

# The same bond at different prices: identifying search frictions and demand pressures <sup>\*</sup>

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## Abstract

I model how corporate bond prices are affected by search frictions and occasional selling pressures, and test my predictions empirically. A key prediction in my model is that in a distressed market with more sellers than buyers, the mid-price (i.e. the average of bid and ask price) paid by institutional investors is lower than that of retail investors. Using a structural estimation, the model is able to identify liquidity crises based on the relative prices of institutional and retail investors. I identify two liquidity crises (i.e. high estimated numbers of forced sellers), namely the downgrade of GM and Ford in 2005 and the current crisis. I also estimate that search costs for institutional investors increase strongly during the subprime crisis, while search costs for retail investors appear stable. Finally, search costs have the highest impact on yields for bonds with short maturities according to the estimation.

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# 1 Introduction

The U.S. corporate bond market is a principal source of financing for U.S. firms. While it is bigger than the U.S. Treasury market measured in amount outstanding, the trading volume is more than twenty times lower.<sup>1</sup> To trade a corporate bond an investor has to sequentially contact one or several dealers over the telephone. Except for the most actively traded bonds, dealers do not "make a market" and a price quote is firm for only a short period of time which limits the ability to obtain multiple quotations before committing to a trade.<sup>2</sup> Hence, prices reflect a bargaining process that depends on the outside options of investors and dealers, including the relative number of investors currently looking to sell or buy.

This paper seeks to capture these phenomena in a search model, and, using a structural estimation of the model, to identify empirically periods of liquidity shortage as captured by large numbers of selling investors and fewer trading opportunities. A key ingredient in the identification is the distribution of prices of the same bond on a given day for buying vs. selling and retail vs. institutional investors. I find two periods where a positive shock to the number of sellers occurred. The first is from March to May 2005 where there was an increasing number of sellers during the three months according to calibration results. This is consistent with results in Acharya, Schaefer, and Zhang (2008) who find that profit warnings of GM and Ford in March and April 2005 and subsequent downgrade to junk bond status in May 2005 caused a large sell-off in their bonds. Interestingly, the difference between institutional and retail mid-prices identifies an excess number of sellers in other bonds with junk bond rating. A possible explanation is that there was a sell-off in junk bonds, because some were being substituted by Ford and GM bonds. The second period with a large number of sellers according to the empirical results is the subprime crisis beginning in August 2007 and lasting to the end of the sample in September 2008. Examining the relation between institutional and retail investor mid-prices in this period shows that there has been a large number of sellers across all ratings apart from the most risky bonds. In the remaining data sample

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<sup>1</sup>Principal outstanding volume by the end of 2007 was \$5,825bn in U.S. corporate bond market and \$4,855bn in the U.S. Treasury market while average daily trading volume in 2007 was \$24.3bn in the U.S. corporate bond market and \$567.1bn in the U.S. Treasury market. Source: Securities and Financial Markets Association ([www.sifma.org](http://www.sifma.org)).

<sup>2</sup>See Bessembinder and Maxwell (2008) for further details about the U.S. corporate bond market.

average mid-prices for institutional investors are higher than those for retail investors.

I examine the extent to which search costs affect average yields by defining the premium due to search costs as the midyield for an average investor minus the midyield for an investor who can instantly find a trading partner. The premium represents average yields in the corporate bond market versus average yields in the ultra-liquid Treasury market. I find that for maturities less than one year the search premium is large for both institutional and retail investors. Illiquidity of the corporate bond market has been suggested as a possible explanation for the 'credit spread puzzle', i.e. the assertion that average yield spreads between corporate bonds and Treasury bonds are larger than what can be explained by the default risk of corporate bonds. The 'puzzle' is particularly severe for short maturities (see for example Huang and Huang (2003)). The impact of search costs on average yields is of a magnitude that explains the 'unexplained' part of credit spreads at short maturities, and the results suggest that search costs are important to take into account when "the credit spread puzzle" at short maturities is investigated.

In the calibration, the estimated search intensity for an investor is determined by the size of average bid-ask spreads at which this investor transacts. I find that search intensities of retail investors are fairly constant throughout the sample, while search intensities of institutional investors decrease strongly after the onset of the subprime crisis, i.e., it has been more expensive for institutional investors to trade during the crisis, while costs of retail investors have not been significantly affected by the crisis.

The search model I set up is a variant of the model in Duffie, Gârleanu, and Pedersen (2005) (DGP05). An asset is traded, and investors meet market makers with an intensity that depends on the sophistication of the investor. A sophisticated (institutional) investor has a high search intensity while an unsophisticated (retail) investor has a low intensity. Investors are randomly hit by liquidity shocks, and gains from trade arise when investors needing liquidity sell to investors with no liquidity need through a market maker. Once an investor and market maker meet, they bargain over the terms of trade. A market maker immediately unloads a bond in the inter-dealer market and therefore has no inventory costs. There are two key differences between my model and that of DGP05. First, there are cross-sectional differences in the search intensities of investors. This cross-sectional variation leads to differences in prices of institutional investors vs. retail investors, key to identifying the relative number of investors looking to buy or sell.

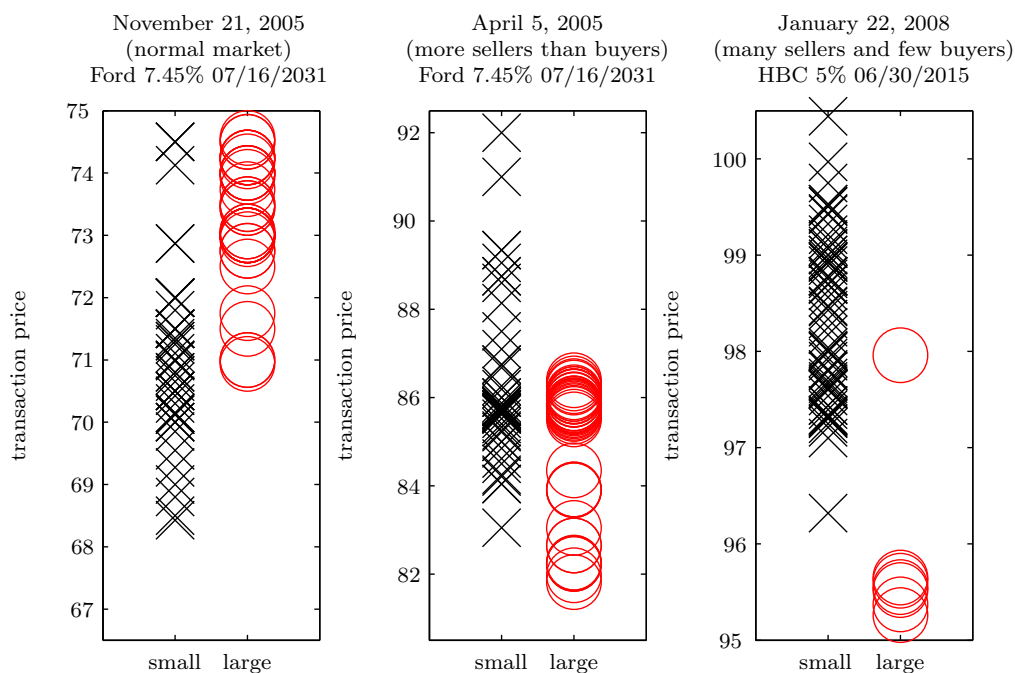
Second, I assume that the asset has a finite maturity, while DGP05 the asset is infinitely lived in DGP05. As assets mature, firms stand ready to issue new assets. The finite maturity of the asset allows me to study search costs at different times-to-maturity, and this turns out to have important implications.

If there is a shock in funding needs of investors, such that there are more sellers than buyers, the model predicts that mid-prices of institutional investors are lower than those of retail investors.<sup>3</sup> Sellers and buyers bargain over prices with marketmakers as intermediaries. With more sellers than buyers, prices are negotiated such that sellers are indifferent between selling or keeping the bond. These prices are lower than before the shock. When a shock occurs, I assume that institutional investors only trade with other institutional investors and likewise for retail investors. When they bargain, buyers are in a strong position and sellers in a weak position, because of the excess number of sellers. Institutional investors have better outside options compared to retail investors, because they can find new counterparties more quickly. The combination of strong buyers and stronger outside options for institutional investors relative to retail investors, leads to a stronger price impact of shocks on institutional bid and ask prices compared to those of retail investors. As an example, we look at Figure 1. The left-hand graph shows how the mid-price of institutional investors is higher than that of retail investors in a normal market. The middle graph illustrates how mid-prices of institutional investors are lower than those of retail investors in a market with more sellers than buyers. The right-hand graph shows how mid-prices of institutional investors are markedly lower than prices of retail investors in a day of crisis.

I derive equilibrium prices and calibrate the parameters of the model using more than 1.5 million corporate bond transactions from the TRACE database for the period October 2004 to September 2008. There are three key ingredients in the estimation. First, roundtrip costs are measured using 'unique roundtrip trades' (URT). If two or three trades in a given bond with the same volume happens within 15 minutes and there are no other trades with the same volume on that day, the transactions are part of a URT. The intuition is that either a customer sells to a dealer who sells it to another customer, or a customer sells to a dealer, who sells it to another dealer, who ultimately

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<sup>3</sup>For an in-depth discussion of funding liquidity, and a model that links market liquidity and trader's funding liquidity, see Brunnermeier and Pedersen (2008).



**Figure 1:** *Small and large trades on a normal day, day of distress, and day of crisis.* This graph shows three examples of all trades for a bond during a day smaller than \$1,000,000 (marked with crosses) and trades of at least \$1,000,000 (marked with circles).

sells it to a customer. I assume the maximal price in a URT is the ask price and the minimal price the bid price. This measure of transaction costs is most closely related to Green, Hollifield, and Schürhoff (2007)'s 'immediate trades', but their measure requires information about the sell and buy side, which is not available in TRACE. Second, I estimate the search intensity of investors who differ in their degree of sophistication by letting trade size proxy for investor type. Third, I fit the model to demeaned yields-to-maturity. Yields are less affected by the coupon of a bond than the price, so fitting to yields rather than prices simplifies the estimation. More importantly, in a given bond on a given day, I define a demeaned bid (ask) yield as a transaction bid (ask) yield part of a URT minus the average yield across all bid and ask yields part of URTs on that day. By fitting to demeaned yields, I focus on the size of bid-ask spreads and the relation between bid and ask yields for different investors.

The outline of the paper is as follows. Section 2 contains the model and my theoretical predictions. Section 3 describes the transaction data and the estimation methodology. Section 4 reports the estimated model and my main empirical findings. Section 5 concludes.

## 2 A search model for the OTC corporate bond market

This section sets up a search model similar to the model in Duffie, Gârleanu, and Pedersen (2005) with two differences that reflect the corporate bond market. First, the asset has a finite maturity since bonds mature and become liquid at maturity. Second, a corporate bond can default and the asset in my model reflects this.

I first derive equilibrium bid and ask prices facing investors with different levels of sophistication in finding trading partners. I then define a liquidity shock as a shock increasing the number of sellers relative to the number of buyers and find how such a shock affects prices. Finally, I derive how a liquidity shock depresses prices faced by sophisticated investors *more* than prices faced by unsophisticated investors.

### 2.1 Model

The economy is populated by two kinds of agents, investors and market makers, who are risk-neutral and infinitely lived. They consume a nonstorable consumption good used as numeraire and their time preferences are given by the discount rate  $r > 0$ .

Investors have access to a risk-free bank account paying interest rate  $r$ . The bank account can be viewed as a liquid security that can be traded instantly. To rule out Ponzi schemes, the value  $W_t$  of an investor's bank account is bounded from below. In addition, investors have access to an over-the-counter corporate bond market for a credit-risky bond paying coupons at the constant rate of 1 unit of consumption per year. The bond has maturity  $T$  and a face value  $F$ , meaning that it matures randomly with constant intensity  $\lambda_T = 1/T$  and pays  $F$  at maturity. The bond defaults with intensity  $\lambda_D$  and pays a fraction  $(1 - f)F$  of face value in default. A bond trade can only occur when an investor finds a market maker in a search process that will be described in a moment.

Investors can hold at most 1 unit of the bond and cannot short-sell. Because agents are risk-neutral, investors hold either 0 or 1 unit of the bond in equilibrium. Investors are heterogeneous in two aspects. First, an investor is of type "high" or "low". The "high" type has no holding cost when owning the asset while the "low" type has a holding cost of  $\delta$  per time unit. The holding cost can be interpreted as a funding

liquidity shock that has hit the investor. An investor switches from "low" to high" with intensity  $\lambda_u$  and from "high" to "low" with intensity  $\lambda_d$  and the switching processes are for all investors pair-wise independent. Second, investors differ in the ease with which they find counterparties to trade with. A sophisticated investor quickly finds a trading partner while an unsophisticated investor spends considerable time finding someone to trade with. There are  $I$  levels of sophistication, which is made precise next.

I assume that there is a unit mass of independent non-atomic market makers who maximize profits. An investor with level of sophistication  $i, i \in \{1, 2, \dots, I\}$  meets a market maker with intensity  $\rho^i$ , which can be interpreted as the sum of the intensity of market makers' search for investors and investors' search for market makers. The search intensity is observable to market makers. Without loss a generality I assume that  $\rho^i < \rho^j$  when  $i < j$ , implying that investors with intensity  $\rho^1$  are the most unsophisticated and those with intensity  $\rho^I$  are the most sophisticated. When an investor and a market maker meet, they bargain over terms of trade. The bargaining will be described in the next section. Market makers immediately unload their positions in an inter-dealer market, so they have no inventory at any time.

I assume that bonds sold by an "i"-investor, can only be bought by an "i"-investor. The motivation for this assumption is that a notional amount of say \$5,000 of a corporate bond sold by a retail investor is likely (through the market maker) to be bought by another retail investor. Likewise a notional amount of say \$5,000,000 sold by an institutional investor is likely to (through the market maker) to be bought by another institutional investor.<sup>4</sup>

The set of investors is  $\Gamma = \{ho^i, hn^i, lo^i, ln^i\}_{i=1}^I$  where  $h/l$  refers to the "high"/"low" type and  $o/n$  to owner/non-owner. There is a continuum of investors and for investors with sophistication level  $i$ ,  $\mu_\sigma^i(t)$  denotes the fraction at time  $t$  of type  $\sigma$ . Since the fractions of "i"-investors add to 1 at any point in time we have

$$\mu_{ho}^i(t) + \mu_{hn}^i(t) + \mu_{lo}^i(t) + \mu_{ln}^i(t) = 1$$

for every  $t$  and any  $i = 1, \dots, I$ . There is also a continuum of identical credit-risky firms who issue bonds. If a firm defaults it is replaced by an identical new firm. They issue

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<sup>4</sup>This assumption can be relaxed when deriving equilibrium prices, but becomes important when examining prices during liquidity shocks.

bonds through market makers when there is an excess demand in the market, i.e. when  $\mu_{hn}^i(t) > \mu_{lo}^i(t)$  for some  $i$ , but the amount they issue is limited by the speed at which market makers meet investors.

## 2.2 Search equilibrium

The model is solved in two steps. First, asset allocations are determined. This is possible without reference to prices because only low-type owners are sellers and high-type non-owners are buyers, and when a low-type owner or high-type non-owner meets a market maker trade occurs immediately. That trade occurs immediately from bargaining theory [Rubinstein (1982)]. In the following,  $i$  defining the level of sophistication is suppressed since the arguments are the same for  $i = 1, \dots, I$ .

The rate of change of mass  $\mu_{lo}(t)$  of low-type owners is

$$\dot{\mu}_{lo}(t) = -\rho\mu_m(t) - (\lambda_T + \lambda_D)\mu_{lo}(t) - \lambda_u\mu_{lo}(t) + \lambda_d\mu_{ho}(t) \quad (1)$$

where  $\mu_m(t) = \min\{\mu_{hn}(t), \mu_{lo}(t)\}$ . Market makers buy from  $lo$  investors and sell to  $hn$  investors instantly through the inter-dealer market. If a  $lo$  investor meets a market maker with intensity  $\rho$  and if  $\mu_{lo}(t) \leq \mu_{hn}(t)$  all meetings lead to a trade and the  $lo$  investor becomes a  $ln$  investor. However, if  $\mu_{lo}(t) > \mu_{hn}(t)$  not all meetings result in a trade. The first term  $-\rho\mu_m(t)$  in (1) reflects this fact. A  $lo$  investor becomes a  $ln$  investor if the owned bond either matures or defaults, and the second term in (1),  $-(\lambda_T + \lambda_D)\mu_{lo}(t)$ , reflects this. The third term is present because  $lo$  investors switch type to  $ho$  with intensity  $\lambda_u$ , and the last term is due to investors switching from type  $ho$  to  $lo$ .

