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Information, Expected Utility, and Portfolio Choice

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Abstract

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We study the consumption-investment problem of an agent with a constant relative risk aversion preference function, who possesses information about the future prospects of a stock. We also solve for the value of information to the agent in closed-form. We find that information can significantly alter consumption and asset allocation decisions. For reasonable parameter ranges, information increases consumption in the vicinity of 25%. Information can shift the portfolio weight on a stock from zero to around 70%. Thus, depending on the stock beta, the weight on the market portfolio can be considerably reduced with information, causing the appearance of under-diversification. The model indicates hat stock holdings of informed agents are positively related to wealth, unrelated to systematic risk, and negatively related to idiosyncratic uncertainty. We also show that the dollar value of information to the agent depends linearly on his wealth and decreases with both the propensity to intermediate consumption and risk aversion.

Information is an important feature of financial market settings. For example, investing clients rely considerably on the signals produced by professional analysts, and the notion that company insiders are likely to have private information about their firms has long been recognized. A few key theoretical questions arise in this context: How does long-lasting private information alter the consumption-investment problem of an agent? How do parameters such as the propensity to consume at intermediate points in time, the degree of risk aversion, stock beta, and the market risk premium affect the consumption-investment problem of an agent with long-lived information about a stock? What is the value of long-term information for a risk averse agent? What is the effect of wealth on the value of private information?

In this paper, we address the preceding questions by studying the value of long-term information in a framework that takes into account wealth effects and can readily yield closed-form solutions. In our framework, the informed agent has a power utility function (i.e., CRRA preferences) over intermediate consumption and terminal wealth, and allocates across the stock, the market portfolio and a risk-free asset within a dynamic (continuous time) economy. Our setting facilitates the understanding of the effect of subjective discount rates, the investment horizon, market risk premium and volatility, and stock beta as well as stock volatility on portfolio choice in the presence of private information.[1]

The framework's tractability comes at a cost. First, we do not consider frictions such as taxes and bid-ask spreads. More importantly, we assume that the trading by the agent

[1]To solve our model, we use the "enlargement of filtration" technique that has been applied elsewhere in various contexts. See, for example, Jacod (1985) Amendinger, Becherer and Schweizer (2003) or Pikovsky and Karatzas (1996). The latter paper also solves the portfolio problem of an investor with logarithmic preferences, who maximizes the utility of terminal wealth with imperfect knowledge of a risky asset's final payoff. We are able to tractably apply this method to CRRA preferences. Our model is related to that of Liu and Longstaff (2004). These authors postulate an arbitrage opportunity which converges to a value of zero at a terminal date. In effect, their model considers the case of an agent who has perfect foresight about the future value of the security. In contrast, we consider a generalized setting in which the informed agent has imperfect information about the future value of the stock.

does not affect the market price. This aspect of our framework is in the spirit of Merton (1971), and various subsequent papers on dynamic portfolio choice (for recent examples, see Chacko and Viceira, 2005, Kahl, Liu, and Longstaff, 2003, and Kim and Omberg, 1996). If the full-fledged equilibrium were tractable, our analysis would correspond to the limiting case where the mass of informed agents goes to zero so that the agent becomes atomistic.[2] This can be interpreted as a scenario where the investment of the agent is very small relative to aggregate trading volume. Alternatively, the value calculated in our paper can be viewed as an upper bound to the value of private information.[3] We justify our framework by observing that our analysis provides a link between the dynamic portfolio choice (and asset pricing) literature, which principally uses power utility functions, and the literature on informed trading, which typically assumes CARA preferences or risk-neutrality. In addition, the fact that our framework can be calibrated to real world data has implications for academics who may be interested in the gains accruing to insiders by way of their access to information about their companies.

We find that information has significant effects on both consumption and portfolio choice. For example, we find that for reasonable parameter ranges, information can increase consumption in the neighborhood of 25%. Further, the allocation to the stock in the portfolio can reach 75% compared to 0% in the absence of information. Depending on the stock's beta, the weight on the market portfolio can be greatly reduced in the presence of private signals. Our analysis also indicates that for agents with a low elasticity of intertemporal substitution (or high risk aversion), the propensity to consume negatively influences their expected initial holding in the risky asset. An agent with low risk aversion,

[2] There is evidence (Cornell and Sirri, 1992, Meulbroek, 1992, and Chakravarty and McConnell, 1999) which indicates that insider trades may not have as strong an effect on the market price as one would expect. This evidence suggests that many insiders make trading decisions under the assumption that they are atomistic.

[3] For our comparative statics on the value of private information, however, we require the interpretation that the computed value corresponds to that for an atomistic agent.

however, wishes to save for the future, and is expected to hold more stock initially to consume relatively more in the future as the propensity to consume increases.

In our setting, the more wealthy the agent, the more valuable is the information in dollar terms. This result is consistent with the casual observation that private information is more valuable for large shareholders; thus they have a greater incentive to collect it. Our analysis further shows that the value of information depends on the propensity to consume at intermediate points in time; informed agents with greater consumption propensities find private information to be less valuable. In addition, the age (or investment horizon) of the agent also has an effect on the value of information. A longer investment horizon trades off the benefit of increased opportunities for intermediate consumption against more uncertain, and highly discounted terminal wealth. This causes the value of information to peak at intermediate values of the time horizon. In addition, we find that less risk averse agents take a more aggressive position in the stock, which increases the value of private information.

We calibrate the model to fit the empirical evidence about stock return performance following earnings surprises. This allows us to determine the value of information about an expected earnings surprise a few weeks prior to its announcement. We find, for example, that an agent with $1 million in wealth, who obtains and trades on even a noisy signal, has the same expected utility as an agent with about $4 million in wealth who does not possess the signal.

The analysis we conduct provides a potential path towards understanding the stockholdings of top executives, given that they are likely to be knowledgeable about the future financial performance of a company. We show that the optimal stock portfolio weight is positively related to wealth but does not depend on β. This is because the effect of β can be undone by taking an offsetting position in the market, Furthermore, the holding is negatively related to idiosyncratic uncertainty (which represents the risk the agent has to bear to take advantage of private information). Our paper also is related to the literature on under-diversification. We find that the holding in the individual stock may approach high values even for relatively low values of the signal precision. This offers a theoretical rationale for why corporate executives may not be as well-diversified as conventional theory would suggest. The paper also is relevant for work on what may appear to be excessive holdings in private investment (Moskowitz and Vissing-Jorgensen, 2002), familiar stocks (Huberman, 2001) and the literature on home bias (Brennan and Cao, 1997, Kang and Stulz, 1997). In each of these cases, strong information about a company or an asset class' performance prospects may cause portfolios to appear considerably under-diversified. The lack of diversification, as we show, can be a rational response to superior (positive) information about assets' future prospects.[4] We also show that changes in stock holdings are positively correlated with future expected returns on the risky asset. This evidence is consistent with the literature that relates insider and institutional holdings to future returns.[5]

The value of private information, of course, has been studied extensively in earlier literature. Specifically, Grossman and Stiglitz (GS) (1980) have stimulated valuable analytical research by providing a tractable closed-form solution to the expected utility of informed agents in a framework with CARA (exponential) utility and normal distributions.[6] Quite aside from the fact that our setting, unlike CARA, permits wealth effects,

[4]The finding of high holdings of an individual stock applies in the case of a positive signal. Even on average, however, short-selling constraints could impede a symmetric negative position (see, e.g., Hong and Stein, 2003, Ofek and Richardson, 2003, and Lamont and Jones, 2002). Hence, portfolios of agents with private investors may appear to be under-diversified in the cross-section.

[5]See Rozeff and Zaman (1988), Seyhun (1992), and Gompers and Metrick (2001).

[6]This basic CARA-normal framework also has been used to analyze a number of important scenarios, for example, the buying and selling of information (Admati and Pfleiderer, 1987, 1990), multiple securities (Admati, 1985), market breakdowns (Bhattacharya and Spiegel, 1991), and diverse information (Verrecchia, 1982, Diamond and Verrecchia, 1981, Hellwig, 1980). For an interaction between insider trading and corporate investment in the context of the Grossman and Stiglitz (1980) setting, see Leland (1992), Ausubel (1991), and Manove (1989).

another feature of GS is that the analysis is done in terms of price levels, not returns. Thus, returns are ratios of normally distributed variables, and therefore the means and variances of returns are not defined. As an empirical matter, however, returns and their moments have been the quantities of interest in cross-sectional settings, and CRRA utility specifications allow the primitive to be returns rather than prices.[7] Second, the GS setting allows for the analysis of dollar, not proportional, holdings, but again in comparing securities and agents, the proportional holdings are of relevance. Thus, potential extensions of our framework facilitate calibrations to market data.[8] While it is true that unlike us, GS and the related literature solve the full rational expectations equilibrium, the offsetting aspect is that our approach provides amenability to closed-form solutions in terms of empirically measurable quantities.

The rest of the paper is organized as follows. Section 1 describes the basic structure of the model. Section 2 describes optimal investment and consumption with private information. Section 3 considers the economic gain from information. Section 4 concludes. All proofs appear in the appendix.

[7]There have been significant and valuable attempts towards calculating the value of information for more general utility functions; however, they have been conducted within static models, and by using approximations to the equilibrium – viz. Peress (2004). While these papers do consider the impact of wealth and private information on portfolio choice, how intermediate consumption and wealth effects influence the value of private information in a dynamic setting with general CRRA preferences is an issue which remains unaddressed in their work.

[8]Previous research also has conducted rich dynamic analyses within the CARA-normal framework, viz. Wang (1994), Brown and Jennings (1989), and Grundy and McNichols (1989). Other than the difference in the preference structure, another distinction between our paper and these other papers relates to the timing of information arrival. While the agent in our paper receives a long-lived signal at the initial date, those in these other papers receive short-lived signals at multiple dates. In particular, in Wang (1993), the signal received is valid only over the next trading period, whereas in the models of Brown and Jennings (1989) and Grundy and McNichols (1989) the signal can be exploited for a maximum of two dates. While Vives (1995) presents a model where agents have information valid over many periods, he also uses the CARA framework and does not consider the value (i.e., expected utility) of the information signals. The models involving strategic traders viz. Kyle (1985) and others (e.g., Admati and Pfleiderer, 1988, Back, 1992, Foster and Viswanathan, 1996) have also adopted risk-neutrality as a special preference structure, and the versions with risk aversion (e.g., Subrahmanyam, 1991, Holden and Subrahmanyam, 1994, Baruch, 2002) have used the same CARA utility function as Grossman and Stiglitz (1980).

1 The Economic Setting

The informed agent has a finite investment horizon $T' < \infty$. We will assume that the agent has a power utility function over intermediate consumption and the final wealth

$$U = \mathrm{E}_0 \left[\int_0^{T'} \alpha'^{\gamma} e^{-\rho t} \frac{c_t^{1-\gamma}}{1-\gamma} dt + e^{-\rho T'} \frac{W_{T'}^{1-\gamma}}{1-\gamma} \right], \tag{1}$$

where c_t and W_t represent consumption and wealth, respectively, at time t. The wealth dynamics are given by

$$dW_t = dV_t - c_t dt$$

where dV_t is the instantaneous dollar return on the agent's portfolio. The parameter γ is a measure of risk aversion as well as an inverse measure of the elasticity of intertemporal substitution, while α' represents the agent's propensity to consume at intermediate time-points. The quantity $T' - t$ may be viewed as the "age" of the agent at time t. We postulate that there are two risky assets: a market portfolio and a stock. The agent allocates wealth across three assets: his own risky stock, the market portfolio, and a riskless asset.

Since the agent is atomistic, he does not influence the market price. Other agents trade the risky assets and determine prices. The market value of the market portfolio, P_t, follows the process

$$dP_t = P_t[(r + \mu)dt + \sigma_m dB_t],$$

whereas that of the individual stock, S_t evolves according to

$$dS_t = S_t[(r + \beta\mu)dt + \beta\sigma_m dB_t + \sigma_s dZ_t],$$

where r is the riskfree rate. We assume that B_t and Z_t are two independent standard Brownian motions. It can be seen that the diffusion processes for the stock and the market portfolio are correlated through the common term involving dB_t, and dZ_t represents stock-specific, or idiosyncratic risk. Further, β describes the systematic risk of the stock.

Note that only the systematic risk dB_t is priced, in the sense that it is associated with a risk premium μ, while there is no risk premium associated with dZ_t. Because of this, the portfolio of an agent without information will consist only of the market portfolio and the riskless asset.

It follows from the Brownian motion specification that the stock price at time T is

$$S_T = S_0 e^{\left(r+\beta\mu-\frac{1}{2}(\beta^2\sigma_m^2+\sigma_s^2)\right)T+\beta\sigma_m B_T+\sigma_s Z_T}.$$

We assume that the agent receives a private signal about the diffusion process Z_t. Specifically, the agent observes a signal L about Z_T (with $T < T'$),

$$L = Z_T + \sigma_\epsilon \epsilon, \tag{2}$$

where ϵ is a standard normal random variable.

