

# The Cross-Section and Time-Series of Stock and Bond Returns\*

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## Abstract

We show that the cross-section of expected returns of stock portfolios sorted along the book-to-market dimension can be explained using a factor, which we call the KLN factor, that is a linear combination of (contemporaneous) forward rates. It has a correlation of 82% with the Cochrane-Piazzesi (2005, CP) factor, itself a linear combination of the same (lagged) forward rates. Since the CP factor is a strong predictor of future excess bond returns, the high correlation between the KLN and the CP factors suggests a tight link between the cross-section of stock returns and bond risk premia. Inspired by this finding, we develop a parsimonious no-arbitrage stochastic discount factor model that can price both the cross-section of stock and bond returns. Featuring only three priced factors, all of which are yields, the model obtains a mean absolute pricing error of 40 basis points per year across 5 maturity-sorted government bond portfolios, 10 book-to-market-sorted stock portfolios, and the aggregate stock market. The CP factor is responsible for pricing the spread between stock portfolios, the dividend yield prices the mean of stock returns, and the level of the term structure prices the cross-section of bond returns. With two additional risk price parameters, the model also replicates the dynamics of bond yields as well as the time-series predictability of stock and bond returns. The value premium seems related to fundamental economic risk. Since the returns on value stocks are high when CP is high, which typically occurs near the end of a recession, value stocks carry high returns exactly when investors need it the least.

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The goal of this paper is to study a model that jointly accounts for stock and bond returns. According to the inter-temporal capital asset pricing model of Merton (1973), assets' exposures to changing investment opportunities are important determinants of average returns, in addition to the market beta. Variables that capture changes in the investment opportunity set should be priced in the cross-section. For bonds, we know that the Cochrane and Piazzesi (2005) factor, henceforth *CP*, is such a variable: it is the single best determinant of changes in future bond returns. A high *CP* signals high future bond returns. Focusing on the cross-section of book-to-market stock portfolios, we find that high book-to-market (value) stocks are assets that have high returns when *CP* is high. Positive *CP* innovations signal improving investment opportunities: *CP* typically reaches its lowest value at the start of a recession, increases sharply during the recession, and reaches its peak as the recession draws to an end and good times are around the corner. Hence, value stocks are a poor hedge against deteriorating investment opportunities; this makes them command a high average return. Growth stocks do well when news comes in that future bond returns are lower, making them desirable for the investor. In equilibrium, they carry a low average return. We use this ICAPM insight as a starting point to build a no-arbitrage stochastic discount factor (SDF) model that can account for the unconditional and conditional moments of stocks and bonds. This unified SDF model is parsimonious in that it only contains three priced risk factors, each of which is a linear combination of yields.

Yields are a natural place to look for information about the cross-section of stock and bond returns. They reflect the state variables of the economy which govern the dynamics of the investment opportunity set. Innovations in these state variables are important sources of risk and risk premia on risky assets provide compensation for bearing that risk. Section 1 spells out the theoretical motivation for our focus on yields. In practice, we show that three priced yield factors, two bond yields and one dividend yield, do a good job in explaining cross-sectional variation in bond and stock returns.

Based on this theoretical motivation, Section 2, finds a linear combination of yields that best fits the cross-section of stock returns. Specifically, we regress the return on the HML portfolio, a zero investment portfolio which goes long in high book-to-market (value) stocks and short in low book-to-market (growth) stocks on the contemporaneous one-year bond yield and the two-through five-year forwards rates. We label the fitted value of this regression the *KLN* factor. The regression  $R^2$  is only 2%, suggesting that the *KLN* factor is hardly correlated with the return on the HML portfolio itself. It is therefore surprising that it captures nearly all of its ability to explain cross-sectional variation in the ten book-to-market portfolios. The MAPE is 42 basis points.

Our main finding is that this *KLN* factor is highly correlated with the Cochrane and Piazzesi (2005) (henceforth *CP*) factor. The latter is the fitted value of a regression of excess bond returns on the lagged one-year bond yield and the two- through five-year forwards rates. It is the best-known

predictor of future bond returns. The correlation between  $KLN_t$  and  $CP_t$  is 82% in monthly data. This close connection between a factor that captures predictability in bond returns and a factor that captures the relevant information for the cross-section of value returns suggests the existence of a parsimonious model tying bond and stock returns together. The  $CP$  factor captures 81% of the cross-sectional variation in the ten book-to-market portfolios and generates a MAPE of 63 basis points per year. It does nearly as well explaining the 25 size and value portfolios with an  $R^2$  of 78% and a MAPE of 97 basis points. While it prices the difference in portfolio returns, the  $CP$  factor fails to capture the average risk premium. Adding a second yield-based factor, the dividend yield on the aggregate stock market, turns out to solve this problem. Finally, we know from the state-of-the-art term structure model of Cochrane and Piazzesi (2006) that it takes one yield-based factor to explain cross-sectional variation in average returns on maturity-sorted bond portfolios. That factor is the level of the term structure. We indeed price five CRSP bond portfolios with the level factor, generating a cross-sectional  $R^2$  of 85% and a MAPE of 8 basis points per year.

These empirical results suggest a parsimonious unified model that can explain both the cross-section of stock and bond returns. Section 3 develops such a model. The SDF takes the same form as in the affine term structure literature. The state dynamics are governed by five factors:  $CP$ , level, slope, and curvature of the term structure, and the dividend yield on the aggregate stock market. We show that three non-zero prices of risk suffice to reduce the mean absolute pricing error (MAPE) on ten book-to-market sorted stock return portfolios, the aggregate market return, and five maturity-sorted government bond portfolios to 40 basis points per year. The price of  $CP$  risk plays the role of pricing the difference between value and growth stock returns, the price of dividend yield risk prices the level of stock returns, and the price of level risk plays the role of pricing the level and cross-sectional variation in bond risk premia. With only one non-zero coefficient in the dynamics of the price of risk vector, this SDF model also generates small pricing errors on bond yields. Finally, the model matches the observed predictability of aggregate stock returns by the dividend yield. The last two properties make it truly a model that captures not only the cross-section but also the time-series of expected bond and stock returns.

Several interesting questions remain: What economic risk does the  $CP$  factor capture? What explains why value stocks have higher exposure to the  $CP$  factor than growth stocks? Can the pattern in  $CP$ -betas be traced back to the underlying dividend growth dynamics? We propose a two-tiered answer to these questions. In Section 4, we study the empirical link between  $CP$ , the business cycle, and value and growth returns. The  $CP$  factor displays interesting cyclical behavior. It is typically very low at the beginning of a recession (or a period of economic turmoil such as the summer of 1998), it increases during the recession, and peaks at the end of a recession. Hence, bond risk premia increase dramatically from a very low (often negative) value to a very high value over the course of a recession. The rewards to a value minus growth strategy differ dramatically

over time. When CP is in its highest quartile of sample observations, the value spread is 16.7% per year, while it is only 1.4% when CP is in the lowest quartile of its sample distribution. Under the ICAPM intuition, times of high CP are times of good investment opportunities. Hence, value stocks have high returns exactly when the marginal value of wealth is low. That makes them risky and command a value risk premium.

In Section 5, we dig deeper and write down an equilibrium asset pricing model. We choose to work in the long-run risk model of Bansal and Shaliastovich (2007) because it provides a tractable laboratory to explore the connections between stock and bond returns. We show that an augmented version of the model can quantitatively replicate the CP betas of and the excess returns on value and growth stocks. The first insight from this model is that the *CP* factor measures economic uncertainty, which increases during recessions. The second insight is that times of increasing uncertainty are also times of higher future long-run growth prospects. This is what makes them good times, and hence what makes the market price of CP risk positive. The last insight is that value stocks' expected long-run dividend growth rates are higher when CP is high and marginal utility is low, which makes them risky assets to hold. Presumably, several other structural models would have similar implications.

The remainder of the introduction discusses the related literature. The last twenty years have seen dramatic improvements in economists' understanding of what determines differences in yields (Cox, Ingersoll, and Ross (1985), Duffie and Kan (1996), Dai and Singleton (2000, 2002)) and returns on bonds, as well as what determines heterogeneity in stock returns which differ by characteristics such as size and book-to-market value (Fama and French (1992)). Yet, these two literatures have developed largely separately and employ largely different asset pricing factors. This is curious from the perspective of a complete markets model because both stock and bond prices equal the expected present discounted value of future cash-flows, discounted by the same stochastic discount factor. This paper contributes to both literatures and helps to bridge the gap.

On the theory side, representative-agent endowment models have been developed that are successful in accounting for many of the features of both stocks and bonds. Examples are the external habit model of Campbell and Cochrane (1999), whose implications for bonds were studied by Wachter (2006) and whose implications for the cross-section of stocks were studied *separately* by Menzly, Santos, and Veronesi (2004) and Santos and Veronesi (2006). Likewise, the implications of the long-run risk model of Bansal and Yaron (2004) for the term structure of interest rates were studied by Piazzesi and Schneider (2006), while Bansal, Dittmar, and Lundblad (2005) and Bansal, Dittmar, and Kiku (2007) *separately* study the implications for the cross-section of equity portfolios. In more recent work, Bansal and Shaliastovich (2007) study some of the joint properties of bond yields and stock returns, although not the ones we will focus on. Other recent work that ties stock and bond markets together in a general equilibrium model is Bekaert, Engstrom, and

Grenadier (2005), Bekaert, Engstrom, and Xing (2008), Lettau and Wachter (2007), Campbell, Sunderam, and Viceira (2008), and Gabaix (2008).