Derivations of the rates of change of mass of the other investor types are very similar. The only slight difference is that although sellers might be rationed, buyers are not. If there are not enough sellers, firms step in and issue more debt. The rates of change are therefore given as

$$\dot{\mu}_{ho}(t) = \rho\mu_{hn}(t) - (\lambda_T + \lambda_D)\mu_{ho}(t) + \lambda_u\mu_{lo}(t) - \lambda_d\mu_{ho}(t) \quad (2)$$

$$\dot{\mu}_{hn}(t) = -\rho\mu_{hn}(t) + (\lambda_T + \lambda_D)\mu_{ho}(t) + \lambda_u\mu_{ln}(t) - \lambda_d\mu_{hn}(t) \quad (3)$$

$$\dot{\mu}_{ln}(t) = \rho\mu_m(t) + (\lambda_T + \lambda_D)\mu_{lo}(t) - \lambda_u\mu_{ln}(t) + \lambda_d\mu_{hn}(t), \quad (4)$$



and in Appendix A the steady state mass of the four investor types are given, that is the state at which the masses are constant.

## 2.3 Equilibrium Prices

Next, we determine the prices that prevail in steady state: a) the bid price  $B_t$  at which investors sell to market makers, b) the ask price  $A_t$  at which investors buy from market makers, and c) the inter-dealer price. Each investor's utility for future consumption depends only on his current type  $\sigma(t) \in \Gamma$  and wealth  $W_t$  in his bank account. Lifetime utility is

$$U(W_t, \sigma(t)) = \sup_{C, \theta} E_t \int_t^\infty e^{-rs} dC_{t+s} \quad (5)$$

subject to

$$dW_t = rW_t dt - dC_t + \theta_t(1 - \delta 1_{\{\sigma^\theta(t)=lo\}})dt - \hat{P}_t d\theta_t \quad (6)$$

where  $C$  is a cumulative consumption process,  $\theta_t \in \{0, 1\}$  is a feasible holding process,  $\sigma^\theta$  is the type process induced by  $\theta$ , and at the time of a possible holding change,  $\hat{P}_t \in \{A_t, B_t, F, (1-f)F\}$  is the "trade price". From (5) and (6) we have that lifetime utility is  $W(t) + V_{\sigma(t)}$  where

$$V_{\sigma(t)}(t) = \sup_{\theta} E_t \left[ \int_t^\infty e^{-r(t-s)} \theta_s (1 - \delta 1_{\{\sigma^\theta(s)=lo\}}) ds - e^{-r(s-t)} \hat{P}_s d\theta_s \right].$$

The appendix shows that the value functions satisfy the equations

$$\dot{V}_{ln} = rV_{ln} - \lambda_u(V_{hn} - V_{ln}) \quad (7)$$

$$\dot{V}_{hn} = rV_{hn} - \lambda_d(V_{ln} - V_{hn}) - \rho(V_{ho} - V_{hn} - A) \quad (8)$$

$$\begin{aligned} \dot{V}_{lo} = rV_{lo} - \lambda_u(V_{ho} - V_{lo}) - \rho(B + V_{ln} - V_{lo}) - \lambda_T(F + V_{ln} - V_{lo}) \\ - \lambda_D((1-f)F + V_{ln} - V_{lo}) - (1 - \delta) \end{aligned} \quad (9)$$

$$\begin{aligned} \dot{V}_{ho} = rV_{ho} - \lambda_d(V_{lo} - V_{ho}) - \lambda_T(F + V_{hn} - V_{ho}) \\ - \lambda_D((1-f)F + V_{hn} - V_{ho}) - 1 \end{aligned} \quad (10)$$

A low-owner investor is willing to sell to a market maker if the price is at least  $\Delta V_l = V_{lo} - V_{ln}$  while a high-nonowner is willing to buy from a market maker if the price is no more than  $\Delta V_h = V_{ho} - V_{hn}$ . Likewise, the market maker is willing to buy if the price is

no more than the inter-dealer price  $M$  (at which he immediately unloads the bond in the inter-dealer market), and willing to sell if the price is no less than  $M$ . If sellers are not constrained the bid price  $B$  can be any price between seller's reservation value  $\Delta V_l$  and the inter-dealer price  $M$ . In contrast, if sellers are constrained the bid price is seller's reservation value, since sellers must be indifferent to trading or not if some trade and others do not. Buyers are per construction not constrained since firms issue bonds if there is a demand, and the ask price  $A$  is between buyer's reservation value  $\Delta V_h$  and the inter-dealer price  $M$ . If sellers are not constrained, Nash bargaining between investor and market maker leads to bid and ask prices

$$A = \Delta V_h z + M(1 - z) \quad (11)$$

$$B = \Delta V_l z + M(1 - z) \quad (12)$$

where  $z$  is the bargaining power of the market maker. Duffie, Gârleanu, and Pedersen (2007) show that a bargaining outcome of this kind can be justified by an explicit bargaining procedure.

If sellers are constrained the bid  $B$  and inter-dealer price  $M$  equals the sellers' reservation price  $\Delta V_l$ . If they are not constrained, any inter-dealer price between  $\Delta V_l$  and  $\Delta V_h$  can be an outcome in the model, and we let  $M = \tilde{q}\Delta V_h + (1 - \tilde{q})\Delta V_l$  where  $0 \leq \tilde{q} \leq 1$ . If  $\tilde{q}$  is close to one, the ask price is close to the reservation value of the buyer and the market can be interpreted as a sellers' market, while a  $\tilde{q}$  close to zero can be interpreted as a buyers' market. In case  $\tilde{q}$  is high, the seller has the most gain and vice versa for a low  $\tilde{q}$ .

A simple example illustrates the effect of  $z$  and  $\tilde{q}$ . Consider a buyer who is willing to buy at a maximal price of  $\Delta V_h = 103$ , a seller who is willing to sell at a minimal price of  $\Delta V_l = 100$ , and a market maker who is intermediating the trade. The gains of trade of  $103 - 100 = 3$  is to be split between the three agents. Assume that  $z = 0.5$  and  $\tilde{q} = 0.75$ . The inter-dealer price is  $M = 0.75 * 103 + 0.25 * 100 = 102.25$ , the bid price is  $100 * 0.5 + 102.25 * 0.5 = 101.125$ , and the ask price is  $103 * 0.5 + 102.25 * 0.5 = 102.625$ . The dealer gains the bid-ask spread  $102.625 - 101.125 = 1.5$  which is equal to  $z * 3$ , so  $z$  measures the part of the gain the dealer is receiving. The seller gains  $101.125 - 100 = 1.125$  while the buyer gains  $103 - 102.625 = 0.375$ , so  $\tilde{q}$  determines how the rest of the gain is split between buyer and seller. Consider now  $\tilde{q} = 0.25$ . In this case the bid price

is 100.375 and the ask price 101.875. The bid-ask spread is the same as before, but bid and ask prices are lower.

The following theorem states the equilibrium bid and ask prices in the economy, and a proof is given in the Appendix. Since I assume that a bond sold by an "i"-investor can only be bought by an "i"-investor, the interdealer prices for bonds traded between investors of different levels of sophistication could be different. However, I assume that the interdealer price is the same for all bonds. Alternatively, I could relax the assumption that bonds can only be traded (through a marketmaker) between "i"-investors. In this case it follows automatically that there is only one interdealer price and the results in the following theorem holds as well.

**Theorem 2.1. (Prices in equilibrium).** *Assume that there are  $I$  investors differing in their search intensities  $\rho_1, \dots, \rho_I$ . The bid  $B^i$  and ask  $A^i$  prices for investor  $i$  with search intensity  $\rho^i$  are given as*

$$\begin{aligned} A^i &= \Delta V_h^i z + M(1 - z) \\ B^i &= \Delta V_l^i z + M(1 - z) \end{aligned}$$

where

$$\begin{aligned} \Delta V_l^i &= \frac{\rho^i(1 - z)M + \lambda_T F + \lambda_D(1 - f)F + 1}{r + (1 - z)\rho^i + \lambda_T + \lambda_D} \\ &\quad - \frac{\delta(r + \lambda_T + \lambda_D + \lambda_d + (1 - z)\rho^i)}{(r + \lambda_T + \lambda_D + \rho^i(1 - z))(\lambda_u + \lambda_d + \rho^i(1 - z) + r + \lambda_T + \lambda_D)} \\ \Delta V_h^i &= \Delta V_l^i + \frac{\delta}{r + (1 - z)\rho^i + \lambda_T + \lambda_D + \lambda_d + \lambda_u} \\ M &= \frac{1 + \lambda_T F + \lambda_D(1 - f)F}{r + \lambda_T + \lambda_D} - \frac{\delta(\lambda_d + (1 - \tilde{q})[r + (1 - z)\rho_0 + \lambda_T + \lambda_D])}{(r + \lambda_T + \lambda_D)(r + (1 - z)\rho_0 + \lambda_d + \lambda_u + \lambda_T + \lambda_D)} \end{aligned}$$

and  $\rho_0 = \min(\rho_1, \dots, \rho_I)$  and  $0 \leq \tilde{q} \leq 1$ .

As an obvious but important consequence of the Theorem we have the following Corollary.

**Corollary 2.1.** *The bid-ask spread for investor  $i$  with search intensity  $\rho_i$  is given as*

$$A^i - B^i = \frac{z\delta}{\lambda_u + \lambda_d + \rho^i(1 - z) + r + \lambda_T + \lambda_D} \quad (13)$$

There are several interesting implications regarding bid-ask spreads at which investors trade at. First, bid-ask spreads decrease with the level of investor sophistication  $\rho$ . Sophisticated investors trade at tight bid-ask spreads while unsophisticated investors

trade at wide bid-ask spreads. Prices are determined through a bargaining process. The threat of cutting off negotiations and finding another trading partner is stronger for sophisticated investors (they find a new trading partner more easily) and therefore they negotiate tighter bid-ask spreads. This insight is also discussed in Duffie, Gârleanu, and Pedersen (2005). Second, bid-ask spreads for a given issuer decrease in the maturity of the bond  $1/\lambda_T$ . When a bond matures, it becomes liquid, and this influences the bargaining power of investors. A seller is almost indifferent between selling or not, since the bond will soon be converted to cash. A buyer is almost indifferent between buying or not, since the buyer will have to search for a new bond to buy when the bond matures. Therefore, as maturity goes to zero, so does the bid-ask spread. Third, the bid-ask spread is decreasing in the default probability of the issuer. The reason for this is the same as that of maturity; as default intensity increases, the firm is more likely to default and a fraction of the notional of the bond is repayed. Thus, the bond becomes "liquid" at default and is similar to a bond with short maturity.

Next, I define a liquidity shock to investors. I assume that the fractions of investors are in steady state and a sudden liquidity shock occurs. If a shock of size  $0 \leq s \leq 1$  occurs a "high"-investor (no liquidity need) becomes a "low"-investor (liquidity need) with probability  $s$ :

**Definition 2.1. (Liquidity shock).** *Assume that the fractions of types of investors with search intensity  $\rho$  are in steady state, denoted  $\mu_{ho}^{ss}, \mu_{hn}^{ss}, \mu_{lo}^{ss}$ , and  $\mu_{ln}^{ss}$ . When a liquidity shock of size  $0 < s \leq 1$  occurs, any high-investor becomes a low-investor with probability  $s$ . The fractions of types immediately after the shock are  $\mu_{ho}(s) = (1-s)\mu_{ho}^{ss}$ ,  $\mu_{hn}(s) = (1-s)\mu_{hn}^{ss}$ ,  $\mu_{lo}(s) = \mu_{lo}^{ss} + s\mu_{ho}^{ss}$ , and  $\mu_{ln}(s) = \mu_{ln}^{ss} + s\mu_{hn}^{ss}$ .*

Prices following a liquidity shock are given in the following theorem and a proof is in the Appendix.

**Theorem 2.2. (Prices after a liquidity shock).** *Assume that a liquidity shock of size  $0 < s \leq 1$  occurs to all investors type and that  $\rho^i + \lambda_T + \lambda_D > \lambda_d + \lambda_u$ . If  $s \leq \frac{\lambda_T + \lambda_D}{\rho^i + \lambda_T + \lambda_D}$  bid and ask prices do not change after the liquidity shock. If  $s > \frac{\lambda_T + \lambda_D}{\rho^i + \lambda_T + \lambda_D}$  bid and ask prices immediately after the shock are*

$$\begin{aligned} B^i(s) &= e^{-t_s^i(r+\lambda_D+\lambda_T)} \Delta V_l^i + (1 - e^{-t_s^i(r+\lambda_D+\lambda_T)}) \Delta V_l^{i,imb} \\ A^i(s) &= B + \frac{z\delta}{\lambda_u + \lambda_d + \rho^i(1-z) + r + \lambda_T + \lambda_D} \end{aligned}$$

where  $\Delta V_l^i$  is given in Theorem 2.1,

$$\Delta V_l^{i,imb} = \frac{1 + \lambda_T F + \lambda_D(1-f)F}{r + \lambda_T + \lambda_D} - \frac{\delta(r + \lambda_T + \lambda_D + \lambda_d + (1-z)\rho^i)}{(r + \lambda_T + \lambda_D)(\lambda_u + \lambda_d + \rho^i(1-z) + r + \lambda_T + \lambda_D)},$$

and  $t_s^i$  is the unique solution to

$$0 = 1 - se^{-(\lambda_u + \lambda_d)t_s^i} - \frac{\rho^i}{\rho^i + \lambda_T + \lambda_D} e^{-(\lambda_T + \lambda_D)t_s^i}.$$

The reservation price  $\Delta V_l^{i,imb}$  in the theorem is the reservation price of an "i"-investor in a situation where there are more sellers than buyers at all points in the future. Thus, it is the price a seller would be willing to sell at, if she knew that she would be constantly constrained. The theorem shows that seller's reservation price is a linear combination of this reservation price and that in equilibrium where sellers are not constrained. We have that  $\Delta V_l^i > \Delta V_l^{i,imb}$  and the weight on  $\Delta V_l^{i,imb}$  depends on the amount of time that sellers are constrained after the shock. The larger the shock  $s$  is, the longer the period of constrained sellers is, and the lower the prices following the shock are.

From the theorem we also see that bid-ask spreads after the shock is the same as those in equilibrium, which is important result and stated in the theorem below. Thus, while prices change following a shock, bid-ask spreads do not. The decrease in prices facing unsophisticated and sophisticated investors is not the same. As the next theorem shows, the midprice (i.e. average of bid and ask price) facing sophisticated investors decreases *more* than the midprice facing unsophisticated investors.

**Theorem 2.3. (Relation between prices after a liquidity shock).** *If a liquidity shock of size  $0 < s \leq 1$  occurs to any two investors of type  $i$  and  $j$  where  $\rho_i < \rho_j$ , the following holds:*

1. *Bid-ask spreads of investors are not affected by the liquidity shock.*
2. *If  $\lambda_u > \frac{1-z}{z}\lambda_d$  and  $\rho_i$  and  $\rho_j$  are sufficiently high<sup>5</sup>, midprices  $M(s) = \frac{1}{2}(A(s)+B(s))$  at which investors trade satisfy*
  - *for any  $0 < s \leq 1$  we have  $M_i(s) - M_j(s) \geq M_i(0) - M_j(0)$ .*
  - *For any  $\frac{\lambda_T + \lambda_D}{\min\{\rho_i, \rho_j\} + \lambda_T + \lambda_D} < s \leq 1$   $M_i(s) - M_j(s)$  is increasing in  $s$ .*

The theorem shows that for any size of liquidity shock the difference between the midprice of unsophisticated investors minus the midprice of sophisticated investors is higher or equal to the difference in equilibrium. It also shows that if shocks are above

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<sup>5</sup>The precise condition for  $\rho_i$  and  $\rho_j$  being "sufficiently high" is that  $(1-z)\rho\lambda_u(\rho + \lambda_T + \lambda_D) - (C + \lambda_u + \lambda_d)^2(\lambda_u + \lambda_d) > 0$ . For realistic parameters this is trivially satisfied except for very short-term bonds (say a maturity of one month or less). For these bonds, prices are not affected by liquidity shocks unless the shock is large anyway.

a minimum threshold, the difference is a monotonously increasing function of the shock size. This threshold is for most cases quite small. For a 10-year bond where the default intensity of the issuing bond is 3%, and the most unsophisticated investor has a search intensity of  $\rho = 40$ , this threshold is a shock size of 0.3%. However, as the maturity of the bond goes to zero ( $\lambda_T \rightarrow \infty$ ), the shock size needed to move prices goes to 1. This underscores the importance of taking into account maturity when modelling search costs in corporate bonds. Not only do bid-ask spreads become smaller as maturity decreases, also the minimum size of a liquidity shock needed to move prices becomes larger.

