market orders, limit orders and cancelation). Clusters in this group have only one profitable triangular arbitrage deviation. In this group, market participants observe an arbitrage opportunity and take instant action to exploit and remove the arbitrage opportunity through market orders, limit orders and cancelation of limit orders. We call this the textbook arbitrage. The remaining clusters fall into the second group where market participants deliberate on their participation in the market to exploit the observed arbitrage opportunity. This caution stems from the risk involved in arbitraging and the existence of market frictions. Rational arbitrageurs will not exploit any risky arbitrage opportunities especially when the arbitrage deviation is insufficient to compensate them for the risky arbitrage. We call this naturally the risky arbitrage.

Insert Table 4 here

Table 4 reports the mean, median, $t$-statistics of the durations for the textbook and risky arbitrage. The median is very close to the mean for the risky arbitrage indicating a fairly symmetric distribution. A typical text book arbitrage has an average duration of about a second while the risky arbitrage takes an average of 2.7 seconds to be eliminated from the market. The duration of risky arbitrage also has a larger variance of 10.15 seconds.

The results from testing the statistical difference between the duration of textbook and risky arbitrage in Table 4 reject the hypothesis that triangular arbitrage is eliminated instantly in the FX market. Arbitrageurs seem to deliberate on their participation of the elimination of arbitrage opportunities in the presence of risk. Arbitrage opportunities are therefore not exploited immediately in the financial market as postulated in most textbooks. This conclusion is consistent with work of De Long et al. (1990), Shleifer and Vishny (1992), Abreu and Brunnermeier (2002) and Kondor (2008) where they argue that exploiting arbitrage opportunities is risky. However, triangular arbitrage is not subjected to traditional impediments to arbitrage as triangular arbitrage does not involves convergence trading. So why do we then observe violations of the triangular arbitrage parity condition?

## 5.2 Arbitraging - Profits or Losses?

We argue that triangular arbitrage is risky because of execution risk. To illustrate execution risk, consider the following example where the rates on euro, pound sterling vis-a-vis the US dollar are quoted as:

| | Bid | Ask |
|---|---|---|
| USD/EUR | 1.5525 | 1.5526 |
| USD/GBP | 1.9859 | 1.9860 |
| GBP/EUR | 0.78165 | 0.78170 |

With €1,000,000 we can buy US$1,552,500, which we can use to buy £781,722.05. We can now sell the pounds for €1,000,028 making a profit of €28. However, this profit is conditional on being able to complete the arbitrage. Consider an arbitrageur successfully purchasing the US dollars and pound sterling at the posted price but failing to purchase the euro at 0.78170. In the presence of competitive arbitrageurs, we might be exposed to execution risk. The demand of these competitive arbitrageurs might drive the £/€$^{bid}$ to 0.78175 eliminating the arbitrage opportunity. We will then be left with an unwanted inventory of £781,722.05 or suffer a loss of €36 if we close our position. We argue that arbitrageurs are exposed to the risk of not completing their arbitrage portfolio at the desired price because of competing arbitrageurs.

**Hypothesis 2: The existence of competitive arbitrageurs induces potential losses in arbitraging.**

We investigate this hypothesis using a Monte Carlo backtesting exercise based on the theoretical model. The exercise is set up with $k$ competitive arbitrageurs competing for limited supplies of three currency pairs ($I = 3$) required to construct an profitable arbitrage portfolio. These competitive arbitrageurs trade on three currency pairs in the spot FX market and are assumed to be able to see the whole limit order book. Thus, arbitrageurs have full information about the price and quantity available. The trading strategy of the these arbitrageurs is to maximize their profits from the deviation of the three currency pairs from the triangular parity.[15] When an arbitrage opportunity arises, all arbitrageurs observe it and compete to obtain the arbitrage profit. In order to do this, they will need to complete a full round of buying and selling of the three currencies in the three different markets. The individual demand $d$ is assumed to be equal to one unit, hence the total demand, $D = d \times k$. They place all three orders simultaneously using limit orders at the best prices. Whether their demand for a particular currency is fulfilled at the best available price, will depend on the demand of the competitors and the supply at the best price. For both arbitrageurs to walk away with a profit, the minimum quantity available at the best price for each currency in the arbitrage portfolio has to be at least $k$. If there are more arbitrageurs than the quantity available for one of the currency and the probability of participation is one, each arbitrageur has a probability of $\mathbf{P} = \frac{n_1^a}{k}$ to get the currency at the best price, where $n_1^a$ is the quantity available at the best ask price. In our Monte Carlo exercise, we first generate the number of participating arbitrageurs of certain participating probability. We use a Bernoulli distribution to determine if an arbitrageur is participating and tabulate the total number of participating arbitrageurs, $|S| + 1$. We then determine whether an arbitrageur gets her currency $i$ at the best price using the success probability of $\mathbf{P} = \frac{n_i^a}{|S|+1}$. Thus, some arbitrageurs will be unsuccessful in acquiring all the required currencies to form a profitable triangular arbitrage portfolio. These arbitrageurs are then assumed to complete the remaining legs of the arbitrage transactions at the next best price or sell their excess

[15] Please see section 3 for explanation of deviation from triangular parity condition.

inventory at the best available price, whichever has the least loss.[16] This problem worsen with increasing number of arbitrageurs and a higher individual demand (that is larger than one unit). Our starting and base currency is GBP. There will be some residual position exposures in the exercise because we assume that trades can only be carried out in multiples of one million units of the base currency. We trade out these residual positions at the market prevailing prices and convert them back to GBP at the end of the day.

The sample size of our data is 2 years and we will repeat the Monte Carlo exercise across the 2-year sample 1000 times. The exercise is repeated with two, eight and sixteen competing arbitrageurs. Arbitrageurs are even allowed to have taken execution or slippage risk into account and establish some thresholds in the observed arbitrage profit as part of the trading rules regarding when to enter the market. We investigate this by repeating the game and imposing different thresholds in the arbitrage trading rules of the competing arbitrageurs. Thus, arbitrageurs will only attempt to eliminate arbitrage opportunities when they observe deviations from the parity condition of a certain size. We have also allowed the arbitrageurs to employ mixed strategies where they participate with some probability. Triangular arbitrage opportunities with transaction costs are identified using binding quotes for three bid and ask cross-rates for three currencies. GBP/USD, EUR/USD and EUR/GBP are the currency pairs used. Bid and ask prices for the three currency pairs are obtained from the reconstructed limit order book. An arbitrage opportunity exists if the purchase of EUR/USD and the sale of GBP/USD (which is short EUR / long GBP) is lower than the sale of EUR/GBP. An arbitrage opportunity also exists if the sale of EUR/USD and the purchase of GBP/USD the purchase is lower than the purchase of EUR/GBP. If an arbitrage opportunity exists, it can only take either one the above ways unless the bid-ask spread is negative. All purchases and sales are carried out using the ask and bid price, respectively. As in Akram et al. (2008), arbitrage opportunities with inter-quote duration for more than two minutes are not considered to minimize the possibility of stale quotes. Moreover, arbitrage opportunities, that are not immediately eliminated from the market or with a duration of more than a second, are only exploited once at the very first moment the arbitrage conditions are violated. Note that this profit is achieved net of any bid-ask spread costs. Our setup tries to provide the best possible scenario to allow our competitive arbitrageurs to profit from the arbitrage.

Tables 5, 6 and 7 present the mean and standard deviation of profits and losses of arbitrageurs with two, eight and sixteen competing arbitrageurs respectively. Table 5 shows that arbitrageurs have positive profits when there are only two participating players in the market. The most significant profit is when an arbitrageur adopts a strategy of one pip threshold and participation probability of one. The arbitrageur would have a handsome average profit of about two million GBP across our sample period. In general, arbitrageurs continue to register a positive profit even when they impose different thresholds in their strategies. However, their profits decrease as their thresholds increase. As shown in Table 3, the number of arbitrage opportunities decreases with increasing threshold. Thus it is not surprising that we find arbitrageurs profiting less as they get more conservative. This is again highlighted by findings in Table 5 where the profit decreases with decreasing probability of participation.

[16] Excess inventory exists when an arbitrageurs fails to buy all three currencies at the best prices. For example, if an arbitrageur only manages to buy or sell currency 1 and 2 at the best available prices but miss out on currency 3 because of excess demand. She is now left with an open position consisting of currency 1 and 2. She can either complete the third leg (currency 3) at the next available price or resell and rebuy currency 1 and 2 (losing out on transaction cost) to close her position. We assume she closes her position using the strategy with the best payoff (the payoff can still be positive if the next best price of currency 3 still yield a positive profit).

Table 6 highlights the importance of execution risk as the number of arbitrageurs increases to eight. Arbitrageurs record negative profits for almost all thresholds (except a threshold of twenty-three pips), if they participate with probability of one. If an arbitrageur were to adopt a strategy with a threshold of one pip and participation probability of one, she will incur an average loss of about twelve million GBP across the two year sample period. This is a sharp contrast with respect to the two million GBP profit she would have made with only two competing arbitrageurs. The results clearly demonstrate that arbitrageurs can incur losses in the presence of other competing arbitrageurs as suggested by our model. With an average breadth of about three million across each currency pair (see Table 1) and a setting of eight arbitrageurs, each with a demand of one million unit, it is clear that there is excess demand for the arbitrage portfolio. The loss of arbitraging increases as it becomes more difficult to complete the arbitrage portfolio at the desired price. However, they still have positive profits if they adopt a mixed strategy. In fact, they have positive profits across all thresholds if they have a probability of participation of less than twenty percent. The results also illustrate why a mixed strategy ($\pi < 1$) might be preferred over a pure strategy ($\pi = 1$) in some circumstances.