On the empirical side, the nominal short rate or the yield spread is routinely used either as a predictor of the aggregate return on the stock market or as a conditioning variable in an estimation of a conditional beta model of the cross-section of stock returns. Cochrane and Piazzesi (2005) show that a tent-shaped function of 5 forward rates forecasts not only nominal returns on bonds, but also has some forecasting ability for future aggregate stock market returns. Ang and Bekaert (2007) find some predictability of nominal short rates for future aggregate stock returns. Ferson and Harvey (1991) study stock and bond returns' sensitivity to aggregate state variables, among which the slope of the yield curve. They conclude that time variation in equity risk premia is important for understanding the cross-sectional variation in size and industry equity portfolios and interest rate risk premia are important for understanding the cross-sectional variation in bond return portfolios. Fama and French (1993) find that three factors (market, size, and book-to-market) account for the variation in stock returns and two bond factors (excess return on long-term bond and a default spread) explain the variation in government and corporate bonds. They find that all of their stocks load in the same way on the term structure factors. Petkova (2006) studies the connection between the Fama-French factors and innovations in state variables such as the default spread, the dividend-price ratio, the yield spread, and the short rate. Lustig, Van Nieuwerburgh, and Verdellhan (2008) price both nominal bond yields and the aggregate stock market return in a no-arbitrage model in order to measure the wealth-consumption ratio in the data; they do not study the cross-section of bond nor stock returns. Finally, Baker and Wurgler (2007) show that government bonds comove most strongly with "bond-like stocks," which are stocks of large, mature, low-volatility, profitable, dividend-paying firms that are neither high growth nor distressed. They propose that stocks and bonds are driven by a common sentiment indicator.

## 1 Motivation

In this section, we argue that it is natural to use yields to price the cross-section of risky asset returns. Yields reflect the state variables  $X_t$  that drive investment opportunities in the economy. The innovations to these state variables  $\varepsilon_{t+1}$  are key sources of risk entering in the risk premium of risky assets. We consider a generic set-up where some  $N$ -dimensional state vector  $X_t$  is Markovian. It drives the conditional mean and volatility of aggregate real consumption growth, inflation, and

real dividend growth (lowercase letters denote logs):

$$\begin{aligned}\Delta c_{t+1} &= \mu_c + g_c(X_t) + \sigma_c(X_t)\eta_{t+1}, \\ \pi_{t+1} &= \bar{\pi} + g_p(X_t) + \sigma_p(X_t)z_{t+1}, \\ \Delta d_{t+1}^i &= \mu_d^i + g_d^i(X_t) + \sigma_d^i(X_t)u_{t+1}^i,\end{aligned}$$

for some generic functions  $g_c(\cdot)$ ,  $g_p(\cdot)$ ,  $g_d^i(\cdot)$  and  $\sigma_c(\cdot)$ ,  $\sigma_p(\cdot)$ ,  $\sigma_d^i(\cdot)$  that map the state variable  $X_t$  into the real line or the positive part of the real line, respectively.

We assume the existence of a unique stochastic discount factor (SDF). Both complete markets models and endogenously incomplete markets models satisfy this assumption. Most utility functions lead to a log stochastic discount factor of the form:

$$m_{t+1} = f(X_t, \varepsilon_{t+1}, \eta_{t+1}), \tag{1}$$

where  $f(\cdot)$  is a generic real-valued function of the state, the innovations in the state, and unexpected consumption growth. The nominal log SDF satisfies  $m_{t+1}^{\$} = m_{t+1} - \pi_{t+1}$ . This class includes the standard consumption-CAPM, external habit preferences, long-run risk preferences, just to name a few. In these models,  $f$  measures the log inter-temporal marginal rate of substitution of the representative agent. Typically,  $f$  is additively separable in its arguments, but we do not impose this.

The price of a  $\tau$ -period nominal zero-coupon bond satisfies:

$$P_t^{\$}(\tau) = E_t \left[ e^{m_{t+1}^{\$} + \dots + m_{t+\tau}^{\$}} \right].$$

The corresponding nominal bond yield is  $y_t^{\$}(\tau) = -\log(P_t^{\$}(\tau))/\tau$ . By the Markovian nature of  $X$  and the law of iterated expectations, bond yields are (potentially non-linear) functions of those state variables in  $X_t$  that affect the dynamics of real consumption growth and inflation. Innovations in yields reflect innovations in these state variables (the relevant partition of  $\varepsilon$ ).

Just like bond yields, the log dividend-price ratio on a stock is a “yield,” which reflects information about the state variables  $X$ . To see this, note that the price of a stock at time  $t$  is the sum of the prices of the claims to the dividend in each of periods  $t + j$ ,  $j \geq 1$ . These are the prices of the equity strips. The price-dividend ratio of each strip is a function of  $X_t$  and so is the price-dividend ratio of their sum. Hence, price-dividend ratios reflect those state variables in  $X_t$  that affect the dynamics of real consumption growth (through the SDF) and real dividend growth. Innovations to the dividend yield reflects innovations to those state variables.

The risk premium on any risky asset  $i$  is the conditional covariance of innovations to  $m$  and innovations to that asset’s return. Both innovations depend on  $\varepsilon_{t+1}$  shocks, but also on unexpected

consumption growth, unexpected inflation, and unexpected dividend growth. Only the orthogonal component of these shocks,  $(\eta_{t+1}, u_{t+1}^i, z_{t+1}) \perp \varepsilon_{t+1}$ , is not reflected in either bond yields or the dividend-price ratio. Hence, as long as this orthogonal component is not too highly priced, yields (bond yields and dividend yields combined) should be very informative for risk premia on risky assets. Below, we explore this intuition, and indeed find that we are able to account for most of the cross-sectional variation in both stock and bond returns with only three yield factors.

The equilibrium asset pricing model of Section 5 provides a concrete example of the setting described above. Both bond yields and the log dividend yield are affine in the model's state variables. Unexpected consumption growth contributes only modestly to the equity risk premium in that model, even with a coefficient of relative risk aversion as high as 10. Hence, yields are very informative to value risky asset returns inside that model.

## 2 Empirical Analysis

The previous section motivates us to explore the link between the returns on stocks and on bonds and the term structure of interest rates. Section 2.2 shows that a single linear combination of forward rates, which we call the *KLN* factor explains most of the variation in average returns between BM sorted stock portfolios. Our main finding is that this factor is highly correlated with the Cochrane-Piazzesi (henceforth *CP*) factor, a measure of the bond risk premium. Section 2.3 shows that the cross-section of bond returns can also be captured by a single yield factor. Hence, both the cross-section of stocks and the cross-section of bonds each have a one-factor structure, and in both cases the factor is a bond market variable. This suggests a unified model for pricing both stocks and bonds, which is what we provide in Section 3. But first, we briefly discuss the data and the construction of the *KLN* and *CP* factors.

### 2.1 The *KLN* and *CP* Projection Factors

We use Fama-Bliss yield data for nominal government bonds of maturities 1- through 5-years. These data are available from June 1953 until December 2008. Following the procedure outlined in Cochrane and Piazzesi (2005), we construct one- through five-year forward rates from the 1-through 5-years bond prices. We do this at monthly, quarterly, or annual nominal frequency. To understand whether the hypothesized link between stock returns and the term structure is present, we focus on stocks sorted along a value dimension. In particular, we regress the return on the HML portfolio, which goes long in value (high book-to-market) stocks and short in growth (low book-to-market) stocks on the contemporaneous one-year bond yield and two- through five-year forward rates. The HML return and all other stock return series we use in this paper are from Kenneth French' data library. This time series regression has an  $R^2$  of 2.0% in monthly data, 3.1%

in quarterly data, and 8.4% in annual data. These low  $R^2$  values show that the linear projection does not merely capture the realized return on HML. We label this projection the KLN factor. The dashed line in Figure 1 plots the time series of this factor.

[Figure 1 about here.]

Interestingly, the KLN factor is highly correlated with another linear combination of forward rates, the one uncovered by Cochrane and Piazzesi (2005). We recall that they regress the equally-weighted average of the one-year excess return on bonds of maturities of two, three, four, and five years on a constant, the one-year yield, and the two- through five-year forward rates. The yields are one-period lagged relative to the return on the left-hand side. The  $CP$  factor is the fitted value of this predictive regression. The contemporaneous correlation between the  $KLN$  factor and the  $CP$  factor is 82% in monthly data, 75% in quarterly, and 87% in annual data. The solid line in the figure indeed confirms the tight link between the  $KLN$  and  $CP$  factor.

We recall that the  $CP$  factor is the best predictor of future bond returns currently known. The  $R^2$  of the Cochrane and Piazzesi (2005) regression on our sample, which runs through the end of 2008, is 20.4% with monthly, 24.6% with quarterly, and 27.8% with annual time-series. The high correlation between the  $KLN$  and the  $CP$  factor therefore suggests a strong link between the (best proxy for) the bond risk premium and the cross-section of stock returns, sorted along the book-to-market dimension. Below, we explore in detail whether the variable that predicts future bond returns also explains the cross-section of stock returns.

Another, often-used proxy for the bond risk premium is the yield spread. We define it as the difference between the nominal yield on a five-year T-bond and the yield on a three-month T-bill.<sup>1</sup> The  $R^2$  of a regression of the equally-weighted average of the one-year excess return of bonds of maturities of two through five years on a constant and the yield spread gives an  $R^2$  of 10.9% with monthly, 12.4% with quarterly, and 16.6% with annual time-series. The correlation between the yield spread and the  $CP$  factor is 0.67 at monthly, 0.61 at quarterly, and also 0.66 at annual frequency. While the yield spread certainly has some predictive power for one-year ahead excess bond returns, it is a much weaker predictor than the  $CP$  factor. Hence, we will exclusively focus on  $CP$  from Section 2 onwards.