### 3 Estimation Methodology

Corporate bond transactions data only recently became available on a large scale. Since January 2001 FINRA<sup>6</sup> members are required to report their secondary over-the-counter corporate bond transactions through TRACE (Trade Reporting and Compliance Engine). Because of the uncertain benefit to investors of price transparency not all trades reported to TRACE were initially disseminated at the launch of TRACE July 1, 2002. The first research papers using TRACE transactions data focus on the effect of enhanced price transparency and find that dissemination of prices lowered transaction costs for investors (Edwards et al. (2007), Goldstein et al. (2007), and Bessembinder et al. (2006)). The dissemination starts in July 1, 2002 with dissemination of a small subset of trades and from October 1, 2004 all trades are disseminated. Trades must be reported within 15 minutes as of July 1, 2005<sup>7</sup>. TRACE covers all trades in the secondary over-the-counter market for corporate bonds and accounts for more than 99% of the total secondary trading volume in corporate bonds. The only trades not covered by TRACE are trades on NYSE which are mainly small retail trades.

I use a sample of non-callable, non-convertible, straight coupon bullet bonds with maturity less than 30 years from October 1, 2004 to September 30, 2008. For each bond the rating from Moody's, Standard & Poors, and Fitch are downloaded from Bloomberg and an average rating is calculated. The rating from Bloomberg is the rating at the end of the sample period. For a bond to be included in the sample it must have a rating

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<sup>6</sup>The Financial Industry Regulatory Authority formerly named National Association of Security Dealers (NASD).

<sup>7</sup>This requirement has gradually been tightened from 1 hour and 15 minutes to 15 minutes. In practice 80% of all transactions are reported within 5 minutes.

from at least one of the rating agencies. The initial sample of bonds in TRACE is 35,124 and the final sample contains 9,532 bonds. For these bonds I collect the trading history from TRACE covering the period from October 1, 2004 to September 30, 2008 and after filtering out erroneous trades 7,272,454 trades are left. I use the filter in Dick-Nielsen (2009) to filter out the error trades.

To estimate the search model outlined in the previous section I need an estimate of roundtrip costs in the dealer market, i.e. the difference between the price at which a dealer sells a bond to a customer and the price at which a dealer buys a bond from a customer. Two main approaches to estimate roundtrip costs exist in the literature. The first is on a given day to average sell prices and subtract average buy prices (Hong and Warga (2000) and Chakravarty and Sarkar (2003)). The second is a regression-based methodology where each transaction price is regressed on a benchmark price and a buy/sell indicator (Bessembinder et al. (2006), Goldstein et al. (2007), and Edwards et al. (2007)). However, both approaches require a buy/sell indicator for each trade, which is not publicly available.

The methodology for estimating roundtrip costs in this paper is based on *unique roundtrip trades* (URT). For a given bond on a given day, if there are exactly 2 or 3 trades for a given volume, I define them to be part of a URT. The intuition is that either 1) a customer sells a bond to a dealer who sells it to another customer, or 2) a customer sells a bond to a dealer, who sells it to another dealer, who ultimately sells it to a customer. For a URT the roundtrip cost in terms of prices is defined as the maximal price minus the minimal price. URTs are closely related to Green, Hollifield, and Schürhoff (2007)'s "immediate matches". An "immediate match" is a pair of trades where a buy from a customer is followed by a sale to a customer in the same bond for the same par amount on the same day with no intervening trades in that bond. However, since there is no information about the sides in the transactions in the TRACE database, "immediate trades" cannot be calculated. Also, URTs allow intervening trades, but restrict the number of trades with the same par value to three. In the model in Section 2 dealers do not have an inventory and therefore I restrict the sample to URTs where the trades occur within 15 minutes. Of the 7,272,454 trades in the full sample, 1,593,052 are part of a 15 minutes URT resulting in a 717,826 URTs.

To assess the accuracy of estimated transaction costs when using URTs, Table 1

compares URT estimates of roundtrip costs with those in Edwards et al. (2007) who use a regression-based methodology and have additional buy/sell information that is not publicly available. For this table only the sample period is January 2003-January 2005 in order to match the period in Edwards et al. (2007). The roundtrip costs using unique roundtrip trades executed within 15 minutes are lower than in Edwards et al. (2007). For small trades the costs are about 60% lower while for large trades the costs are around 10% lower. While transaction costs based on 15 minutes URTs likely reflect only search costs, transaction costs for roundtrips for a longer period of time might also reflect inventory costs and/or asymmetric information and thus be higher. Therefore, the table also shows roundtrip costs based on URTs that occur within the same day, and the cost estimates based on these trades are indeed higher and match those in Edwards et al. (2007) fairly well<sup>8</sup>. The roundtrip costs for trade sizes of 100,000 are the same, small trades are underestimated by around 25%, while large trades are overestimated by roughly 10%. The deviations in trading costs can possibly be explained by differences in the sample of bonds. Overall, the numbers in the table suggest that transaction costs based on URTs are reasonable.

While transaction costs are most often measured in percentage of the price, liquidity premia in corporate bonds are usually measured in terms of yields (Huang and Huang (2003), Longstaff et al. (2005), and Covitz and Downing (2007) among others). In order to be consistent with this literature, I will in the empirical section measure roundtrip costs in terms of yield-to-maturity. This has the additional advantage that it is less relevant to include information about the coupon of a bond, since size of coupon has a smaller effect on yield-to-maturity than on prices. Roundtrip cost defined in terms of yields is the yield-to-maturity based on the dealer's buying price minus the yield-to-maturity based on the dealer's selling price.

Next, I explain the maximum likelihood procedure used to estimate model parameters and how yields are demeaned before estimation. Furthermore, I describe how investor sophistication is proxied by trade size, and finally identification of the parameters is

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<sup>8</sup>Another possible explanation for higher estimated costs for all-day URTs compared to 15 minutes URTs is that all-day URTs are upward biased because prices move during the day. However, Green, Hollifield, and Schürhoff (2007) find that transaction costs for "immediate trades" in the municipal bond market are the same whether or not they control for market movements, so this explanation is unlikely to cause the difference between 15 minutes and all-day URT costs.



discussed.

One of the predictions of the search model in Section 2 is that investors with the different degrees of sophistication have different roundtrip costs. Thus, it is tempting to find the parameters of the model by fitting actual roundtrip costs to fitted roundtrip costs. However, the model also predicts that the dispersion of bid yields for different type investors might be very different from the dispersion of ask yields for those investors, so there is important information lost if only roundtrip costs are fitted and instead I fit the model to *demeaned yields*. Any bid or ask yield for a given bond on a given day is demeaned with the average of all bid and ask yields for this bond on this day. That is, if there are  $N_{tb}$  URTs on bond  $b$  on day  $t$ , and  $A_{tbi}$  is the  $i$ 'th ask yield and  $B_{tbi}$  the corresponding bid yield, the demeaned ask yield is defined as  $A_{tbi} - \overline{AB}_{tb}$  and demeaned bid yield as  $B_{tbi} - \overline{AB}_{tb}$  where  $\overline{AB}_{tb} = \frac{1}{2N_{tb}} \sum_{i=1}^{N_{tb}} (A_{tbi} + B_{tbi})$ .

For day  $t$  and bond  $b$  all demeaned bid and ask yields are denoted  $y_{tb}^1, y_{tb}^2, \dots, y_{tb}^{2N_{tb}-1}, y_{tb}^{2N_{tb}}$  (the sorting does not matter). The demeaned fitted yields are denoted  $\hat{y}_{tb}^1, \hat{y}_{tb}^2, \dots, \hat{y}_{tb}^{2N_{tb}-1}, \hat{y}_{tb}^{2N_{tb}}$  and they are calculated using the prices in Theorem 2.1. I assume that each fitting error is normally distributed with zero mean and a standard deviation that depends on the maturity of the bond

$$\begin{aligned} y_{tb}^i - \hat{y}_{tb}^i &\sim N(0, w_{tb}\sigma^2), \\ w_{tb} &= \frac{1}{\min(1, T_{tb}^2)}, \end{aligned}$$

where  $T_{tb}$  is the maturity of bond  $b$  on day  $t$ . For a bond with maturity shorter than one year, the standard deviation is increasing as the time-to-maturity shortens. This is due to the fact that small price deviations in bonds that are about to mature lead to large changes in yields, and the standard deviation scaling avoids that too much weight is put on transactions on bonds with maturity less than one year. We then have that

$$\epsilon_{tb}^i = \frac{y_{tb}^i - \hat{y}_{tb}^i}{\sqrt{w_{tb}}} \sim N(0, \sigma^2).$$

I define  $\Theta$  as a vector with the parameters of the model and assume that errors are independent, such the likelihood function is given as

$$L(\Theta, \sigma | Y) = \prod_{t=1}^T \prod_{b=1}^{N_b} \prod_{i=1}^{2N_{tb}} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(\epsilon_{tb}^i)^2}{2\sigma^2}\right)$$

where  $N_b$  is the number of bonds in the sample. We therefore have that

$$-2 \log L(\Theta, \sigma | Y) = \frac{1}{\sigma^2} \sum_{t=1}^T \sum_{b=1}^{N_b} \sum_{i=1}^{2N_{tb}} (\epsilon_{tb}^i)^2 + \sum_{t=1}^T \sum_{b=1}^{N_b} \sum_{i=1}^{2N_{tb}} [\log(\sigma^2) + 2\pi] \quad (14)$$

and maximizing the likelihood function therefore amounts to minimizing the sum of squared weighted errors  $\sum_{t=1}^T \sum_{b=1}^{N_b} \sum_{i=1}^{2N_{tb}} (\epsilon_{tb}^i)^2$ . Standard errors are calculated using the outer product of gradients estimator.

The Appendix shows that the sum of squared errors in (14) equals

$$\sum_{t=1}^T \sum_{b=1}^{N_b} w_{tb} \left[ \sum_{i=1}^{N_{tb}} [(A_{tbi} - \bar{A}_{tb}) - (A_{tbi}^M - \bar{A}_{tb}^M)]^2 \right] \quad (15)$$

$$+ \sum_{i=1}^{N_{tb}} [(B_{tbi} - \bar{B}_{tb}) - (B_{tbi}^M - \bar{B}_{tb}^M)]^2 \quad (16)$$

$$+ \frac{N_{tb}}{2} [(\bar{A}_{tb} - \bar{B}_{tb}) - (\bar{A}_{tb}^M - \bar{B}_{tb}^M)]^2. \quad (17)$$

where the superscript  $M$  refers to fitted yields,  $\bar{A}_{tb} = \frac{1}{N_{tb}} \sum_{m=1}^{N_{tb}} A_{tbm}$ , and  $\bar{B}_{tb} = \frac{1}{N_{tb}} \sum_{m=1}^{N_{tb}} B_{tbm}$ . This shows that the fitting procedure of fitting bid and ask yields demeaned by the average mid-yield can be regarded as fitting ask prices demeaned by average ask price, bid prices demeaned by average bid price, and the average bid-ask spread. In particular, if there is exactly one observation for each bond on each day ( $N_{tb} = 1$  for all  $t$  and  $b$ ) the expression (15) reduces to  $\sum_{t=1}^T \sum_{b=1}^{N_b} w_{tb} \frac{1}{2} [(A_{tb} - B_{tb}) - (A_{tb}^M - B_{tb}^M)]$ , i.e. fitting bid-ask spreads.

Investors in the search model presented in Section 2 have different speeds with which they meet a market maker. Unsophisticated investors have a low search intensity while sophisticated have a high search intensity. I use trade size as a proxy for investor sophistication. Specifically, there are six investor classes, who differ in their search intensity  $\rho$ , and they trade in par values of \$0–5,000, \$5,000–20,000, \$20,000–100,000, \$100,000 – 500,000, \$500,000 – 1,000,000, and more than \$1,000,000. In addition, I split the bonds up in three rating classes,  $AAA - A$ ,  $BBB$ , and speculative grade. The bonds differ in their default intensity  $\lambda_D$ .

There are a number of parameters in the model and many of them are not separately identified. The following parameters are fixed before estimation. The riskless rate is set to  $r = 0.05$ , which is close to the average 10-year swap rate of 4.94% in the estimation period. The bond pays a dividend stream of one, and the face value of a bond is set to

$F = 20$ , such that it corresponds to a bond with par value 100 paying a coupon of 5%. The recovery rate on the bond in case of default is set to zero such that  $f = 1$  and the bargaining power of the market maker is 0.5. Finally,  $\lambda_d = 0.1$  and  $\lambda_u = 1$  such that an investor is a "high" type 91% of the time.

## 4 Empirical results

In this section I discuss parameter estimates and the ability of the model to fit actual yield roundtrip costs. Then, I examine time variations in search intensities and shocks to funding liquidity, and finally I assess the impact of search costs on average yields at different maturities.

### 4.1 Parameter estimates and model fit

Table 2 shows the parameter estimates. We see that search intensities increase as investor sophistication increases (proxied by trade size). Differences in roundtrip costs are main determinants of differences in search costs between investors, and roundtrip costs decrease in search intensity according to (13). Therefore, increasing search intensities are consistent with results in Chakravarty and Sarkar (2003), Bessembinder, Maxwell, and Venkaraman (2006) Edwards, Harris, and Piwowar (2007), who show that roundtrip costs decrease in trade size. This fact is evident in Panel A of Table 3 where actual and fitted roundtrip costs across trade sizes are shown. The model fits actual costs well except for the largest trade sizes, where costs are estimated to be larger than actual costs. The reason that search intensities of the most sophisticated investors are not higher, such that roundtrip costs in Panel A are fit better is due to a tension between fitting roundtrip costs and differences in roundtrip costs between large and small investors. A larger search intensity for institutional investors lead to larger differences in midyield between institutional and retail investors investors shown in Panel D. In Panel B we see that roundtrip costs are fitted well across maturity.

The most unsophisticated investors (trading in sizes between 0 and \$5,000) have a search intensity of 47. This implies that they need a week on average before they find a market maker with whom to trade with. This can be viewed as the time it takes a non-professional to learn how to trade in the corporate bond market and keep up-to-date

about information relevant for trading. The most sophisticated investors (trading sizes of more than \$1,000,000) have a search intensity of 141 implying that it takes 1-2 days to complete trades of large size.

We see in the table that default intensities are increasing as credit quality decreases. The default intensity is 0.046 for AAA-A bonds, 0.076 for BBB bonds, and 0.145 for speculative grade bonds. For comparison, Longstaff, Mithal, and Neis (2005) report a default intensity of 0.0098 for AAA-A, 0.0288 for BBB, and 0.0956 for BB. Although default intensities in this paper are somewhat higher, they are in the same order of magnitude, and the higher estimates might reflect high default risk during the subprime crisis. Note that although the model is highly stylized and default intensity estimates are based on size of bid-ask spreads and not on level of yield spreads, the estimates are reasonable. It might be surprising that default intensities are well identified given that they affect bid-ask spreads in prices in a minor way since price spreads are given as  $\frac{z\delta}{\lambda_u + \lambda_d + \rho(1-z) + r + \lambda_T + \lambda_D}$  according to equation (13). In this expression, the effect of  $\lambda_D$  is small compared to  $\rho(1-z)$ . However, the default intensity effects the price level of bonds, and since parameters are estimated by fitting bid-ask spreads in yields, given by  $(A-B)\frac{1-F\lambda_T}{AB}$ , we see that default intensity affect bid-ask yield spreads through the denominator  $AB$ . Higher default intensity leads to lower prices which in turn causes higher bid-ask yield spreads. Panel C in Table 3 shows that the model captures variations in transaction costs well across ratings. Edwards et al. (2007) argue that the influence of credit risk and maturity on transaction costs might be due to adverse selection and inventory costs. The results in this paper show that taking into account the nonlinear relations between investors heterogeneity, maturity, and the risk of default, search costs can explain the variation. For example, the high average roundtrip cost in yields of 63 basis points for BB rated bonds partly reflects that more than 4% of the URTs in BB bonds are in bonds with less than 6 months to maturity while it is less than 1% for all other rating classes.