**Hypothesis 3: Losses increases with increasing number of competing arbitrageurs.**

Table 7 demonstrates how execution risk increases as the number of competing arbitrageurs increases. This can be seen by the increase in magnitude of losses in a strategy with a probability of participation of one across all thresholds. The losses increase to an incredible thirty five million GBP when there are sixteen arbitrageurs. Figure 2 presents plots of arbitrageurs' profits and losses with respect to the number of arbitrageurs when the strategy has a probability of participation of one. All plots show monotonically increasing losses with increasing number of arbitrageurs. With the increasing use of algorithmic trading in arbitrage in recent years and hundreds of competing arbitrageurs in the real world, our results show that arbitrage can be a very risky business because of execution risk.

We further test this hypothesis by estimating the following linear regression:

$$PL = x_0 + x_1 \times k. \tag{8}$$

where $PL$ is the average profit and loss of $k$ number of arbitrageurs in our backtesting. The results are shown in Table 8. The estimates and the $t$-statistics show that all the estimated parameters are highly significant. The results report a strong negative relation between the average profit and losses and the number of competing arbitrageurs. The negative relation remains for different probabilities of participation and thresholds.

In summary, the Monte Carlo backtesting exercise using quotes and depth from the LOB demonstrates the importance of execution risk. Arbitrageurs are found to incur losses in the presence of competition. These losses increase with the number of competing arbitrageurs. Thus, our results are in favor of the stated hypotheses and consistent with the model predictions.

### 5.3 Arbitrage Deviation and Market Illiquidity

The equilibrium results postulate a positive relation between execution risk and market illiquidity. Execution risk is more severe in illiquid markets as the impact of trade on prices increases. A recent and growing body of literature points out that market liquidity can affect financial asset prices (see, inter alia, Stoll (1978), O'Hara and Oldfield (1986), Kumar and Seppi (1994), Chordia, Roll and Subrahmanyam (2002) and references therein). More specifically, Roll et al. (2007) shows that market illiquidity affects deviations from the law of one price in the US stock market. Our theoretical conclusion supports this literature and develops a relation between arbitrage deviation and market liquidity. Specifically, we ask the question of whether arbitrage deviation is higher when the market is more illiquid given a fixed number of competitive arbitrageurs in the market.

**Hypothesis 4: Arbitrage deviations are proportional to market illiquidity.**

We test this hypothesis by estimating the linear regression relation in equation (4). In the case of triangular arbitrage ($I = 3$) we obtain

$$A = a_0 + a_1 \times \Delta_{GBP/USD}(w_{GBP/USD}) + a_2 \times \Delta_{EUR/USD}(w_{EUR/USD}) + a_3 \times \Delta_{EUR/GBP}(w_{EUR/GBP}), \quad (9)$$

where $\Delta_i(w_i)$ is the price difference between the first and second best bid price and $i = GBP/USD$, $EUR/USD$, $EUR/GBP$. The corresponding estimates of coefficients can be interpreted as the implied probabilities of not getting the best prices at the $i$th market. Thus, the risk premium demanded for execution risk is the expected loss of failing to acquire each asset at the desired price that generates a positive return. $a_0$ can either be interpreted as the compensation for monitoring the market in the spirit of Grossman and Stiglitz (1976) or Grossman and Stiglitz (1980) the risk premium paid to a risk adverse arbitrageur for the uncertainty of the execution risk. We estimate the parameters using generalized method of moments (GMM, Hansen (1982)) with a Newey-West correction for autocorrelation and heteroscedasticity.

The results are shown in Panel A of Table 9. The estimates and $t$-statistics show that all the estimated parameters are significantly different from zero, with $p$-values less than 0.0001. From the estimated parameters, the implied probability of not being able to purchase GBP and EUR with USD at the desired price is 30.08% and 16.13%, respectively, conditional on the number of arbitrageurs and breadth of the limit order book. The implied probability of not being able to purchase EUR with GBP at the desired price is 16.01%. The results establish a positive statistical relations between the price impact of trade and arbitrage deviations.

We further test the relation between market illiquidity and arbitrage deviation by regressing the observed deviation against the slopes of demand and supply schedules of the three currency pairs. We test this hypothesis based on Equation 5 using the following regression:

$$A = b_0 + b_1 \times \lambda_{GBP/USD}(w_{GBP/USD}) + b_2 \times \lambda_{EUR/USD}(w_{EUR/USD}) + b_3 \times \lambda_{EUR/GBP}(w_{EUR/GBP}), \quad (10)$$

where $\lambda_i(w_i)$ is the slope of the corresponding demand or supply schedules and $i = GBP/USD$, $EUR/USD$, $EUR/GBP$. Results from the regression are reported in Panel B of Table 9. The estimates and $t$-statistics show that all the estimated parameters are significantly different from zero, with $p$-values less than 0.0001. The results indicate that the evidence is strongly in support of our hypothesis. There is a positive relation between illiquidity and arbitrage profits. The conclusion is congruent with Roll et al. (2007) in that the more liquid the markets, the smaller the deviations from the law of one price. However, we argue that the economic reason behind this relation, in the case of triangular arbitrage, is the presence of execution risk. As the price impact of trade increases with market illiquidity, the cost of execution failure increases as well.

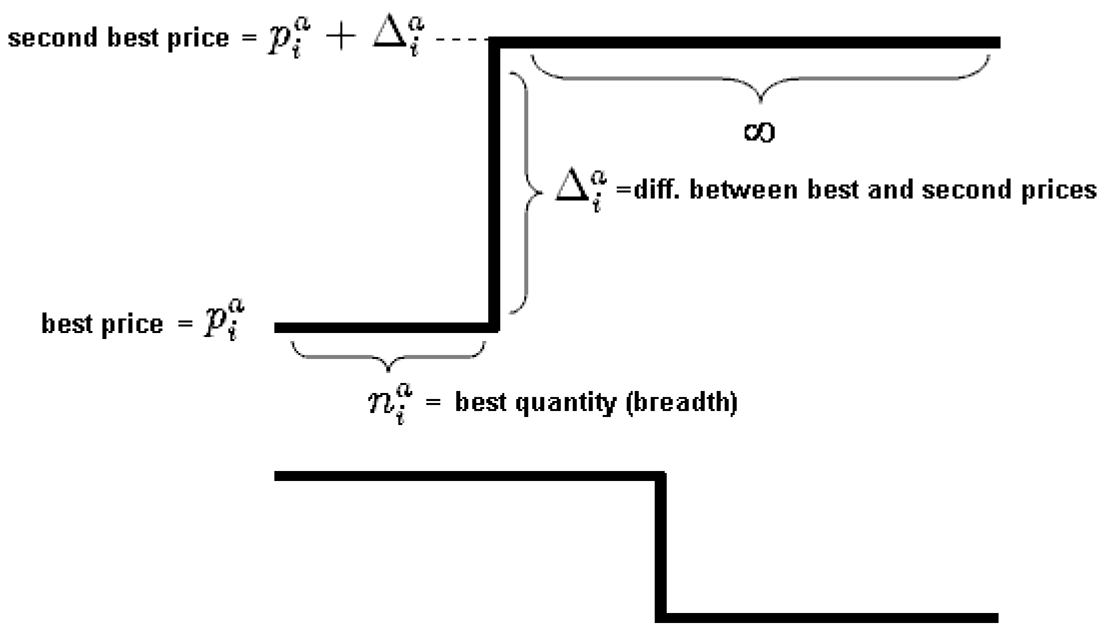
# 6 Conclusion

The paper presents a new impediment to arbitrage, which we call execution risk. The distinctive feature of execution risk compared to previous limits of arbitrage is that it is not related to the existence of behavioral traders nor the convergence uncertainty of mispriced securities. We provide a theoretical model where execution risk stems from the arbitrageur's uncertainty in completing the arbitrage portfolio in the presence of other competing arbitrageurs. As a result, there can be significant and long departures from efficient prices, as arbitrageurs condition their probability of engaging in arbitrage activities on the market illiquidity and actions of other arbitrageurs. Hypotheses from our theoretical model are supported by our empirical evidence that triangular arbitrage opportunities are not eliminated from the market instantly, as arbitrageurs take into account the potential of execution risk in the FX market. Economic evaluation of various triangular arbitrage strategies show that arbitrageurs suffer from losses in the presence of other competitive arbitrageurs. These losses increase with the increasing number of competing arbitrageurs. Our empirical results also establish a positive relation between the size of the arbitrage deviation and market illiquidity.

# Tables and Figures

### Figure 1: Structure of Limit Order Book

The figure represents a structure of the limit order book in the market $i$. There are only two layers in the discrete demand and supply schedules. The first layer consists of the best bid and ask prices and the quantity available at this prices. $p_i^b$ and $p_i^a$ are the best bid and ask prices of asset $i$, respectively. $n_i^b$ and $n_i^a$ denote the corresponding quantity available at the best bid and ask prices. The next best available bid price of the asset is $p_i^b - \Delta_i^b$ and the next best ask price is $p_i^a + \Delta_i^a$ at the second layer. Prices of all assets at the second layer are assumed to be available with infinite supply.