As an aside, the yield spread plays a second role, which makes it more different from the  $CP$  factor as the previous paragraph suggests. In particular, we ran a regression of the SMB return, which goes long small (low market capitalization) stocks and short large (high market capitalization stocks), on the contemporaneous one-year yield and the four forward rates. This regression has a time-series  $R^2$  of only 1% in monthly data, making the SMB projection a poor proxy for the SMB return itself. Interestingly, this SMB projection has a high correlation with the yield spread of 76%

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<sup>1</sup>The results are very similar if we define the yield spread as the difference between the five-year yield and the 1-year T-note.



in monthly data (69% in quarterly and 57% in annual data). Hence, while the value dimension in stock returns seems to be associated with  $CP$ , the size dimension seems associated with the yield spread. Because there is a much smaller return spread in size portfolios than in value portfolios, and much of the difference between small and large company returns is situated in the month of January, the size dimension is of less economic interest and we will mostly focus on the value dimension.

## 2.2 Pricing the Cross-Section of Stocks with a Bond Factor

We now study nominal stock return data on 10 book-to-market sorted portfolios and on 25 size and book-to-market portfolios. We use monthly, quarterly, and annual data. For each of the portfolio returns  $i \in \{1, \dots, N\}$  in the cross-section, we construct log excess returns corrected for a Jensen term:  $r_{t+1}^{i,e} \equiv r_{t+1}^i - y_t^s(1) + .5Var[r_{t+1}^i]$ , where the lowercase letters denote logs. We use log excess returns with Jensen adjustment for ease of comparison with the model in Section 3. All our results are quantitatively similar for gross excess returns and for gross returns. Our benchmark set of test assets are 10 value-weighted decile portfolios sorted alongside the book-to-market dimension. The value spread, measured as  $r_{t+1}^{10,e} - r_{t+1}^{1,e}$ , is 5.12% per year. For comparison, the value premium for gross returns is 5.16% per year. We use a standard two-step estimation procedure; see Fama and MacBeth (1973) or Lettau and Ludvigson (2001). In the first step, we run  $N$  univariate time-series regressions of returns on the  $KLN$  factor, the  $CP$  factor, or the yield spread and a constant in order to estimate the factor beta. In the second step we run one univariate cross-sectional regression of the  $N$  average returns on the betas from the first step and a constant in order to estimate the market price of risk.

### 2.2.1 First-Stage Betas

The left panels of Figure 2 show that value firms are exposed more heavily to the  $KLN$  factor. Each bar represents the slope coefficient (beta) of a time-series regression of a portfolio return on the  $KLN$  factor. The data are monthly (1952.6-2008.12) in the top panel, quarterly in the middle panel (1952.II-2008.IV), and annual in the bottom panel (1953-2008). The book-to-market ratio increases reading from the most left bar to the most right bar. There is a clear pattern in these betas: they are much larger for value than for growth firms. At monthly and annual frequency, the  $KLN$ -beta for extreme growth is negative whereas it is high and positive at every frequency for the extreme value portfolio. Even though the  $KLN$  factor is constructed as the projection of the HML return on forward rates, it is nevertheless surprising how well the  $KLN$ -betas of the stock returns line up. After all, the  $KLN$  factor is quite distinct from the HML return itself; their correlation is only 14% and the  $R^2$  of the projection is only 2%. Maybe more interesting from an economic point of view is that the pattern in  $CP$  betas, plotted in the right columns, is very similar to that

of the  $KLN$  betas. This lends interpretation to the  $KLN$  factor as being closely related to the bond risk premium.

[Figure 2 about here.]

The same pattern of exposures arises when we add a size dimension to the cross-section. Figure 3 shows the contemporaneous  $KLN$  (left) and  $CP$  betas (right) for the 25 market equity- and book-to-market equity-sorted portfolios (5-by-5 sort). The first five bars are for the smallest quintile of firms; the last five bars are for the biggest quintile of firms. Within each group of five, the book-to-market ratio increases. There is the same clear pattern in these betas: they are much larger for value than for growth firms. While the difference in exposure between the highest and lowest book-to-market group is more pronounced for small firms than for big firms, there is less variation in betas along the size dimension.

[Figure 3 about here.]

Finally, a similar pattern in betas also arises when we use the yield spread as an alternative proxy for the bond risk premium. For both the 10 book-to-market portfolios and the 25 size and value portfolios, we find that value stocks have a high positive loading on the yield spread, whereas growth stocks have a much lower exposure. The difference is larger for small stocks. We summarize these findings in Figure 4, which shows betas for these two sets of test assets at all three frequencies.

[Figure 4 about here.]

### 2.2.2 Second-Stage Regression

In a second stage, we run a cross-sectional regression of average excess stock returns on the first-stage beta and a constant in order to estimate the market price of risk  $\tilde{\Lambda}$ . Table 1 shows the results; the top panel is for the 10 BM portfolios, the bottom panel for the 25 size and BM portfolios. The left panel uses the  $KLN$  factor, the middle panel uses  $CP$ , and the right panel uses the yield spread. In each of the four panels, we report results at monthly (M), quarterly (Q), and annual (A) frequency. For ease of comparison, the second-stage intercept  $\alpha$  is expressed as a percent per year by multiplying the monthly intercept by 12 and the quarterly intercept by 4. Likewise, the mean absolute pricing error (MAPE) and the root mean squared pricing error (RMSE) are expressed as a percent per year.

The price of  $KLN$  risk is consistently estimated to be positive, consistent with a positive risk price attached to value exposure. Furthermore, this  $KLN$  factor explains between 79 and 94% of the cross-sectional variation in book-to-market portfolio returns. This is about as much as what the return on HML itself explains, despite the fact that the correlation between that return and

the  $KLN$  factor is very low. Hence, the  $KLN$  factor turns out to capture all the relevant pricing information that is in the HML return. Pricing errors on the 10 BM portfolios are as low as 40 basis points per year. The  $KLN$  factor also does a good job pricing the 25 size and BM portfolios with MAPE and RMSE around 110 basis points. The monthly  $R^2$  is 67%, lower than for the 10 BM portfolios, but higher than the 40% one would obtain using the HML return itself to price the 25 size and BM portfolios.

Most importantly, the  $CP$  factor, a proxy for the bond risk premium, does about as good or better a job at pricing the cross-section as the  $KLN$  factor. Using  $CP$ , we find a highly significant market price of risk estimate between 1.3 (M) and 2.4 (A) for the 10 BM portfolios and between 2.1 (M) and 2.9 (A) for the 25 size and BM portfolios. Point estimates for this price of risk are directly comparable across frequencies because of how  $CP$  is constructed, and indeed end up being similar across regressions. The second-stage regression has a high  $R^2$  between 74 and 81%. The pricing errors are small: the MAPE across the 10 portfolios is between 63 basis points per year (M) and 70 basis points (A) for the 10 BM portfolios, the RMSE between 71 (M) and 87 (A) basis points per year. For the 25 portfolios, the MAPE is between 94 and 116 and the RMSE between 108 and 138 basis points per year. Figure 5 plots average realized excess returns against predicted excess returns, formed as  $\alpha + \beta\lambda$ , where  $\alpha$  is the second-stage regression intercept. The left row is for the 10 BM, the right row for the 25 FF portfolios. The sampling frequency is monthly in the top panels, quarterly in the middle panels, and annual in the bottom panels. It confirms visually that a large fraction of the cross-sectional variation in stock returns is explained by their exposure to the bond risk premium, as proxied by  $CP$ . Combining results from the two stages, value firms (BM10) and growth firms (BM1) have a differential exposure to  $CP$  of .33 in the monthly data. With a market price of risk of 1.30, this translates into an annual value premium of 5.16%, which is essentially the entire value premium observed in the data.

[Table 1 about here.]

[Figure 5 about here.]

We repeat the analysis with the yield spread, in the right columns of Table 1. There too, we find significantly positive market prices of risk. The pricing errors are somewhat larger at monthly and quarterly frequencies, but somewhat smaller at annual frequency. The regression  $R^2$  vary between 47% and 82%. Because exposure to the yield spread carries a positive risk price, the differential exposure of value and growth stocks leads to a value spread. The higher explanatory power for the 25 size and BM than for the 10 BM portfolios is partially due to the fact that the yield spread has a high correlation with the size dimension (the SMB projection factor).

One advantage of the yield spread is that the second stage regression no longer has a significant intercept  $\alpha$  in the monthly data. The positive  $\alpha$ 's in all other specifications, implies that not all

cross-sectional variation is explained by the yield spread or the  $CP$  factor. In particular, there seems to be a factor missing that increases the returns on all portfolios. We will introduce such a “market level factor” in Section 3. Nevertheless, the small pricing errors throughout show that a single bond factor, linked to the bond risk premium, can explain most of the variation around a common market return. We consider the regression analysis of this section as preliminary evidence. Section 3 explores the links between stock returns, bond returns, and bond yields in much more detail.

### 2.2.3 Robustness

**Sub-sample Analysis** As a first robustness check, we repeat the analysis with returns instead of excess returns. The results are very similar and are available upon request. As a second robustness check, we conduct a sub-sample analysis and focus on  $CP$  as a pricing factor. We present two sets of results in Table 2 for the 10 BM portfolios (top panel) and the 25 size and BM portfolios at monthly frequency (bottom panel). The first set of results uses the  $CP$  factor constructed from the full sample (left panel); the second set of results re-estimates the  $CP$  factor over the sub-sample in question (right panel). If we start the analysis in 1963, an often used starting point for cross-sectional equity analysis (e.g., Fama and French (1993)), we find somewhat *stronger* results. The pattern in the betas is similar as in the full sample, but the market price of risk estimate increases and so does the second-stage  $R^2$ . For the 10 BM portfolios, the mean absolute pricing errors drop from 63 to 26 basis points per year and the  $R^2$  increases from 80% to 96%. For the 25 size and BM portfolios, the MAPE drops 12 basis points. We also split the full sample into two equally-sized sub-samples: 1952-1980 and 1980-2008. The results remain strong in both sub-samples: pricing errors generally remain below 1.5% per year and the cross-sectional  $R^2$ s remain substantial. The market price of risk estimates are remain significantly positive in every instance.