The parameter  $\tilde{q}$  is estimated to be close to 1, which implies that in a trade the gains of trade is split between seller and market maker, while the buyer receives very little. The identification of  $\tilde{q}$  is through two channels. The first is the size of bid-ask yield spreads. Price spreads in equation (13) are not affected by  $\tilde{q}$  but bid-ask yield spreads are, because prices are affected by  $\tilde{q}$  (similar to how default intensities affect bid-ask

yield spreads). The second channel is through the relation between bid-ask spreads of different investors. If  $\tilde{q} = 1$  seller receives the gains of trade relative to buyer and prices are high. In this case, both retail and institutional buyers are almost indifferent between trading or not, so they trade at similar prices, since their outside option - not to trade - have similar value. Sellers are eager to sell because of high prices, so in bargaining, the "threat" of buyers to cut off negotiations and let the seller wait until he meets another counterparty, is particularly strong for retail investors and they accept a lower price compared to the institutional investor. Overall, this means that mid-prices of retail investors are lower than mid-prices of institutional investors. If  $\tilde{q} = 0$  buyer receives the gains of trade relative to seller and prices are low. In this case sellers are almost indifferent between trading or not, because the price is so low that sellers might as well keep the bond and incur the extra cost of holding the bond. In contrast, buyers are eager to buy, which means that retail investors are in a worse bargaining position and buy at a higher price. In this case the mid-price of retail investors is higher than that of institutional investors. Both search intensities, default intensities, and  $\tilde{q}$  affect bid-ask spreads, while only  $\tilde{q}$  determine how bid-ask spreads of different investors are related to each other, so it is likely that the latter channel of identification is the dominant for determining the value of  $\tilde{q}$ . In Panel D in Table 3 we see that the model matches that roundtrip costs in yields decreases in maturity and that the midyield of large trades are smaller than midyields of small trades. The model cannot match the magnitude of large-small differences at maturities less than six months, and at longer maturities the model overestimates the average differences somewhat. The discrepancies between estimated and actual differences at longer maturities are partly due to periods of distress where the relation between small and large mid-prices are reversed, and we examine this more in detail in the next Section.

The liquid market for credit default swaps might be a reason for a  $\tilde{q}$  close to one. There is an approximate arbitrage relation between corporate bonds and credit default swaps: one can approximately create a bond with no default risk by buying a corporate bond and insuring against default by buying protection through a credit default swap (Duffie (1999), Longstaff, Mithal, and Neis (2005), and Blanco, Brennan, and Marsh (2005)). If corporate bond prices are low relative to credit default swaps, one can buy the bond and buy protection through a credit default swap contract and earn an

abnormal profit. If prices are high relative to credit default swaps, one needs to short the bond and sell protection through a credit default swap. It can be difficult and expensive to short the bond as shown in Nashikkar and Pedersen (2007), so the arbitrage is easier to carry out when corporate bond prices are low compared to when they are high.

## 4.2 Time variation in search intensities and shocks to funding liquidity

The parameter estimates of the calibration reflect the average market over the estimation period. However, the period has seen several significant events in the corporate bond market, evidence for example by the fact that the 10-year spread between highly rated AAA corporate bonds and Treasury bonds has ranged from a low of 64 basis points to a high of 200 basis<sup>9</sup>. To examine the importance of shifting conditions in the market, I estimate the time-variation in search intensities and possible shocks to the number of investors in need of liquidity.

To simultaneously assess time variation in search intensities and estimate a possible shock to the number of sellers, I do the following. For month  $m$ , I multiply all search intensities by  $c_m$  and assume a liquidity shock of size  $s_m$  as defined in Definition 2.1 occurs. Holding all other parameters fixed, I find the optimal values of  $c_m$  and  $s_m$  according to the likelihood procedure explained earlier. Prices are calculated according to Theorem 2.2. I do this for every month in the sample.

Search intensities are determined primarily by the size of roundtrip costs, so variations over time in search intensities measure the cost of buying and selling the bond. If there is a shock in funding needs of investors, such that there are more sellers than buyers, the model predicts that mid-prices of institutional investors are lower than those of retail investors. Bid and ask prices decrease in the size of the shock for both investor types, since the bargaining position of buyers improves relative to that of sellers, and bid and ask prices decrease more for institutional investors. Sellers are constrained and have no bargaining power, so prices are set such that they are indifferent between trading or keeping the bond. The outside option of waiting to buy is more valuable to institutional buyers when they bargain, since they can quickly find a new counterparty, so institu-

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<sup>9</sup>The yield spread is based on monthly data from the Federal Reserve's H.15 data. The AAA yield is "Moody's seasoned Aaa" while Treasury yield is the 10-year constant maturity rate.

tional buyers benefit more from the shock than retail buyers. Consequently, bid and ask prices of institutional investors decrease more in the shock size compared to bid and ask prices of retail investors. Thus, shocks to funding liquidity is detected through a higher than usual difference between retail mid-prices and institutional mid-prices, and for significant shocks retail mid-prices are higher than institutional mid-prices.

Since conditions in the corporate bond market might affect retail and institutional investors differently, I estimate the time variation in search intensities and liquidity shocks for each investor type separately. that is, I split the data sample up in trades of \$50,000 or less (retail) and more than \$50,000 (institutional) and carry out the aforementioned time series estimation on each data set.

Figure 4 graphs the estimated liquidity shocks for institutional and retail investors. For retail investors shocks occur in July and September of 2008. In this period Reuters /University of Michigan index of consumer sentiment recorded the lowest values in more than two decades. For institutional investors there are two periods with shocks occurring. The first is in 2005, where a shock starts in March 2005 and peaks in May 2005, whereafter the excess number of sellers disappears in June 2005. The shock can likely be contributed to events on the automobile companies GM and Ford. Since GM issued a steep profit warning in March 16, 2005, GM and Ford had been trading at or near junk levels. Downgrade was imminent and many insurance companies, pension funds, and other investment funds were restricted from investing in junk bonds and were forced to liquidity GM/Ford bonds. Acharya, Schaefer, and Zhang (2008) find an increasing imbalance between bid and offer quotes on GM and Ford bonds after the profit warning in March lasting until a month after the downgrade to junk bond status in May 5.

Table 4 shows the differences in mid-price paid by institutional investors and retail investors. The table supports the intuition that shocks are identified through this difference. Excluding the two crisis periods, differences are positive for all ratings except the statistically insignificant coefficient for AAA. During the GM/Ford crisis mid-prices for institutional investors are lower than those for retail investors for partly BB and particularly B rating classes, higher for BBB while the rest of the rating classes have similar magnitudes as those in a normal market. The rating of bonds is registered at the end of the sample, and most GM/Ford bonds has a registered B rating, which explains the strong sell signal that the institutional/retail mid-price difference implies.

The second crisis according to the estimation results starts in August 2007 and lasts until the end of the data sample in September 2008. Table 4 shows that in this period mid-prices for institutional investors are smaller than those of retail investors for all but the lowest rated bonds and with high statistical significance.

Time variations in intensities are shown in Figure 2. We see that search intensities of retail investors are largely unaffected by the subprime crisis. In contrast, institution investors see their ease of finding a trading partner decrease throughout the crisis. This leads to higher transaction costs due to harder search efforts. Dick-Nielsen, Feldhütter, and Lando (2008) find that market depth, measured by the Amihud measure, decrease strongly after the onset of the subprime crisis. This is consistent with Figure 2 showing that small trades can be carried out at normal roundtrip costs, while larger trades are harder to execute and therefore reflect larger roundtrip costs.

### 4.3 Search premium

One of the most widely employed frameworks of credit risk, structural models, was developed in the seminal work of Merton (1974). Structural models take as given the dynamics of the value of a firm and values corporate bonds as contingent claims on the firm value. In this model the spread between the yield on a corporate bond and the riskless rate goes to zero as maturities shortens. However, yield spreads are typically positive, also at very short maturities, and this has given rise to the "credit spread puzzle", namely that corporate yield spreads, particularly at very short maturities, are too high to be explain by the corporate bond issuer's default risk (see for example Huang and Huang (2003))<sup>10</sup>. This paper offers a possible explanation for the credit risk puzzle, namely search costs<sup>11</sup>. To examine the impact of search costs on credit spreads, I define the search premium for an investor as the midyield paid by this investor minus the yield of an investor who can instantly find a trading partner ( $\rho = \infty$ ) in which case the bid-ask spread is zero. This mimics a trade in the corporate bond market versus a trade in the ultra-liquid Treasury market. I do this for an average retail investor, where the

<sup>10</sup>Extensions to structural models that can explain the short-maturity credit spread puzzle include jumps in firm value (Zhou (2001)) and incomplete accounting information (Duffie and Lando (2001)).

<sup>11</sup>The model in this paper is related to reduced-form models of credit risk, where there is an intensity process governing the risk of default. Thus, it does not predict a near zero contribution of default risk to spreads at very short maturities as structural models, but nevertheless the implications of search costs can be examined in the model.



search intensity is the average of the search intensities for the three most unsophisticated investors,  $\rho_1, \rho_2$ , and  $\rho_3$  in Table 2, and for an average institutional investor, where the search intensity is the average of  $\rho_4, \rho_5$ , and  $\rho_6$  in the same table. Figures 3 graphs the term structure of search premia. The figure shows that while the search premium is modest at long maturities, it is high for both retail and institutional investors for maturities less than one year and is in the three-digits range for very short maturities.

Covitz and Downing (2007) examine very short-term spreads in the commercial paper market for the roles of credit risk and liquidity. The average trade size in their data set is USD15 million, so the trades are a magnitude larger than those examined in this paper. Consistent with the results here, they find that trade size plays a role in the determination of commercial paper spreads and that the effect of trade size decreases with maturity.

The following calculations provide an estimate of the additional cost due to search that investors in the corporate bond market incur compared to that of the Treasury market. The average maturity in the data sample is 5.5 years, so a 5-year is the most representative bond for the corporate market. An estimate of the average bid-ask spread as a percentage of the par value for a 5-year bond in the Treasury market is 0.0122 according to Fleming (2003). For an average investor and average bond, i.e. an investor with an average search intensity and a bond with an average default intensity, the corresponding estimate for a 5-year bond in the corporate bond market is 0.0506 according to the parameter estimates and equation (13). Thus, an estimate of the cost of search on a roundtrip in the corporate bond market relative to the Treasury market is 0.0384 percent of par value. The yearly trading volume in the corporate bond market was \$6196.5 billion in 2007, so an estimate of the additional yearly costs investors bear in the corporate bond market compared to the Treasury market is  $\$6196.5 * 0.000384 = \$2.38$  billion<sup>12</sup>.

Overall, the results in this section shows that search costs affects average yield spreads strongly at short maturities, and can explain the credit spread puzzle where it is reported to be most severe.

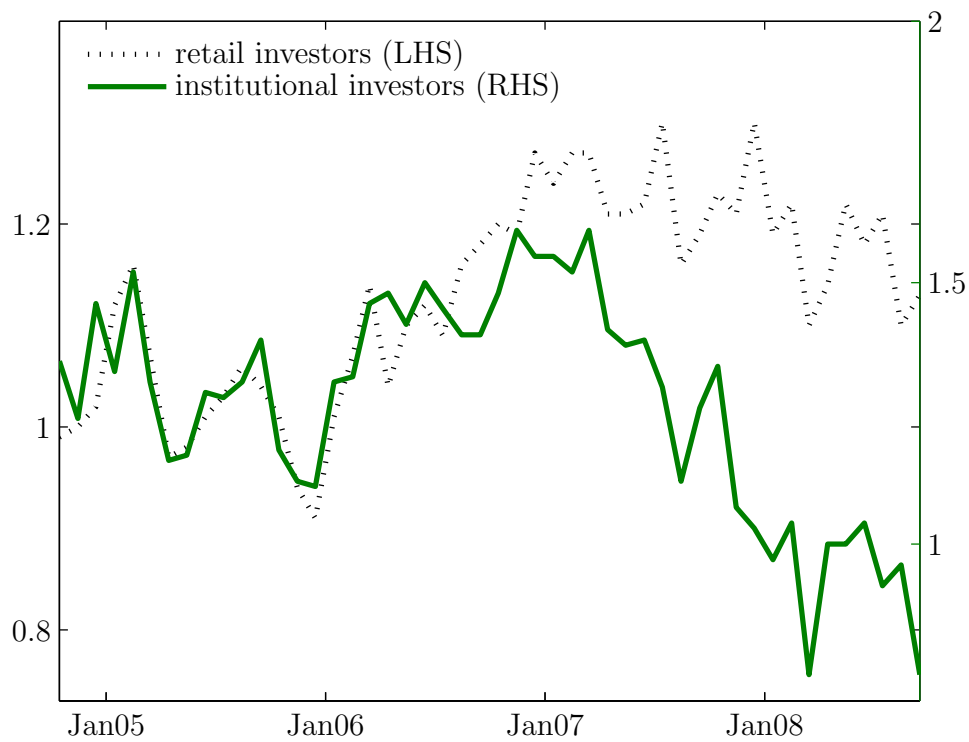
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<sup>12</sup>Average daily volume was \$24.3 billion according to footnote 1, so yearly was  $255 * \$24.3$  billion.

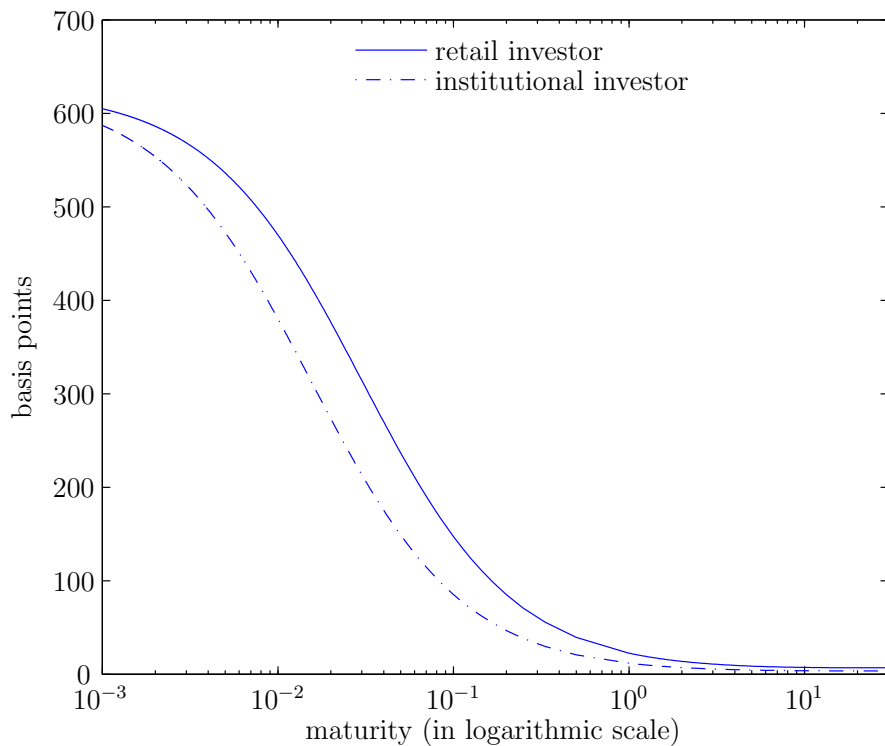
## 5 Conclusion

The corporate bond market is an over-the-counter market where investors search for counterparties and once there is a match, prices are determined through bargaining. I set up a search model that captures the over-the-counter features, and estimate parameters using transaction data from October 2004 to September 2008. The model is stylized but captures well the empirical features of transaction costs across investors, maturity, and rating.