## Figure 2: Expected profit vs. number of arbitrageurs

These figures present the relation between the mean of arbitrageurs' profits and losses and the number of arbitrageurs in the market. Profits are in millions GBP. The arbitrage strategy is based on the triangular arbitrage condition. Arbitrageurs can participate in exploiting arbitrage opportunities when they observed any deviation. Each panel represents the arbitrage strategy corresponding to a particular threshold, which is the self-imposed minimum absolute deviation that arbitrageurs will participate in arbitraging. Arbitrageurs are assumed to use only pure strategies and will participate when the observed deviation exceeds the threshold. They will not participate if the deviation is below the threshold. All quotes, depth and breadth of the market is based in the reconstructed limit order book. The sample period is from January 2, 2003 to December 30, 2004.

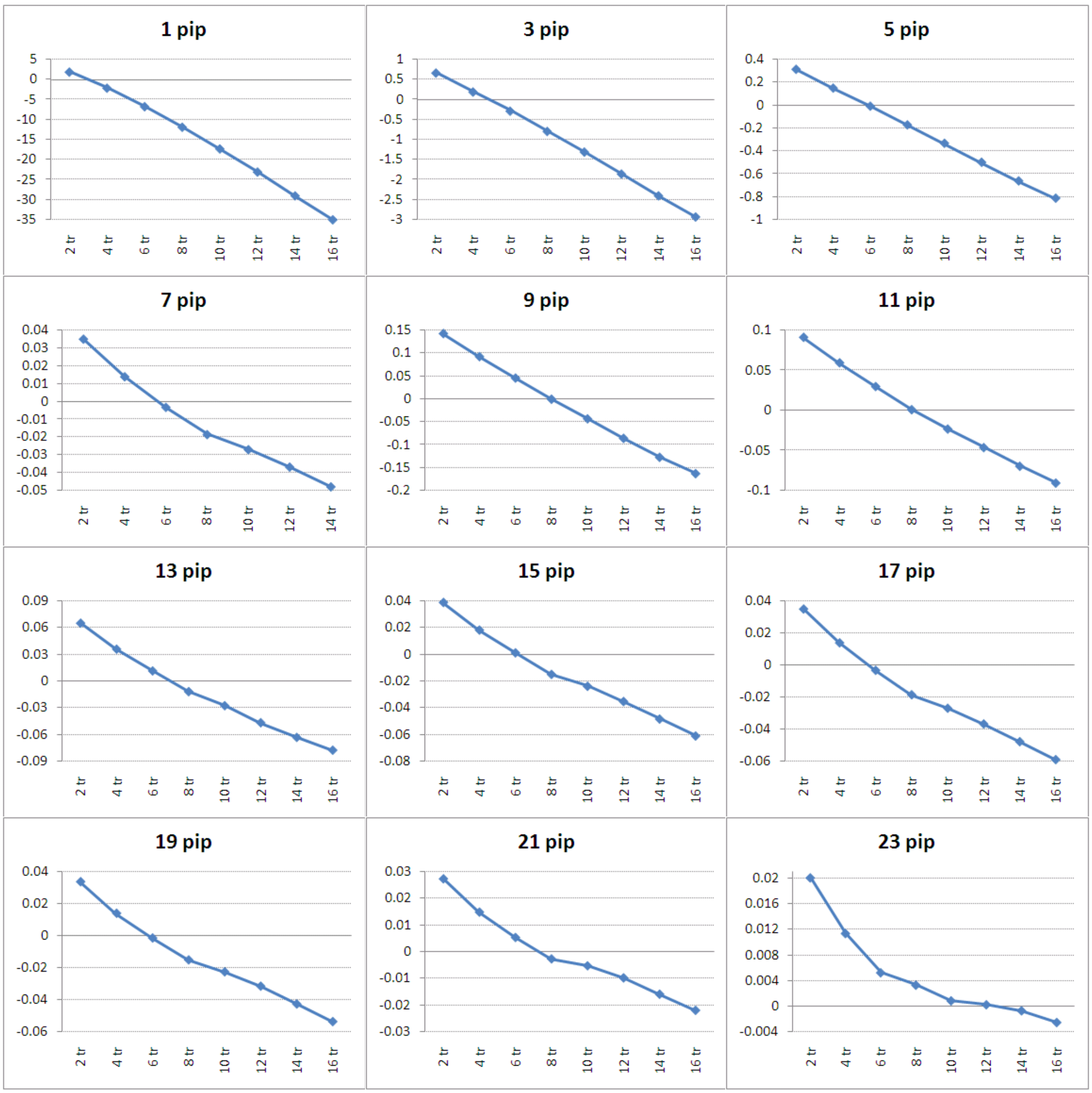


**Table 1: Preliminary data analysis of liquidity**

This table provides descriptive statistics on average inter-quote duration (in seconds), average quoted spread (in pips), average slope of the demand and supply schedule in basis points per billion of the base currency and the average depth (in million of base currency) for the USD/EUR, USD/GBP and GBP/EUR exchange rates. The sample period is from January 2, 2003 to December 30, 2004.

| Exchange rate | GBP/EUR | USD/EUR | USD/GBP |
|---|---|---|---|
| Average inter-quote duration | 1.31 | 1.05 | 1.71 |
| Average bid-ask spread | 1.03 | 2.13 | 2.07 |
| Average slope of demand schedules | 31.37 | 85.73 | 68.37 |
| Average slope of supply schedules | 36.41 | 99.07 | 74.60 |
| Average depth of demand schedules | 41.29 | 29.46 | 45.08 |
| Average depth of supply schedules | 49.68 | 32.80 | 48.78 |
| Average breadth of demand schedules | 3.33 | 2.79 | 2.72 |
| Average breadth of supply schedules | 3.25 | 2.88 | 2.78 |

**Table 2: Preliminary data analysis**

This table provides the summary statistics on deviations of triangular arbitrage parity. Triangular arbitrage opportunities are identified by comparing the three most recent quotes for each set of three currencies. An arbitrage opportunity exists if there is a mismatch among these three currencies. Panel A reports the mean, standard deviation and the maximum arbitrage deviation (in pips), average trade duration (in seconds) and the numbers of arbitrage opportunities in an arbitrage cluster. Panel B contains results for the statistics tests used to determine if the mean is statistically different from zero. The null hypothesis of no difference from zero is tested using the $t$-test. The sample period is from January 2, 2003 to December 30, 2004.

| Panel A | | | |
|---|---|---|---|
| | Mean | Standard deviation | Maximum |
| Average arbitrage profit (pips) | 1.53 | 1.78 | 53.00 |
| Arbitrage duration (seconds) | 1.37 | 6.56 | 307.60 |
| Number of arbitrage in a cluster | 4.33 | 4.10 | 45.00 |
| Panel B | | | |
| $t$-stats (average profit =zero) | | 141.82 | |
| $t$-stats (arbitrage duration =zero) | | 539.44 | |
| Number of profitable clusters | | 5055 | |

## Table 3: Number of arbitrage

This table provides the number of deviations of triangular arbitrage parity of different sizes. Triangular arbitrage opportunities are identified by comparing the three most recent quotes for each set of three currencies. An arbitrage opportunity exists if there is a mismatch among these three currencies. Deviations from parity are measured in pips. The sample period is from January 2, 2003 to December 30, 2004.

| Deviation | 2003 | % 2003 of Total | 2004 | % 2004 of Total | Total |
|---|---|---|---|---|---|
| 1 | 16822 | 50.48 | 12401 | 49.52 | 29223 |
| 3 | 1151 | 42.74 | 1129 | 57.26 | 2280 |
| 5 | 256 | 43.95 | 343 | 56.05 | 599 |
| 7 | 109 | 37.50 | 139 | 62.50 | 248 |
| 9 | 48 | 39.47 | 80 | 60.53 | 128 |
| 11 | 30 | 25.49 | 46 | 74.51 | 76 |
| 13 | 13 | 19.44 | 38 | 80.56 | 51 |
| 15 | 7 | 13.79 | 29 | 86.21 | 36 |
| 17 | 4 | 3.85 | 25 | 96.15 | 29 |
| 19 | 1 | 6.25 | 25 | 93.75 | 26 |
| 21 | 1 | 0 | 15 | 100.00 | 16 |
| 23 | 0 | 0 | 8 | 100.00 | 8 |
| $\geq$25 | 0 | 0 | 3 | 100.00 | 3 |

## Table 4: Preliminary data analysis

This table presents descriptive statistics and $t$-statistics of the duration of arbitrage clusters. Triangular arbitrage opportunities are identified by comparing the three most recent quotes for each set of three currencies. An arbitrage opportunity exists if there is a mismatch between these three currencies. The textbook arbitrage column reports statistics of arbitrage clusters that are eliminated by any next incoming order (market orders, limit orders and cancelation), indicating that clusters in this group have only one profitable triangular arbitrage deviation. The textbook arbitrage column consists of arbitrage clusters that are consistent with a textbook example of an efficient elimination of arbitrage opportunities. The remaining clusters will fall into the risky arbitrage group where market participants deliberate on the participation of exploiting the observed arbitrage opportunity. The $t$-statistics compares if the mean of textbook arbitrage duration is statistically different from the mean of risky arbitrage duration. The sample period is from January 2, 2003 to December 30, 2004.

| | Textbook Arbitrage | Risky Arbitrage |
|---|---|---|
| Mean | 0.966 | 2.690 |
| $t$-stats | 5.66 | |
| Standard deviation | 0.993 | 10.153 |
| Median | 1.3 | 2.6 |
| Number of obs. | 3941 | 1114 |