[Table 2 about here.]

**Comparison with Three-Factor Model** As a last robustness exercise, we compare our results to the celebrated Fama-French three-factor and Carhart four-factor models. For brevity, we focus on the 25 size and BM portfolios and on monthly data. The results for quarterly and annual data are similar. The factors are the log excess return on the market including a Jensen adjustment (5.94% per year on average), the log return on SMB (1.71% per year on average), the log return on the HML (4.25% per year on average), and the log return on the UMD portfolio (9.56% per year on average). The first two columns of Table 3 repeat the results for the models with only  $CP$  or the yield spread as pricing factor. Columns 3 and 4 present the results from the standard three-factor (3-f) and four-factor model (4-f). The main observation is that the pricing errors are similar to, and if anything *larger* than, the pricing errors for our one-factor model with  $CP$  as

factor, reported in Column 1. Additionally, the three-factor model in Column 3 has a negative price of market risk, inconsistent with theory, and the cross-sectional intercept is very large. The latter two issues are mitigated by including the momentum factor in Column 4.

Does the bond risk premium survive the inclusion of the three- or four-factor model’s risk factors? Columns 5 through 8 show that both the *CP* factor and the yield spread remain significantly priced risk factors once the market, SMB, HML, and UMD are added. Adding the *CP* factor to the 3-factor model lowers the MAPE from .91% per year to .80% per year and increases the cross-sectional  $R^2$  from 74.9% (Column 3) to 83.3% (Column 5), a substantial improvement in fit. Adding the *CP* factor to the 4-factor model lowers the MAPE from .89% to .79% and increases the  $R^2$  from 80.6% (Column 4) to 85.9% (Column 7). Adding the yield spread leads to even bigger gains in  $R^2$  and bigger reductions in pricing errors. The MAPE of the resulting 4-factor and 5-factor models in Columns 6 and 8 are .54% and .52% per year, about 40 basis points lower than in the corresponding models without the yield spread factor. The  $R^2$ s are 93.5% and 94.1%.

[Table 3 about here.]

**Results for Other Portfolios** We briefly investigate the results for other cross-sections of equity portfolios. We focus on monthly returns and the *CP* factor. First, the *CP* factor is able to explain 79% of the variation in the 10 size (market equity sorted) decile portfolios. The MAPE is 36 basis points per annum and the RMSE 50 basis points. Moreover, the price of *CP* risk is estimated to be 2.29 with a t-stat of 5.1. This estimate is similar to the one for the twenty-five portfolios. Small stocks have a higher *CP* exposure than big stocks. Second, we look at the 100 size and book-to-market portfolios (10 by 10 sort). The market price of *CP* risk is estimated to be 1.36 with t-stat of 7.14. The MAPE is 1.65% (RMSE is 2.26%) and the  $R^2$  is 34%.<sup>2</sup> Finally, the *CP* factor is able to explain 79% of the variation in earnings-price decile portfolios, with MAPE of .94% per annum (RMSE of 1.11%). The market price of risk is significantly positive at 2.61 with a t-stat of 5.52. High EP portfolios have a higher exposure than low EP portfolios. The difference in the betas between the extreme portfolios multiplied by the estimated price of risk delivers a spread of 6.27% per year, very close to the observed spread of 7.03%.

## 2.3 Pricing the Cross-Section of Bonds with a Bond Factor

While the *CP* factor is both a strong *predictor* of future bond returns and a strong explanatory variable of differences between average returns of value and growth stocks, it is well known that it

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<sup>2</sup>Some data are missing (240 observations out of 67,900). Since our analysis requires a full data set, we fill in the missing data by interpolation. This introduces some measurement error, especially in the results for S1B1, S1B2, and S10B9, and S10B10. Without these 4 portfolios, the  $R^2$  increases to 41% and the MAPE reduces to 1.50% (RMSE of 1.99%).

is not a good explanatory variable of differences between average returns on long- and short-term bonds. Cochrane and Piazzesi (2006) show that a different bond factor, the *level* of the term structure, has significant ability to explain the cross-section of bond returns. As a first measure of the level factor, we use the five-year Fama-Bliss bond yield. As a second measure, we use the first principal component of yields as the level factor, extracted from the cross-section of 1- through 5-year Fama-Bliss yields. We use CRSP excess bond returns on portfolios of maturities 1, 3, 5, 7, and 10 years at monthly, quarterly, or annual frequency. Average excess returns are respectively 1.08%, 1.24%, 1.53%, 1.75%, and 1.38% per year. The sample is again June 1952 until December 2007.

Figure 6 shows the level betas of excess bond returns on the 5 maturity-sorted bond portfolios. They are decreasing from short to long maturities and generally negative. Table 4 shows the corresponding market prices of risk using  $y^s(20)$  (on the left) and using the first principal component (on the right). The market price of risk is estimated to be negative and significant, as predicted by theory. Intuitively, investors are willing to pay for assets with high returns when yields increase, which are times in which wealth decreases and marginal utility increases. Long bonds have more negative level betas: their returns fall by more when yields increase and hence they earn higher average returns. The pricing errors on the 5 bond portfolios are small; the MAPE is 8 basis points and the RMSE 12 basis points per year. The explanatory power of the 20-quarter yield and the standard level factor are similar. The cross-sectional  $R^2$  is around 85%. The negative betas combined with the negative market price of risk allow this one-factor model to capture 0.70% of the observed 0.51% spread between the excess return on the 10-year and the 1-year bonds.

Combined with the previous section, these results suggest that two term structure variables, a proxy for the bond risk premium  $CP$  and a level factor, can help explain both the cross-section of stock and bond returns. Section 3 will propose a model that can fit both. In addition to matching cross-sectional variation, it will also seek to match the *levels* of the risk premia on stocks and bonds. It will do so for a single market price of risk vector  $\tilde{\Lambda}$ , not a different one for stocks and bonds.

[Figure 6 about here.]

[Table 4 about here.]

### 3 A Unified Model of Stock and Bond Returns

The previous analysis suggests that the level and the the cross-section of expected stock returns are driven by the  $CP$  factor and a market factor, while the level, the cross-section, and the time-variation of expected bond returns are driven by the level factor and the  $CP$  factor. In this

section, we develop a parsimonious stochastic-discount factor (SDF) model that captures all of these features. It also accommodates the time-variation in expected stock returns driven by the dividend yield.<sup>3</sup> We show that this model is also able to match the time-series and cross-sectional properties of bond yields, thereby providing a unified pricing framework for stocks and bonds. For brevity, we focus on the 10 book-to-market sorted stock portfolios and study returns at the monthly frequency. Section 5 develops a general equilibrium model that starts from preferences and from dividend growth rates on the stock portfolios. There, returns are generated inside the model, while here, they are taken as given.

### 3.1 Setup

We model the monthly dynamics of the state vector  $X$  as:

$$X_{t+1} = \mu_X + \Gamma X_t + \varepsilon_{t+1}^X, \quad (2)$$

with Gaussian innovations  $\varepsilon_{t+1}^X \sim \mathcal{N}(0, \Sigma_X)$ , i.i.d. The state vector contains five yield variables: the  $CP$  factor, the level factor, the slope factor, the curvature factor, and the log dividend yield on the aggregate stock market portfolio in order of appearance. To construct the level, slope, and curvature factors, we regress Fama-Bliss forward rates on bonds of maturities 1-5 years (the same rates that were used in the construction of  $CP$ ) on the  $CP$  factor. Level, slope, and curvature are the first three principal components of the residuals of this regression, and therefore orthogonal to  $CP$ . The first four factors have been suggested by Cochrane and Piazzesi (2006) to price the nominal term structure of interest rates. Even though we use five factors, only three of them will carry a non-zero price of risk in our preferred specification. The other factors are only relevant for the expectations' formation of future state variables (see equation 2).

The stochastic discount factor is modeled as in the affine term structure literature:

$$M_{t+1} = \exp \left( -y_t^{\$}(1) - \frac{1}{2} \Lambda_t' \Sigma_X \Lambda_t - \Lambda_t' \varepsilon_{t+1}^X \right), \quad (3)$$

with  $y_t^{\$}(1)$  the 1-month short rate and market prices of risk that are affine in the state  $\Lambda_t = \Lambda_0 + \Lambda_1 X_t$ .

The no-arbitrage condition for each asset is given by:

$$\log E_t [M_{t+1} R_{t+1}] = 0,$$

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<sup>3</sup>See for instance Fama and French (1988), Cochrane (2006), Lettau and Van Nieuwerburgh (2007), and Binsbergen and Koijen (2007).

which implies, with  $r_{t+1}^e = E_t[r_{t+1}^e] + \varepsilon_{t+1}^R$ :

$$E_t[r_{t+1}^e] = -cov_t(r_{t+1}^e, m_{t+1}) = cov(\varepsilon_{t+1}^R, \varepsilon_{t+1}^{X'}) \Lambda_t = \Sigma_{XR}(\Lambda_0 + \Lambda_1 X_t). \quad (4)$$

Unconditional expected excess returns are computed by taking the unconditional expectation of (4):

$$E[r_{t+1}^e] = \Sigma_{XR}(\Lambda_0 + \Lambda_1 E[X_t]).$$

In what follows, we first show that a parsimonious model is able to price the cross-section of *unconditional* expected stock and bond *returns*. We then show that this does not come at the expense of pricing the time series and cross-section of bond *yields*.