We now characterize the stochastic process for the evolution of the stock's value from the standpoint of the agent with private information. Note that at time t, dZ_t is a mean-zero normal random variable with variance dt. Equation (2) implies that

$$L = Z_t + dZ_t + (Z_T - Z_{t+dt}) + \sigma_\epsilon \epsilon.$$

Therefore, the above equation implies that $L - Z_t$ is a signal on dZ_t, with noise $(Z_T - Z_{t+dt}) + \sigma_\epsilon \epsilon$ which has a variance $T - (t + dt) + \sigma_\epsilon^2 \approx T - t + \sigma_\epsilon^2$. Standard filtering theory involving normal random variables implies that

$$d\hat{Z}_t = dZ_t - \frac{(L - Z_t)}{T - t + \sigma_\epsilon^2} dt$$

is a standard normal random variable conditional on L and Z_t. Thus, the original Brownian motion is an Ornstein-Uhlenbeck process in the information set of the agent,

$$dZ_t = \frac{(L - Z_t)}{T - t + \sigma_\epsilon^2} dt + d\hat{Z}_t,$$

with $\hat{Z}_t$ being the standard Brownian motion in the information set of the informed agent. While the above arguments are descriptive, formal derivations of this result are provided in the mathematics literature.

Let us define a process we term the "spread" as

$$\Lambda_t \equiv L - Z_t. \tag{3}$$

This process satisfies

$$d\Lambda_t = \frac{-\Lambda_t}{T - t + \sigma_\epsilon^2} dt - d\hat{Z}_t. \tag{4}$$

Note that the spread has a mean of 0 and a time-varying mean reversion coefficient of $\frac{1}{T-t+\sigma_\epsilon^2}$. The mean reversion coefficient decreases deterministically with time t and the mean reversion is highest at $t = T$.

The spread Λ_t can be expressed as a weighted average of past $d\hat{Z}_t$ realizations,

$$\Lambda_t = \frac{T - t + \sigma_\epsilon^2}{T + \sigma_\epsilon^2} L - \int_0^t \frac{T - t + \sigma_\epsilon^2}{T - u + \sigma_\epsilon^2} d\hat{Z}_u.$$

To the informed agent, the evolution of the stock price is given by

$$\frac{dS_t}{S_t} = \left(r + \beta\mu + \frac{\Lambda_t \sigma_s}{T - t + \sigma_\epsilon^2} \right) dt + \beta\sigma_m dB_t + \sigma_s d\hat{Z}_t, \tag{5}$$

where the evolution of Λ is given by equation (4). It is clear from the above expression that the instantaneous expected return on the stock, conditional on the information signal, is directly related to Λ_t.

Since Λ_t is determined given the paths of dB_t and $d\hat{Z}_t$ up to time t, $d\hat{Z}_t$ is determined by $\frac{dS_t}{S_t}$ and $\frac{dP_t}{P_t}$:

$$d\hat{Z}_t = \frac{1}{\sigma_s} \left(\frac{dS_t}{S_t} - \beta \frac{dP_t}{P_t} \right) - \frac{\Lambda_t}{T - t + \sigma_\epsilon^2} dt.$$

We adopt the standard stochastic control approach to solve the asset allocation problem of the informed agent. Let ϕ_t and ϕ_t^m denote the time t proportional holdings in the stock and the market, respectively. The wealth dynamics are given by

$$\begin{aligned} dW_t &= W_t\left(r + \mu\phi_t^m + \left(\beta\mu + \frac{\Lambda_t\sigma_s}{T - t + \sigma_\epsilon^2}\right)\phi_t\right)dt - c_t dt \\ &\quad + W_t(\phi_t^m\sigma_m dB_t + \phi_t(\beta\sigma_m dB_t + \sigma_s d\hat{Z}_t)). \end{aligned}$$

Note that the expected evolution of the wealth of the individual depends on Λ. This indicates that Λ is expected to play a key role in determining the individual's portfolio holdings, and it is to this issue we will now turn.

To account for a terminal date that is later than the information horizon (i.e., T' is greater than T), it is convenient to reformulate the problem by noting that Equation (1) can be written as

$$U = \alpha'^{\gamma}\mathrm{E}_0\left[\int_0^T \alpha^{\gamma} e^{-\rho t}\frac{c_t^{1-\gamma}}{1-\gamma}dt + e^{-\rho T}\frac{W_T^{1-\gamma}}{1-\gamma}\right], \tag{6}$$

where

$$\alpha = \left\{\frac{1}{A} + \left(\frac{1}{\alpha'} - \frac{1}{A}\right)e^{-A(T'-T)}\right\}^{-1}, \tag{7}$$

with

$$A = \frac{\rho - (1-\gamma)\left(r + \frac{\mu^2}{2\gamma\sigma_m^2}\right)}{\gamma}. \tag{8}$$

Following Merton (1971), we define the indirect utility function J by

$$J(W, \Lambda, t) = \max_{c_t, \phi_t, \phi_t^m} E_t[U].$$

It is well known that the indirect utility function has the following form

$$J(W, \Lambda, t) = e^{-\rho t}\frac{W^{1-\gamma}}{1-\gamma}\left[f(t, \Lambda)\right]^{\gamma}.$$

The appendix proves the following proposition, which gives the function f.

Proposition 1 *The function f in the indirect utility function J is given by*

$$f(t, \Lambda; T) = \alpha\int_t^T e^{a(t;s,T) + \frac{1}{2}b(t;s,T)\Lambda^2}ds + e^{a(t;T,T) + \frac{1}{2}b(t;T,T)\Lambda^2}, \tag{9}$$

where a and b are given by

$$
\begin{aligned}
\gamma a(t;s,T) &= \left(-\rho + (1-\gamma)\left(r + \frac{\mu^2}{2\gamma\sigma_m^2}\right)\right)(s-t) + \frac{1}{2}\ln\left(\frac{T-t+\sigma_\epsilon^2}{T-s+\sigma_\epsilon^2}\right) \\
&- \frac{1}{2}\gamma\ln\left(\frac{s-t}{\gamma\left(T-s+\sigma_\epsilon^2\right)} + 1\right), \\
\gamma b(t;s,T) &= \frac{1-\gamma}{\left(T-t+\sigma_\epsilon^2\right)}\frac{s-t}{\left[s-t+\gamma\left(T-s+\sigma_\epsilon^2\right)\right]}.
\end{aligned}
$$

Note that when $\sigma_\epsilon \to \infty$, $\gamma a = \left(-\rho + (1-\gamma)\left(r + \frac{\mu^2}{2\gamma\sigma_m^2}\right)\right)(s-t)$ and $b = 0$; in this case, we recover Merton's (1971) standard results, which are derived in the absence of private information. Also, the dependence of function f on $(r, \rho, \mu, \sigma_m^2)$ is very similar to that in Merton. In fact, it can be shown when $\alpha = 0$, the dependence is identical. As such, the variation of optimal consumption and portfolio choice and the value function with $(r, \rho, \mu, \sigma_m^2)$ is isomorphic to that in Merton's model. In the rest of the paper, our main focus will be on the dependence of consumption, portfolio choice, and expected utility on the variables that characterize the information signal, namely, Λ_t and σ_ϵ.

One striking property of $a(t)$ and $b(t)$ is that they are finite and well-defined for γ and t. Kim and Omberg (1996) show that if the stock return is predictable by way of an Ornstein-Uhlenbeck process, the functions $a(t)$ and $b(t)$ can be infinite for $\gamma < 1$ for a finite t. As such, the trading opportunity offered by such a return dynamics can be so great that the value function is infinity. However, the mean-reversion coefficient in the Ornstein-Uhlenbeck process is constant, while the spread in our paper has a time-varying mean-reversion coefficient that increases with t, as shown in equation (4). Even though the mean-reversion coefficient is still bounded in our paper, the fact that it increases with time reduces the stochastic investment opportunity and thus leads to a finite value function.

2 Optimal Investment and Consumption Policies with Information

In this section, we analyze how information affects the investor's consumption and portfolio choice.

2.1 The General Case

Our next proposition derives the optimal consumption policy and the portfolio weights chosen by the informed agent.

Proposition 2 *1. The optimal consumption is given by*

$$c_t^* = \alpha W f^{-1}. \tag{10}$$

2. The optimal portfolio weights are given by

$$\phi_t^* = \frac{1}{\sigma_s}\left(\frac{1}{T-t+\gamma\sigma_\epsilon^2} + \frac{\alpha\int_t^T (b(t;T,T)-b(t;s,T))e^{a(t;s,T)+\frac{1}{2}b(t;s,T)\Lambda_t^2}ds}{\alpha\int_t^T e^{a(t;s,T)+\frac{1}{2}b(t;s,T)\Lambda_t^2}ds + e^{a(t;T,T)+\frac{1}{2}b(t;T,T)\Lambda_t^2}}\right)\Lambda_t, \tag{11}$$

and

$$\phi_t^{m*} = \frac{\mu}{\gamma\sigma_m^2} - \beta\phi_t^*. \tag{12}$$

Qualitatively, the optimal consumption rate is proportional to the wealth, as expected for a power utility maximizer. For a given Λ, the more the agent invests today, the more the advantage he can take of the information and consume more later, which is the so-called substitution effect. On the other hand, with private information, the agent in effect has more resources by being better able to predict stock price movements, and thus may want to consume more in earlier periods, which is the so-called wealth effect. When $\gamma < 1$, the substitution effect dominates, the agent will invest more in the stock and thus consume less. On the other hand, for an agent with $\gamma > 1$, the wealth effect dominates, so that spreading consumption over the whole period is more important and the agent will consume more. From the same intuition, the consumption rate increases (decreases) with $|\Lambda|$ for $\gamma > 1$ ($\gamma < 1$).

We can also obtain an interpretation of the parameter α in the utility function represented by (6). Specifically, note from (10) that

$$\frac{\partial c^*}{\partial W} = \frac{\alpha}{f}.$$

It can be seen from (9) that α/f is increasing in α (since a and b do not involve α), so that α is a measure of the propensity to consume at intermediate time points. Later, we will see how α influences the holdings of the informed agents.

To compute the optimal consumption-to-wealth ratio and later on the optimal portfolio weights, we will assume the following benchmark case: $\Lambda = 0.5$, $\sigma_\epsilon = 30\%$, $T = 1$ year, $\gamma = 3$, $\alpha = 1$, $\sigma_s = 40\%$, $r = 4\%$, $\rho = 0.2$, $\mu = 6\%$, $\sigma_m = 15\%$, and $\beta = 1$ (only needed for computing the portfolio weight of the market). As we discussed earlier, the effects due to r, ρ, μ, and σ_m are small. $\alpha = 1$ is used in most literature. $\gamma = 3$ is standard. For this computation and all the following calibrations and calculations until section 3.4 we will assume the investment horizon, T', is equal to T. Note that uncertainty in $\hat{Z}_1$ is 1, so $\sigma_\epsilon = 30\%$ implies a reduction in volatility of $1 - \sqrt{1 - 0.3^2} = 4.6\%$, which is quite small. Also observe that the contribution to the expected return from Λ is $\frac{\Lambda_t \sigma_s}{T-t+\sigma_\epsilon^2} = \frac{0.5\times 0.4}{1+0.3^2} = 18\%$. The benchmark case is indicated by $*$ in all the figures to follow.

For the benchmark case, computations show that the agent will increase the optimal consumption by 25% relative to the case of no information. This is quite a significant increase. As a comparison, the consumption to wealth ratio is just a deterministic function of time for a CRRA agent without information, such an agent would increase his consumption by 25% only if his wealth is increased by 25%.

Figure 1 plots the consumption to wealth ratio as a function of Λ for $\gamma = 3$ and $\gamma = 1/2$, with the rest of the parameters the same as the benchmark case. Note that the consumption is an even function of Λ_t, thus the agent will increase the consumption irrespective of whether he receives good news or bad news. This is due to the fact that if Λ_t is negative, the agent will just short the stock. Even at $\Lambda = 0$, the informed agent still increases his consumption by 21% relative to the uninformed. This is due to the fact that there is information content in a signal with $\Lambda = 0$ relative to the case when there is no information at all.

Figure 2 plots the consumption to wealth ratio (at time 0) of an informed agent relative to that of an uninformed one as a function of the risk aversion γ. The parameter values used are otherwise the same as the benchmark case. The figure shows that the informed consume relatively less than the uninformed for γ smaller than 1, while the reverse is true for γ larger than 1. These results can be explained by noting that the parameter γ in the utility function is inversely related to the elasticity of intertemporal substitution. For small γ, the agent has a stronger tendency to substitute intertemporally, and consequently the consumption to wealth ratio is low. The reverse is true for high γ. Note, however, that the ratio is not strictly increasing in γ. For very high γ investors are too risk averse to make any use of the uncertain information and the consumption to wealth ratio of the informed asymptotes to that of the uninformed.