## Table 5: Arbitrage Strategy Average Profits: 2 traders

This table presents the mean and standard deviation (in parentheses) of arbitrageurs' profits and losses in the market with only two arbitrageurs. Profits are in millions GBP. The arbitrage strategy is based on the triangular arbitrage condition. Arbitrageurs can participate in exploiting arbitrage opportunities when they observe any price discrepancy. Each arbitrageur is allowed to trade 1 million GBP. Threshold represents the minimum absolute deviation that needs to be observed for the arbitrageur to participate in arbitraging. Arbitrageurs employ a mixed strategy where they participate with probability $\pi$ when the deviation is below the threshold. 1 pip means one basis point (0.0001). The values in the table are computed based on 1000 simulations. All quotes, depth and breadth of the market is based on the reconstructed limit order book. The sample period is from January 2, 2003 to December 30, 2004.

| Threshold (pips) | $\pi = 0.1$ | $\pi = 0.2$ | $\pi = 0.3$ | $\pi = 0.4$ | $\pi = 0.5$ | $\pi = 0.6$ | $\pi = 0.7$ | $\pi = 0.8$ | $\pi = 0.9$ | $\pi = 1.0$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.33947 (0.18619) | 0.64560 (0.28609) | 0.91804 (0.28609) | 1.16652 (0.30126) | 1.37350 (0.33067) | 1.55203 (0.30977) | 1.69676 (0.29473) | 1.81233 (0.26527) | 1.89100 (0.19004) | 1.94221 (0.02390) |
| 3 | 0.08415 (0.05727) | 0.16417 (0.08722) | 0.24165 (0.08722) | 0.31373 (0.09610) | 0.38194 (0.09313) | 0.44734 (0.09682) | 0.50812 (0.08687) | 0.56407 (0.07920) | 0.61705 (0.05686) | 0.66577 (0.00801) |
| 5 | 0.03838 (0.03206) | 0.07642 (0.04927) | 0.11140 (0.04927) | 0.14517 (0.05137) | 0.17734 (0.05534) | 0.20891 (0.05333) | 0.23755 (0.05046) | 0.26482 (0.04280) | 0.29093 (0.03396) | 0.31540 (0.00618) |
| 7 | 0.00473 (0.00759) | 0.00930 (0.01155) | 0.01330 (0.01155) | 0.01717 (0.01226) | 0.02101 (0.01286) | 0.02446 (0.01250) | 0.02746 (0.01133) | 0.03033 (0.01004) | 0.03274 (0.00812) | 0.03516 (0.00325) |
| 9 | 0.01719 (0.01623) | 0.03353 (0.02489) | 0.04913 (0.02489) | 0.06474 (0.02727) | 0.07962 (0.02776) | 0.09352 (0.02680) | 0.10610 (0.02437) | 0.11903 (0.02209) | 0.13100 (0.01663) | 0.14226 (0.00438) |
| 11 | 0.01096 (0.01214) | 0.02174 (0.01817) | 0.03194 (0.01817) | 0.04156 (0.01949) | 0.05085 (0.02029) | 0.05999 (0.01871) | 0.06835 (0.01875) | 0.07616 (0.01585) | 0.08423 (0.01209) | 0.09128 (0.00381) |
| 13 | 0.00805 (0.01059) | 0.01571 (0.01553) | 0.02337 (0.01553) | 0.03014 (0.01692) | 0.03699 (0.01709) | 0.04352 (0.01715) | 0.04958 (0.01520) | 0.05543 (0.01370) | 0.06052 (0.01081) | 0.06566 (0.00358) |
| 15 | 0.00507 (0.00875) | 0.00978 (0.01326) | 0.01420 (0.01326) | 0.01866 (0.01431) | 0.02247 (0.01545) | 0.02629 (0.01449) | 0.02991 (0.01429) | 0.03320 (0.01206) | 0.03623 (0.00945) | 0.03896 (0.00291) |
| 17 | 0.00473 (0.00759) | 0.00930 (0.01155) | 0.01330 (0.01155) | 0.01717 (0.01226) | 0.02101 (0.01286) | 0.02446 (0.01250) | 0.02746 (0.01133) | 0.03033 (0.01004) | 0.03274 (0.00812) | 0.03516 (0.00325) |
| 19 | 0.00432 (0.00759) | 0.00865 (0.01159) | 0.01270 (0.01159) | 0.01655 (0.01238) | 0.01990 (0.01191) | 0.02310 (0.01282) | 0.02625 (0.01128) | 0.02906 (0.01042) | 0.03158 (0.00807) | 0.03385 (0.00299) |
| 21 | 0.00328 (0.00688) | 0.00645 (0.01063) | 0.00949 (0.01063) | 0.01259 (0.01159) | 0.01530 (0.01179) | 0.01812 (0.01111) | 0.02049 (0.01064) | 0.02301 (0.00904) | 0.02519 (0.00733) | 0.02745 (0.00236) |
| 23 | 0.00221 (0.00503) | 0.00445 (0.00797) | 0.00663 (0.00797) | 0.00862 (0.00807) | 0.01069 (0.00898) | 0.01271 (0.00827) | 0.01480 (0.00810) | 0.01649 (0.00677) | 0.01831 (0.00511) | 0.02012 (0.00177) |

## Table 6: Arbitrage Strategy Average Profits: 8 traders

This table presents the mean and standard deviation (in parentheses) of arbitrageurs' profits and losses in the market with eight arbitrageurs. Profits are in millions GBP. The arbitrage strategy is based on the triangular arbitrage condition. Arbitrageurs can participate in exploiting arbitrage opportunities when they observe any price discrepancy. Each arbitrageur is allowed to trade 1 million GBP. Threshold represents the minimum absolute deviation that needs to be observed for the arbitrageur to participate in arbitraging. Arbitrageurs employ a mixed strategy where they participate with probability $\pi$ when the deviation is below the threshold. 1 pip means one basis point (0.0001). The values in the table are computed based on 1000 simulations. All quotes, depth and breadth of the market is based on the reconstructed limit order book. The sample period is from January 2, 2003 to December 30, 2004.

| Threshold (pips) | $\pi = 0.1$ | $\pi = 0.2$ | $\pi = 0.3$ | $\pi = 0.4$ | $\pi = 0.5$ | $\pi = 0.6$ | $\pi = 0.7$ | $\pi = 0.8$ | $\pi = 0.9$ | $\pi = 1.0$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.23766 | 0.21885 | -0.08691 | -0.69713 | -1.64117 | -2.93223 | -4.58440 | -6.61474 | -9.03048 | -11.83620 |
| | (0.24480) | (0.32921) | (0.37723) | (0.39911) | (0.42128) | (0.40232) | (0.38691) | (0.34521) | (0.26985) | (0.13726) |
| 3 | 0.07197 | 0.11368 | 0.12235 | 0.09804 | 0.03970 | -0.05474 | -0.18435 | -0.34993 | -0.55302 | -0.79005 |
| | (0.07407) | (0.10030) | (0.11409) | (0.12368) | (0.12468) | (0.12357) | (0.11650) | (0.10383) | (0.08062) | (0.03356) |
| 5 | 0.03399 | 0.05664 | 0.06793 | 0.06827 | 0.05680 | 0.03467 | 0.00052 | -0.04551 | -0.10303 | -0.17174 |
| | (0.04252) | (0.05624) | (0.06520) | (0.07007) | (0.07193) | (0.07064) | (0.06636) | (0.05936) | (0.04682) | (0.02033) |
| 7 | 0.00394 | 0.00631 | 0.00708 | 0.00686 | 0.00509 | 0.00245 | -0.00127 | -0.00616 | -0.01192 | -0.01851 |
| | (0.00967) | (0.01283) | (0.01465) | (0.01575) | (0.01614) | (0.01607) | (0.01550) | (0.01407) | (0.01183) | (0.00783) |
| 9 | 0.01534 | 0.02676 | 0.03486 | 0.03930 | 0.04097 | 0.03929 | 0.03368 | 0.02554 | 0.01399 | -0.00048 |
| | (0.02070) | (0.02761) | (0.03162) | (0.03393) | (0.03492) | (0.03408) | (0.03231) | (0.02888) | (0.02265) | (0.01140) |
| 11 | 0.00992 | 0.01710 | 0.02244 | 0.02542 | 0.02648 | 0.02531 | 0.02236 | 0.01720 | 0.01002 | 0.00102 |
| | (0.01506) | (0.02049) | (0.02304) | (0.02449) | (0.02567) | (0.02526) | (0.02368) | (0.02152) | (0.01712) | (0.00986) |
| 13 | 0.00701 | 0.01199 | 0.01509 | 0.01635 | 0.01562 | 0.01325 | 0.00940 | 0.00372 | -0.00311 | -0.01158 |
| | (0.01290) | (0.01710) | (0.01989) | (0.02177) | (0.02245) | (0.02225) | (0.02102) | (0.01934) | (0.01586) | (0.00965) |
| 15 | 0.00428 | 0.00708 | 0.00836 | 0.00835 | 0.00738 | 0.00506 | 0.00163 | -0.00283 | -0.00823 | -0.01485 |
| | (0.01142) | (0.01498) | (0.01741) | (0.01864) | (0.01902) | (0.01917) | (0.01802) | (0.01635) | (0.01310) | (0.00797) |
| 17 | 0.00394 | 0.00631 | 0.00708 | 0.00686 | 0.00509 | 0.00245 | -0.00127 | -0.00616 | -0.01192 | -0.01851 |
| | (0.00967) | (0.01283) | (0.01465) | (0.01575) | (0.01614) | (0.01607) | (0.01550) | (0.01407) | (0.01183) | (0.00783) |
| 19 | 0.00377 | 0.00605 | 0.00699 | 0.00680 | 0.00540 | 0.00314 | -0.00020 | -0.00436 | -0.00939 | -0.01512 |
| | (0.00955) | (0.01264) | (0.01458) | (0.01558) | (0.01600) | (0.01597) | (0.01542) | (0.01382) | (0.01179) | (0.00765) |
| 21 | 0.00297 | 0.00504 | 0.00620 | 0.00666 | 0.00645 | 0.00565 | 0.00426 | 0.00245 | 0.00020 | -0.00268 |
| | (0.00880) | (0.01188) | (0.01332) | (0.01437) | (0.01493) | (0.01450) | (0.01387) | (0.01259) | (0.01049) | (0.00645) |
| 23 | 0.00205 | 0.00416 | 0.00536 | 0.00614 | 0.00633 | 0.00638 | 0.00589 | 0.00543 | 0.00453 | 0.00331 |
| | (0.00650) | (0.00997) | (0.01137) | (0.01258) | (0.01299) | (0.01325) | (0.01252) | (0.01173) | (0.01039) | (0.00812) |