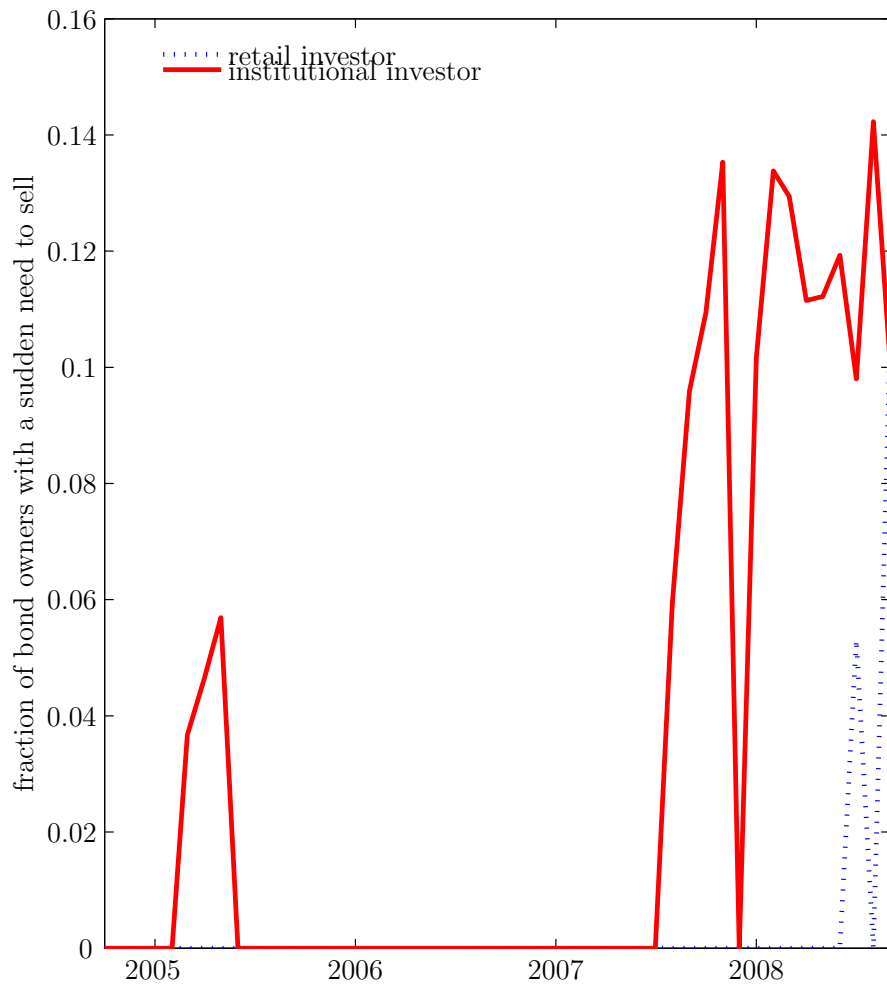
The calibration results show that in a "normal" market, mid-prices of institutional investors are higher than those of retail investors. In distressed periods, where there is an excess number of sellers, the relation is reversed. The model is thus able to identify two distressed periods. The first period is in Spring 2005 where Ford and GM were downgraded from investment grade to junk status. The second period begins in August 2007, where the subprime crisis starts, and lasts until the end of the data sample. The first crisis was mainly concentrated in Ford/GM and bonds with similar credit quality, while the second crisis has hit bonds across all but the lowest ratings. Results also show that it has become more expensive for institutional investors to trade in the corporate bond market after the onset of the subprime crisis, while costs of retail investors have remained fairly stable. Finally, I find that the impact of search costs on average yields is high for maturities less than one year. This suggests that search costs are important for understanding the determination of spreads in the short end of the yield curve.



**Figure 2:** *Time-variation in ease of finding a counterparty.* This graph shows the monthly time-variation in search intensity  $\rho$  for institutional and retail investors. Retail investors are assumed to trade par amounts of \$50,000 or less while institutional investors trade more than this amount.



**Figure 3:** *Search premium in yields for retail and institutional investors.* This graph shows the search premium in yields for retail and institutional investors. The two figures are identical except that the x-axis in the top figure is in logs while it is not in the bottom figure. The search premium for an investor for a given maturity is defined as the average yield of a buy and sell transaction for this investor minus the average yield of a buy and sell transaction for an investor that can instantly find a trading partner. In the latter case, the buy and sell yields are identical. The transacted bond is assumed to have default intensity  $\lambda_D = 0.089$ , a retail investor is assumed to have a search intensity  $\rho = 64.5$ , and an institutional investor a search intensity of  $\rho = 127.3$ .



**Figure 4:** *Fraction of bond owners experiencing a negative shock in their funding liquidity.* The number of investors with high liquidity in steady state are shocked, bid and ask yields immediately after the shock are derived, and the most likely shock is found as explained in the text. This is done for each month in the sample. Retail investors are assumed to trade par amounts of \$50,000 or less while institutional investors trade more than this amount.

Trade size	5K	10K	20K	50K	100K	200K	500K
Edwards, Harris, and Piwowar (2007)	1.50	1.42	1.24	0.92	0.68	0.48	0.28
Unique Roundtrip Trades (15mins)	0.91	0.82	0.78	0.61	0.56	0.44	0.25
Unique Roundtrip Trades (15mins), observations	8,385	14,819	9,943	10,114	6,566	2,189	2,907
Unique Roundtrip Trades (all day)	1.14	1.10	1.04	0.79	0.68	0.53	0.31
Unique Roundtrip Trades (all day), observations	19,017	32,298	20,470	23,454	15,678	4,829	7,147

**Table 1:** *Roundtrip costs.* This table compares roundtrip costs using this paper’s *unique roundtrip trade* measure with those in Edwards, Harris, and Piwowar (2007)’s Table IV. A trade is defined as a unique roundtrip if for a given bond on a given date 2 or 3 trades with a given trade size occurs. The roundtrip cost is the largest minus the smallest trade price. The table shows the roundtrip costs for both unique roundtrips that occur during a trading day and those that occur within 15 minutes. The URT roundtrip estimates in this table is based on the sample period January 2003-January 2005 which matches that in Edwards, Harris, and Piwowar (2007).

$\tilde{q}$	$\rho_1$	$\rho_2$	$\rho_3$	$\rho_4$	$\rho_5$	$\rho_6$	$\lambda_1$	$\lambda_2$	$\lambda_3$
1.000 (0.006)	47 (0.2)	67 (0.2)	80 (0.2)	110 (0.7)	131 (2.7)	141 (1.6)	0.046 (0.001)	0.076 (0.001)	0.145 (0.001)

**Table 2:** *Parameter estimates for a search model for the corporate bond market.* This table shows the estimated parameters of the search model presented in Section 2. The model parameters are estimated by maximum likelihood and standard errors are calculated using the outer product of gradients estimator. Corporate bond data used in estimation are actual transactions from TRACE for the period October 1, 2004 to September 30, 2008.

Panel A: Trade size

	0-5K	6-10K	11-20K	21-50K	51-100K	101-200K	201-500K	501-1000K	>1000K
fitted (in bps)	38.4 (0.1)	27.2 (0.1)	26.7 (0.1)	22.6 (0.1)	22.8 (0.1)	16.7 (0.1)	16.8 (0.1)	13.0 (0.1)	11.2 (0.1)
actual (in bps)	39.4 (0.5)	29.1 (0.3)	27.4 (0.3)	24.1 (0.2)	20.5 (0.2)	16.2 (0.2)	11.6 (0.2)	7.6 (0.2)	4.4 (0.1)

Panel B: Maturity

	0-3m	3-6m	6m-1y	1-3y	3-5y	5-10y	10-20y	>20y
fitted (in bps)	211.0 (1.1)	91.2 (0.2)	51.4 (0.1)	26.1 (0.0)	15.4 (0.0)	11.6 (0.0)	13.1 (0.0)	13.5 (0.0)
actual (in bps)	225.4 (7.1)	77.0 (0.8)	46.7 (0.3)	27.6 (0.1)	16.6 (0.0)	12.0 (0.0)	12.3 (0.1)	10.5 (0.1)

Panel C: Rating

	AAA	AA	A	BBB	BB	B	CCC
fitted (in bps)	15.6 (0.1)	21.3 (0.1)	22.3 (0.1)	24.2 (0.1)	52.1 (0.2)	28.8 (0.1)	22.8 (0.1)
actual (in bps)	14.6 (0.1)	19.4 (0.2)	21.5 (0.1)	24.4 (0.3)	60.9 (1.1)	31.2 (0.1)	20.2 (0.2)

Panel D: Midyield for large trades minus midyield for small trades across maturity

	0-3m	3m-6m	6m-9m	9m-1y	1y-2y	>2y
fitted (in bps)	-28.1 (0.9)	-13.4 (0.3)	-9.0 (0.1)	-7.2 (0.2)	-5.4 (0.2)	-2.6 (0.0)
actual (in bps)	-94.4 (21.5)	-22.7 (6.1)	-12.0 (4.4)	-5.8 (1.9)	-1.2 (1.1)	0.2 (0.2)

**Table 3:** *Estimated round-trip costs measured in yield spreads.* This table reports the fitted roundtrip costs in yields for the search model presented in Section 2. Parameters in the model are those in Table 2. Below the fitted roundtrip costs are actual roundtrip costs. In panel D the differences in midyield are calculated for all roundtrip pairs on a given day for a given bond for which a roundtrip occurred with volume both smaller and larger than \$50,000. If more than one roundtrip occurred. Corporate bond data used in estimation as well as in calculation of actual roundtrip costs are transactions from TRACE for the period October 1, 2004 to September 30, 2008.



	subprime crisis (large crisis)	GM/Ford downgrade (moderate crisis)	no crisis
All ratings	-0.10*** (0.01)	-0.03* (0.01)	0.07*** (0.01)
AAA	-0.09*** (0.02)	-0.01 (0.06)	-0.01 (0.01)
AA	-0.06*** (0.01)	-0.02 (0.02)	0.02*** (0.01)
A	-0.18*** (0.02)	0.03 (0.02)	0.02* (0.01)
BBB	-0.29*** (0.04)	0.17** (0.05)	0.00 (0.02)
BB	-0.26** (0.08)	-0.10** (0.03)	0.06** (0.02)
B	0.10*** (0.03)	-0.22*** (0.04)	0.24*** (0.02)
C	0.39*** (0.08)	0.25* (0.12)	0.30*** (0.03)

**Table 4:** *Difference in mid-prices between large and small trades.* The search model identifies two periods with an excess number of sellers relative to buyers, namely the period leading up to the downgrade of GM and Ford to junk status (March-May 2005) and after the onset of the subprime crisis (beginning in August 2007). This table splits the data sample up in these two periods (on a monthly basis) and a third period which are the residual months Oct 2004-Feb 2005 and June 2005-July 2007. In each period all incidences where a bond on a given day has a roundtrip trade below and above par value USD50,000 are recorded and the differences in mid-yields between large roundtrip trades and small roundtrip trades are shown in the table. Note that the rating of a bond is recorded at the end of the data sample and most GM and Ford bonds had a B rating at that time. Standard errors are in parenthesis and '\*' denotes significance at 5% level, '\*\*' at 1% level, and '\*\*\*' at 0.1% level.

## A Equilibrium Allocations

The rate of change of mass of owners is

$$\dot{\mu}_{lo}(t) = -\rho\mu_m(t) - (\lambda_T + \lambda_D)\mu_{lo}(t) - \lambda_u\mu_{lo}(t) + \lambda_d\mu_{ho}(t) \quad (18)$$

$$\dot{\mu}_{ho}(t) = \rho\mu_{hn}(t) - (\lambda_T + \lambda_D)\mu_{ho}(t) + \lambda_u\mu_{lo}(t) - \lambda_d\mu_{ho}(t) \quad (19)$$

$$\dot{\mu}_{hn}(t) = -\rho\mu_{hn}(t) + (\lambda_T + \lambda_D)\mu_{ho}(t) + \lambda_u\mu_{ln}(t) - \lambda_d\mu_{hn}(t) \quad (20)$$

$$\dot{\mu}_{ln}(t) = \rho\mu_m(t) + (\lambda_T + \lambda_D)\mu_{lo}(t) - \lambda_u\mu_{ln}(t) + \lambda_d\mu_{hn}(t) \quad (21)$$

where  $\mu_m(t) = \min\{\mu_{hn}(t), \mu_{lo}(t)\}$ .

If  $\mu_m = \mu_{hn}$  in steady state then the sum of (18) and (19) yields  $(\lambda_T + \lambda_D)(\mu_{lo} + \mu_{ho}) = 0$  which cannot be the case, so  $\mu_m = \mu_{lo}$  in steady state. Inserting  $\mu_{hn} = 1 - (\mu_{ho} + \mu_{ln} + \mu_{lo})$  into (21) and letting  $\dot{\mu}_{ln}(t) = 0$  yields

$$-\lambda_d = (\rho + \lambda_T + \lambda_D - \lambda_d)\mu_{lo} - (\lambda_u + \lambda_d)\mu_{ln} - \lambda_d\mu_{ho}$$

so in steady state we have

$$\begin{pmatrix} \mu_{lo} \\ \mu_{ho} \\ \mu_{hn} \\ \mu_{ln} \end{pmatrix} = A^{-1} \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\lambda_d \end{pmatrix}$$

where

$$A = \begin{pmatrix} -(\rho + \lambda_T + \lambda_D + \lambda_u) & \lambda_d & 0 & 0 \\ \lambda_u & -(\lambda_T + \lambda_D + \lambda_d) & \rho & 0 \\ 0 & \lambda_T + \lambda_D & -(\lambda_d + \rho) & \lambda_u \\ \rho + \lambda_T + \lambda_D - \lambda_d & -\lambda_d & 0 & -(\lambda_u + \lambda_d) \end{pmatrix}.$$

## B Equilibrium prices

This Appendix derives the value functions stated in the text and the prices stated in Theorem 2.1.