Figure 3 presents the same quantity as in Figure 1 as a function of the noisiness of private information (σ_ϵ^2). The figure shows that for log utility ($\gamma = 1$), the consumption to wealth ratio for an informed investor equals that for an uninformed investor. In this case, myopia dictates that the informed investor is only concerned about the one-step ahead investment opportunity. On the other hand, for $\gamma > 1$, the bigger the precision, the more the informed agents consume relative to the uninformed. In this case, the low elasticity of intertemporal substitution dictates that the more precise information will be employed to increase current consumption. For $\gamma < 1$, the informed are more patient and choose to consume less and to exploit the more precise private information later on in the trading process. Regardless of the risk aversion, as the precision of the information vanishes the consumption to wealth ratio of the informed goes to that of the uninformed.

We now turn to the portfolio weights chosen by the informed agent as determined in Proposition 2. Note that neither the J function nor the optimal stock portfolio weight depends on β. This is because the effect of β can be undone by taking an offsetting position in the market and the optimal combined exposure to the market risk is completely determined by the market volatility σ_m and market risk premium μ. Furthermore, the holding of the stock is determined completely by the information advantage Λ_t and the idiosyncratic risk σ_s (which represents the risk the agent has to bear to take advantage of private information).

On other hand, the market portfolio weight ϕ_t^{m*} depends on β linearly, due to the fact that the market needs to offset the market risk exposure in the stock position. Note that the dependence of the market portfolio weight on Λ_t and σ_ϵ is opposite to that of the stock portfolio weight as shown in equation (12) (assuming $\beta > 0$). So, we will mainly focus on discussions of the stock portfolio weight below. This weight is proportional to Λ_t; thus, the agent will hold more stock with a larger positive Λ_t and short more stock with a larger negative Λ, as one might intuitively expect.

For the benchmark case, the optimal stock portfolio weight is 74%. In contrast, for the uninformed agent, the holding of the stock should be zero. The market portfolio weight in this case is 15% whereas without information it is 89%. Thus information dramatically alters the agent's portfolio. With such an order of magnitude difference, information effect can potentially be used to explain why investor hold undiversified portfolios.

In Figure 4, the initial (time 0) holding is plotted as a function of the propensity to consume, α. For highly risk averse informed agents, the propensity to consume negatively influences their holding in the risky asset. In this case, the agent wishes to hold less stock at time 0 and consume more if α is large. An agent who is less risk averse than log utility, however, has a greater tendency to postpone consumption for the future, and in this case holds more stock to consume relatively more in the future as α increases. Observe that an informed agent with low risk aversion ($\gamma = 0.5$) initially chooses to invest more than 100% of his wealth in the stock. The intuition is that the agent takes a more aggressive position to consume more in the future when the risk aversion is low.

In Figure 5, we present the initial holdings in the the stock and the market as a function of the noisiness parameter σ_ϵ^2. We find that as the information becomes more imprecise, the holding in the stock decreases, while the holdings in the market increase. This finding is intuitive. It is noteworthy that the proportion allocated to the individual stock can approach a quantity as high as 70% even for moderate values of σ_ϵ^2. The paper is thus related to the literature on investing in the familiar (Huberman, 2001) as well as that on home bias (Brennan and Cao, 1997, Kang and Stulz, 1997). In each of these cases, strong positive information about a company or an asset class' performance prospects may cause portfolios to appear considerably underdiversified. The under-diversification, as we show, can be a rational response to superior (positive) information about assets' future prospects.

Note that for high values of σ_ϵ^2, the holding in the risky stock dips below the holding in the market. Thus, insiders with highly imprecise information will place greater emphasis on diversification than on holdings in their own stock. A prediction of this part of the analysis is that for companies where good information is hard to come by, such as the high tech sector, will have better-diversified insiders. Also note that $\phi_t^* \to 0$ as $\sigma_\epsilon \to \infty$; this is expected since the signal becomes completely uninformative in this limit.

2.2 The Case of Logarithmic Utility

The expressions for consumption and portfolio weights can be further simplified in the case of logarithmic utility ($\gamma = 1$), as shown in the appendix (within the proof of Proposition 3 to follow). Under this preference structure, the investor's utility can be written as

$$U = \lim_{\gamma \to 1} E_0 \left[\int_0^T \alpha^\gamma e^{-\rho t} \frac{c_t^{1-\gamma} - 1}{1-\gamma} dt + e^{-\rho T} \frac{W_T^{1-\gamma} - 1}{1-\gamma} \right]$$

$$= E_0 \left[\int_0^T \alpha e^{-\rho t} \ln(c_t) dt + e^{-\rho T} \ln(W_T) \right] \qquad (13)$$

Further, in this setting, the consumption to wealth ratio does not depend on the signal and is given by

$$\frac{c}{W} = \frac{\alpha}{f_1}$$

where

$$f_1 = \frac{\alpha}{\rho} \left[1 + \left(\frac{\rho}{\alpha} - 1 \right) e^{-\rho(T-t)} \right]$$

From the first order conditions, the portfolio holdings are given by

$$\phi_t^{m*} = \frac{\mu}{\sigma_m^2} - \beta \phi_t^*, \qquad (14)$$

$$\phi_t^* = \frac{1}{\sigma_s} \frac{\Lambda_t}{T - t + \sigma_\epsilon^2}. \qquad (15)$$

As can be seen, the consumption-to-wealth ratio is a non-stochastic function of the various parameters that do not involve the information signal. The myopic behavior implied by logarithmic utility dictates that the agent ignore the long-term value of the private signal in designing his optimal consumption policy. From (15), we see, however, that the holdings of the risky stock depends directly on Λ. From the definition of Λ_t in (3), it is evident that the expected long-run return on the stock, i.e., $\ln(S_T/S_0)$, is correlated with the informed agent's initial holding of the risky asset.[9] This accords with

[9] Of course, this finding holds for the general CRRA case as well, but the expressions are more complicated.

the empirical literature (e.g., Seyhun, 1986, 1991, Hadlock, 1998) that documents the relation of insider holdings with future stock returns.

3 Utility Gains from Private Information

In this section, we present a series of comparative statics results that build intuition on the economic impact of various parameters on the consumption and investment of the informed, as well as the value of private information. We define the value of private information as the ratio of certainty equivalents with and without the signal, for a given set of parameter values.

Let us define

$$a_0 \equiv \left(-\rho + (1-\gamma)\left(r + \frac{\mu^2}{2\gamma\sigma_m^2}\right)\right)(s-t)$$

and let $f_0(t;T)$ be the value of $f(t;\Lambda,T)$ that corresponds to $a = a_0$ and $b = 0$. Note that the function f_0 does not depend on Λ because $b = 0$. Then, $J_0 = e^{-\rho t}\frac{W^{1-\gamma}}{1-\gamma}f_0^\gamma$ is the indirect utility of an agent without information. In effect, the value of information at time t, denoted by $R(t)$, is that return on wealth which equates the indirect utility of the agent without information to that with information. We obtain the following proposition.

Proposition 3 *The value of private information at time t, defined as the ratio of the certainty equivalent with the information signal to that without the signal, is given by*

$$R(t) = \left(\frac{f(t,\Lambda;T)}{f_0(t;T)}\right)^{\frac{\gamma}{1-\gamma}}, \tag{16}$$

and is always greater than unity.

Note that the ex ante value of private information (before the signal is realized), which we denote R_v, is given by $R_v \equiv \left(\mathrm{E}\left[R(t)^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}$.

3.1 Numerical Illustrations

For the benchmark case from the previous section, the ex ante value of information, i.e., R_v (at time 0), is 1.56. If the signal is more precise, i.e. σ_ϵ is 0.10, the value jumps to 2.22. Thus the expected utility of an agent who has a wealth of \$1 million and receives a signal with $\sigma_\epsilon = 0.10$ is the same as that of an agent with about \$2.2 million in wealth but who receives no signal.[10]

We present the ex post value of information (i.e., after realization of the private signal) in Figure 6 as a function of the time horizon. As can be seen, the value of information first increases, and then decreases in the time horizon. The intuition is that increasing the time horizon has two effects: there are more opportunities to trade, but it is also more risky to hold a position in the stock. Hence, for small values of T, the former effect dominates, whereas for large values of T, the latter feature takes over. The figure also indicates that information is more valuable for less risk averse agents. This is because agents with low risk aversion are able to take a more aggressive position in the stock.

In Figure 7 we plot the ex ante value of information (before realization of the signal) as a function of the propensity to consume, α, using the same base parameters as before. As can be seen, the greater is the propensity to consume, the smaller is the value of information. In addition the value of information is greater for low risk aversion. The drop in the value of information as a function of α is steeper for the low risk aversion ($\gamma = 0.5$) case. In this case, the agent wishes to exploit private information by consuming relatively less and saving more for the future. A high α shifts relatively more consumption early on in the trading process and thus sharply reduces the ex ante value of information. The basic notion is that agents who wish to consume more at intermediate time points

[10]For a highly wealthy top manager in a company like Microsoft or Google, the assumption of an atomistic agent is less reasonable (unless perhaps the stock position is accumulated slowly and anonymously). In such cases, as we mentioned in the introduction, our computed quantity can be viewed as an upper bound to the value of information for such agents.

find long-term information to be less valuable.

3.2 A Specific Calibration

To get a feel for the (ex post) value of information given the realization of a specific information signal with a tangible empirical interpretation, we calibrate our model to the case of earnings surprises. Campbell, Lo, and MacKinlay (1997) show that the 21 day cumulative abnormal return in advance of a positive earnings surprise is 1.966%. The analogous number for a negative surprise is -1.539%. Since the CAR can be viewed as the cumulative realization of $\sigma_s dZ_t$, the signal L can be calibrated as the cumulative abnormal return scaled by σ_s, the idiosyncratic volatility of the stock. We choose $\sigma_s = 0.25$ based on the findings of Campbell, Lettau, Malkiel, and Xu (2004) about the idiosyncratic volatility of individual stocks. Then, for a positive surprise, L can be calibrated as 0.01966*250/(21*0.25)=0.9362, and the corresponding number for a negative surprise is -0.7329. Considering current Treasury Bill yields, we choose an annual risk-free interest rate of 4%, and, based on Siegel (1998), an equity risk premium of 8%. Somewhat subjectively, we set $\gamma = 3$, which is within (but towards the lower end of) the range considered by Prescott and Mehra (1997). The subjective discount rate (ρ) and the propensity to consume (α) are set to be 0.1 and 1, respectively. The β is 0.8 and T is 0.084 (21 days).

Table 1 reports the ex-post value of information for the cases of a good news surprise and of a bad news surprise. Even when the signal is relatively noisy, such as in the case where σ_ϵ equals 20%, the value of information is 4.31 in the case of good news, implying that an agent with $1 million in wealth has the same utility from the signal as an agent with $4.31 million in wealth, but without the signal. For a less precise signal, e.g., when σ_ϵ is 30%, the value of information drops to 1.83, but it increases dramatically when σ_ϵ drops to 10%, i.e., as the signal becomes more precise.

Figure 8 plots how calibrated holdings in the stock vary with the precision of information, σ_ϵ, in response to a positive or a negative earnings signal. The weight on the stock can be very high for the case of precise signals, and diminishes as the signal becomes less precise. The figure demonstrates how private information can significantly impact the holdings of informed agents.

3.3 The Special Case of No Intermediate Consumption

Consider the case where $\alpha = 0$. In this case the ratio of the utility equivalent of the informed agent to that of the uninformed is given by

$$R(t) = \frac{\left(\frac{T-t}{\sigma_\epsilon^2}+1\right)^{\frac{1}{2(1-\gamma)}}}{\left(\frac{T-t}{\gamma\sigma_\epsilon^2}+1\right)^{\frac{\gamma}{2(1-\gamma)}}} \exp\left(\frac{(T-t)\Lambda_t^2}{2(T-t+\sigma_\epsilon^2)(T-t+\gamma\sigma_\epsilon^2)}\right).$$

Note that $R > 1$ even if $\Lambda = 0$. Knowing that Z_T will equal Z_t is still more valuable than knowing nothing. The increased value is due to trading before T. Even though Λ_t may be zero today, the future spread Λ may become non-zero and the informational advantage can thereby be exploited between t and T. The value of information depends positively on Λ_t^2, which is intuitive.

Using the explicit expression above, we can verify the following. When there is no information, i.e., $\sigma_\epsilon \to \infty$, $R(t) \to 1$. In the special case where $\sigma_\epsilon = 0$, the informed agent knows the stock price at time T precisely and the return of the stock follows a Brownian bridge process. As $\sigma_\epsilon \to 0$, $R(t) \to \infty$, implying that utility with information approaches infinity when the noise in information approaches 0. Liu and Longstaff (2004) study a version of this special case by modeling an arbitrage opportunity whose final value converges to zero. However, they obtain a finite utility level because, in their paper, there are margin requirements which place restrictions on the position the agent can take.