## Table 7: Arbitrage Strategy Average Profits: 16 traders

This table presents the mean and standard deviation (in parentheses) of arbitrageurs' profits and losses in the market with sixteen arbitrageurs. Profits are in millions GBP. The arbitrage strategy is based on the triangular arbitrage condition. Arbitrageurs can participate in exploiting arbitrage opportunities when they observe any price discrepancy. Each arbitrageur is allowed to trade 1 million GBP. Threshold represents the minimum absolute deviation that needs to be observed for the arbitrageur to participate in arbitraging. Arbitrageurs employ a mixed strategy where they participate with probability $\pi$ when the deviation is below the threshold. 1 pip means one basis point (0.0001). The values in the table are computed based on 1000 simulations. All quotes, depth and breadth of the market is based on the reconstructed limit order book. The sample period is from January 2, 2003 to December 30, 2004.

| Threshold (pips) | $\pi = 0.1$ | $\pi = 0.2$ | $\pi = 0.3$ | $\pi = 0.4$ | $\pi = 0.5$ | $\pi = 0.6$ | $\pi = 0.7$ | $\pi = 0.8$ | $\pi = 0.9$ | $\pi = 1.0$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.08801 | -0.44541 | -1.70720 | -3.76546 | -6.67332 | -10.47320 | -15.19870 | -20.86090 | -27.46770 | -34.99620 |
| | (0.25456) | (0.33980) | (0.39239) | (0.42270) | (0.43880) | (0.43917) | (0.42689) | (0.40132) | (0.36697) | (0.32197) |
| 3 | 0.05448 | 0.03853 | -0.05043 | -0.21679 | -0.46394 | -0.79085 | -1.20423 | -1.69990 | -2.27462 | -2.92262 |
| | (0.07658) | (0.10332) | (0.11952) | (0.12680) | (0.13103) | (0.13077) | (0.12724) | (0.11637) | (0.09947) | (0.06918) |
| 5 | 0.02761 | 0.03079 | 0.00989 | -0.03572 | -0.10586 | -0.20134 | -0.32123 | -0.46431 | -0.63010 | -0.81382 |
| | (0.04398) | (0.05770) | (0.06732) | (0.07234) | (0.07483) | (0.07499) | (0.07120) | (0.06530) | (0.05573) | (0.03819) |
| 7 | 0.00301 | 0.00305 | 0.00058 | -0.00378 | -0.00940 | -0.01626 | -0.02430 | -0.03400 | -0.04559 | -0.05911 |
| | (0.00986) | (0.01329) | (0.01521) | (0.01653) | (0.01722) | (0.01722) | (0.01692) | (0.01556) | (0.01386) | (0.01113) |
| 9 | 0.01316 | 0.01882 | 0.01755 | 0.00935 | -0.00493 | -0.02569 | -0.05236 | -0.08456 | -0.12138 | -0.16217 |
| | (0.02128) | (0.02867) | (0.03303) | (0.03524) | (0.03637) | (0.03641) | (0.03504) | (0.03238) | (0.02787) | (0.02055) |
| 11 | 0.00849 | 0.01205 | 0.01149 | 0.00683 | -0.00175 | -0.01365 | -0.02846 | -0.04642 | -0.06720 | -0.09048 |
| | (0.01547) | (0.02049) | (0.02401) | (0.02608) | (0.02700) | (0.02702) | (0.02564) | (0.02410) | (0.02103) | (0.01635) |
| 13 | 0.00595 | 0.00760 | 0.00573 | 0.00071 | -0.00694 | -0.01684 | -0.02900 | -0.04359 | -0.05985 | -0.07766 |
| | (0.01359) | (0.01829) | (0.02119) | (0.02306) | (0.02377) | (0.02401) | (0.02344) | (0.02176) | (0.01949) | (0.01569) |
| 15 | 0.00340 | 0.00394 | 0.00186 | -0.00218 | -0.00770 | -0.01445 | -0.02280 | -0.03354 | -0.04631 | -0.06093 |
| | (0.01169) | (0.01562) | (0.01792) | (0.01945) | (0.02016) | (0.01981) | (0.01921) | (0.01795) | (0.01556) | (0.01174) |
| 17 | 0.00301 | 0.00305 | 0.00058 | -0.00378 | -0.00940 | -0.01626 | -0.02430 | -0.03400 | -0.04559 | -0.05911 |
| | (0.00986) | (0.01329) | (0.01521) | (0.01653) | (0.01722) | (0.01722) | (0.01692) | (0.01556) | (0.01386) | (0.01113) |
| 19 | 0.00298 | 0.00311 | 0.00113 | -0.00275 | -0.00779 | -0.01382 | -0.02098 | -0.02981 | -0.04092 | -0.05379 |
| | (0.00985) | (0.01306) | (0.01494) | (0.01654) | (0.01720) | (0.01697) | (0.01664) | (0.01540) | (0.01385) | (0.01107) |
| 21 | 0.00244 | 0.00315 | 0.00255 | 0.00099 | -0.00108 | -0.00337 | -0.00638 | -0.01044 | -0.01582 | -0.02203 |
| | (0.00908) | (0.01202) | (0.01399) | (0.01511) | (0.01570) | (0.01563) | (0.01509) | (0.01404) | (0.01215) | (0.00950) |
| 23 | 0.00179 | 0.00244 | 0.00227 | 0.00161 | 0.00095 | 0.00049 | 0.00019 | -0.00031 | -0.00113 | -0.00253 |
| | (0.00674) | (0.00891) | (0.01032) | (0.01147) | (0.01194) | (0.01207) | (0.01168) | (0.01092) | (0.00974) | (0.00771) |

## Table 8: Arbitrage Strategy Average Profits vs. number of traders

This table presents the regression estimates $x_1$ of Equation 8, $PL = x_0 + x_1 \times k$. It also reports the t-statistics (in parentheses) and $R^2$ coefficients. $k$ is the number of competing arbitrageurs and $PL$ is the average profit of these arbitrageurs from the backtesting exercise. $PL$ is in millions GBP. Arbitrageurs are assumed to use mixed strategies with the probability of participation given by $\pi$ when the observed deviation exceeds the threshold. Each row represents the arbitrage strategy corresponding to a particular threshold, which is the self-imposed minimum absolute deviation that arbitrageurs will participate in arbitraging. They will not participate if the deviation is below the threshold. The backtesting exercise is repeated 1000 times and $PL$ is the average profit across the repeated exercise. All quotes, depth and breadth of the market is based in the reconstructed limit order book. The sample period is from January 2, 2003 to December 30, 2004.