From (5) and (6) we have that lifetime utility is  $W(t) + V_{\sigma(t)}$  where

$$V_{\sigma(t)}(t) = \sup_{\theta} E_t \left[ \int_t^{\infty} e^{-r(t-s)} \theta_s (1 - \delta 1_{\{\sigma^\theta(s)=lo\}}) ds - e^{-r(s-t)} \hat{P}_s d\theta_s \right].$$

In order to calculate  $V_\sigma$  and the bid/ask prices, we consider a particular agent and a particular time  $t$ . Let  $\tau_l$  be the next stopping time at which the agent's type changes,  $\tau_m$  the next time a market maker is met, (in case of an owner)  $\tau_T$  the time at which the bond matures and  $\tau_D$  the time at which the bond defaults. Furthermore, let  $\bar{\tau} = \min\{\tau_l, \tau_m\}$ ,  $\tilde{\tau} = \min\{\tau_l, \tau_T, \tau_D\}$ , and  $\tau = \min\{\tau_l, \tau_m, \tau_T, \tau_D\}$ . Then

$$\begin{aligned}
V_{ln}(t) &= E_t[e^{-r(\tau_l-t)}V_{hn}(\tau_l)] \\
V_{hn}(t) &= E_t[e^{-r(\tau_l-t)}V_{ln}(\tau_l)\mathbf{1}_{\{\tau_l=\bar{\tau}\}} + e^{-r(\tau_m-t)}(V_{ho}(\tau_m) - A_{\tau_m})\mathbf{1}_{\{\tau_m=\bar{\tau}\}}] \\
V_{lo}(t) &= E_t\left[\int_t^\tau e^{-r(u-t)}(1-\delta)du + e^{-r(\tau_l-t)}V_{ho}(\tau_l)\mathbf{1}_{\{\tau_l=\tau\}}\right. \\
&\quad + e^{-r(\tau_m-t)}(V_{ln}(\tau_m) + B_{\tau_m})\mathbf{1}_{\{\tau_m=\tau\}} \\
&\quad + e^{-r(\tau_T-t)}(V_{ln}(\tau_T) + F)\mathbf{1}_{\{\tau_T=\tau\}} \\
&\quad \left. + e^{-r(\tau_D-t)}(V_{ln}(\tau_D) + (1-f)F)\mathbf{1}_{\{\tau_D=\tau\}}\right] \\
V_{ho}(t) &= E_t\left[\int_t^{\tilde{\tau}} e^{-r(u-t)}du + e^{-r(\tau_l-t)}V_{lo}(\tau_l)\mathbf{1}_{\{\tau_l=\tilde{\tau}\}}\right. \\
&\quad + e^{-r(\tau_T-t)}(V_{hn}(\tau_T) + F)\mathbf{1}_{\{\tau_T=\tilde{\tau}\}} \\
&\quad \left. + e^{-r(\tau_D-t)}(V_{hn}(\tau_D) + (1-f)F)\mathbf{1}_{\{\tau_D=\tilde{\tau}\}}\right]
\end{aligned}$$

Suppressing the dependence on time, the value functions satisfy the (HJB) equations

$$\dot{V}_{ln} = rV_{ln} - \lambda_u(V_{hn} - V_{ln}) \quad (22)$$

$$\dot{V}_{hn} = rV_{hn} - \lambda_d(V_{ln} - V_{hn}) - \rho(V_{ho} - V_{hn} - A) \quad (23)$$

$$\begin{aligned}
\dot{V}_{lo} &= rV_{lo} - \lambda_u(V_{ho} - V_{lo}) - \rho(B + V_{ln} - V_{lo}) - \lambda_T(F + V_{ln} - V_{lo}) \\
&\quad - \lambda_D((1-f)F + V_{ln} - V_{lo}) - (1-\delta)
\end{aligned} \quad (24)$$

$$\begin{aligned}
\dot{V}_{ho} &= rV_{ho} - \lambda_d(V_{lo} - V_{ho}) - \lambda_T(F + V_{hn} - V_{ho}) \\
&\quad - \lambda_D((1-f)F + V_{hn} - V_{ho}) - 1
\end{aligned} \quad (25)$$

To see this, we explicitly derive equation (25) and note that the derivation of (22)-(24)

is very similar. We have that

$$\begin{aligned}
E_t\left[\int_t^{\tilde{\tau}} e^{-r(u-t)} du\right] &= \int_t^\infty \int_0^{\tilde{\tau}-t} e^{-ru} du d\tilde{\tau} \\
&= \int_t^\infty \frac{1}{r} (1 - e^{-r(\tilde{\tau}-t)}) d\tilde{\tau} \\
&= \int_0^\infty \frac{1}{r} (1 - e^{-r\tilde{\tau}}) d\tilde{\tau} \\
&= \int_0^\infty \frac{1}{r} (1 - e^{-rx}) (\lambda_d + \lambda_T + \lambda_D) e^{-x(\lambda_d + \lambda_T + \lambda_D)} dx \\
&= \frac{\lambda_d + \lambda_T + \lambda_D}{r} \left[ \frac{-1}{\lambda_d + \lambda_T + \lambda_D} e^{-x(\lambda_d + \lambda_T + \lambda_D)} \right. \\
&\quad \left. + \frac{1}{r + \lambda_d + \lambda_T + \lambda_D} e^{-x(r + \lambda_d + \lambda_T + \lambda_D)} \right]_0^\infty \\
&= \frac{\lambda_d + \lambda_T + \lambda_D}{r} \left( \frac{1}{\lambda_d + \lambda_T + \lambda_D} - \frac{1}{r + \lambda_d + \lambda_T + \lambda_D} \right) \\
&= \frac{1}{r + \lambda_d + \lambda_T + \lambda_D}
\end{aligned}$$

and for  $\tau = \min\{\tau_1, \tau_2\}$  that

$$\begin{aligned}
E_t[e^{-r(\tau_1-t)} 1_{\{\tau_1=\tau\}} V(\tau_1)] &= \int_t^\infty \int_t^\infty e^{-r(x-t)} V(x) \lambda_1 e^{-\lambda_1(x-t)} \lambda_2 e^{-\lambda_2(y-t)} 1_{\{x < y\}} dx dy \\
&= \int_t^\infty \int_x^\infty e^{-r(x-t)} V(x) \lambda_1 e^{-\lambda_1(x-t)} \lambda_2 e^{-\lambda_2(y-t)} dy dx \\
&= \int_t^\infty e^{-r(x-t)} V(x) \lambda_1 e^{-\lambda_1(x-t)} [-e^{-\lambda_2(y-t)}]_x^\infty dx \\
&= \int_t^\infty e^{-r(x-t)} V(x) \lambda_1 e^{-\lambda_1(x-t)} e^{-\lambda_2(x-t)} dx \\
&= \int_t^\infty \lambda_1 e^{-(r+\lambda_1+\lambda_2)(x-t)} V(x) dx
\end{aligned}$$

Such that<sup>13</sup>

$$\begin{aligned}
\dot{V}_{ho} &= \frac{\partial}{\partial t} E_t \left[ \int_t^{\bar{\tau}} e^{-r(u-t)} du + e^{-r(\tau_l-t)} V_{lo} 1_{\{\tau_l=\tau\}} \right. \\
&\quad \left. + e^{-r(\tau_T-t)} (V_{hn} + F) 1_{\{\tau_T=\tau\}} + e^{-r(\tau_D-t)} (V_{hn} + (1-f)F) 1_{\{\tau_D=\tau\}} \right] \\
&= \frac{\partial}{\partial t} E_t \left[ e^{-r(\tau_l-t)} V_{lo} 1_{\{\tau_l=\tau\}} + e^{-r(\tau_T-t)} (V_{hn} + F) 1_{\{\tau_T=\tau\}} \right. \\
&\quad \left. + e^{-r(\tau_D-t)} (V_{hn} + (1-f)F) 1_{\{\tau_D=\tau\}} \right] \\
&= \int_t^\infty \lambda_d (r + \lambda_d + \lambda_T + \lambda_D) e^{-(r+\lambda_d+\lambda_T+\lambda_D)(x-t)} V_{lo} dx - \lambda_d V_{lo} \\
&\quad + \int_t^\infty \lambda_T (r + \lambda_d + \lambda_T + \lambda_D) e^{-(r+\lambda_d+\lambda_T+\lambda_D)(x-t)} (V_{hn} + F) dx - \lambda_T (V_{hn} + F) \\
&\quad + \int_t^\infty \lambda_D (r + \lambda_d + \lambda_T + \lambda_D) e^{-(r+\lambda_d+\lambda_T+\lambda_D)(x-t)} (V_{hn} + (1-f)F) dx \\
&\quad - \lambda_D (V_{hn} + (1-f)F) \\
&= (r + \lambda_d + \lambda_T + \lambda_D) E_t [e^{-r(\tau_l-t)} V_{lo} 1_{\{\tau_l=\bar{\tau}\}}] - \lambda_d V_{lo} \\
&\quad + (r + \lambda_d + \lambda_T + \lambda_D) E_t [e^{-r(\tau_T-t)} (V_{hn} + F) 1_{\{\tau_T=\bar{\tau}\}}] - \lambda_T (V_{hn} + F) \\
&\quad + (r + \lambda_d + \lambda_T + \lambda_D) E_t [e^{-r(\tau_D-t)} (V_{hn} + (1-f)F) 1_{\{\tau_D=\bar{\tau}\}}] \\
&\quad - \lambda_D (V_{hn} + (1-f)F) \\
&= (r + \lambda_d + \lambda_T + \lambda_D) \left[ V_{ho} - \frac{1}{r + \lambda_d + \lambda_T + \lambda_D} \right] - \lambda_d V_{lo} - \lambda_T (V_{hn} + F) \\
&\quad + \lambda_D (V_{hn} + (1-f)F) \\
&= (r + \lambda_d + \lambda_T + \lambda_D) V_{ho} - \lambda_d V_{lo} - \lambda_T (V_{hn} + F) - \lambda_D (V_{hn} + (1-f)F) - 1.
\end{aligned}$$

In steady state  $\dot{V}_\sigma = 0$  and hence the HJB equations imply the following equations for the value functions and prices:

$$V_{ln} = \frac{\lambda_u V_{hn}}{r + \lambda_u} \quad (26)$$

$$V_{hn} = \frac{\lambda_d V_{ln} + \rho V_{ho} - \rho A}{r + \lambda_d + \rho} \quad (27)$$

$$V_{lo} = \frac{\lambda_u V_{ho} + \rho B + \lambda_T F + \lambda_D (1-f)F + (\rho + \lambda_T + \lambda_D) V_{ln} + 1 - \delta}{r + \lambda_u + \rho + \lambda_T + \lambda_D} \quad (28)$$

$$V_{ho} = \frac{\lambda_d V_{lo} + (\lambda_T + \lambda_D) V_{hn} + \lambda_T F + \lambda_D (1-f)F + 1}{r + \lambda_d + \lambda_T + \lambda_D} \quad (29)$$

Bilateral bargaining between investors and market makers determines bid and ask prices. A low-type owner demands at least  $\Delta V_l = V_{lo} - V_{ln}$  for the bond while a high-type nonowner will not pay more than  $\Delta V_h = V_{ho} - V_{hn}$ . Nash bargaining between investors and market makers in which the outside option of the market maker is to trade in the inter-dealer market results in the bid and ask prices

$$\begin{aligned}
A &= \Delta V_h z + M(1-z) \\
B &= \Delta V_l z + M(1-z)
\end{aligned}$$

<sup>13</sup>According to Leibnitz's rule  $\frac{\partial}{\partial \alpha} \int_\alpha^\infty F(x, \alpha) dx = \int_\alpha^\infty \frac{\partial F}{\partial \alpha} dx - F(\alpha, \alpha)$ .

where  $z$  is the bargaining power of the market maker. Thus,

$$\begin{aligned}
(r + \lambda_u)V_{ln} &= \lambda_u V_{hn} \\
(r + \lambda_d)V_{hn} &= \lambda_d V_{ln} + \rho \Delta V_h - \rho[\Delta V_h z + M(1 - z)] \\
(r + \lambda_u)V_{lo} &= \lambda_u V_{ho} + \rho[\Delta V_l z + M(1 - z)] + \lambda_T F + \lambda_D(1 - f)F \\
&\quad - (\rho + \lambda_T + \lambda_D)\Delta V_l + 1 - \delta \\
(r + \lambda_d)V_{ho} &= \lambda_d V_{lo} - (\lambda_T + \lambda_D)\Delta V_h + \lambda_T F + \lambda_D(1 - f)F + 1.
\end{aligned}$$

These equations reduce to

$$\begin{aligned}
(r + \lambda_u + (1 - z)\rho + \lambda_T + \lambda_D)\Delta V_l &= \lambda_u \Delta V_h + \rho(1 - z)M + \psi_c - \delta \\
(r + \lambda_d + (1 - z)\rho + \lambda_T + \lambda_D)\Delta V_h &= \lambda_d \Delta V_l + \rho(1 - z)M + \psi_c
\end{aligned}$$

where  $\psi_c = \lambda_T F + \lambda_D(1 - f)F + 1$ . This implies that

$$\begin{pmatrix} \Delta V_l \\ \Delta V_h \end{pmatrix} = \frac{\rho(1 - z)M + \psi_c}{C} \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \frac{\delta}{C(C + \lambda_d + \lambda_u)} \begin{pmatrix} C + \lambda_d \\ \lambda_d \end{pmatrix} \quad (30)$$

where  $C = r + (1 - z)\rho + \lambda_T + \lambda_D$ . The agent faces a bid-ask spread of

$$z(\Delta V_h - \Delta V_l) = \frac{z\delta}{r + (1 - z)\rho + \lambda_d + \lambda_u + \lambda_T + \lambda_D}$$

For agents with different search intensities the same arguments can be applied. Next, we now find the range of inter-dealer prices for which all investors trade. Define  $\rho_0 = \min_{i=1, \dots, I} \rho^i$ . For all  $\rho \geq \rho_0$  the inter-dealer price has to satisfy

$$M \leq \Delta V_h = \frac{\rho(1 - z)M + \psi_c}{C} - \frac{\delta \lambda_d}{C(C + \lambda_d + \lambda_u)}$$

according to (30) and rearranging this inequality yields

$$M \leq \frac{\psi_c}{r + \lambda_T + \lambda_D} - \frac{\delta \lambda_d}{(r + \lambda_T + \lambda_D)(r + (1 - z)\rho + \lambda_d + \lambda_u + \lambda_T + \lambda_D)}.$$

If this holds for all  $\rho \geq \rho_0$  then

$$M \leq \frac{\psi_c}{r + \lambda_T + \lambda_D} - \frac{\delta \lambda_d}{(r + \lambda_T + \lambda_D)(r + (1 - z)\rho_0 + \lambda_d + \lambda_u + \lambda_T + \lambda_D)}.$$

Likewise we find that

$$M \geq \frac{\psi_c}{r + \lambda_T + \lambda_D} - \frac{\delta(C + \lambda_d)}{(r + \lambda_T + \lambda_D)(C + \lambda_d + \lambda_u)}$$

which holds for all  $\rho$  if

$$M \geq \frac{\psi_c}{r + \lambda_T + \lambda_D} - \frac{\delta(r + (1 - z)\rho_0 + \lambda_d + \lambda_T + \lambda_D)}{(r + \lambda_T + \lambda_D)(r + (1 - z)\rho_0 + \lambda_d + \lambda_u + \lambda_T + \lambda_D)}.$$

## C Proof of Theorem 2.2

In this section I provide a proof of Theorem 2.2. I first find the length of time with more sellers than buyers following a liquidity shock. I then find bid and ask prices prevailing immediately after a liquidity shock.

If we define  $\mu_l = \mu_{lo} + \mu_{ln}$  and  $\mu_h = \mu_{ho} + \mu_{hn}$  we have according to equations (1)-(4) that

$$\begin{pmatrix} \dot{\mu}_l \\ \dot{\mu}_h \end{pmatrix} = \begin{bmatrix} -\lambda_u & \lambda_d \\ \lambda_u & -\lambda_d \end{bmatrix} \begin{pmatrix} \mu_l \\ \mu_h \end{pmatrix}.$$

The solution to these ODEs is

$$\begin{pmatrix} \mu_l(t) \\ \mu_h(t) \end{pmatrix} = (1 - e^{-t(\lambda_u + \lambda_d)}) \begin{pmatrix} \frac{\lambda_d}{\lambda_d + \lambda_u} \\ \frac{\lambda_u}{\lambda_d + \lambda_u} \end{pmatrix} + e^{-t(\lambda_u + \lambda_d)} \begin{pmatrix} \mu_l(0) \\ \mu_h(0) \end{pmatrix} \quad (31)$$

This result will be useful in a short moment. We now look at  $\mu_{hn} - \mu_{lo}$  since sellers are constrained if this difference is negative. We have that

$$\begin{aligned} \dot{\mu}_{hn} - \dot{\mu}_{lo} &= \rho(\mu_m - \mu_{hn}) + (\lambda_T + \lambda_D)(\mu_{ho} + \mu_{lo}) + \lambda_u(\mu_{ln} + \mu_{lo}) - \lambda_d(\mu_{hn} + \mu_{ho}) \\ &= \rho(\mu_m - \mu_{hn}) - (\lambda_T + \lambda_D)(\mu_{hn} - \mu_{lo}) + (\lambda_T + \lambda_D - \lambda_d - \lambda_u)\mu_h + \lambda_u \end{aligned}$$

If sellers are constrained we have  $\mu_m = \mu_{hn}$  and

$$\dot{\mu}_{hn} - \dot{\mu}_{lo} = -(\lambda_T + \lambda_D)(\mu_{hn} - \mu_{lo}) + (\lambda_T + \lambda_D - \lambda_d - \lambda_u)\mu_h + \lambda_u.$$

Assume that  $\mu_{hn}(0) - \mu_{lo}(0) < 0$ . Using the result in equation (31), the solution for  $\mu_{hn}(t) - \mu_{lo}(t)$  for any  $t$  where  $\mu_{hn} - \mu_{lo}$  has not yet been positive is

$$\begin{aligned} \mu_{hn}(t) - \mu_{lo}(t) &= \frac{\lambda_u}{\lambda_d + \lambda_u} + e^{-(\lambda_u + \lambda_d)t} \left( \mu_h(0) - \frac{\lambda_u}{\lambda_d + \lambda_u} \right) \\ &\quad - (\mu_{ho}(0) + \mu_{lo}(0)) e^{-(\lambda_T + \lambda_D)t}. \end{aligned} \quad (32)$$