Note that, when $\alpha = 0$, with the exception of γ, R only depends on the characteristics of the signal, such as time to revelation of information, $T - t$, the signal precision, σ_ϵ, and the spread Λ_t. The ratio R does not depend on the interest rate, market risk premium, market volatility, stock beta, and stock volatility. One might expect otherwise because the riskless asset and the market portfolio are the alternative to investment in the stock. This happens because the information affects the indirect utility function J multiplicatively when $\alpha = 0$, as can be seen from Proposition 1. That is, the indirect utility function is the indirect utility for the no information case multiplied by a factor that is affected by private information.

It is also of interest the calculate the value of information at the time the information is received (i.e., time 0). We have that

$$R(0) = \frac{\left(\frac{T}{\sigma_\epsilon^2} + 1\right)^{\frac{T}{2(1-\gamma)}}}{\left(\frac{1}{\gamma\sigma_\epsilon^2} + 1\right)^{\frac{\gamma}{2(1-\gamma)}}} \exp\left(\frac{T \cdot L^2}{2(T + \sigma_\epsilon^2)(T + \gamma\sigma_\epsilon^2)}\right).$$

As can be seen, the ratio of the utility equivalents is related to the square of the signal.

We now present an expression for the ex-ante value of the information.

Proposition 4 *The ex-ante value of information at any given time t is*

$$R_v = \left(\mathrm{E}\left[R(t)^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}} = \sqrt{1 + \frac{T-t}{\gamma\sigma_\epsilon^2}}. \quad (17)$$

As can be seen, the ex ante value of information is always greater than 1. At any time t, it is increasing in the ratio of the variance of the brownian motion of the stock return $(T - t)$ over the variance of the signal (σ_ϵ^2) and decreasing in the risk aversion. Even if the signal noise has greater variance than the underlying stock the additional information is still valuable, because the signal helps reduce uncertainty about the stock's terminal value.

3.4 Differing Information and Investment Horizons

Note that in our model, the agent has information about the stock value at time T but has an investment horizon until time $T' > T$.[11] From (6)-(8), this problem is equivalent to that of an investor who has an investment horizon of T and a propensity to consume $\alpha' = \frac{1}{K_0}$, where K_0 is given by

$$K_0 = \frac{1}{A} + \left(\frac{1}{\alpha} - \frac{1}{A}\right) e^{A(T'-T)},$$

and where

$$A = \frac{\rho - (1-\gamma)\left(r + \frac{\mu^2}{2\gamma\sigma_m^2}\right)}{\gamma}.$$

The value of a longer investment horizon is a tradeoff between additional intermediate consumption and a farther out, and therefore riskier and highly discounted, terminal wealth. In Figure 9 we plot the ex ante value of information as a function of the investment horizon for various levels of risk aversion. In all the plotted cases the propensity to consume is large enough that the effect of additional opportunities to consume dominates, so that the ex ante value of information monotonically increases with the investment horizon.

To build more intuition in this case, Figure 10 shows the holdings in stock at time zero as a function of the investment horizon. Since in this case an opportunity for more intermediate consumption is more valuable than a terminal wealth that is nearer in the future, the consumption stream tends to become smoother with a longer investment horizon. Therefore, as can be seen in the figure, highly risk averse investors will increase their time zero holding in stock and postpone their consumption as their investment

[11] To account for a terminal date that is earlier than the information horizon (i.e., T' is smaller than T,) recall that the informed agent holds information about the diffusion process at time T, Z_T. This information is also a noisy signal about $Z_{T'}$ only with a higher noise variance. In that case the solution is similar to the case of T'=T only that the noise variance has to be adjusted for the time difference, $T' - T$.

horizon increases. Conversely, the time zero holding in stock decreases for the low risk aversion individuals as they face a longer investment horizon. Myopic log-utility investors are of course indifferent to the investment horizon.

4 Conclusion

We analyze the consumption-investment problem of an agent with CRRA preferences in a continuous time setting. For tractability, we assume that the agent is atomistic, which leads to an analytic expression for consumption, portfolio weights, and the ex ante value of private information for such an agent. Our analysis provides a link between the literature on dynamic portfolio choice, which principally uses power utility functions, and that on informed investment, which typically assumes CARA preferences or risk-neutrality. In addition, our model allows the characterization of the value of information in terms of empirically measurable quantities. This allows a calibration of the model to real-world data, which has implications for academics (and policymakers) who are interested in the magnitude of gains from information in equity markets.

Our analysis indicates that information is worth more in dollar terms, the greater is the wealth of agents, unlike in the case of exponential preferences, in which instance the value of information is independent of wealth. Since we explicitly model the propensity to consume at intermediate time points, we are able to examine how consumption alters the value of private information. Thus, we find that informed agents who have greater propensities to consume at intermediate times find long-term private information to be less valuable since they are less able to fully obtain the long-term benefits of trading on such information.

We also show that information can have significant effects on consumption as well as asset allocation. For reasonable parameter ranges, information can increase the consumption in the vicinity of 25%. An investor's holding in the stock may approach 75% with information compared to 0% were he not to have any information. Thus, agents with private information about an investment opportunity may appear to be substantially overinvested in that opportunity, which sheds light on the under-diversification phenomenon documented in various settings. We conduct a calibration exercise and find that even a noisy signal about a good or bad earnings surprise can significantly amplify expected utility. Further, insider holdings in the risky asset are related to future expected returns on that stock, which is consistent with the analyses of Seyhun (1986, 1992), and Rozeff and Zaman (1988).

The model suggests that portfolio holdings are positively related to wealth, inversely related to idiosyncratic uncertainty, and unrelated to systematic risk. We note that there are aspects of our theoretical analysis that could be extended to other settings. First, adapting our framework explicitly to multiple, correlated assets would be interesting and allow for predictions about insider holdings in related stocks, possibly those in the same industry. Second, while this is a difficult analytical issue, a solution to the full rational expectations setting where the insider is not atomistic remains elusive. A search for such a solution is clearly a predominant part of the agenda for future work on the subject.

Appendix

Proof of Propositions 1-3: We follow Merton (1971) in defining the indirect utility function J by

$$J(t, W_t, \Lambda) = \max_{c_t, \phi_t, \phi_t^m} E_t[U(W_T)]$$

$$\begin{aligned} dW_t &= W_t \left(r + \mu\phi_t^m + \left(\beta\mu + \frac{\Lambda_t \sigma_s}{T - t + \sigma_\epsilon} \right) \phi_t \right) dt - c_t dt \\ &\quad + W_t(\phi_t^m \sigma_m dB_t + \phi_t(\beta\sigma_m dB_t + \sigma_s d\hat{Z}_t)) \\ &= W_t \left(r + \mu(\phi_t^m + \beta\phi_t) + \frac{\Lambda_t \sigma_s}{T - t + \sigma_\epsilon} \phi_t \right) dt - c_t dt \\ &\quad + W_t((\phi_t^m + \beta\phi_t)\sigma_m dB_t + \phi_t \sigma_s d\hat{Z}_t) \\ &= W_t \left(r + \mu\varphi_t^m + \frac{\Lambda_t}{T - t + \sigma_\epsilon} \varphi_t \right) dt - c_t dt + W_t(\varphi_t^m \sigma_m dB_t + \varphi_t d\hat{Z}_t), \end{aligned}$$

where $\varphi_t^m = \phi_t^m + \beta\phi_t$ and $\varphi_t = \sigma_s \phi_t$. Note from above that the expected evolution of the wealth of the individual depends on the filtration parameter Λ, which represents the amount of information the agent has at any given time. This indicates that Λ is expected to play a key role in determining the individual's portfolio holdings, and it is to this issue we now turn.

From the Hamilton-Jacobi-Bellman equation, we obtain the following:

$$\begin{aligned} \max\nolimits_{c, \varphi^m, \varphi} \quad & \alpha^\gamma e^{-\rho t} \frac{c^{1-\gamma}}{1-\gamma} + \frac{\partial}{\partial t} J + W \left(\left(r + \mu\varphi^m + \varphi \frac{\Lambda}{T - t + \sigma_\epsilon^2} \right) - c \right) J_W \\ & + \frac{1}{2}(\varphi^{m2}\sigma_m^2 + \varphi^2) W^2 J_{WW} - \frac{\Lambda}{T - t + \sigma_\epsilon^2} J_\Lambda + \frac{1}{2} J_{\Lambda\Lambda} - W\varphi J_{W\Lambda} = 0, \end{aligned}$$

with the terminal condition

$$J(T, W_T, \Lambda) = e^{-\rho T} \frac{W_T^{1-\gamma}}{1-\gamma}.$$

We solve for the optimal portfolio strategy by conjecturing that the indirect utility function should have the form

$$J(t, W_t, \Lambda_t) = e^{-\rho t} \frac{W_t^{1-\gamma}}{1-\gamma} f^{\gamma}(t, \Lambda_t).$$

The first order condition for consumption c is given by

$$\alpha^{\gamma} c^{-\gamma} = W^{-\gamma} f^{\gamma},$$

so that

$$c = \alpha \frac{W}{f}. \tag{18}$$

As can be seen from the above expression, the consumption of the agent is a known proportion of current wealth.

The first order conditions for the portfolio weights are

$$\mu W J_W + \varphi^m \sigma_m^2 W^2 J_{WW} = 0; \tag{19}$$

$$\frac{\Lambda}{T - t + \sigma_\epsilon^2} W J_W + \varphi W^2 J_{WW} - W J_{W\Lambda} = 0. \tag{20}$$

This gives

$$\varphi_t^m = \frac{\mu}{\gamma \sigma_m^2}; \tag{21}$$

$$\varphi_t = \frac{\Lambda}{\gamma(T - t + \sigma_\epsilon^2)} - (\ln f)_\Lambda. \tag{22}$$

It can be seen from above that the optimal holding in the stock depends directly on the current Λ. The bigger is Λ, the greater is the value of information and the more aggressive is the position taken in the stock.

The HJB equation can be rewritten as

$$\begin{aligned} & \alpha f^{-1} - \rho + \gamma f^{-1} \frac{\partial}{\partial t} f + r(1-\gamma) - \alpha(1-\gamma) f^{-1} + \frac{1}{2}(1-\gamma) \frac{\mu^2}{\gamma \sigma_m^2} \\ & + \frac{(1-\gamma)\gamma}{2} \left(\frac{\Lambda}{\gamma(T - t + \sigma_\epsilon^2)} - (\ln f)_\Lambda \right)^2 - \frac{\Lambda}{T - t + \sigma_\epsilon^2} \gamma f^{-1} f_\Lambda \\ + \; & \frac{1}{2}(\gamma f^{-1} f_{\Lambda\Lambda} + \gamma(\gamma - 1) f^{-2} f_\Lambda^2) = 0, \end{aligned}$$

or

$$\begin{aligned}
&\alpha\gamma - \rho f + \gamma\frac{\partial}{\partial t}f + r(1-\gamma)f + \frac{1}{2}(1-\gamma)\frac{\mu^2}{\gamma\sigma_m^2}f \\
&+\frac{1-\gamma}{2\gamma}\left(\frac{\Lambda}{T-t+\sigma_\epsilon^2}\right)^2 f - (1-\gamma)\left(\frac{\Lambda}{T-t+\sigma_\epsilon^2}\right) f_\Lambda \\
&-\frac{\Lambda}{T-t+\sigma_\epsilon^2}\gamma f_\Lambda + \frac{1}{2}\gamma f_{\Lambda\Lambda} = 0,
\end{aligned}$$

$$\begin{aligned}
&\alpha\gamma - \rho f + \gamma\frac{\partial}{\partial t}f + r(1-\gamma)f + \frac{1}{2}(1-\gamma)\frac{\mu^2}{\gamma\sigma_m^2}f \\
&+\frac{1-\gamma}{2\gamma}\left(\frac{\Lambda}{T-t+\sigma_\epsilon^2}\right)^2 f - \left(\frac{\Lambda}{T-t+\sigma_\epsilon^2}\right) f_\Lambda + \frac{1}{2}\gamma f_{\Lambda\Lambda} = 0,
\end{aligned}$$

The PDE can be written as

$$\alpha\gamma + \mathcal{L}f(t,\Lambda;T) = 0; \tag{23}$$

$$f(T,\Lambda;T) = 1, \tag{24}$$

where

$$\begin{aligned}
\mathcal{L}f \;=\; & -\rho f + \gamma\frac{\partial}{\partial t}f + r(1-\gamma)f + \frac{1}{2}(1-\gamma)\frac{\mu^2}{\gamma\sigma_m^2}f \\
&+\frac{1-\gamma}{2\gamma}\left(\frac{\Lambda}{T-t+\sigma_\epsilon^2}\right)^2 f - \left(\frac{\Lambda}{T-t+\sigma_\epsilon^2}\right) f_\Lambda + \frac{1}{2}\gamma f_{\Lambda\Lambda}.
\end{aligned} \tag{25}$$

Proposition 5 *Suppose that $g(t,\Lambda;s,T)$ satisfies*

$$\mathcal{L}g(t,\Lambda;s,T) = 0; \tag{26}$$

$$g(s,\Lambda;s,T) = 1, \tag{27}$$

then

$$f(t,\Lambda;T) = \alpha\int_t^T g(t,\Lambda;s,T)ds + g(t,\Lambda;T,T).$$

Proof. It is obvious that $\alpha \int_T^T g(t,\Lambda;s,T)ds + g(T,\Lambda;T,T) = g(T,\Lambda;T) = 1$ so that the terminal condition is satisfied. Furthermore,

$$\begin{aligned}
&\mathcal{L}\left(\alpha \int_t^T g(t,\Lambda;s,T)ds + g(t,\Lambda;T,T)\right) \\
= \quad & -\alpha\gamma g(t,\Lambda;t,T) + \alpha \int_t^T \mathcal{L}g(t,\Lambda;s,T)ds + \mathcal{L}g(t,\Lambda;T,T) - \alpha\gamma, \qquad (28)
\end{aligned}$$

where the first term is from $\gamma\frac{\partial}{\partial t}$ on the lower integration limit.