| Threshold (pips) | $\pi = 0.1$ | $\pi = 0.2$ | $\pi = 0.3$ | $\pi = 0.4$ | $\pi = 0.5$ | $\pi = 0.6$ | $\pi = 0.7$ | $\pi = 0.8$ | $\pi = 0.9$ | $\pi = 1.0$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1pip | -2.6644 | -2.1152 | -1.6321 | -1.2157 | -0.8646 | -0.5781 | -0.3538 | -0.1882 | -0.0781 | -0.01798 |
| | (-38.84) | (-38.01) | (-37.90) | (-38.15) | (-38.97) | (-40.18) | (-42.64) | (-47.18) | (-57.52) | (-89.29) |
| | 99.60% | 99.59% | 99.58% | 99.59% | 99.61% | 99.63% | 99.67% | 99.73% | 99.82% | 99.92% |
| 3pip | -0.2576 | -0.2072 | -0.1620 | -0.1226 | -0.0887 | -0.0606 | -0.0381 | -0.0209 | -0.0090 | -0.00212 |
| | (-90.12) | (-84.77) | (-83.28) | (-88.54) | (-94.30) | (-91.85) | (-98.26) | (-109.3) | (-80.71) | (-124.7) |
| | 99.93% | 99.92% | 99.91% | 99.92% | 99.93% | 99.93% | 99.94% | 99.95% | 99.91% | 99.96% |
| 5pip | -0.0810 | -0.0659 | -0.0521 | -0.0399 | -0.0293 | -0.0203 | -0.0129 | -0.0073 | -0.0032 | -0.00077 |
| | (-295.0) | (-560.1) | (-523.6) | (-462.4) | (-577.0) | (-487.5) | (-511.5) | (-772.1) | (-126.5) | (-97.12) |
| | 99.99% | 99.99% | 99.99% | 99.99% | 99.99% | 99.99% | 99.99% | 99.99% | 99.96% | 99.94% |
| 7pip | -0.0064 | -0.0054 | -0.0045 | -0.0037 | -0.0029 | -0.0022 | -0.0015 | -0.0009 | -0.0004 | -0.00012 |
| | (-16.03) | (-15.52) | (-15.45) | (-16.48) | (-18.30) | (-22.75) | (-30.75) | (-38.86) | (-36.07) | (-49.61) |
| | 97.72% | 97.57% | 97.55% | 97.84% | 98.24% | 98.85% | 99.37% | 99.60% | 99.54% | 99.76% |
| 9pip | -0.0218 | -0.0180 | -0.0145 | -0.0113 | -0.0085 | -0.0060 | -0.0039 | -0.0023 | -0.0011 | -0.00029 |
| | (-54.30) | (-64.56) | (-72.07) | (-76.40) | (-71.86) | (-66.86) | (-68.42) | (-80.04) | (-71.06) | (-55.78) |
| | 99.80% | 99.86% | 99.88% | 99.90% | 99.88% | 99.87% | 99.87% | 99.91% | 99.88% | 99.81% |
| 11pip | -0.0129 | -0.0108 | -0.0087 | -0.0069 | -0.0052 | -0.0037 | -0.0025 | -0.0015 | -0.0007 | -0.00018 |
| | (-33.90) | (-36.36) | (-40.88) | (-44.03) | (-50.56) | (-58.16) | (-58.91) | (-57.04) | (-47.39) | (-56.64) |
| | 99.48% | 99.55% | 99.64% | 99.69% | 99.77% | 99.82% | 99.83% | 99.82% | 99.73% | 99.81% |
| 13pip | -0.0101 | -0.0085 | -0.0070 | -0.0056 | -0.0043 | -0.0031 | -0.0021 | -0.0013 | -0.0006 | -0.00016 |
| | (-21.20) | (-22.87) | (-23.74) | (-25.47) | (-28.54) | (-35.13) | (-47.56) | (-51.85) | (-73.41) | (-30.52) |
| | 98.68% | 98.87% | 98.95% | 99.08% | 99.27% | 99.52% | 99.74% | 99.78% | 99.89% | 99.36% |
| 15pip | -0.0068 | -0.0057 | -0.0046 | -0.0037 | -0.0029 | -0.0022 | -0.0015 | -0.0009 | -0.0004 | -0.00012 |
| | (-21.07) | (-20.13) | (-19.40) | (-19.49) | (-22.24) | (-28.46) | (-32.80) | (-48.02) | (-47.44) | (-32.33) |
| | 98.67% | 98.54% | 98.43% | 98.44% | 98.80% | 99.26% | 99.45% | 99.74% | 99.73% | 99.43% |
| 17pip | -0.0064 | -0.0054 | -0.0045 | -0.0037 | -0.0029 | -0.0022 | -0.0015 | -0.0009 | -0.0004 | -0.00012 |
| | (-16.03) | (-15.52) | (-15.45) | (-16.48) | (-18.30) | (-22.75) | (-30.75) | (-38.86) | (-36.07) | (-49.61) |
| | 97.72% | 97.57% | 97.55% | 97.84% | 98.24% | 98.85% | 99.37% | 99.60% | 99.54% | 99.76% |
| 19pip | -0.0059 | -0.0049 | -0.0041 | -0.0033 | -0.0026 | -0.0020 | -0.0014 | -0.0008 | -0.0004 | -0.00010 |
| | (-16.93) | (-15.93) | (-15.32) | (-16.01) | (-18.51) | (-22.38) | (-27.13) | (-32.54) | (-48.16) | (-37.44) |
| | 97.95% | 97.69% | 97.51% | 97.71% | 98.28% | 98.82% | 99.19% | 99.44% | 99.74% | 99.57% |
| 21pip | -0.0033 | -0.0027 | -0.0023 | -0.0019 | -0.0015 | -0.0012 | -0.0008 | -0.0005 | -0.0002 | -0.00006 |
| | (-12.01) | (-11.78) | (-11.35) | (-12.15) | (-13.84) | (-18.28) | (-26.68) | (-44.60) | (-59.67) | (-32.38) |
| | 96.01% | 95.85% | 95.55% | 96.09% | 96.96% | 98.24% | 99.16% | 99.70% | 99.83% | 99.43% |
| 23pip | -0.0014 | -0.0013 | -0.0012 | -0.0010 | -0.0009 | -0.0007 | -0.0005 | -0.0003 | -0.0002 | -0.00003 |
| | ( -5.43) | ( -5.79) | ( -6.50) | ( -7.66) | (-10.34) | (-14.29) | (-16.14) | (-14.78) | ( -8.92) | (-16.33) |
| | 83.08% | 84.82% | 87.56% | 90.73% | 94.69% | 97.15% | 97.75% | 97.33% | 92.98% | 97.80% |

**Table 9: The Effect of illiquidity on arbitrage deviation**

This table presents the regression estimates and statistics of the equilibrium profit Equation (9), $A = a_0 + a_1 \times \Delta_{GBP/USD} + a_2 \times \Delta_{EUR/USD} + a_3 \times \Delta_{EUR/GBP}$. Triangular arbitrage opportunities are identified by comparing the three most recent quotes for each set of three currencies. An arbitrage opportunity exists if there is a mismatch between these three currencies. $A$ denotes the arbitrage deviation size. Variables $\Delta_{GBP/USD}$, $\Delta_{EUR/USD}$ and $\Delta_{EUR/GBP}$ are the price difference between the best and second best quotes of the corresponding exchange rates. $\lambda_{GBP/USD}$, $\lambda_{EUR/USD}$ and $\lambda_{EUR/GBP}$ are the slopes of the demand or supply schedules calculated as difference between the best and the second best quotes divided by the quantity of the best quote of the corresponding exchange rate. We use either demand or supply of the limit order book which corresponds to the necessary transaction to exploit the arbitrage opportunity (depending on whether a purchase or sales of the direct currency price is involved in the transaction). The sample period is from January 2, 2003 to December 30, 2004.

| *Panel A: Regression of deviation on price difference of best and second best bid-ask price* | | | | |
|---|---|---|---|---|
| | $\Delta_{GBP/USD}$ | $\Delta_{EUR/USD}$ | $\Delta_{EUR/GBP}$ | |
| Para. estimates | 0.30863 | 0.16126 | 0.16011 | |
| Std. errors | 0.0020 | 0.0022 | 0.0080 | |
| $p$-value | <0.0001 | <0.0001 | <0.0001 | |
| $R^2$ | | | | 0.32 |

| *Panel B: Regression of deviation on the slope of demand and supply schedules* | | | | |
|---|---|---|---|---|
| | $\lambda_{GBP/USD}$ | $\lambda_{EUR/USD}$ | $\lambda_{EUR/GBP}$ | |
| Para. estimates | 0.00804 | 0.00659 | 0.00470 | |
| Std. errors | 0.00005 | 0.00004 | 0.00017 | |
| $p$-values | <0.0001 | <0.0001 | <0.0001 | |
| $R^2$ | | | | 0.27 |
| Num. obs. | | | | 139580 |

# Appendix

**Proof of Theorem 1**

We will prove the statement of the theorem by induction. Let us first check that the statement is true for $I = 2$. If an arbitrageur $j$ wins the competition and gets the best prices in both markets, she earns the profit $A$. The probability of winning the best prices in both markets is $\mathbf{P}_1^j\mathbf{P}_2^j$ because of the independence of the two markets. The probability of the arbitrageur failing to get the best price in the market 1 (in the market 2) but winning the best price in the market 2 (in the market 1) is $\mathbf{P}_2^j(1-\mathbf{P}_1^j)$ (respectively, $\mathbf{P}_1^j(1-\mathbf{P}_2^j)$). She earns, in this case, the profit $A-\Delta_1(w_1)$ ($A-\Delta_2(w_2)$, respectively). With the probability $(1-\mathbf{P}_1^j)(1-\mathbf{P}_2^j)$, the arbitrageur fails to get best prices in both markets and earns $A-\Delta_1(w_1)-\Delta_2(w_2)$. It is easy to check that the expected profit of the arbitrageur from the "trade "strategy is

$$\begin{aligned} E(U^j) &= A\mathbf{P}_1^j\mathbf{P}_2^j + (A-\Delta_1(w_1))\mathbf{P}_2^j(1-\mathbf{P}_1^j) + (A-\Delta_2(w_2))\mathbf{P}_1^j(1-\mathbf{P}_2^j) \\ &+ (A-\Delta_1(w_1)-\Delta_2(w_2))(1-\mathbf{P}_1^j)(1-\mathbf{P}_2^j) = A-\Delta_1(w_1)(1-\mathbf{P}_1^j)-\Delta_2(w_2)(1-\mathbf{P}_2^j). \end{aligned}$$

We assume that the statement of the theorem is satisfied for $I-1$ markets, that is, $E(U^j) =$

$A - \sum_{i=1}^{I-1} \Delta_i(w_i)(1 - \mathbf{P}_i^j)$. Let us show that the corresponding statement is also true in the case of $I$ markets.

Let us denote by $\mathcal{I}$ the set $\{1, ..., I\}$ of the markets indices and let $J$ be a non-empty subset of $\mathcal{I}$. Similarly to the case of two markets, arbitrageur $j$ earns the observed profit $A$ if she gets the best prices in all $I$ markets. If she fails to get the best prices in each market from $J$ and gets the best prices in all the rest $\mathcal{I} \setminus J$ markets, her payoff will be $A - \sum_{i \in J} \Delta_i(w_i)$. The probability of the event that the trader fails exactly in each of $J$ markets and wins the best prices all other markets is equal to

$$\prod_{i \in J} \mathbf{P}_i^j \cdot \prod_{i \in \mathcal{I} \setminus J} \bar{\mathbf{P}}_i^j.$$

The expected payoff of the arbitrageur is equal to the weighted sum of all possible payoffs where the weights are the corresponding probabilities.