Our test assets are the 10 portfolios sorted on their book-to-market ratio, the value-weighted market return from CRSP, and bond returns with maturities 1, 2, 5, 7, and 10 years from CRSP. Our estimation procedure estimates the dynamics of the state by OLS in a first step. It then finds the market price of risk coefficients  $\Lambda_0$  that minimize the pricing errors on the test assets in a second step. All 16 moments are weighted equally. In a third step, we estimate the parameters in  $\Lambda_1$  to fit bond yields.

### 3.2 The Cross-section of Unconditional Expected Returns

We now focus on the cross-section of *unconditional*, i.e., average, expected stock and bond returns. We switch off all time variation in risk prices ( $\Lambda_1 = 0$ ) and optimize over a subset of  $\Lambda_0$ . As a point of reference, we start by pricing the 16 securities with a risk-neutral SDF  $M_{t+1} = \exp\{-y_t^{\$}(1)\}$ . That is, all prices of risk are zero:  $\Lambda_0 = 0$ . The first column of Table 5, labeled SDF1, thus recovers the risk premia we want to explain. The value spread (BM10-BM1) is 5.1% per year. Bond risk premia are not monotone in maturity and smaller than equity risk premia. The MAPE across all securities equals 5.5%.

Our first candidate SDF to explain the stock and bond risk premia is the one proposed by Cochrane and Piazzesi (2006). This is a natural candidate because their model does a great job pricing the nominal term structure of bond yields. Following Cochrane and Piazzesi (2006), we only allow the market price of level risk to be non-zero. This implies one non-zero element in  $\Lambda_0$ . The second column of Table 5 shows that the best-fitting SDF, SDF2, is unable to jointly explain the cross-section of stock and bond returns. The MAPE is 2.68%. All pricing errors on the stock portfolios are large and positive, while all pricing errors on the bond portfolios are large and negative. The reason that this bond pricing model does not do better for the bond return portfolios is that the stock return risk premia command most attention because they are much larger in magnitude. Consequently, the estimation concentrates its efforts on reducing the pricing



errors of stocks. To illustrate that this bond kernel *is* able to price the cross-section of bonds, we estimate the same model by minimizing only the bond pricing errors. The third column of Table 5 illustrates that the bond pricing errors are small in this case, but this pricing kernel SDF2 is not able to price the cross-section of expected stock returns nor the expected return on the aggregate stock market. The MAPE increases to 4.82%.

Our second candidate SDF to explain stock and bond risk premia is a version of the canonical equity pricing model, the Capital Asset Pricing Model. The fourth column of Table 5 reports pricing errors for the CAPM. The only non-zero price of risk is the one corresponding to the dividend yield.<sup>4</sup> This SDF3 model is again unable to jointly price stock and bond returns. The MAPE is 1.50%. One valuable feature is that pricing errors of book-to-market portfolio returns go through zero. This means that the model gets the level of all expected stock returns right. However, their pattern clearly shows the value spread. Pricing errors on bond portfolios are sizeable as well and are all positive.

Having concluded that both the bond SDF and the stock SDF offer crucial ingredients to price bond returns and stock returns, but that neither is able to satisfactorily price both cross-sections, we now turn to our unified model. We allow the price of risks of the *CP* factor, the level factor, and the dividend yield to be non-zero. For pricing purposes, this is a three-factor model. The three elements in  $\Lambda_0$  are estimated to minimize the 16 pricing errors. As before, there is no time variation in the prices of risk:  $\Lambda_1 = 0$ . We call this model SDF4; the fifth column shows the results. There hardly is a value spread in the pricing errors: The spread between the extreme portfolios is only 25bp per year. We also price the market portfolio well, and the bond pricing errors are much lower than in the other models SDF1, SDF2, and SDF3. The MAPE falls to a low 40bp. This is essentially the same magnitude pricing error we found in Section 2 for the 10 stock portfolios. But now, we also price bonds, and all with one set of risk prices. Using one kernel with three price of risk parameters only, we are able to match the level and the pattern in stock and bond risk premia jointly. Finally, the price of CP risk is estimated to be positive, while the price of level factor risk and dividend-yield risk are negative. These are the signs predicted by theory.

[Table 5 about here.]

### 3.3 Implications for the Yield Curve and Return Predictability

**Prices of risk and estimation procedure** The previous section showed that a parsimonious SDF model was able to price unconditional excess returns on both stocks and bonds. An important question is how well the same SDF4 model does in pricing the term structure of nominal bond yields and matching return predictability of both bonds and stocks. It is well known that three

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<sup>4</sup>We also considered a model in which the dividend yield is replaced by the aggregate stock market return. The pricing errors are very similar to the results for SDF3.

factors (level, slope, and curvature) are necessary to adequately describe bond yield dynamics, e.g., Dai and Singleton (2000, 2002). While the slope and curvature factor were unnecessary to price average returns, as shown above, we included them in the state vector  $X$  because they are necessary to adequately describe yields. They enter through the expectations formation in the state variable dynamics in equation (2). Matching yield dynamics and return predictability requires three non-zero elements in  $\Lambda_1$ , only *two* of which are freely estimated. First, to match the predictability of bond returns and the yield dynamics, we allow  $\Lambda_{1(2,1)}$  to be non-zero, following Cochrane and Piazzesi (2006). Second, we exactly match the predictability of the aggregate stock market return by the dividend yield  $dp_t$ . That is, geometric excess returns (including Jensen adjustment) on the aggregate stock market,  $r_{t+1}^{e,m}$ , are modeled as:

$$r_{t+1}^{e,m} = r_{t+1}^m - y_t^s(1) + \frac{1}{2}var_t[r_{t+1}^m] = \delta_0 + \delta_1 dp_t + \varepsilon_{t+1}^m, \quad (5)$$

with Gaussian innovations  $\varepsilon_{t+1}^m \sim \mathcal{N}(0, \sigma_m^2)$  and covariance matrix  $cov(\varepsilon_{t+1}^m, \varepsilon_{t+1}^{X'}) = \sigma_{Xm}$ .<sup>5</sup> This predictive regression restricts  $\Lambda_1$  in the following way:

$$\sigma'_{Xm} \Lambda_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & \delta_1 \end{pmatrix}. \quad (6)$$

If we combine the predictability of bond and stock returns,  $\Lambda_1$  takes the following form:

$$\Lambda_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ \Lambda_{1(2,1)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -\Lambda_{1(2,1)} \frac{\sigma_{Xm(2)}}{\sigma_{Xm(5)}} & 0 & 0 & 0 & \Lambda_{1(5,5)} \end{pmatrix}, \quad (7)$$

where  $\Lambda_{1(5,5)}$  is implied by  $\delta_1$  and the correlation between innovations to aggregate stock market returns and the dividend yield:

$$\Lambda_{1(5,5)} = \delta_1 / \sigma_{Xm(5)}. \quad (8)$$

The element  $\Lambda_{1(5,1)}$  is chosen in such a way the conditional expectation of aggregate excess stock market returns does not depend on  $CP$ . We can relax this assumption, but it does not significantly improve the model fit.

Our SDF model implies an essentially affine term structure of yields (Duffee (2002)). We model

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<sup>5</sup>We will assume that the Jensen adjustment  $\frac{1}{2}var_t(r_{t+1})$  is a constant. This would arise under homoscedasticity assumptions on return innovations. This adjustment is tiny in practise.

the nominal short rate as

$$y_t^{\$}(1) = \xi_0 + \xi_1' X_t.$$

Then, the SDF implies that the price of a nominal bond of maturity  $\tau$  is exponentially affine in the state variables  $X$ :

$$P_t^{\$}(\tau) = \exp \{A^{\$}(\tau) + B^{\$'}(\tau)X_t\}.$$

By no-arbitrage, we have:

$$\begin{aligned} P_t^{\$}(\tau) &= E_t [M_{t+1} P_{t+1}^{\$}(\tau - 1)] \\ &= \exp \left\{ -\xi_0 - \xi_1' X_t + A^{\$}(\tau - 1) + B^{\$'}(\tau - 1)\mu_X + \right. \\ &\quad \left. B^{\$'}(\tau - 1)\Gamma X_t - \Lambda_t' \Sigma B^{\$}(\tau - 1) + \frac{1}{2} B^{\$'}(\tau - 1)\Sigma_X B^{\$}(\tau - 1) \right\}, \end{aligned}$$

which implies that  $A^{\$}(\tau)$  and  $B^{\$}(\tau)$  follow from the recursions:

$$\begin{aligned} A^{\$}(\tau) &= -\xi_0 + A^{\$}(\tau - 1) + B^{\$'}(\tau - 1)\mu_X - \Lambda_0' \Sigma_X B^{\$}(\tau - 1) + \frac{1}{2} B^{\$'}(\tau - 1)\Sigma_X B^{\$}(\tau - 1), \\ B^{\$'}(\tau) &= -\xi_1' + B^{\$'}(\tau - 1)\Gamma - B^{\$'}(\tau - 1)\Sigma_X \Lambda_1, \end{aligned}$$

with initial conditions  $A^{\$}(0) = 0$  and  $B^{\$}(0) = 0_{5 \times 1}$ .