Now we need to solve the following PDE

$$\begin{aligned}
&-\rho g(t,\Lambda;s,T) + \gamma\frac{\partial}{\partial t}g(t,\Lambda;s,T) + r(1-\gamma)g(t,\Lambda;s,T) \\
&+\frac{1}{2}(1-\gamma)\frac{\mu^2}{\gamma\sigma_m^2}g(t,\Lambda;s,T) + \frac{1-\gamma}{2\gamma}\left(\frac{\Lambda}{T-t+\sigma_\epsilon^2}\right)^2 g(t,\Lambda;s,T) \\
&-\left(\frac{\Lambda}{T-t+\sigma_\epsilon^2}\right)g_\Lambda(t,\Lambda;s,T) + \frac{1}{2}\gamma g_{\Lambda\Lambda}(t,\Lambda;s,T) = 0; \\
&g(s,\Lambda;s,T) = 1. \qquad (29)
\end{aligned}$$

Let $g(t,\Lambda;s,T) = e^{a(t;s,T)+\frac{1}{2}b(t;s,T)\Lambda_t^2}$. This reduces to the following ODE

$$\begin{aligned}
&-\rho + \gamma\frac{\partial}{\partial t}a + r(1-\gamma) + \frac{1}{2}(1-\gamma)\frac{\mu^2}{\gamma\sigma_m^2} + \frac{1}{2}\gamma b = 0; \\
&\gamma\frac{\partial}{\partial t}b + \frac{1-\gamma}{\gamma}\left(\frac{1}{T-t+\sigma_\epsilon^2}\right)^2 - \frac{2b}{T-t+\sigma_\epsilon^2} + \gamma b^2 = 0; \\
&a(s;s,T) = 0; \\
&b(s;s,T) = 0.
\end{aligned}$$

Let $d = (T-t+\sigma_\epsilon^2)\gamma b$ and $\tau = \ln(T-t+\sigma_\epsilon^2)$. We have

$$-\frac{\partial}{\partial\tau}d + \frac{1-\gamma}{\gamma} + \left(1-\frac{2}{\gamma}\right)d + \frac{d^2}{\gamma} = 0.$$

The solution is given by

$$\gamma a(t;s,T) =$$

$$\left(-\rho + (1-\gamma)\left(r + \frac{\mu^2}{2\gamma\sigma_m^2}\right)\right)(s-t) + \frac{1}{2}\ln\left(\frac{T-t+\sigma_\epsilon^2}{T-s+\sigma_\epsilon^2}\right) - \frac{1}{2}\gamma\ln\left(\frac{s-t}{\gamma\left(T-s+\sigma_\epsilon^2\right)}+1\right);$$

$$\gamma b(t;s,T) = \frac{1-\gamma}{(T-t+\sigma_\epsilon^2)} \frac{s-t}{[s-t+\gamma(T-s+\sigma_\epsilon^2)]}.$$

The function f is given by

$$f(t,\Lambda;T) \;=\; \alpha \int_t^T e^{a(t;s,T)+\frac{1}{2}b(t;s,T)\Lambda_t^2} ds + e^{a(t;T,T)+\frac{1}{2}b(t;T,T)\Lambda_t^2}. \tag{30}$$

The optimal portfolio weight is given by

$$\begin{aligned}
\varphi_t^* &= \left(\frac{1}{\gamma(T-t+\sigma_\epsilon^2)} - \frac{\alpha \int_t^T b(t;s,T) e^{a(t;s,T)+\frac{1}{2}b(t;s,T)\Lambda_t^2} ds + b(t;T,T) e^{a(t;T,T)+\frac{1}{2}b(t;T,T)\Lambda_t^2}}{\alpha \int_t^T e^{a(t;s,T)+\frac{1}{2}b(t;s,T)\Lambda_t^2} ds + e^{a(t;T,T)+\frac{1}{2}b(t;T,T)\Lambda_t^2}} \right) \Lambda_t \\
&= \left(\frac{1}{\gamma(T-t+\sigma_\epsilon^2)} - b(t;T,T) + \frac{\alpha \int_t^T (b(t;T,T)-b(t;s,T)) e^{a(t;s,T)+\frac{1}{2}b(t;s,T)\Lambda_t^2} ds}{\alpha \int_t^T e^{a(t;s,T)+\frac{1}{2}b(t;s,T)\Lambda_t^2} ds + e^{a(t;T,T)+\frac{1}{2}b(t;T,T)\Lambda_t^2}} \right) \Lambda_t \\
&= \left(\frac{1}{T-t+\gamma\sigma_\epsilon^2} + \frac{\alpha \int_t^T (b(t;T,T)-b(t;s,T)) e^{a(t;s,T)+\frac{1}{2}b(t;s,T)\Lambda_t^2} ds}{\alpha \int_t^T e^{a(t;s,T)+\frac{1}{2}b(t;s,T)\Lambda_t^2} ds + e^{a(t;T,T)+\frac{1}{2}b(t;T,T)\Lambda_t^2}} \right) \Lambda_t. \qquad (31) \\
\varphi_t^{m*} &= \frac{\mu}{\gamma\sigma_m^2} - \beta\phi_t^*.
\end{aligned}$$

For proving Part 2 of Proposition 2, note that without private information,

$$a \equiv a_0 = \left(-\rho + (1-\gamma) \left(r + \frac{\mu^2}{2\gamma\sigma_m^2} \right) \right) (s-t)$$

and $b = 0$. Let f_0 be the value of f that corresponds to $a = a_0$ and $b = 0$. Note that when $\gamma < 1$, $f(t,\Lambda;T) > f_0(t;T)$. Therefore, $\frac{\alpha W}{f} < \frac{\alpha W}{f_0}$. When $\gamma > 1$, $f(t,\Lambda;T) < f_0(t;T)$.Therefore, $\frac{\alpha W}{f} > \frac{\alpha W}{f_0}$ Thus the informed agent with $\gamma < 1$ will consume a greater fraction of his wealth than the agent with $\gamma > 1$. This completes the proofs of Propositions 1 through 2. □

Proof of Proposition 3: First, suppose that $\gamma < 1$, then, it can be easily proved that $a(t) > 0$ and $b(t) > 0$.

$$f(t,\Lambda;T) = \alpha \int_t^T e^{a(t;s)+\frac{1}{2}b(t;s)\Lambda_t^2} ds + e^{a(t;T)+\frac{1}{2}b(t;T)\Lambda_t^2} > \alpha \int_t^T e^{a_0(t;s)} ds + e^{a_0(t;T)} = f_0(t;T).$$

Therefore,

$$R = \left(\frac{f(t,\Lambda;T)}{f_0(t;T)} \right)^{\frac{\gamma}{1-\gamma}} > 1.$$

Second, consider the case $\gamma > 1$. In this case, it can be easily proved that $a(t) < 0$ and $b(t) < 0$.

$$f(t,\Lambda;T) = \alpha \int_t^T e^{a(t;s)+\frac{1}{2}b(t;s)\Lambda_t^2} ds + e^{a(t;T)+\frac{1}{2}b(t;T)\Lambda_t^2} < \alpha \int_t^T e^{a_0(t;s)} ds + e^{a_0(t;T)} = f_0(t;T).$$

Therefore,

$$R = \left(\frac{f(t,\Lambda;T)}{f_0(t;T)} \right)^{\frac{\gamma}{1-\gamma}} > 1.$$

The Case of Logarithmic Utility: In the case of $\gamma = 1$, we have logarithmic utility, which requires special mathematical treatment. The utility function can be written as

$$\begin{aligned} U &= \lim_{\gamma \to 1} \mathrm{E}_0 \left[\int_0^T \alpha^\gamma e^{-\rho t} \frac{c_t^{1-\gamma} - 1}{1-\gamma} dt + e^{-\rho T} \frac{W_T^{1-\gamma} - 1}{1-\gamma} \right] = \mathrm{E}_0 \left[\int_0^T \alpha e^{-\rho t} \ln c_t dt + e^{-\rho T} \ln W_T \right] \\ &= \lim_{\gamma \to 1} \mathrm{E}_0 \left[\int_0^T \alpha^\gamma e^{-\rho t} \frac{c_t^{1-\gamma}}{1-\gamma} dt + e^{-\rho T} \frac{W_T^{1-\gamma}}{1-\gamma} \right] - \int_0^T \frac{\alpha^\gamma e^{-\rho t}}{1-\gamma} dt - \frac{e^{-\rho T}}{1-\gamma}. \end{aligned}$$

So

$$\begin{aligned} J &= \lim_{\gamma \to 1} \frac{W_t^{1-\gamma}}{1-\gamma} \left(\int_t^T \alpha e^{a(t;s)+\frac{1}{2}b(t;s)\Lambda_t^2} ds + e^{a(t;T)+\frac{1}{2}b(t;T)\Lambda_t^2} \right)^\gamma - \int_t^T \frac{\alpha^\gamma e^{-\rho(s-t)}}{1-\gamma} ds - \frac{e^{-\rho(T-t)}}{1-\gamma} \\ &= g(t) lnW + h(t,\Lambda). \end{aligned}$$

Using a Taylor expansion around $\gamma = 1$ and denoting

$$f_1 = f(\gamma = 1) = \alpha \int_t^T e^{-\rho(s-t)} ds + e^{-\rho(T-t)}.$$

The indirect utility is

$$J = -\left(f_1 \ln f_1 + \frac{\partial f}{\partial \gamma} (\gamma = 1) \right) + f_1 \ln W + \alpha \ \ \ln \alpha \int_t^T e^{-\rho(s-t)} ds.$$

Noting that

$$\begin{aligned} \lim_{\gamma \to 1} a(t,s;T) &= -\rho(s-t) + (1-\gamma) \left(r + \frac{\mu^2}{2\sigma_m^2} - \rho \right) (s-t) \\ &+ \frac{1}{2}(1-\gamma) \left[\ln \left(\frac{T-t+\sigma_\epsilon^2}{T-s+\sigma_\epsilon^2} \right) - \frac{s-t}{T-t+\sigma_\epsilon^2} \right] \end{aligned}$$

$$
\begin{aligned}
& = & -\rho(s-t)+(1-\gamma)A_0+(1-\gamma)A_1 \\
\lim_{\gamma\to 1} b(t,s;T) & = & (1-\gamma)\frac{s-t}{(T-t+\sigma_\epsilon^2)^2}=(1-\gamma)B_1 \\
\frac{\partial f}{\partial\gamma}(\gamma=1) & = & \alpha\int_t^T\left(-A_0(t,s;T)-A_1(t,s;T)-\frac{1}{2}B_1(t,s;T)\Lambda^2\right)e^{-\rho(s-t)}ds \\
& + & \left(-A_0(t,T;T)-A_1(t,T;T)-\frac{1}{2}B_1(t,T;T)\Lambda^2\right)e^{-\rho(T-t)},
\end{aligned}
$$

results in the indirect utility being

$$
\begin{aligned}
J &= f_1\ln W - f_1\ln f_1+\alpha\int_t^T\left(A_0(t,s;T)+A_1(t,s;T)+\frac{1}{2}B_1(t,s;T)\Lambda^2\right)e^{-\rho(s-t)}ds+ \\
&+ \left(A_0(t,T;T)+A_1(t,T;T)+\frac{1}{2}B_1(t,T;T)\Lambda^2\right)e^{-\rho(T-t)}+\alpha\ \ \ln\alpha\int_t^T e^{-\rho(s-t)}ds.
\end{aligned}
$$

A similar result can be obtained by directly solving the HJB equation for the log utility case under the conjecture that $h(t,\Lambda)=a(t)+b(t)\Lambda^2$.