The expected value of the profit is

$$\begin{aligned}
E(U^j) &= \sum_{J \in 2^{\mathcal{I}}} \left( \left( A - \sum_{i \in J} \Delta_i(w_i) \right) \cdot \prod_{i \in J} \left( 1 - \mathbf{P}_i^j \right) \cdot \prod_{\iota \in \mathcal{I} \setminus J} \mathbf{P}_\iota^j \right) \\
&= A \sum_{J \in 2^{\mathcal{I}}} \left( \prod_{i \in J} \left( 1 - \mathbf{P}_i^j \right) \cdot \prod_{\iota \in \mathcal{I} \setminus J} \mathbf{P}_\iota^j \right) - \sum_{J \in 2^{\mathcal{I}}} \left( \left( \sum_{i \in J} \Delta_i(w_i) \right) \cdot \prod_{i \in J} \left( 1 - \mathbf{P}_i^j \right) \cdot \prod_{\iota \in \mathcal{I} \setminus J} \mathbf{P}_\iota^j \right) \\
&= A - \sum_{J \in 2^{\mathcal{I}}} \left( \left( \sum_{i \in J} \Delta_i(w_i) \right) \cdot \prod_{i \in J} \left( 1 - \mathbf{P}_i^j \right) \cdot \prod_{\iota \in \mathcal{I} \setminus J} \mathbf{P}_\iota^j \right). \qquad (11)
\end{aligned}$$

In the above expression we used the equality

$$\sum_{J \in 2^{\mathcal{I}}} \prod_{i \in J} \mathbf{P}_i^j \cdot \prod_{\iota \in \mathcal{I} \setminus J} (1 - \mathbf{P}_\iota^j) = \prod_{i=1}^{I} \left( \mathbf{P}_i^j + (1 - \mathbf{P}_i^j) \right) = 1.$$

Let us decompose the last sum of the equality (11) into the term containing $\Delta_I(w_I)$ and not containing $\Delta_I(w_I)$. This gives

$$\begin{aligned}
E(U^j) &= A - \mathbf{P}_I^j \sum_{J \in 2^{\mathcal{I} \setminus \{I\}}} \left( \left( \Delta_I(w_I) + \sum_{i \in J} \Delta_i(w_i) \right) \cdot \prod_{i \in J} \left( 1 - \mathbf{P}_i^j \right) \cdot \prod_{\iota \in \mathcal{I} \setminus J} \mathbf{P}_\iota^j \right) \\
&- (1 - \mathbf{P}_I^j) \sum_{J \in 2^{\mathcal{I} \setminus \{I\}}} \left( \left( \sum_{i \in J} \Delta_i(w_i) \right) \cdot \prod_{i \in J} \left( 1 - \mathbf{P}_i^j \right) \cdot \prod_{\iota \in \mathcal{I} \setminus J} \mathbf{P}_\iota^j \right) \\
&= A - \Delta_I(w_I) \mathbf{P}_I^j - \mathbf{P}_I^j \left( \sum_{i=1}^{I-1} \Delta_i(w_i) \mathbf{P}_i^j \right) - (1 - \mathbf{P}_I^j) \left( \sum_{i=1}^{I-1} \Delta_i(w_i) \mathbf{P}_i^j \right) \\
&= A - \sum_{i=1}^{I} \Delta_i(w_i) \mathbf{P}_i^j.
\end{aligned}$$

Q.E.D.

**Proof of Theorem 2**

(i) The expression for the probability $\bar{\mathbf{P}}^j_{i|n_i,k,\Pi_{-j}}$ of failing to get the best price in the market $i$ by trader $j$ can be derived from the law of total probability. Consider the set of mutually exclusive and exhaustive events $X_S$ with $S \in 2^{\mathcal{K}_{-j}}$ each of which means that all opponents of trader $j$ from the subset $S$ participate in the market with certainly and the rest $\mathcal{K}_{-j} \setminus S$ does not. The probability of the event $X_S$ occurring is equal to

$$\mathbf{P}(X_S) = \prod_{s\in S}\pi_s \prod_{s\in\mathcal{K}_{-j}\setminus S}(1-\pi_s)$$

and the probability of failing to get the best price price in the market $i$ for trader $j$ *conditional* on $X_S$ is

$$\bar{\mathbf{P}}^j_{i|n_i,|X_S|} = \begin{cases} 1, & |S| \le n_i - 1 \\ \frac{n_i}{|S|+1}, & |S| > n_i - 1 \end{cases}$$

since there are only $|S|$ opponents are in the market. By the law of total probability we get

$$\bar{\mathbf{P}}^j_{i|n_i,k,\Pi_{-j}} = \sum_{S\in 2^{\mathcal{K}_{-j}}} \bar{\mathbf{P}}^j_{i|n_i,|S|}\cdot \mathbf{P}(X_S) = \sum_{S\in 2^{\mathcal{K}_{-j}}}\prod_{s\in S}\pi_s \prod_{s\in\mathcal{K}_{-j}\setminus S}(1-\pi_s)\bar{\mathbf{P}}^j_{i|n_i,|S|}.$$

(ii) Let us now add one more arbitrageur $k+1$ into the market who plays her mixed strategy $\pi_{k+1}$. The new set of arbitrageurs is now denoted by $\mathcal{K}' = \{1,\ldots,k+1\}$. Let the new mixed strategy profile be $\Pi' = \{\pi_1,\ldots,\pi_k,\pi_{k+1}\}$. In order to prove the second statement of the theorem, we need to show that $\bar{\mathbf{P}}^j_{i|n_i,k+1,\Pi'_{-j}} > \bar{\mathbf{P}}^j_{i|n_i,k,\Pi_{-j}}$ for each $j \in \mathcal{K}$ and $i \in \mathcal{I}$.

By the statement (i) of this theorem, we know that

$$\bar{\mathbf{P}}^j_{i|n_i,k+1,\Pi'_{-j}} = \sum_{S\in 2^{\mathcal{K}'_{-j}}}\prod_{s\in S}\pi_s \prod_{s\in\mathcal{K}'_{-j}\setminus S}(1-\pi_s)\bar{\mathbf{P}}^j_{i|n_i,|S|}.$$

Let us decompose the sum into the part with subsets $S$ containing the arbitrageur $k+1$ and not containing her.

$$\begin{aligned}
\bar{\mathbf{P}}^j_{i|n_i,k+1,\Pi'_{-j}} &= \sum_{S\in 2^{\mathcal{K}_{-j}}}\prod_{s\in S\cup\{k+1\}}\pi_s \prod_{s\in\mathcal{K}_{-j}\setminus S}(1-\pi_s)\bar{\mathbf{P}}^j_{i|n_i,|S|+1} + \sum_{S\in 2^{\mathcal{K}_{-j}}}\prod_{s\in S}\pi_s \prod_{s\in\mathcal{K}'_{-j}\cup\{k+1\}\setminus S}(1-\pi_s)\bar{\mathbf{P}}^j_{i|n_i,|S|} \\
&= \pi_{k+1}\sum_{S\in 2^{\mathcal{K}_{-j}}}\prod_{s\in S}\pi_s \prod_{s\in\mathcal{K}_{-j}\setminus S}(1-\pi_s)\bar{\mathbf{P}}^j_{i|n_i,|S|+1} + (1-\pi_{k+1})\sum_{S\in 2^{\mathcal{K}_{-j}}}\prod_{s\in S}\pi_s \prod_{s\in\mathcal{K}'_{-j}\setminus S}(1-\pi_s)\bar{\mathbf{P}}^j_{i|n_i,|S|} \\
&> \pi_{k+1}\sum_{S\in 2^{\mathcal{K}_{-j}}}\prod_{s\in S}\pi_s \prod_{s\in\mathcal{K}_{-j}\setminus S}(1-\pi_s)\bar{\mathbf{P}}^j_{i|n_i,|S|} + (1-\pi_{k+1})\sum_{S\in 2^{\mathcal{K}_{-j}}}\prod_{s\in S}\pi_s \prod_{s\in\mathcal{K}'_{-j}\setminus S}(1-\pi_s)\bar{\mathbf{P}}^j_{i|n_i,|S|} \\
&= \bar{\mathbf{P}}^j_{i|n_i,k,\Pi_{-j}}.
\end{aligned}$$

The strict inequality appears due to Assumption 3.

Q.E.D.

**Proof of Theorem 3**

(i) Let the mixed strategy profile $\Pi$ form a Nash equilibrium of the game. The expected profit of trader $j$ is given by $\pi_j E\left(U^j|\Pi_{-j}\right)$. If $E\left(U^j|\Pi_{-j}\right) > 0$, it contradicts to the definition of Nash equilibrium since trader $j$ can always choose a strategy $\pi'_j > \pi_j$. This will lead to $\pi_j E\left(U^j|\Pi_{-j}\right) < \pi'_j E\left(U^j|\Pi_{-j}\right)$. On the other hand, condition $E\left(U^j|\Pi_{-j}\right) < 0$ can not be true in equilibrium as the strategy "not to trade "with $\pi^j = 0$ provides better off for the trader.