This equation has a unique  $t_s$  where  $\mu_{hn}(t_s) - \mu_{lo}(t_s) = 0$ . This implies that if sellers are constrained at time 0 they become unconstrained at time  $t_s$ . What remains to show is that they stay unconstrained after time  $t_s$ . If sellers are not constrained we have  $\mu_m = \mu_{lo}$  and

$$\dot{\mu}_{hn} - \dot{\mu}_{lo} = -(\rho + \lambda_T + \lambda_D)(\mu_{hn} - \mu_{lo}) + (\lambda_T + \lambda_D - \lambda_d - \lambda_u)\mu_h + \lambda_u$$

The solution is (assuming that  $\mu_{hn}(0) - \mu_{lo}(0) = 0$ )

$$\begin{aligned} \mu_{hn}(t) - \mu_{lo}(t) &= \frac{\lambda_u}{\lambda_d + \lambda_u} \frac{\lambda_T + \lambda_D}{\rho + \lambda_T + \lambda_D} \\ &\quad + \frac{\lambda_T + \lambda_D - \lambda_d - \lambda_u}{\rho + \lambda_T + \lambda_D - \lambda_d - \lambda_u} \left( \mu_h(0) - \frac{\lambda_u}{\lambda_d + \lambda_u} \right) e^{-(\lambda_u + \lambda_d)t} \\ &\quad - \left[ \left( \mu_h(0) - \frac{\lambda_u}{\lambda_d + \lambda_u} \right) \frac{\lambda_T + \lambda_D - \lambda_d - \lambda_u}{\rho + \lambda_T + \lambda_D - \lambda_d - \lambda_u} + \frac{\lambda_u}{\lambda_d + \lambda_u} \frac{\lambda_T + \lambda_D}{\rho + \lambda_T + \lambda_D} \right] e^{-(\rho + \lambda_T + \lambda_D)t}. \end{aligned}$$

Assume that  $\mu_h(0) = (1 - s) \frac{\lambda_u}{\lambda_u + \lambda_d}$ . Then we have

$$\begin{aligned} \mu_{hn}(t) - \mu_{lo}(t) = & \frac{\lambda_u}{\lambda_d + \lambda_u} \left[ \frac{\lambda_T + \lambda_D}{\rho + \lambda_T + \lambda_D} \right. \\ & - s \frac{\lambda_T + \lambda_D - \lambda_d - \lambda_u}{\rho + \lambda_T + \lambda_D - \lambda_d - \lambda_u} e^{-(\lambda_u + \lambda_d)t} \\ & \left. - \left[ \frac{(1 - s)(\lambda_T + \lambda_D)(\rho + \lambda_T + \lambda_D - \lambda_u - \lambda_d) + s\rho(\lambda_T + \lambda_D)}{(\rho + \lambda_T + \lambda_D)(\rho + \lambda_T + \lambda_D - \lambda_u - \lambda_d)} \right] e^{-(\rho + \lambda_T + \lambda_D)t} \right]. \end{aligned}$$

If  $\rho + \lambda_T + \lambda_D > \lambda_d + \lambda_u$  we have that  $\mu_{hn}(t) - \mu_{lo}(t) \geq 0$  for all  $t$ . What I have now shown is that if the fractions of investors are such that sellers are constrained, the sellers are constrained a period of time  $t_s$  and unconstrained thereafter. Next, I find  $t_s$ . The steady state value of  $\mu_{hn} - \mu_{lo}$  is  $\frac{\lambda_u}{\lambda_d + \lambda_u} \frac{\lambda_T + \lambda_D}{\rho + \lambda_T + \lambda_D}$ . This, along with equations (1) and (2) yielding

$$\dot{\mu}_{lo} + \dot{\mu}_{ho} = \rho(\mu_{hn} - \mu_{lo}) - (\lambda_T + \lambda_D)(\mu_{ho} + \mu_{lo})$$

gives the steady state value of  $\mu_{lo} + \mu_{ho}$  as  $\frac{\lambda_u}{\lambda_d + \lambda_u} \frac{\rho}{\rho + \lambda_T + \lambda_D}$ . Since the steady state value of  $\mu_h$  is  $\frac{\lambda_u}{\lambda_d + \lambda_u}$  we have according to (32) that the time,  $t$ , an investor is constrained after a shock of  $s$  solves

$$0 = 1 - s e^{-(\lambda_u + \lambda_d)t} - \frac{\rho}{\rho + \lambda_T + \lambda_D} e^{-(\lambda_T + \lambda_D)t}. \quad (33)$$

If  $s \leq \frac{\lambda_T + \lambda_D}{\rho + \lambda_T + \lambda_D}$  the investor is not constrained at any time after the shock and prices do not change.

Next, I find bid and ask prices immediately after the liquidity shock. Rewriting equations (22)-(25) we have that

$$\begin{aligned} \dot{\Delta V}_h &= (C_1 + \lambda_d + \rho)\Delta V_h - \lambda_d \Delta V_l - \psi_C - \rho A \\ \dot{\Delta V}_l &= (C_1 + \lambda_u + \rho)\Delta V_l - \lambda_u \Delta V_h - (\psi_C - \delta) - \rho B \\ C_1 &= r + \lambda_D + \lambda_T \\ \psi_C &= \lambda_T F + \lambda_D(1 - f)F + 1. \end{aligned}$$

If sellers are constrained we have that the interdealer price is  $M = \Delta V_l$ , so  $A = \Delta V_h z + \Delta V_l(1 - z)$  and  $B = \Delta V_l$ . Thus,

$$\begin{aligned} \dot{\Delta V}_h &= (C_1 + \lambda_d + (1 - z)\rho)\Delta V_h + (-\lambda_d - (1 - z)\rho)\Delta V_l - \psi_C \\ \dot{\Delta V}_l &= (C_1 + \lambda_u)\Delta V_l - \lambda_u \Delta V_h - (\psi_C - \delta) \end{aligned}$$

which can be rewritten as

$$\begin{pmatrix} \dot{\Delta V}_h \\ \dot{\Delta V}_l \end{pmatrix} = \begin{bmatrix} C_1 + \lambda_d + (1 - z)\rho & -\lambda_d - (1 - z)\rho \\ -\lambda_u & C_1 + \lambda_u \end{bmatrix} \begin{pmatrix} \Delta V_h \\ \Delta V_l \end{pmatrix} - \begin{pmatrix} \psi_C \\ \psi_C - \delta \end{pmatrix} \quad (34)$$

The reservation values immediately after the shock is found by solving these ODEs backwards from the steady state reservation values for the period of time sellers are



constrained. That is, the reservation values after the shock is the time  $t_s$  solution to

$$\begin{pmatrix} \dot{\Delta V}_h \\ \dot{\Delta V}_l \end{pmatrix} = - \begin{bmatrix} C_1 + \lambda_d + (1-z)\rho & -\lambda_d - (1-z)\rho \\ -\lambda_u & C_1 + \lambda_u \end{bmatrix} \begin{pmatrix} \Delta V_h \\ \Delta V_l \end{pmatrix} + \begin{pmatrix} \psi_C \\ \psi_C - \delta \end{pmatrix}$$

$$\begin{pmatrix} \Delta V_h(0) \\ \Delta V_l(0) \end{pmatrix} = \begin{pmatrix} \Delta V_h^{ss} \\ \Delta V_l^{ss} \end{pmatrix}.$$

The steady state solution of this system is

$$\begin{pmatrix} \Delta V_h^{\text{imb}} \\ \Delta V_l^{\text{imb}} \end{pmatrix} = - \begin{bmatrix} -(C_1 + \lambda_d + (1-z)\rho) & \lambda_d + (1-z)\rho \\ \lambda_u & -(C_1 + \lambda_u) \end{bmatrix}^{-1} \begin{pmatrix} \psi_C \\ \psi_C - \delta \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\psi_C}{C_1} - \delta \left( \frac{\sqrt{\gamma} - \lambda_u}{C_1(\sqrt{\gamma} + C_1)} \right) \\ \frac{\psi_C}{C_1} - \delta \left( \frac{\sqrt{\gamma} - \lambda_u}{C_1(\sqrt{\gamma} + C_1)} \right) - \frac{\delta}{\sqrt{\gamma} + C_1} \end{pmatrix}$$

where  $\sqrt{\gamma} = \lambda_d + \lambda_u + (1-z)\rho$ . If sellers were always constrained, the reservation values of investors would be  $\begin{pmatrix} \Delta V_h^{\text{imb}} \\ \Delta V_l^{\text{imb}} \end{pmatrix}$ . Tedious calculations and the use of Corollary 11.3.3 in Bernstein (2005) show that

$$\begin{pmatrix} \Delta V_h(t) \\ \Delta V_l(t) \end{pmatrix} = e^{-tC_1} \begin{pmatrix} \Delta V_h^{ss} \\ \Delta V_l^{ss} \end{pmatrix} + (1 - e^{-tC_1}) \begin{pmatrix} \Delta V_h^{\text{imb}} \\ \Delta V_l^{\text{imb}} \end{pmatrix}$$

## D Proof of Theorem 2.3

This section provides a proof of Theorem 2.3. Part 1 of the theorem - that bid-ask spreads are unaffected by a liquidity shock - follows immediately from Theorem 2.1 and 2.2. Consequently, any relation between midprices of investors with different levels of sophistication can be shown by showing the relation for bid prices.

I first assume that  $\frac{\lambda_T + \lambda_D}{\min\{\rho_i, \rho_j\} + \lambda_T + \lambda_D} < s \leq 1$  and prove the second half of part 2 in the theorem by showing that the bid price of an unsophisticated investor minus that of a sophisticated investor,  $B_i(s) - B_j(s)$ , is increasing in  $s$ . I show this by proving that  $\frac{\partial B(s)}{\partial \rho \partial s} < 0$ . Implicit differentiation in equation (33) yields

$$\frac{\partial t}{\partial s} = \frac{1}{s(\lambda_u + \lambda_d) + \frac{\rho(\lambda_T + \lambda_D)}{\rho + \lambda_T + \lambda_D} e^{(\lambda_u + \lambda_d - \lambda_T - \lambda_D)t}}$$

so we have

$$\frac{\partial B(s)}{\partial s} = (r + \lambda_D + \lambda_T) \frac{\partial t}{\partial s} e^{-t(r + \lambda_D + \lambda_T)} (\Delta V_l^{ss} - \Delta V_l^{imb}).$$

Since

$$\begin{aligned} \Delta V_l^{imb} - \Delta V_l^{ss} &= \frac{\rho(1-z)}{C(r + \lambda_T + \lambda_D)} \left[ \frac{(1-\tilde{q})C_0 + \lambda_d}{C_0 + \lambda_u + \lambda_d} - \frac{C + \lambda_d}{C + \lambda_u + \lambda_d} \right] \\ C_0 &= r + (1-z)\rho_0 + \lambda_T + \lambda_D \\ C &= r + (1-z)\rho + \lambda_T + \lambda_D \end{aligned}$$

we have that

$$\frac{\partial B(s)}{\partial s} = \frac{\frac{\rho(1-z)}{C} \left[ \frac{(1-\tilde{q})C_0 + \lambda_d}{C_0 + \lambda_u + \lambda_d} - \frac{C + \lambda_d}{C + \lambda_u + \lambda_d} \right] e^{-rt}}{s(\lambda_u + \lambda_d) e^{(\lambda_T + \lambda_D)t} + \frac{\rho(\lambda_T + \lambda_D)}{\rho + \lambda_T + \lambda_D} e^{(\lambda_u + \lambda_d)t}}.$$

Now

$$\begin{aligned} \frac{\partial t}{\partial \rho} &= \frac{\frac{\lambda_T + \lambda_D}{(\rho + \lambda_T + \lambda_D)^2} e^{-(\lambda_T + \lambda_D)t}}{(\lambda_u + \lambda_d) s e^{-(\lambda_u + \lambda_d)t} + \frac{\rho(\lambda_T + \lambda_D)}{\rho + \lambda_T + \lambda_D} e^{-(\lambda_T + \lambda_D)t}} \\ \frac{\partial \left( \frac{\rho(C + \lambda_d)}{C(C + \lambda_u + \lambda_d)} \right)}{\partial \rho} &= \frac{\rho(1-z)C\lambda_u + (r + \lambda_T + \lambda_D)(C + \lambda_d)(C + \lambda_d + \lambda_u)}{C^2(C + \lambda_u + \lambda_d)^2} \\ \frac{\partial \left( \frac{\rho(1-z)}{C} \right)}{\partial \rho} &= \frac{(1-z)(r + \lambda_T + \lambda_D)}{C^2} \end{aligned}$$

so the numerator in  $\frac{\partial B(s)}{\partial \rho \partial s}$  is

$$\begin{aligned}
& \left[ \frac{(1-z)(r+\lambda_T+\lambda_D)}{C^2} \frac{(1-\bar{q})C_0+\lambda_d}{C_0+\lambda_u+\lambda_d} - (1-z) \frac{\rho(1-z)C\lambda_u+(r+\lambda_T+\lambda_D)(C+\lambda_d)(C+\lambda_d+\lambda_u)}{C^2(C+\lambda_u+\lambda_d)^2} \right] \times \\
& \left[ s(\lambda_u + \lambda_d)e^{(\lambda_T+\lambda_D)t} + \frac{\rho(\lambda_T+\lambda_D)}{\rho+\lambda_T+\lambda_D} e^{(\lambda_u+\lambda_d)t} \right] - \frac{\rho(1-z)}{C} \left[ \frac{(1-\bar{q})C_0+\lambda_d}{C_0+\lambda_u+\lambda_d} - \frac{C+\lambda_d}{C+\lambda_u+\lambda_d} \right] \times \\
& \left[ (se^{(\lambda_T+\lambda_D)t} + \frac{\rho}{\rho+\lambda_T+\lambda_D} e^{(\lambda_d+\lambda_u)t}) \frac{(\lambda_u+\lambda_d) \frac{(\lambda_T+\lambda_D)^2}{(\rho+\lambda_T+\lambda_D)^2} e^{-(\lambda_D+\lambda_T)t}}{(\lambda_u+\lambda_d)se^{-(\lambda_u+\lambda_d)t} + \frac{\rho(\lambda_T+\lambda_D)}{\rho+\lambda_T+\lambda_D} e^{-(\lambda_T+\lambda_D)t}} + \frac{(\lambda_T+\lambda_D)^2}{(\rho+\lambda_T+\lambda_D)^2} e^{(\lambda_u+\lambda_d)t} \right] = \\
& \left[ \frac{(1-\bar{q})C_0+\lambda_d}{C_0+\lambda_u+\lambda_d} - \frac{C+\lambda_d}{C+\lambda_u+\lambda_d} \right] \frac{1-z}{C} \left[ \frac{r+\lambda_T+\lambda_D}{C} \left[ s(\lambda_u + \lambda_d)e^{(\lambda_T+\lambda_D)t} + \frac{\rho(\lambda_T+\lambda_D)}{\rho+\lambda_T+\lambda_D} e^{(\lambda_u+\lambda_d)t} \right] - \right. \\
& \left. \rho \left[ (se^{(\lambda_T+\lambda_D)t} + \frac{\rho}{\rho+\lambda_T+\lambda_D} e^{(\lambda_d+\lambda_u)t}) \frac{(\lambda_u+\lambda_d) \frac{(\lambda_T+\lambda_D)^2}{(\rho+\lambda_T+\lambda_D)^2} e^{-(\lambda_D+\lambda_T)t}}{(\lambda_u+\lambda_d)se^{-(\lambda_u+\lambda_d)t} + \frac{\rho(\lambda_T+\lambda_D)}{\rho+\lambda_T+\lambda_D} e^{-(\lambda_T+\lambda_D)t}} + \frac{(\lambda_T+\lambda_D)^2}{(\rho+\lambda_T+\lambda_D)^2} e^{(\lambda_u+\lambda_d)t} \right] \right] \\
& - \frac{(1-z)^2 \rho \lambda_u}{C(C+\lambda_u+\lambda_d)^2} \left[ s(\lambda_u + \lambda_d)e^{(\lambda_T+\lambda_D)t} + \frac{\rho(\lambda_T+\lambda_D)}{\rho+\lambda_T+\lambda_D} e^{(\lambda_u+\lambda_d)t} \right] = \\
& \left[ \frac{(1-\bar{q})C_0+\lambda_d}{C_0+\lambda_u+\lambda_d} - \frac{C+\lambda_d}{C+\lambda_u+\lambda_d} \right] \frac{1-z}{C} \times \\
& \left[ \left( \frac{r+\lambda_T+\lambda_D}{C} + \frac{(1-z)\rho\lambda_u}{(C+\lambda_u+\lambda_d)^2} \left( \frac{C+\lambda_d}{C+\lambda_u+\lambda_d} - \frac{(1-\bar{q})C_0+\lambda_d}{C_0+\lambda_u+\lambda_d} \right)^{-1} \right) \left[ s(\lambda_u + \lambda_d)e^{(\lambda_T+\lambda_D)t} + \frac{\rho(\lambda_T+\lambda_D)}{\rho+\lambda_T+\lambda_D} e^{(\lambda_u+\lambda_d)t} \right] - \right. \\
& \left. \rho \left[ (se^{(\lambda_T+\lambda_D)t} + \frac{\rho}{\rho+\lambda_T+\lambda_D} e^{(\lambda_d+\lambda_u)t}) \frac{(\lambda_u+\lambda_d) \frac{(\lambda_T+\lambda_D)^2}{(\rho+\lambda_T+\lambda_D)^2} e^{-(\lambda_D+\lambda_T)t}}{(\lambda_u+\lambda_d)se^{-(\lambda_u+\lambda_d)t} + \frac{\rho(\lambda_T+\lambda_D)}{\rho+\lambda_T+\lambda_D} e^{-(\lambda_T+\lambda_D)t}} + \frac{(\lambda_T+\lambda_D)^2}{(\rho+\lambda_T+\lambda_D)^2} e^{(\lambda_u+\lambda_d)t} \right] \right] \leq \\
& \left[ \frac{(1-\bar{q})C_0+\lambda_d}{C_0+\lambda_u+\lambda_d} - \frac{C+\lambda_d}{C+\lambda_u+\lambda_d} \right] \frac{1-z}{C} \times \\
& \left[ s(\lambda_u + \lambda_d)e^{(\lambda_T+\lambda_D)t} \left( \frac{r+\lambda_T+\lambda_D}{C} + \frac{(1-z)\rho\lambda_u}{(C+\lambda_u+\lambda_d)^2} - \frac{\lambda_T+\lambda_D}{\rho+\lambda_T+\lambda_D} \right) + \right. \\
& \left. \frac{\rho(\lambda_T+\lambda_D)}{\rho+\lambda_T+\lambda_D} e^{(\lambda_u+\lambda_d)t} \left( \frac{r+\lambda_T+\lambda_D}{C} + \frac{(1-z)\rho\lambda_u}{(C+\lambda_u+\lambda_d)^2} - \frac{\lambda_T+\lambda_D}{\rho+\lambda_T+\lambda_D} - \frac{\rho}{\rho+\lambda_T+\lambda_D} \frac{(\lambda_u+\lambda_d) \frac{\lambda_T+\lambda_D}{\rho+\lambda_T+\lambda_D} e^{-(\lambda_D+\lambda_T)t}}{(\lambda_u+\lambda_d)se^{-(\lambda_u+\lambda_d)t} + \frac{\rho(\lambda_T+\lambda_D)}{\rho+\lambda_T+\lambda_D} e^{-(\lambda_T+\lambda_D)t}} \right) \right].
\end{aligned}$$