We estimate the parameters in three steps. The first two steps are the same as in the unconditional analysis described above. The third step estimates  $\xi_0$ ,  $\xi_1$ , and the first non-zero entry of  $\Lambda_1$ ,  $\Lambda_{1(2,1)}$ , to match nominal bond yields. In particular, we minimize the distance between the five annual yields from the Fama-Bliss data set (maturities 1 through 5 years) and the corresponding yields implied by the SDF:

$$\min_{\xi_0, \xi_1, \Lambda_{1(2,1)}} \left\{ \sum_{n=1}^5 \sum_{t=1}^T \left( y_t^{\$}(12 \times n) + \frac{A^{\$}(12 \times n)}{12 \times n} + \frac{1}{12 \times n} B^{\$'}(12 \times n)X_t \right)^2 \right\},$$

where the factor “12” is introduced because we have a monthly model and we price annual yields. In optimizing over  $\Lambda_1$ , we modify  $\Lambda_0$  to  $\bar{\Lambda}_0 \equiv \Lambda_0 - \Lambda_1 E[X_t]$  to ensure that the unconditional asset pricing results from the previous section are unaffected. In related work, Adrian and Moench (2008) match bond returns first and then study the implications for yields.

**Yield curve dynamics** The model SDF4 does a very nice job matching yields. The annualized standard deviation of the pricing errors of yields equal 19bp, 9bp, 12bp, 22bp, and 28bp for 1-5 year yields. Figure 7 plots this model’s implications for the yields of maturities one through five years. The parameter  $\Lambda_{1(2,1)}$  is estimated to be negative ( $\hat{\Lambda}_{1(2,1)} = -714.67$ ), consistent with the results in Cochrane and Piazzesi (2006). A negative sign means that an increase in  $CP$  leads to

higher bond risk premia because the price of level risk is negative and a higher  $CP$  makes it more negative.

[Figure 7 about here.]

## 4 The Timing of Value and Growth Returns

The previous analysis suggests a key role for the  $CP$  factor as a predictor of bond returns and an explanatory variable for the cross-section of value and growth returns. The unified SDF model of Section 3 tied together the bond and stock returns. Since it admits no arbitrage opportunities, arbitrageurs cannot earn the value premium and perfectly hedge out the risk with a position in bonds. However, it leaves open a few important questions. What source of economic risk does the  $CP$  factor capture? Why are value stocks more exposed to this risk? We provide a two-tiered answer to these important questions. In the next section, we write down a simple equilibrium asset pricing model that ties the  $CP$  factor to fundamental sources of risk (the marginal utility of consumption). By specifying how value and growth firms' cash flows are linked to these sources of risk, it provides the connection between the bond risk premium and the value premium. This connection is crucial because there is not a lot of evidence yet about the link between the value premium and economic risk. Lakonishok, Schleifer, and Vishny (1994) study the timing of value-growth returns and conclude that "It is impossible to conclude from this that value strategies did particularly badly in recessions, when the marginal utility of consumption is especially high." In this section, we take a fresh look at the timing of value-growth returns in the data. We find that value-growth returns are especially high at the end of a recession, when  $CP$  is high, investment opportunities are great, and marginal utility of consumption is low. That makes them risky, and suggests a risk-based explanation of the value premium.

### 4.1 $CP$ and the State of the Economy

The bond risk premium proxies are clearly related to the state of the economy. Figure 8 plots the  $CP$  factor (top panel) and the yield spread (bottom panel) against the NBER recession dates. Both variables tend to be low at the beginning of a recession, increase substantially during a recession, and tend to peak at the end of a recession. During the average recession in the sample,  $CP$  increases by 128 basis points (1.11 standard deviations), typically from a negative to a positive value. Likewise, the yield spread increases by 143 basis points (1.39 standard deviations) from a negative to a positive value. The fact that the  $CP$  factor and the yield spread peak at the end of a recession (the through of the business cycle), when good economic times are around the corner and marginal utility growth for the representative investor is presumably low, can explain the positive

price of CP or yield spread risk we found above. Note that by December 2008, the yield spread already increased substantially (up 126 basis points from December 2007), while the  $CP$  factor has not (yet) increased and is in fact at a very low value of -240 basis points in December 2008. In fact, bond returns This suggests some interesting differences in the precise timing of the two measures.

[Figure 8 about here.]

Returns on value and growth stocks are also linked to the cycle. An investment strategy which is long value and short growth stocks has low returns when entered at the start of a recession and high returns when entered at the end of a recession. Average monthly holding period returns on the value-minus-growth strategy are only 2.39% during NBER recessions (15% of the months in the sample) but 5.85% the rest of the time. So, the fortunes of a value-growth strategy reverse over the course of a recession.

Not all periods with a high (low)  $CP$  factor or yield spread observations occur at the end (beginning) of recessions. However, several of these low-to-high swings in the  $CP$  factor are also associated with times of economic uncertainty, such as 1987 (stock market crash), 1998 (Asian and Russian crisis, LCTM), or 2003 (jobless recovery). An even starker difference between the returns on value and growth stock emerges when we condition on the top quartile of the  $CP$  observations or the top quartile of yield spread observations. The  $CP$  factor cutoffs are -0.44% (25<sup>th</sup> percentile), 0.57% (median), and 1.58% (75<sup>th</sup> percentile), while the yield spread cutoffs are 0.28% (25<sup>th</sup> percentile), 0.96% (median), and 1.68% (75<sup>th</sup> percentile). Figure 9 shows that the value spread is highest when  $CP$  is high (first row, left panel) or when the yield spread is high (first row, right panel). In that highest quartile, the value spread is a *striking* 16.7% for  $CP$  and 12.9% for the yield spread. As the other rows show, there is little spread in the other quartiles. The value spread is almost exclusively concentrated in periods of high  $CP$  or yield spread realizations.

[Figure 9 about here.]

This non-linear contemporaneous relationship between the value spread and the  $CP$  factor naturally suggests a refinement of the cross-sectional stock return regression we studied in Section 2.2. Instead of using  $CP_{t+1}$  as an explanatory variable, it suggests using  $CP_{t+1}\mathbb{I}_{CP_{t+1}>.0158}$ , where  $\mathbb{I}$  denotes an indicator variable and .0158 is the cutoff for the fourth quartile of  $CP$ . In such a specification, innovations to  $CP$  are only priced when  $CP$  is high. This occurs typically at the end of a recession, when good times are around the corner. Using the ten book-to-market sorted portfolios, the resulting market price of risk parameter in the second-stage Fama-McBeth regression is 0.96 with a t-stat of 6.15. The pricing errors are 59 (MAPE) and 67 (RMSE) basis points and the cross-sectional  $R^2$  is 82.5%. This model further reduces pricing errors by 4 (MAPE

and RMSE) basis points and further increases the  $R^2$  by 2 percentage points over the linear specification. A bigger improvement occurs if we use the same non-linear function of the yield spread,  $yspr_{t+1}\mathbb{I}_{yspr_{t+1}>.0168}$ . The MAPE goes down by 19 and the RMSE by 15 basis points, while the cross-sectional  $R^2$  improves by 12 percentage points.

These quartile results show that the link between the bond risk premium and the value premium is especially strong when the bond risk premium is high. The positive price of  $CP$  risk and the economic intuition from the previous section indicate that these are good times, i.e., times of low marginal utility. That makes value stocks risky assets, because they have high returns exactly when the investor needs it the least.

## 4.2 Value-Growth Return Predictability

The fact that value-growth returns are so high when  $CP$  is high raises the question of whether the value premium is predictability. After all, the covariance of the bond risk premium proxies  $CP_{t+1}$  or  $yspr_{t+1}$  with excess returns  $r_{t+1}^e$  is the weighted average of two components: a covariance of  $r_{t+1}^e$  with *innovations* in the bond risk premium proxy and a covariance of  $r_{t+1}^e$  with the *lagged* bond risk premium proxy. So far we studied the overall covariance and the innovation covariance. Predictability is about the covariance with the  $CP_t$  or  $yspr_t$ . To investigate this question, we use monthly data and form cumulative, discounted holding period returns of horizons of 3 months up to 5 years.<sup>6</sup> The first 59 months are discarded so that all cumulative returns are computed over the same sample. All cumulative results are expressed on a per annum basis so that the slope coefficients have comparable magnitudes across holding periods. In *linear* regressions of value-growth returns on  $CP_t$  or  $yspr_t$ , we found little evidence of such predictability. Only the 2-year horizon V-G return is predicted significantly by  $CP_t$  (Newey-West t-stat of 2.05). At all other horizons, we found a positive relationship but not enough significance for  $CP$ . For the yield spread, the signs were only positive at horizons of 3-quarters to 2 years, and never significant.

However, the results above suggest that there only is a value premium in the months in which  $CP$  is in the top quartile of the sample. This raises the question of whether the predictability of the value minus growth return may also be non-linear. We indeed find strong evidence for predictability of the  $CP$  factor for future value minus growth returns, but only in months in which  $CP$  is in its highest quartile. Table 6 shows results from regressing the cumulative value-growth returns on a constant and a *non-linear* function of the  $CP$  factor,  $CP_t\mathbb{I}_{CP_t>.0158}$ , the lagged version of the variable that did such a good job explaining the cross-section of book-to-market portfolio returns. The left panel shows a positive coefficient which is statistically significant at horizons of 1 quarter up to 3 years. The  $R^2$  are respectable, peaking at 5% at the 2-year horizon. For

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<sup>6</sup>The discounting of future returns on each asset  $i$  is done by the constant  $(\kappa_1^i)^{-1} = \exp(A_0^i)/(1 + \exp(A_0^i))$ , where  $A_0^i$  is the mean log price-dividend ratio on portfolio  $i$ .

completeness, we repeat the exercise with a similar non-linear function of the yield spread (right panel). We find similar improvements in the predictability of value minus growth compared to the linear case, but the results are somewhat weaker. This is an important reason why we prefer the  $CP$  measure. The main conclusion is that the same variable that explains the cross-section of book-to-market portfolio returns also has time-series predictability for the high minus low book-to-market portfolio. In future work, we plan to augment the stochastic discount factor model of Section 3 to generate the non-linearity in the relationship between realized value-growth returns and the  $CP$  factor.