The indirect utility for an informed investor is

$$
J_0=f_1\ln W-f_1\ln f_1+\alpha\int_t^T A_0(t,s;T)e^{-\rho(s-t)}ds+A_0(t,T;T)e^{-\rho(T-t)}+\alpha\ln\alpha\int_t^T e^{-\rho(s-t)}ds.
$$

The (transformed) value of information in this case will be

$$
\ln(R)=\frac{\alpha\int_t^T\left(A_1(t,s;T)+\frac{1}{2}B_1(t,s;T)\Lambda^2\right)e^{-\rho(s-t)}ds+\left(A_1(t,T;T)+\frac{1}{2}B_1(t,T;T)\Lambda^2\right)e^{-\rho(T-t)}}{f_1},
$$

and is always positive. □

Proof of Proposition 4: The ratio of the utilities from being informed and uninformed is given by

$$
R(t)^{1-\gamma} = \frac{\left(\frac{T-t}{\sigma_\epsilon^2}+1\right)^{\frac{1}{2}}}{\left(\frac{T-t}{\gamma\sigma_\epsilon^2}+1\right)^{\frac{\gamma}{2}}}\exp\left(\frac{(1-\gamma)(T-t)\Lambda_t^2}{2(T-t+\sigma_\epsilon^2)(T-t+\gamma\sigma_\epsilon^2)}\right).
$$

Noting that at time t, Λ_t has a mean of zero and a variance of $T-t+\sigma_\epsilon^2$,

$$
E\left[R(t)^{1-\gamma}\right] = \int_{-\infty}^{\infty}\frac{\left(\frac{T-t+\sigma_\epsilon^2}{\sigma_\epsilon^2}\right)^{\frac{1}{2}}}{\left(\frac{T-t+\gamma\sigma_\epsilon^2}{\gamma\sigma_\epsilon^2}\right)^{\frac{\gamma}{2}}\sqrt{2\pi\left(T-t+\sigma_\epsilon^2\right)}}e^{\left(\frac{(1-\gamma)(T-t)\Lambda_t^2}{2(T-t+\sigma_\epsilon^2)(T-t+\gamma\sigma_\epsilon^2)}-\frac{\Lambda_t^2}{2(T-t+\sigma_\epsilon^2)}\right)}d\Lambda_t
$$

$$
\begin{aligned}
&= \frac{\left(\gamma\sigma_\epsilon^2\right)^{\frac{\gamma}{2}}}{\sqrt{2\pi}\sigma_\epsilon\left(T-t+\gamma\sigma_\epsilon^2\right)^{\frac{\gamma}{2}}}\int_{-\infty}^{\infty} exp\left(-\frac{\gamma}{2\left(T-t+\gamma\sigma_\epsilon^2\right)}\Lambda_t^2\right)d\Lambda_t \\
&= \left(\frac{T-t}{\gamma\sigma_\epsilon^2}+1\right)^{\frac{1-\gamma}{2}} \qquad (32)
\end{aligned}
$$

Equation (17) follows directly from the above. □

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Table 1:

Value of Insider Information About Earnings Surprises

This table reports the value of private information in the case of insiders anticipating a firm's earnings surprise. The value of private information is defined in the text as the ratio of the certainty equivalent with the information signal to that without the signal. Campbell, Lo, and MacKinlay (1997) show that the 21 day cumulative abnormal return (CAR) in advance of a positive earnings surprise is 1.966%. The analogous number for a negative surprise is -1.539%. Since the CAR can be viewed as the cumulative realization of $\sigma_s dZ_t$, the signal L can be calibrated as the cumulative abnormal return scaled by σ_s, the idiosyncratic volatility of the stock. We choose $\sigma_s = 0.25$ in line with Campbell et al. (2001). For a positive surprise, L is calibrated as 0.01966*250/(21*0.25)=0.9362, and the corresponding number for a negative surprise is -0.7329. The risk-free rate is set to 4%, and, based on Siegel (1998), the equity risk premium is 8%. We set $\gamma = 3$, which is within (but towards the lower end of) the range considered by Prescott and Mehra (1997). The subjective discount rate, ρ, and the propensity to consume , α, are set to 0.1 and 1, respectively. β is 0.8. The value of private information is defined in the text as the ratio of the certainty equivalent with the information signal to that without the signal.

Signal Noise (σ_ϵ)	Good News	Bad News
0.10	27.51	9.81
0.20	4.31	2.57
0.30	1.83	1.47
0.40	1.31	1.19
0.50	1.14	1.09
0.60	1.07	1.05
0.70	1.04	1.03
0.80	1.03	1.02
0.90	1.02	1.01
1.00	1.01	1.01

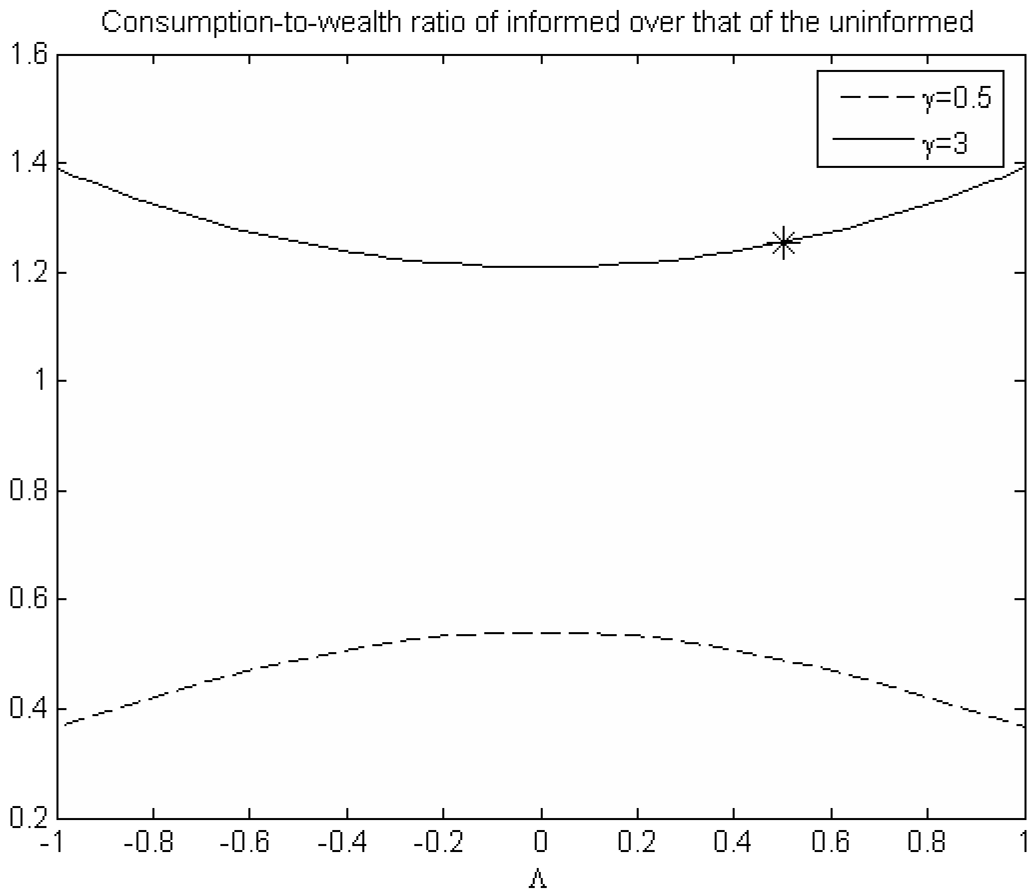

Figure 1: **Consumption-to-wealth ratio of informed over that of the uninformed as a function of the initial signal spread, Λ.** We assume that the propensity to consume, α, is 1; the volatility of the signal noise, σ_ϵ, is 0.3; the market risk premium, μ, is 6%; the market volatility, σ_m, is 15%; the risk free rate, r, is 4%; the time to the event, T, is 1; the idiosyncratic volatility of the stock, σ_s, is 40%; the subjective discount rate, ρ, is 0.2; and the β is 1.

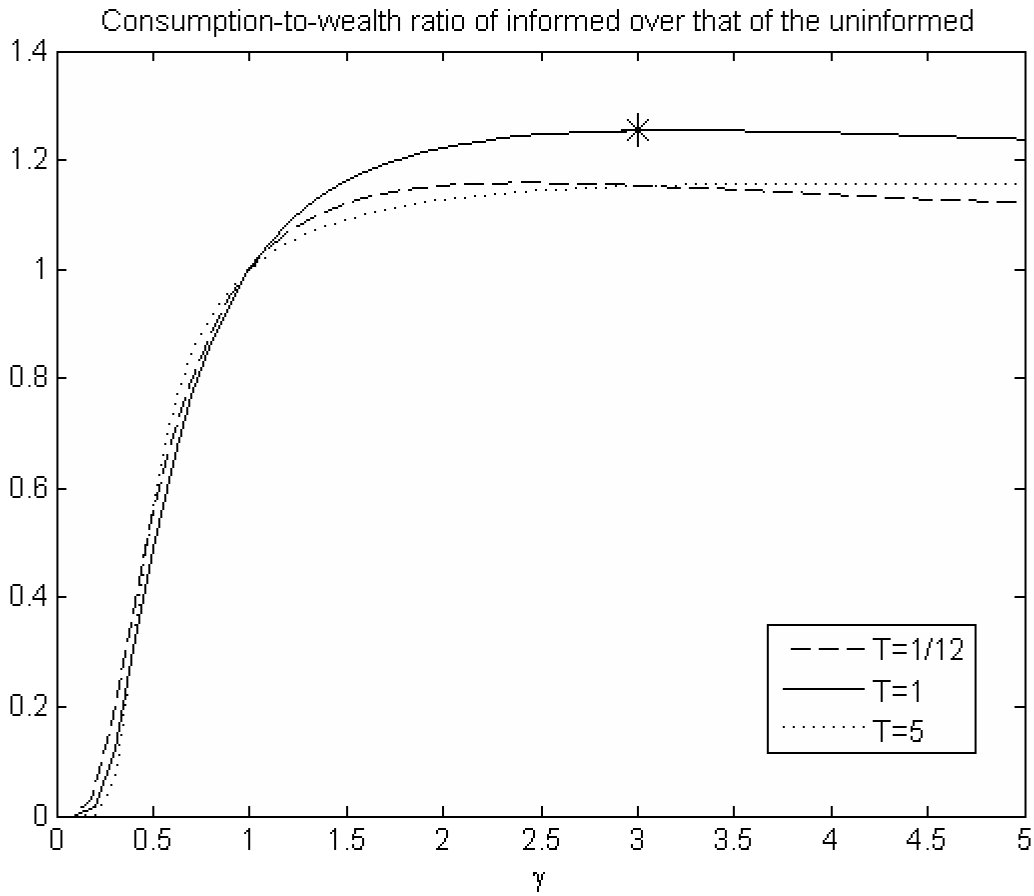

Figure 2: **Consumption-to-wealth ratio of informed over that of the uninformed as a function of the risk aversion, γ.** We assume that the propensity to consume, α, is 1; the volatility of the signal noise, σ_ϵ, is 0.3; the initial signal spread, Λ, is 0.5; the market risk premium, μ, is 6%; the market volatility, σ_m, is 15%; the risk free rate, r, is 4%; the time to the event, T, is 1; the idiosyncratic volatility of the stock, σ_s, is 40%; the subjective discount rate, ρ, is 0.2; and the β is 1.

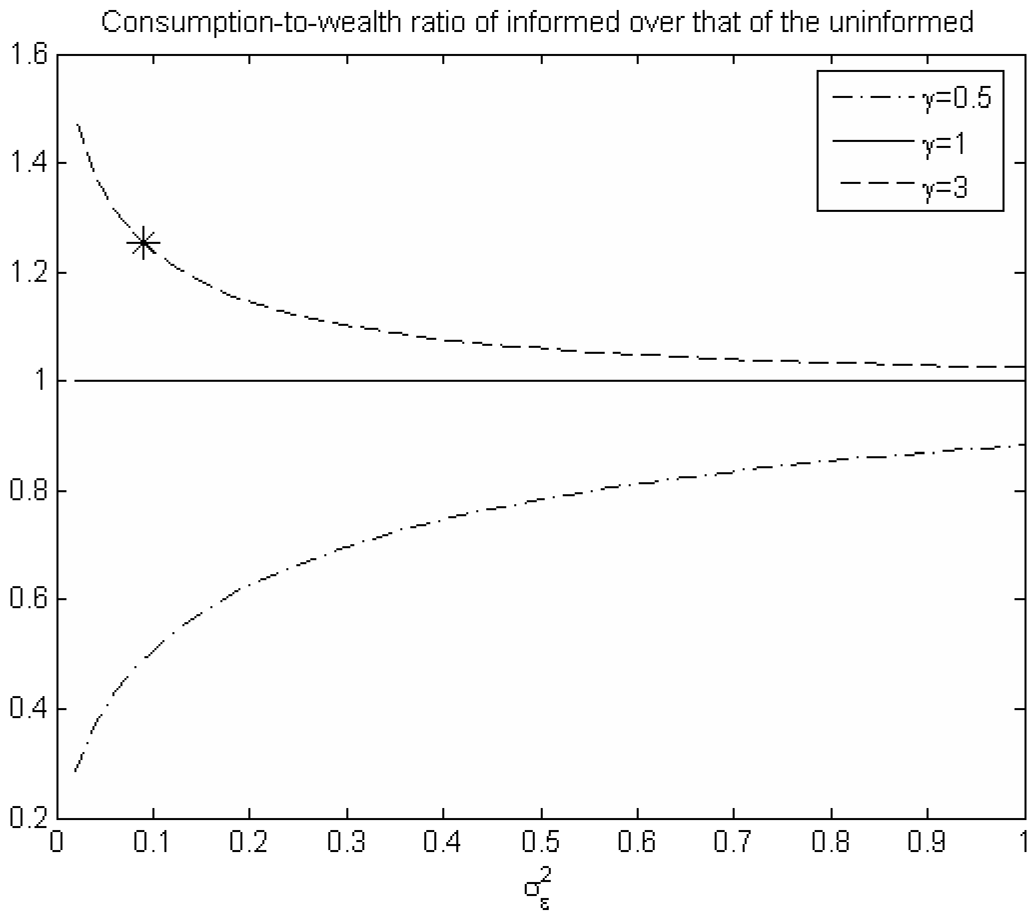

Figure 3: **Consumption-to-wealth ratio of informed over that of the uninformed as a function of the signal noise variance, σ_ϵ^2.** We assume that the propensity to consume, α, is 1; the initial signal spread, Λ, is 0.5; the market risk premium, μ, is 6%; the market volatility, σ_m, is 15%; the risk free rate, r, is 4%; the time to the event, T, is 1; the idiosyncratic volatility of the stock, σ_s, is 40%; the subjective discount rate, ρ, is 0.2; and the β is 1.