We have that  $\frac{(1-\bar{q})C_0+\lambda_d}{C_0+\lambda_u+\lambda_d} - \frac{C+\lambda_d}{C+\lambda_u+\lambda_d} < 0$  and  $\frac{r+\lambda_T+\lambda_D}{C} > \frac{\lambda_T+\lambda_D}{\rho+\lambda_T+\lambda_D}$ . Also

$$\begin{aligned}
& \frac{(1-z)\rho\lambda_u}{(C+\lambda_u+\lambda_d)^2} - \frac{\rho}{\rho+\lambda_T+\lambda_D} \frac{(\lambda_u+\lambda_d) \frac{\lambda_T+\lambda_D}{\rho+\lambda_T+\lambda_D} e^{-(\lambda_D+\lambda_T)t}}{(\lambda_u+\lambda_d)se^{-(\lambda_u+\lambda_d)t} + \frac{\rho(\lambda_T+\lambda_D)}{\rho+\lambda_T+\lambda_D} e^{-(\lambda_T+\lambda_D)t}} \\
& = \frac{(1-z)\rho\lambda_u}{(C+\lambda_u+\lambda_d)^2} - \frac{1}{\frac{(\rho+\lambda_T+\lambda_D)^2}{(\lambda_T+\lambda_D)\rho} se^{(\lambda_T+\lambda_D-\lambda_u-\lambda_d)t} + \frac{\rho+\lambda_T+\lambda_D}{\lambda_u+\lambda_d}}.
\end{aligned}$$

A sufficient condition for  $\frac{\partial B}{\partial \rho \partial s} < 0$  is that  $\frac{(1-z)\rho\lambda_u}{(C+\lambda_u+\lambda_d)^2} > \frac{1}{\frac{(\rho+\lambda_T+\lambda_D)^2}{(\lambda_T+\lambda_D)\rho} se^{(\lambda_T+\lambda_D-\lambda_u-\lambda_d)t} + \frac{\rho+\lambda_T+\lambda_D}{\lambda_u+\lambda_d}}$ .

This is the case if  $f(s) > 0$  for every  $0 \leq s \leq 1$  where  $f$  is defined as

$$f(s) = \frac{(1-z)\rho\lambda_u}{(C+\lambda_u+\lambda_d)^2} \left[ \frac{(\rho+\lambda_T+\lambda_D)^2}{(\lambda_T+\lambda_D)\rho} se^{(\lambda_T+\lambda_D-\lambda_u-\lambda_d)t} + \frac{\rho+\lambda_T+\lambda_D}{\lambda_u+\lambda_d} \right] - 1.$$

Since  $f(s) > f(0)$  for  $s > 0$  we need to show that  $f(0) > 0$ . We have that

$$\begin{aligned}
(1-z)\rho\lambda_u(\rho+\lambda_T+\lambda_D) - (C+\lambda_u+\lambda_d)^2(\lambda_u+\lambda_d) &= \\
&= [(1-z)(z\lambda_u - \lambda_d(1-z))] \rho^2 - \\
(1-z)[(2\lambda_d+\lambda_u)(\lambda_T+\lambda_D) + 2(\lambda_u+\lambda_d)^2] \rho - & \\
(\lambda_u+\lambda_d)(\lambda_T+\lambda_D+\lambda_u+\lambda_d)^2. &
\end{aligned}$$

The last expression is a second-order polynomial in  $\rho$  and if this polynomial is larger than 0, then so is  $f(0)$ . If  $(1-z)(z\lambda_u - \lambda_d(1-z)) > 0$  then the polynomial is positive for  $\rho$  sufficiently large. If  $\lambda_u > \frac{1-z}{z}\lambda_d$  then  $(1-z)(z\lambda_u - \lambda_d(1-z)) > 0$ . Thus, if  $\rho$  is sufficiently large and  $\lambda_u > \frac{1-z}{z}\lambda_d$  then  $\frac{\partial B(s)}{\partial \rho \partial s} < 0$  as needed to be proven.

With the second half of part 2 proven, the first half follows easily. If  $s \leq \frac{\lambda_T + \lambda_D}{\rho_j + \lambda_T + \lambda_D}$  prices are not affected by the liquidity shock, so in this range of  $s$  we have  $M_i(s) - M_j(s) = M_i(0) - M_j(0)$ . If  $\frac{\lambda_T + \lambda_D}{\rho_j + \lambda_T + \lambda_D} < s \leq \frac{\lambda_T + \lambda_D}{\rho_i + \lambda_T + \lambda_D}$  prices of investor  $i$  is affected but not prices of investor  $j$ . Since the potential effect of a liquidity shock is a price decrease, we have in this range of  $s$  that  $M_i(s) - M_j(s) > M_i(0) - M_j(0)$

## E Additional details on estimation methodology

### E.1 Squared errors in likelihood estimation

In the text  $A_{tbi}$  is defined as the  $i$ 'th ask yield on bond  $b$  and day  $t$  and  $B_{tbi}$  as the corresponding bid yield. Fitted yields follow the same notation with an addition superscript  $M$ . The sum of squared errors in the likelihood in equation (14) equal

$$\sum_{t=1}^T \sum_{b=1}^{N_b} w_{tb} \sum_{i=1}^{N_{tb}} \left[ [(A_{tbi} - \overline{AB}_{tb}) - (A_{tbi}^M - \overline{AB}_{tb}^M)]^2 + [(B_{tbi} - \overline{AB}_{tb}) - (B_{tbi}^M - \overline{AB}_{tb}^M)]^2 \right]$$

where I use the notation

$$\begin{aligned} \overline{A}_{tb} &= \frac{1}{N_{tb}} \sum_{m=1}^{N_{tb}} A_{tbm} \\ \overline{B}_{tb} &= \frac{1}{N_{tb}} \sum_{m=1}^{N_{tb}} B_{tbm} \\ \overline{AB}_{tb} &= \frac{1}{2} (\overline{A}_{tb} + \overline{B}_{tb}). \end{aligned}$$

We have that

$$\begin{aligned} &\sum_{i=1}^{N_{tb}} [(A_{tbi} - \overline{AB}_{tb}) - (A_{tbi}^M - \overline{AB}_{tb}^M)]^2 \\ &= \sum_{i=1}^{N_{tb}} [(A_{tbi} - A_{tbi}^M) - (\overline{A}_{tb} - \overline{A}_{tb}^M)]^2 + N_{tb} [(\overline{A}_{tb} - \overline{A}_{tb}^M) - (\overline{AB}_{tb} - \overline{AB}_{tb}^M)]^2 \end{aligned}$$

and

$$\begin{aligned} &\sum_{i=1}^{N_{tb}} [(B_{tbi} - \overline{AB}_{tb}) - (B_{tbi}^M - \overline{AB}_{tb}^M)]^2 \\ &= \sum_{i=1}^{N_{tb}} [(B_{tbi} - B_{tbi}^M) - (\overline{B}_{tb} - \overline{B}_{tb}^M)]^2 + N_{tb} [(\overline{B}_{tb} - \overline{B}_{tb}^M) - (\overline{AB}_{tb} - \overline{AB}_{tb}^M)]^2. \end{aligned}$$

Since

$$\begin{aligned} & N_{tb} \left[ (\bar{A}_{tb} - \bar{A}_{tb}^M) - (\overline{AB}_{tb} - \overline{AB}_{tb}^M) \right]^2 + N_{tb} \left[ (\bar{B}_{tb} - \bar{B}_{tb}^M) - (\overline{AB}_{tb} - \overline{AB}_{tb}^M) \right]^2 \\ &= \frac{N_{tb}}{2} \left[ (\bar{A}_{tb} - \bar{B}_{tb}) - (\bar{A}_{tb}^M - \bar{B}_{tb}^M) \right]^2 \end{aligned}$$

the squared errors equal

$$\begin{aligned} & \sum_{t=1}^T \sum_{b=1}^{N_b} w_{tb} \left[ \sum_{i=1}^{N_{tb}} [(A_{tbi} - \bar{A}_{tb}) - (A_{tbi}^M - \bar{A}_{tb}^M)]^2 + \sum_{i=1}^{N_{tb}} [(B_{tbi} - \bar{B}_{tb}) - (B_{tbi}^M - \bar{B}_{tb}^M)]^2 \right. \\ & \left. + \frac{N_{tb}}{2} [(\bar{A}_{tb} - \bar{B}_{tb}) - (\bar{A}_{tb}^M - \bar{B}_{tb}^M)]^2 \right]. \end{aligned}$$

## References

- Acharya, V., S. Schaefer, and Y. Zhang (2008). Liquidity risk and correlation risk: A clinical study of the General Motors and Ford downgrade of May 2005. *Working Paper, London Business School*.
- Bernstein, D. S. (2005). *Matrix Mathematics*. 41 William Street, Princeton, New Jersey 08540: Princeton University Press.
- Bessembinder, H. and W. Maxwell (2008). Transparency and the corporate bond market. *Journal of Economic Perspectives* 22(2), 217–234.
- Bessembinder, H., W. Maxwell, and K. Venkaraman (2006). Market Transparency, Liquidity Externalities, and Institutional Trading Costs in Corporate Bonds. *Journal of Financial Economics* 82(2), 251–288.
- Blanco, R., S. Brennan, and I. W. Marsh (2005). An Empirical Analysis of the Dynamic Relationship between Investment Grade Bonds and Credit Default Swaps. *Journal of Finance* 60(5), 2255–2281.
- Brunnermeier, M. K. and L. H. Pedersen (2008). Market liquidity and funding liquidity. *forthcoming, Review of Financial Studies*.
- Chakravarty, S. and A. Sarkar (2003). Trading costs in three U.S. bond markets. *Journal of Fixed Income* 13, 39–48.
- Covitz, D. and C. Downing (2007). Liquidity or credit risk? The determinants of very short-term corporate yield spreads. *Journal of Finance* 62(5), 2303–2328.
- Dick-Nielsen, J. (2009). Liquidity biases in TRACE. *Working Paper*.
- Dick-Nielsen, J., P. Feldhütter, and D. Lando (2008). Corporate bond liquidity before and after the onset of the subprime crisis. *Working Paper*.
- Duffie, D. (1999). Credit Swap Valuation. *Financial Analysts Journal* 55, 73–85.
- Duffie, D., N. Gârleanu, and L. H. Pedersen (2005). Over-the-Counter Markets. *Econometrica* 73(6), 1815–1847.
- Duffie, D., N. Gârleanu, and L. H. Pedersen (2007). Valuation in Over-the-Counter Markets. *Review of Financial Studies* 20(6), 1865–1900.
- Duffie, D. and D. Lando (2001, May). Term Structures of Credit Spreads with Incomplete Accounting Information. *Econometrica*, 633–664.
- Edwards, A. K., L. E. Harris, and M. S. Piwowar (2007). Corporate Bond Market Transaction Cost and Transparency. *Journal of Finance* 62(3), 1421–1451.
- Fleming, M. J. (2003). Measuring Treasury Market Liquidity. *Federal Reserve Bank of New York Economic Policy Review* 9, 83–108.
- Goldstein, M. A., E. S. Hotchkiss, and E. R. Sirri (2007). Transparency and Liquidity: A Controlled Experiment on Corporate Bonds. *The Review of Financial Studies* 20(2), 235–273.
- Green, R., B. Hollifield, and N. Schürhoff (2007). Financial intermediation and the costs of trading in an opaque market. *Review of Financial Studies* 20, 275–314.
- Hong, G. and A. Warga (2000). An empirical study of bond market transactions. *Financial Analysts Journal* 56, 32–46.

- Huang, J. and M. Huang (2003). How Much of the Corporate-Treasury Yield Spread is due to Credit Risk? *Working Paper, Stanford University*.
- Jotikasthira, C. (2007). Does search friction really matter? Evidence from the corporate bond market. *Working Paper*.
- Longstaff, F., S. Mithal, and E. Neis (2005). Corporate yield spreads: default risk or liquidity? New evidence from the credit-default swap market. *Journal of Finance* 60(5), 2213–2253.
- Merton, R. (1974). On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *Journal of Finance* 29, 449–470.
- Nashikkar, A. and L. H. Pedersen (2007). Corporate bond specialness. *Working paper, New York University*.
- Rubinstein, A. (1982). Perfect equilibrium in a bargaining model. *Econometrica* 50, 97–109.
- Vayanos, D. and T. Wang (2007). Search and endogenous concentration of liquidity in asset markets. *Journal of Economic Theory* 136, 66–104.
- Vayanos, D. and P.-O. Weill (2008). A search-based theory of the on-the-run phenomenon. *Journal of Finance* 63, 1361–1398.
- Weill, P.-O. (2008). Liquidity premia in dynamic bargaining markets. *Journal of Economic Theory* 140, 66–96.
- Zhou, C. (2001). The term structure of credit spreads with jump risk. *Journal of Banking and Finance* 25, 2015–2040.