[Table 6 about here.]

## 5 Equilibrium Model

In this final section, we study the fundamental determinants of the yield curve and the value premium in a consumption-based asset pricing framework. Such an approach imposes additional discipline: it requires that the  $CP$  betas of stock returns are linked to the  $CP$  betas of bond returns and to the  $CP$  betas of dividend growth rates. We start by documenting the properties of contemporaneous  $CP$  betas for the dividend growth rates on the various stock portfolios in Section 5.1, and show that they have a distinct pattern as well. Section 5.2 then introduces the equilibrium asset pricing model which is able to match the value premium, the  $CP$  betas of stock returns, the  $CP$  betas for dividend growth rates, and the predictability of bond returns by the  $CP$  factor. We emphasize that our choice of the long-run risk framework is one of analytical convenience. It allows for closed-form expressions of bond- and equity risk premia, as well as an affine term-structure model, and has been shown to have good quantitative properties for stocks and bonds. Presumably, several other structural asset pricing models would have similar implications.

### 5.1 Cash-flow Betas in the Data

For each of the ten book-to-market portfolios, we collect data on cum- and ex-dividend returns. They allow us to construct a dividend growth series for each portfolio (see Appendix A for details). Figure 10 plots the dividend growth betas obtained from regressing dividend growth on the contemporaneous bond risk premium. In the left column, we use the  $CP$  factor as a proxy for the bond risk premium, in the right panel the yield spread. While the pattern is not perfectly monotone, it does seem to be the case that growth firm's dividend growth has a positive correlation with contemporaneous  $CP$  and value firms' dividend growth a strong negative correlation. We find a similar pattern when we use the yield spread as proxy for the bond risk premium. Dividend growth of value stocks is contemporaneously more negatively correlated with the yield spread than

dividend growth of growth stocks. The patterns are similar for monthly, quarterly, and annual frequencies.

[Figure 10 about here.]

This pattern of *contemporaneous* dividend growth betas deepens the puzzle. The high (low) exposure of value (growth) stock returns to the CP factor does not seem to be coming from high (low) exposure of dividend growth to the CP factor. In fact, the opposite seems to be true. When CP is high (for example, at the end of recession), current cash-flow growth on value stocks is low, while that of growth stocks is high. The model in the following sections will reconcile these facts by appealing to the behavior of *future* dividend growth.

## 5.2 Equilibrium Asset Pricing Model

We now present a long-run risk (LRR) model along the lines of Bansal and Yaron (2004). The three questions we want to ask of this model are, first, whether it is able to generate the cross-sectional variation in the exposures of excess returns on book-to-market-sorted stock portfolios to the *CP* factor. We showed in Section 2 that there was a clearly increasing pattern in this exposure from growth to value. Since this is an equilibrium model, stock returns are endogenous, and only cash-flow growth can be specified exogenously. This leads us to ask whether, second, the model can match also the cross-sectional variation in the exposures of dividend growth rates of book-to-market-sorted stock portfolios to the *CP* factor. Section 5.1 showed dividend growth CP betas that were decreasing from growth (mildly positive) to value (strongly negative). Third, the *CP* factor is a strong predictor of bond returns in the data; we ask whether the model can capture this predictability quantitatively.

The LRR model combines Epstein and Zin (1989) preferences, which impute a concern for the timing of the resolution of uncertainty, with a consumption growth specification that features a small, but very persistent component as well as heteroscedasticity. The two state variables  $x_t$  and  $\sigma_t^2 - \bar{\sigma}^2$  capture time-varying growth rates and time-varying economic uncertainty, and each follow an AR(1). We prefer to think of  $x_t$  as a low-frequency variable and  $\sigma_t^2 - \bar{\sigma}^2$  as a variable that operates at business-cycle frequency.<sup>7</sup> Our main departure from the canonical LRR model is that we introduce *positive correlation* between the two state variables  $x_t$  and  $\sigma_t^2 - \bar{\sigma}^2$ . This new feature will play an important role in the model's ability to account for the CP betas. To think about nominal bonds, we model inflation as in Wachter (2006), Piazzesi and Schneider (2006), and Bansal and Shaliastovich (2007). This structure gives rise to a three-factor nominal term structure model; the factors are  $x_t$ ,  $\sigma_t^2 - \bar{\sigma}^2$ , and demeaned expected inflation  $\bar{\pi}_t - \mu_\pi$ .

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<sup>7</sup>This is different from Bansal and Yaron (2004) who make both state variables very persistent. In Bansal and Shaliastovich (2007), economic uncertainty is actually more persistent than long-run growth.



The separate appendix shows that both the real and the nominal bond risk premium in the model are affine in the state variable  $\sigma_t^2 - \bar{\sigma}^2$ . That is, the only source of time variation in nominal bond risk premia is economic uncertainty:  $\sigma_t^2$  is a *perfect* predictor of future excess bond returns. Therefore, it is natural to associate the *CP* factor with  $\sigma_t^2$ . To see the connection with the construction of the *CP* factor (see Section ??), note that forward rates, like yields, are affine functions of these three state variables. Hence, any three forward rates identify the three state variables exactly at each time  $t$ . A linear combination of them identifies  $CP_t$ :

$$CP_t = c_1 f_t^{\$}(4) + c_3 f_t^{\$}(12) + c_5 f_t^{\$}(20) = b_{CP}(\sigma_t^2 - \bar{\sigma}^2),$$

for some constants  $(c_1, c_3, c_5)$ . In order to pin down the ratio of the standard deviation of  $CP_t$  relative to that of  $\sigma_t^2$ ,  $b_{CP}$ , we form annual expected excess returns on nominal bonds of horizons 2-, 3-, 4-, and 5-years in the model. They only depend on  $\sigma_t^2$ . We set the *CP* factor equal to the equally-weighted average of these four returns, as we did in the data. Hence  $b_{CP}$  is not a free parameter but rather a function of the structural parameters of the model.

To construct equity risk premia, we specify a process for dividend growth rates for 10 book-to-market portfolios as well as one for the aggregate stock market. We let expected dividend growth rates for portfolio  $i$  depend on both  $x_t$  and on  $\sigma_t^2 - \bar{\sigma}^2$ , with loadings that we will estimate. Equity risk premia contain four parts: a compensation for bearing temporary consumption growth shocks, shocks to long-run risk consumption growth, shocks to economic uncertainty, and a term arising from the correlation between long-run growth and economic uncertainty. The market price of temporary cash-flow risk equals the coefficient of relative risk aversion, which is positive. The market price of long-run risk is also positive, while the market price of uncertainty risk is negative. These signs make sense: when long-run risk is low (economic uncertainty is high), the marginal utility growth for the representative investor is high, and she is willing to pay a high price for a security that has high dividend growth in such states of the world. The new, fourth term lowers the equity risk premium when the state variables are positively correlated. Indeed, when high uncertainty goes hand in hand with high future growth, the world is a less risky place. Finally, as for bond risk premia, the only source of time variation in equity risk premia is economic uncertainty  $\sigma_t^2$ .

With the definition of *CP* in hand, we can derive expressions for the *CP* betas of excess returns and dividend growth rates for each stock portfolio. Because our goal is to understand the patterns in *CP* betas and return, we refrain from a wholesale re-calibration or estimation of the model. Instead, we estimate how expected dividend growth of each of the 10 book-to-market portfolios and the aggregate market loads on the two state variables  $x_t$  and  $\sigma_t^2 - \bar{\sigma}^2$  (22 parameters), we estimate the correlation between  $x_t$  and  $\sigma_t^2 - \bar{\sigma}^2$ , and we estimate the persistence of  $\sigma_t^2 - \bar{\sigma}^2$ . All other parameters are identical to those in Bansal and Shaliastovich (2007). These 24 parameters using

non-linear least squares by minimizing the distance between model and data along the following 38 dimensions: 11 contemporaneous log excess stock return CP betas, 11 contemporaneous log real dividend growth CP betas, 11 equity risk premia, and 5 lagged bond return CP betas for maturities 1-, 2-, 5-, 7-, and 10-years. We estimate positive loadings on  $x_t$  and negative loadings on  $\sigma_t^2 - \bar{\sigma}^2$  for expected dividend growth. They increase in absolute value from low to high book-to-market portfolios. Importantly, we estimate a positive correlation between the two state variables of about 20%, and we estimate less persistence in  $\sigma_t^2$ . The lower persistence we estimate in economic uncertainty facilitates the interpretation of  $\sigma_t^2$  (or  $CP_t$ ) as a business cycle variable. Matching these moments does not come at the expense of matching the moments of consumption, interest rates, or inflation. This is because we kept all other parameters at their standard values. The separate appendix lists the point estimates and tabulates the moments of consumption, inflation, and interest rates.

Figure 11 shows that our model does a good job matching the patterns and magnitudes of the CP exposures of excess stock returns (top left panel), dividend growth rates (top right), the predictability of excess nominal bond returns by lagged  $CP$  (bottom left), while maintaining the good fit for the value premium (bottom right).

[Figure 11 about here.]