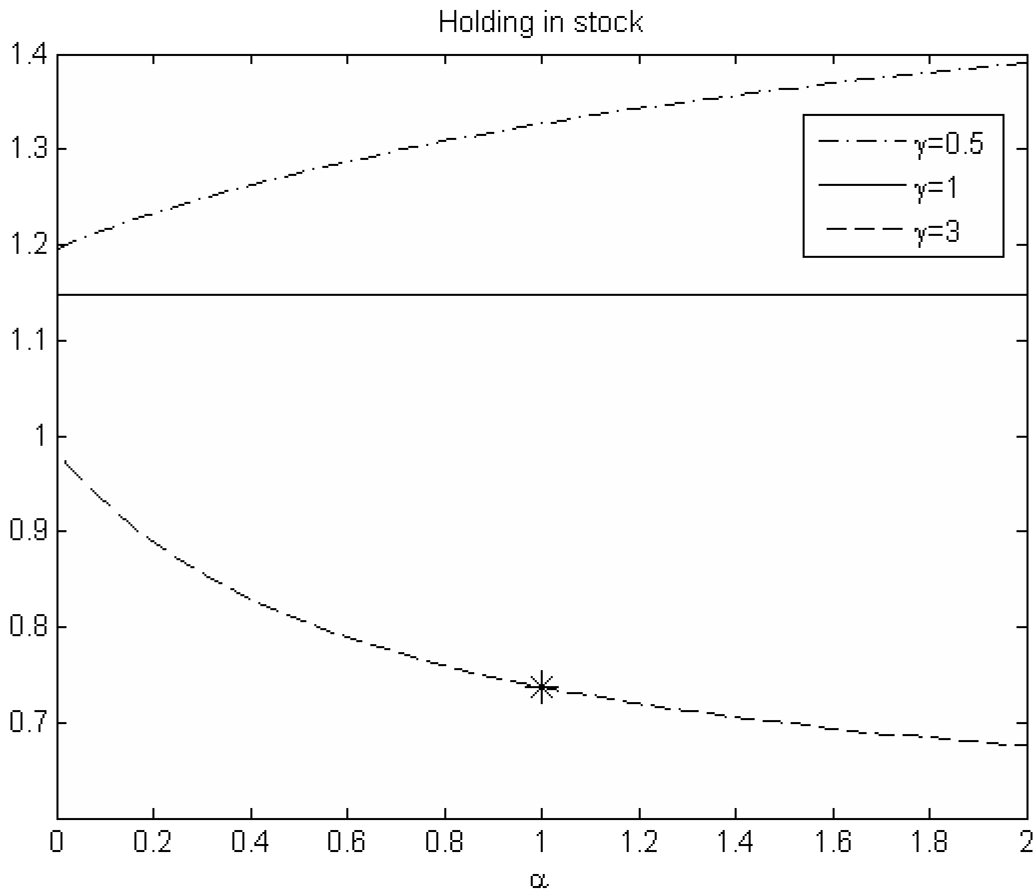

Figure 4: **Holding in stock as a function of the propensity to consume,** α. We assume that the volatility of the signal noise, σ_ϵ, is 0.3; the initial signal spread, Λ, is 0.5; the market risk premium, μ, is 6%; the market volatility, σ_m, is 15%; the risk free rate, r, is 4%; the time to the event, T, is 1; the idiosyncratic volatility of the stock, σ_s, is 40%; the subjective discount rate, ρ, is 0.2; and the β is 1.

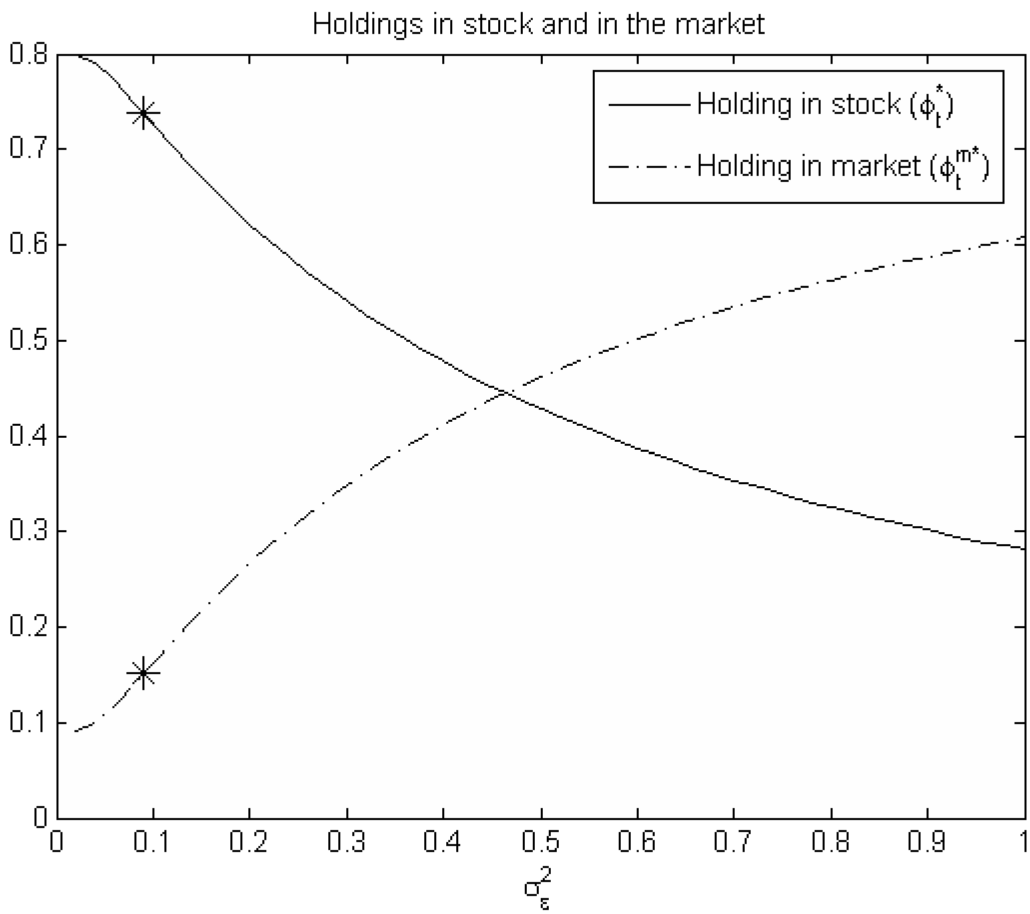

Figure 5: **Holding in stock and in the market as a function of the signal noise variance, σ_ϵ^2.** We assume that the propensity to consume, α, is 1; the initial signal spread, Λ, is 0.5; the market risk premium, μ, is 6%; the market volatility, σ_m, is 15%; the risk free rate, r, is 4%; the time to the event, T, is 1; the idiosyncratic volatility of the stock, σ_s, is 40%; the subjective discount rate, ρ, is 0.2; and the β is 1.

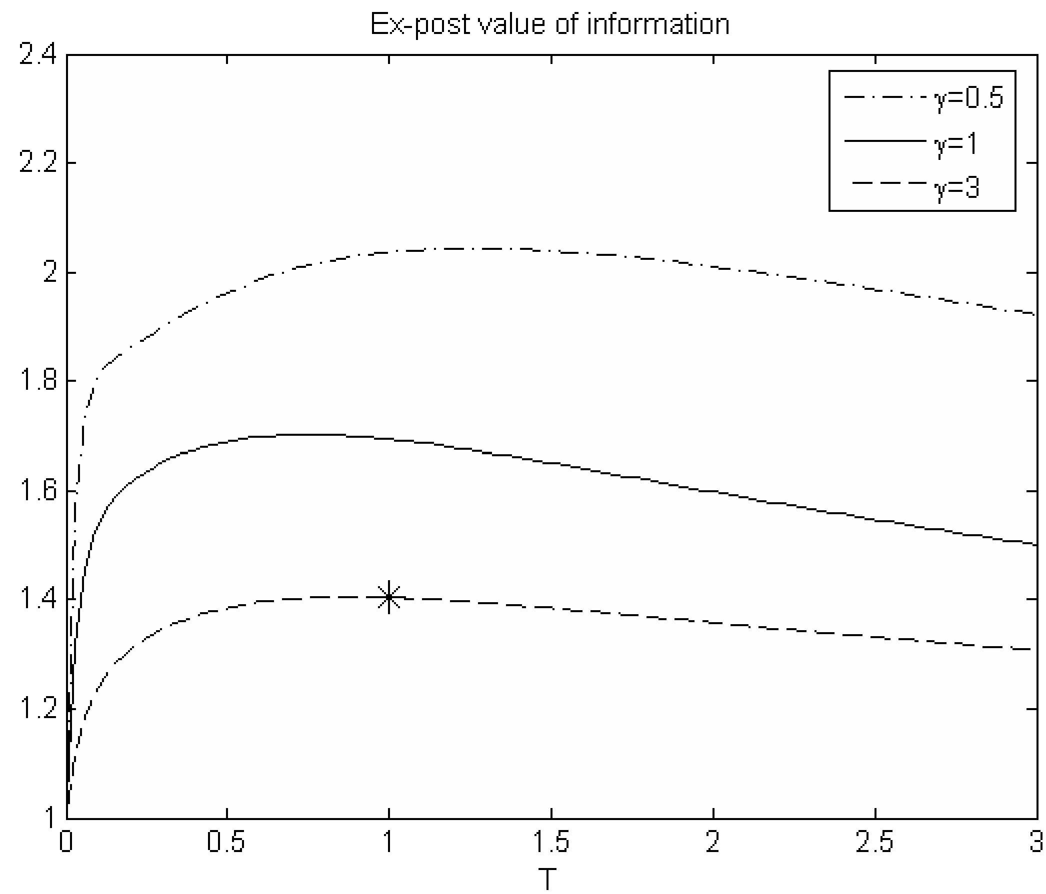

Figure 6: **Ex-post value of information as a function of the time to the event, T.** We assume that the propensity to consume, α, is 1; the volatility of the signal noise, σ_ϵ, is 0.3; the initial signal spread, Λ, is 0.5; the market risk premium, μ, is 6%; the market volatility, σ_m, is 15%; the risk free rate, r, is 4%; the idiosyncratic volatility of the stock, σ_s, is 40%; the subjective discount rate, ρ, is 0.2; and the β is 1.