(ii) Let us consider a $2\times 2$ subgame played by two arbitrarily chosen traders $j$ and $j'$. The subgame has a form

*TRADER* $j'$

| | | 0 | 1 |
|---|---|---|---|
| TRADER $j$ | 0 | $0,0$ | $0,y$ |
| | 1 | $y,0$ | $z,z$ |

$y$ denotes the payoff of the trader choosing "trade" ($\pi = 1$) and the other trader in the subgame choosing "not trade" ($\pi = 0$) while the remaining $k-2$ arbitrageurs stick to the mixed strategy profile $\Pi_{-j,j'}$. $z$ denotes the payoff of traders $j$ and $j'$ when they both choose to "trade ". According to Theorem 2, the payoff $z$ is smaller than $y$ as there is one more participating opponent with trader $j'$ participating. In equilibrium, each trader must be indifferent between using "trade" or "not trade" strategies, so

$$\begin{array}{c} \pi_j z + (1-\pi_j)\, y = 0 \\ \pi_{j'} z + \left(1-\pi_{j'}\right) y = 0 \end{array},$$

which implies $\left(\pi_j - \pi_{j'}\right)(z-y) = 0$. Since $z - y > 0$, we get $\pi_j = \pi_{j'}$. As arbitrageurs $j$ and $j'$ were chosen arbitrarily, this implies that all traders use the same mixed strategy.

Q.E.D.

**Proof of Corollary 4**

(i) According to Theorem 3, in equilibrium all arbitrageurs play the same mixed strategy $\pi$ and their expected profit $\pi E(U|\pi) = 0$. On the other hand, Theorem 1 claims that $E(U|\pi) = A - \sum_{i=1}^{I} \Delta_i(w_i)\mathbf{P}_{i|n_i,k,\pi}$. Equating these two equations leads to the statement of the Corollary

$$A = \sum_{i=1}^{I} \Delta_i(w_i)\mathbf{P}_{i|n_i,k,\pi}$$

(ii) The statement can be directly obtained from Equation 4 by straightforward substitution $\Delta_i(w_i) = \lambda_i(w_i)n_i(w_i)$ for $i \in \{1, ..., I\}$.

Q.E.D.

# References


Abreu, D. and Brunnermeier, M.: 2002, Synchronization risk and delayed arbitrage, *Journal of Financial Economics* **66**, 341–360.

Aiba, Y. and, H. N., Takayasu, H., Marumo, K. and Shimizu, T.: 2002, Triangular arbitrage as an interaction among foreign exchange rates, *Physica A* **310**, 467–479.

Aiba, Y. and, H. N., Takayasu, H., Marumo, K. and Shimizu, T.: 2003, Triangular arbitrage and negative auto-correlation of foreign exchange rates, *Physica A* **324**, 253–257.

Akram, F., Rime, D. and Sarno, L.: 2008, Arbitrage in the foreign exchange market: Turning on the microscope, *Journal of International Economics* . Forthcoming.

Baker, M. and Savaşoglu, S.: 64, Limited arbitrage in mergers and acquisitions, *Journal of Financial Economics* pp. 91–115.

Barberis, N. and Thaler, R.: 2003, *A survey of behavioral finance*, Handbook of the Economics of Finance, North-Holland, Amsterdam, pp. 1051–1121.

Basak, S. and Croitoru, B.: 2000, Equilibrium mispricing in a capital market with portfolio constraints, *Review of Financial Studies* **13**, 715–748.

Bollerslev, T. and Domowitz, I.: 1993, Trading patterns and prices in the interbank foreign exchange market, *Journal of Finance* **48**, 1421–1443.

Branson, W.: 1969, The minimum covered interest differential needed for international arbitrage activity, *Journal of Political Economy* **77**, 1028–1035.

Chordia, T., Roll, R. and Subrahmanyam, A.: 2002, Order imbalance, liquidity and market returns, *Journal of Financial Economics* **65**, 111–130.

Daniel, K., Hirshleifer, D. and Teoh, S.: 2002, Investor psychology in capital markets: Evidence and policy implications, *Journal of Monetary Economics* **49**, 139–209.

De Jong, A., Rosenthal, L. and Van Dijk, M.: 2008, The risk and return of arbitrage in dual-listed companies, *Review of Finance* . Forthcoming.

De Long, J., Shleifer, A., Summers, L. and Waldmann, R.: 1990, Noise trader risk in financial markets, *Journal of Political Economy* **98**, 703–738.

Deville, L. and Riva, F.: 2007, Liquidity and arbitrage in options markets: A survival analysis approach, *Review of Finance* **11**, 497–525.

Edwards, F.: 1999, Hedge funds and the collapse of long-term capital management, *Journal of Economic Perspectives* **13**, 189–210.

Engle, R. and Russell, J.: 1998, Autoregressive conditional duration: A new model for irregularly spaced transaction data, *Econometrica* **66**, 1127–1162.

Fong, W.-M., Valente, G. and Fung, J.: 2008, Market liquidity and FX arbitrage in emerging markets: Evidence from Hong Kong tick data. Working Papers 082008, Hong Kong Institute for Monetary Research.

Frenkel, J.: 1973, Elasticities and the interest parity theory, *Journal of Political Economy* **81**, 741–747.

Frenkel, J. and Levich, R.: 1975, Covered interest arbitrage: Unexploited profits, *Journal of Political Economy* **83**, 325–338.

Frenkel, J. and Levich, R.: 1977, Transaction costs and interest arbitrage: Tranquil versus turbulent periods, *Journal of Political Economy* **85**, 1209–1226.

Gagnon, L. and Karolyi, G.: 2004, Multi-market trading and arbitrage. Working Paper, Ohio State University.

Glosten, L.: 1994, Is the electronic open limit order book inevitable?, *Journal of Finance* **49**(4), 1127–1161.

Gromb, D. and Vayanos, D.: 2002, Equilibrium and welfare in markets with financially constrained arbitrageurs, *Journal of Financial Economics* **66**, 361–407.

Grossman, S. and Stiglitz, J.: 1976, Information and competitive price systems, *American Economic Review* **66**, 246–252.

Grossman, S. and Stiglitz, J.: 1980, On the impossibility of informationally efficient markets, *American Economic Review* **70**, 393–408.

Hansen, L.: 1982, Large sample properties of generalized method of moments estimators, *Econometrica* **50**, 1029–1054.

Hendershott, T., Jones, C. and Menkveld, A.: 2007, Does algorithmic trading improve liquidity?, *Technical report*, UC Berkeley.

Hendershott, T. and Moulton, P.: 2007, The shrinking new york stock exchange floor and the hybrid market, *Technical report*, UC Berkeley.

Kondor, P.: 2008, Risk in dynamic arbitrage: Price effects of convergence trading, *Journal of Finance* . Forthcoming.

Kumar, P. and Seppi, D.: 1994, Information and index arbitrage, *Journal of Business* **67**, 481–509.

Kyle, A. and Xiong, W.: 2001, Contagion as a wealth effect, *Journal of Finance* **56**, 1401–1440.

Lamont, O. and Thaler, R.: 2003, Can the market add and subtract? mispricing in tech stock carve-outs, *Journal of Political Economy* **111**, 227–268.

Liu, J. and Longstaff, F.: 2004, Losing money on arbitrage: optimal dynamic portfolio choice in markets with arbitrage opportunities, *Review of Financial Studies* **17**, 611641.

Loewenstein, R.: 2000, *When genius failed: the rise and fall of long-term capital management*, New York, Random House.

Lyons, R.: 2001, *The microstructure approach to the exchange rate economics*, MIT Press.

Marshall, B., Treepongkaruna, S. and Young, M.: 2008, Exploitable arbitrage opportunities exist in the foreign exchange market, *American Finance Association Annual Meeting, New Orleans.*

Oehmke, M.: 2008, Dynamic arbitrage in illiquid market. Mimeo.

Ofek, E., Richardson, M. and Whitelaw, R.: 2004, Limited arbitrage and short sales restrictions. Evidence from the options markets, *Journal of Financial Economics* .

O'Hara, M. and Oldfield, G.: 1986, The microeconomics of market making, *Journal of Financial and Quantitative Analysis* **21**, 361–376.

Osler, C.: 2008, *Foreign Exchange Microstructure: A Survey*, Springer Encyclopedia of Complexity and System Science, Springer.

Rime, D.: 2003, *New Electronic Trading Systems in the Foreign Exchange Market*, New Economy Handbook, Academic Press.

Roll, R., Schwartz, E. and Subrahmanyam, A.: 2007, Liquidity and the law of one price: The case of future-cash basis, *Journal of Finance* **62**, 2201–2234.

Shleifer, A. and Summers, L.: 1990, The noise trader approach to finance, *Journal of Economic Perspectives* **4**, 19–33.

Shleifer, A. and Vishny, R.: 1992, The limits of arbitrage, *Journal of Finance* **52**, 35–55.

Stoll, H.: 1978, The supply of dealer services in securities markets, *Journal of Finance* **33**, 1133–1151.

Taylor, M.: 1987, Covered interest parity: A high-frequency, high-quality data study, *Economica* **54**, 429–438.

Taylor, M.: 1989, Covered interest arbitrage and market turbulence, *Economic Journal* **99**, 376–391.

Tham, W.: 2008, Macroeconomic announcements, price discovery, liquidity in FX market. Mimeo.

Xiong, W.: 2001, Convergence trading with wealth effects, *Journal of Financial Economics* **62**, 247–292.

Zigrand, J.-P.: 2004, General equilibrium analysis of strategic arbitrage, *Journal of Mathematical Economics* **40**, 923–952.