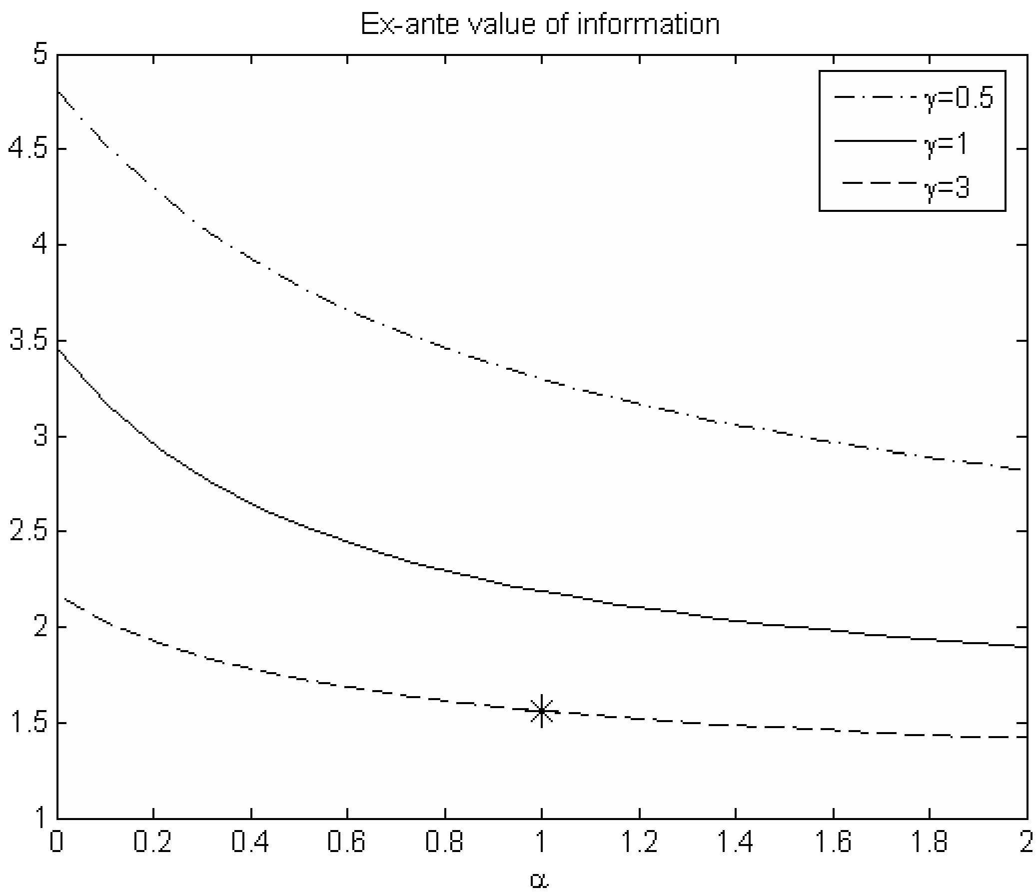

Figure 7: **Ex-ante value of information as a function of the propensity to consume, α.** We assume that the volatility of the signal noise, σ_ϵ, is 0.3; the initial signal spread, Λ, is 0.5; the market risk premium, μ, is 6%; the market volatility, σ_m, is 15%; the risk free rate, r, is 4%; the time to the event, T, is 1; the idiosyncratic volatility of the stock, σ_s, is 40%; the subjective discount rate, ρ, is 0.2; and the β is 1.

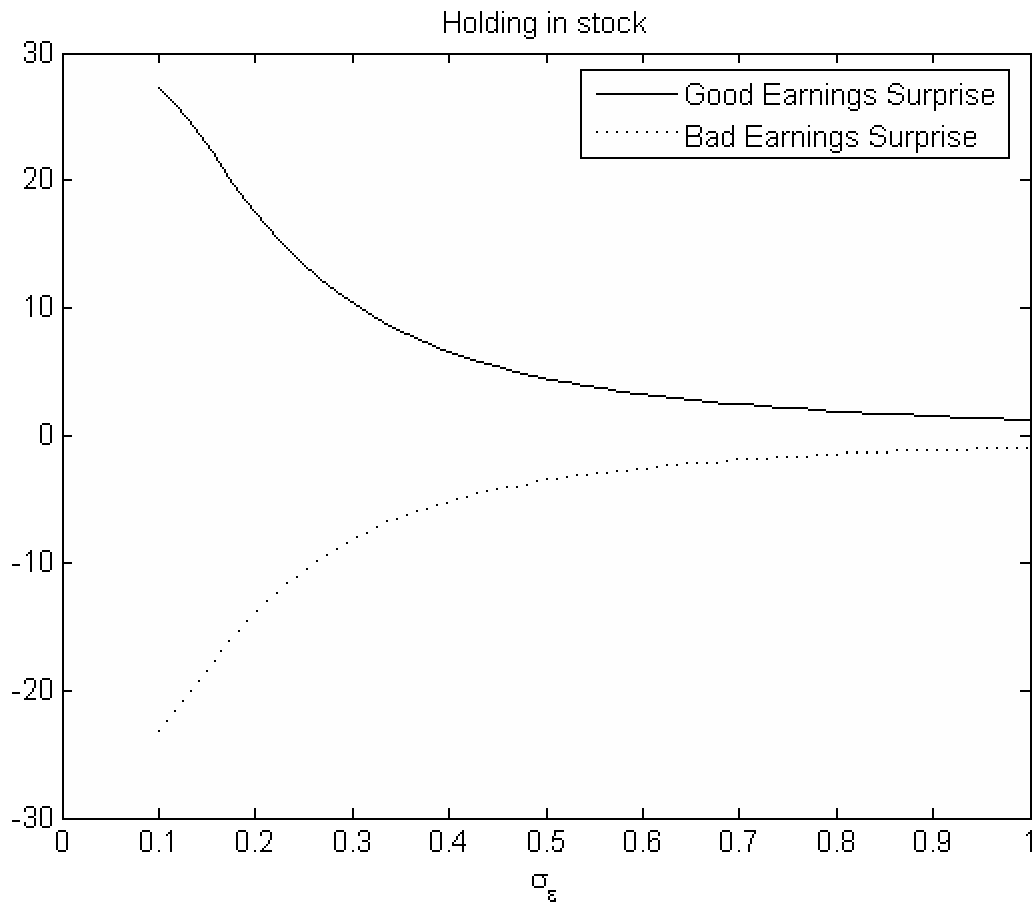

Figure 8: **Initial holdings in stock, in response to private information on earnings surprise, as a function of the volatility of the signal noise, σ_ϵ.** Calibrated to Campbell, Lo and MacKinlay (1997): The time to the event is 21 days (T=21/250), and L is based on a 21-day CAR for good news: L = 0.01966*250/(21*0.25) = 0.9362 and on a 21-day CAR for bad news: L = -0.01539*250/(21*0.25) = -0.7329. We assume that the propensity to consume, α, is 1; the market risk premium, μ, is 8%; the market volatility, σ_m, is 12%; the risk free rate, r, is 4%; the idiosyncratic volatility of the stock, σ_s, is 25%; the subjective discount rate, ρ, is 0.1; and the β is 1.

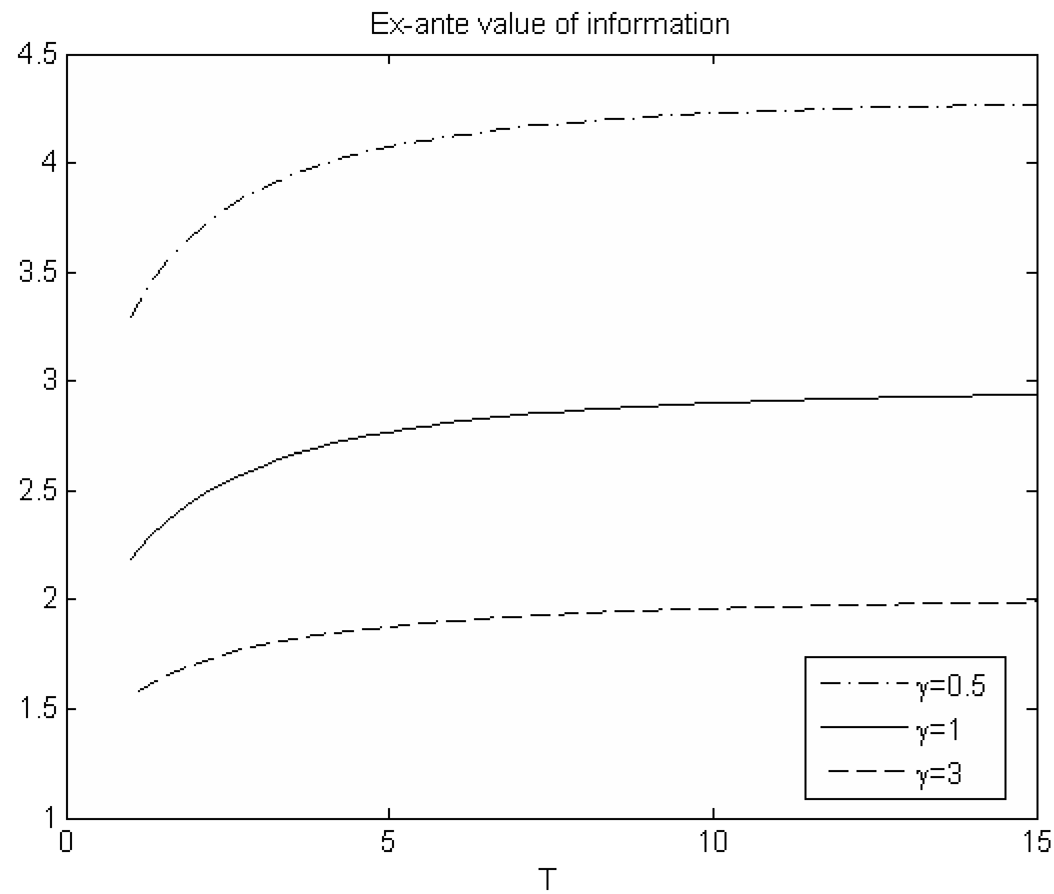

Figure 9: **Ex-ante value of information as a function of the investment horizon, T'.** We assume that the propensity to consume, α, is 1; the signal noise, σ_ϵ, is 0.3; the initial signal spread, Λ, is 0.5; the market risk premium, μ, is 6%; the market volatility, σ_m, is 15%; the risk free rate, r, is 4%; the time to the event, T, is 1; the idiosyncratic volatility of the stock, σ_s, is 40%; the subjective discount rate, ρ, is 0.2; and the β is 1.

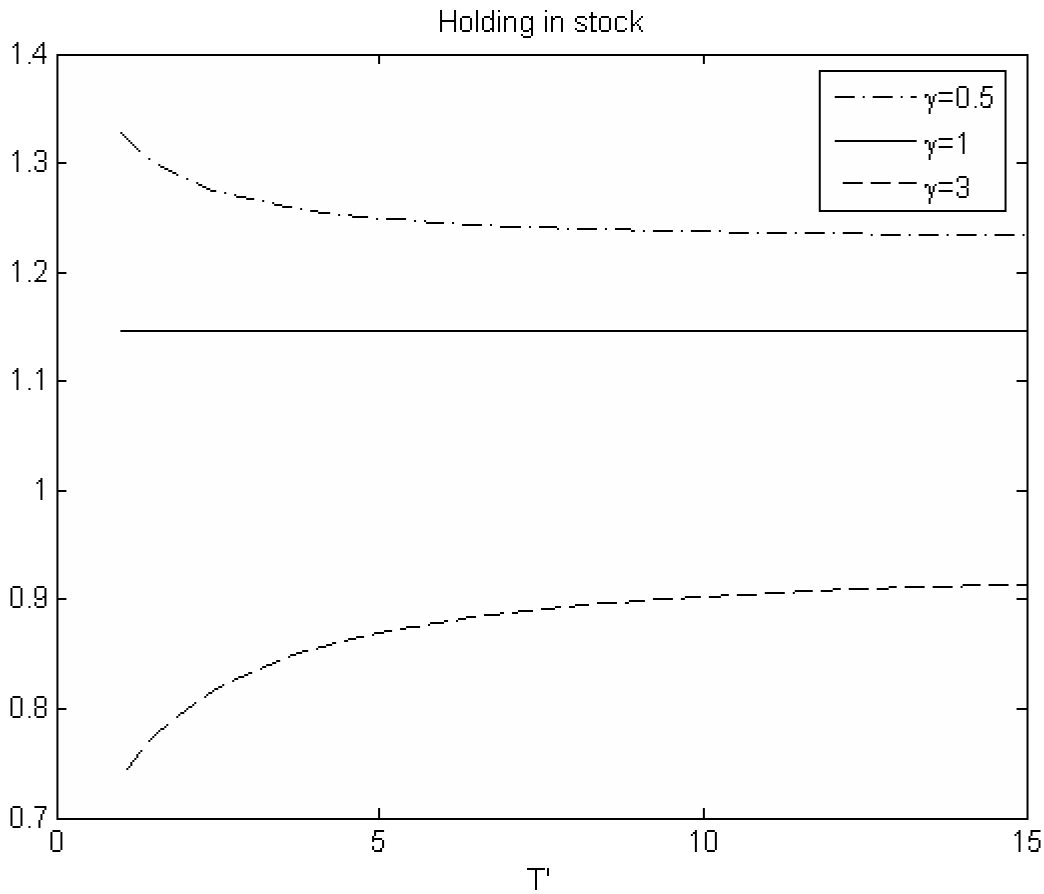

Figure 10: **Holding in stock as a function of the investment horizon, T'.** We assume that the propensity to consume, α, is 1; the signal noise, σ_ϵ, is 0.3; the initial signal spread, Λ, is 0.5; the market risk premium, μ, is 6%; the market volatility, σ_m, is 15%; the risk free rate, r, is 4%; the time to the event, T, is 1; the idiosyncratic volatility of the stock, σ_s, is 40%; the subjective discount rate, ρ, is 0.2; and the β is 1.