What drives these results? First, economic uncertainty, and hence  $CP$  predicts future excess returns, including nominal bond returns. Second, the pattern of negative and declining dividend growth betas results from the negative loading of expected dividend growth on economic uncertainty. According to this channel, stocks, and in particular value stocks, have lower expected dividend growth when economic uncertainty, and therefore  $CP$ , is high. In addition, expected dividend growth loads more heavily on  $x_t$  for value than for growth. Hence, value stocks are risky because their dividend growth is low in states of the world where long-run consumption growth  $x_t$  is low and/or economic uncertainty is high. These are states of high marginal utility growth for the representative investor. Taken together, that explains the value premium. The positive correlation between the two state variables we estimate is crucial to generate that times of high  $CP$  are good times. We recall the positive market price of risk we found for  $CP$  in the data. The positive correlation implies that times with higher economic uncertainty (which carries a negative price of risk) are also times of higher future long-run growth (which carries a positive risk price).<sup>8</sup> Because of the higher persistence of expected growth, the latter effect dominates, and high  $CP$  times are times of good future investment opportunities. The positive correlation also helps to-

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<sup>8</sup>Such a positive correlation arises endogenously in models of information production (Van Nieuwerburgh and Veldkamp 2006). It also increases the correlation between expected returns and expected dividend growth. (Binsbergen and Koijen 2007) estimate this correlation for the aggregate stock market and conclude it is positive.

wards matching the excess return CP betas: the model with no correlation generates betas that are too high and without discernible difference between value and growth.

Finally, an informative quantity is the difference between the return (not excess return) CP beta and the dividend growth CP beta. The Campbell-Shiller decomposition implies that it equals the difference between the CP beta of *news* about all *future* expected discounted dividend growth rates  $\Delta d_{t+1}^{i,H}$  and the CP beta of *news* about all *future* expected discounted stock returns  $r_{t+1}^{i,H}$ :

$$\beta^{rs} - \beta^{\Delta d} = \beta \left[ CP_{t+1}, \Delta d_{t+1}^{i,H} - \kappa_1^i \Delta d_t^{i,H} \right] - \beta \left[ CP_{t+1}, r_{t+1}^{i,H} - \kappa_1^i r_t^{i,H} \right] \quad (9)$$

The appendix provides an expression for each in terms of the structural parameters of the model. In the data, the left hand side is positive for all portfolios and strongly increasing from growth to value. This is the result of increasing contemporaneous return betas ( $\beta^{rs}$ ) and decreasing contemporaneous dividend growth betas ( $\beta^{\Delta d}$ ). Our model also predicts a strongly increasing pattern in the left-hand side of equation (9). The reason is that the first beta on the right-hand-side is positive and increasing from growth to value. A high bond risk premium ( $CP$  is high, at the end of a recession/the beginning of a boom) implies good news about future dividend growth, and more so for value stocks than for growth stocks. The second term on the right-hand-side is also positive and increasing. High future excess bond returns (at the end of a recession) coincide with high future excess stock returns, and more so for value than for growth stocks. The cash-flow effect dominates the discount rate effect, delivering the desired pattern. Hence value stocks are risky because there is good news about future cash-flow growth exactly when investors need it the least. The standard model with no correlation between the state variables has a zero first term on the right-hand-side of equation (9) and a (counter-intuitively) negative second term. In addition, there is no clear pattern in this beta difference as opposed to the strongly increasing pattern in the data.

## 6 Conclusion

We document that differential exposure to the Cochrane-Piazzesi (CP) factor, a strong predictor of future excess bond returns and hence a proxy for the bond risk premium, can help explain why value stocks have higher average returns than value stocks. This connection between a bond market and the cross-section of stocks leads us to develop a unified pricing model for stocks and bonds. We propose a no-arbitrage stochastic discount factor model that combines the canonical three-factor affine term structure model (level, slope, and curvature) with the CP factor, and a dividend yield factor. Only three factors need to have non-zero prices of risk to account for average returns on the book-to-market decile portfolios, the market portfolio, and maturity-sorted government bond portfolios. The price of market risk captures the common level of equity risk premia, the price of

level risk captures the cross-sectional spread between long-term and short-term bond returns, and the price of CP risk captures the cross-sectional spread between value and growth stock returns. This model also captures the dynamics of bond yields and expected excess stock returns well when the market prices of risk change with the CP factor. We explore what sources of underlying economic risk the CP factor captures in the context of the long-run risk model. We show that this equilibrium asset pricing model is able to replicate the differential exposure of excess stock returns to the CP factor of value and growth stocks. The CP factor is high at the end of a recession, periods that are characterized by high economic uncertainty but also good long-term growth prospects. A value-minus-growth strategy has high realized returns in periods when CP is high because value stocks receive good news about future cash flow growth in such periods.

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## A Constructing Quarterly Dividend Growth

To avoid seasonality in monthly (quarterly) data, we form the dividend-price ratio as the ratio of the equally-weighted average of the current dividend and the dividends over the past 11 months (3 quarters) and the current price. We define dividend growth correspondingly so that the adjusted dividend growth rate and change in the dividend price ratio gives back the original cum-dividend return. In symbols, cum-dividend returns equal  $R_{t+1} = DG_{t+1} \frac{PD_{t+1+1}}{PD_t}$ , where  $DG_{t+1} = D_{t+1}/D_t$  and  $PD_t = P_t/D_t$ . Define the ex-dividend return as  $R_{t+1}^{ex}$  and construct the dividend yield as the difference between the cum- and ex-dividend return:  $DY_{t+1} = R_{t+1} - R_{t+1}^{ex}$ . This leads to the quarterly dividend  $D_{t+1} = DY_{t+1} P_t^{ex}$ . The ex-dividend price index is updated recursively:  $P_{t+1}^{ex} = P_t^{ex}(1 + R_{t+1}^{ex})$ . For quarterly data, the adjusted variables, denoted with a tilde, are constructed as follows. The adjusted capital gain is defined as  $\tilde{R}_{t+1}^{ex} = R_{t+1}^{ex} - (D_{t-2}/4 + D_{t-1}/4 + D_t/4 - 3D_{t+1}/4) / P_t^{ex}$ . The adjusted ex-dividend price index is updated recursively:  $\tilde{P}_{t+1}^{ex} = \tilde{P}_t^{ex}(1 + \tilde{R}_{t+1}^{ex})$ . The adjusted dividend yield is  $\tilde{D}Y_{t+1} = R_{t+1} - \tilde{R}_{t+1}^{ex}$ . Finally, the adjusted quarterly dividend is  $\tilde{D}_{t+1} = \tilde{D}Y_{t+1} \tilde{P}_t^{ex}$ , dividend growth rate is  $\tilde{D}G_{t+1}$ , and the adjusted price-dividend ratio is  $\tilde{D}P_{t+1} = \tilde{D}Y_{t+1} \tilde{P}_t^{ex} / \tilde{P}_{t+1}^{ex}$ . Of course,  $\tilde{P}D_t = (\tilde{D}P_t)^{-1}$ . One can verify that this construction leaves gross returns unchanged:  $R_{t+1} = \tilde{D}G_{t+1} \frac{\tilde{P}D_{t+1+1}}{\tilde{P}D_t}$ . The construction of monthly data is similar.



Table 1: Market Prices of Risk in the Data

This table reports slope and intercept estimates from the second-stage estimation of average returns on the 10 value portfolios (top panel) or the 25 size- and value portfolios (bottom panel) on the bond risk factor exposures (from the first-stage estimation). The intercept is in percent per year. The bond risk factor is the Cochrane-Piazzesi  $CP$  factor in the left panel and the 20-1-quarter yield spread in the right panel. The intercept is expressed as a percent per year. The last four rows denote the mean absolute pricing error (MAPE) and the root mean squared pricing error (RMSE) across the portfolios, also expressed as a percent per year. We also report the cross-sectional  $R^2$  from the second stage estimation, and the sample length from the first-stage estimation. The different columns denote different sampling frequencies. The sample runs from June 1952 until December 2008.

<b>10 Value Portfolios (1952-2008)</b>									
predictor:	<i>KLN</i> factor			<i>CP</i> factor			Yield Spread		
	<i>M</i>	<i>Q</i>	<i>A</i>	<i>M</i>	<i>Q</i>	<i>A</i>	<i>M</i>	<i>Q</i>	<i>A</i>
slope	<b>0.27</b>	<b>0.86</b>	<b>4.34</b>	<b>1.30</b>	<b>2.18</b>	<b>2.38</b>	<b>1.47</b>	<b>1.48</b>	<b>1.39</b>
$[t - stat]$	[7.57]	[11.44]	[5.52]	[5.74]	[5.32]	[5.55]	[2.69]	[4.01]	[4.78]
intercept	5.27	7.51	9.18	4.63	3.97	7.15	2.60	3.59	6.73
$[t - stat]$	[15.29]	[54.65]	[21.04]	[8.49]	[5.58]	[22.96]	[1.42]	[3.52]	[17.85]
MAPE	0.42	0.31	0.72	0.63	0.64	0.70	0.96	0.81	0.77
RMSE	0.56	0.39	0.87	0.71	0.76	0.87	1.16	0.93	0.97
$R^2$	<b>87.7</b>	<b>94.2</b>	<b>79.2</b>	<b>80.5</b>	<b>78.0</b>	<b>79.4</b>	<b>47.6</b>	<b>66.8</b>	<b>74.0</b>
T	678	226	56	678	226	56	678	226	56

<b>25 Size- and Value Portfolios (1952-2008)</b>									
predictor:	<i>KLN</i> factor			<i>CP</i> factor			Yield Spread		
	<i>M</i>	<i>Q</i>	<i>A</i>	<i>M</i>	<i>Q</i>	<i>A</i>	<i>M</i>	<i>Q</i>	<i>A</i>
slope	<b>0.43</b>	<b>1.16</b>	<b>4.47</b>	<b>2.05</b>	<b>2.79</b>	<b>2.85</b>	<b>1.40</b>	<b>1.29</b>	<b>1.73</b>
$[t - stat]$	[6.85]	[8.65]	[8.24]	[9.02]	[10.11]	[8.02]	[5.56]	[7.27]	[10.13]
intercept	4.33	7.22	9.83	3.09	3.06	7.49	2.01	3.26	6.06
$[t - stat]$	[6.52]	[23.73]	[31.36]	[4.87]	[5.14]	[23.34]	[1.68]	[4.10]	[17.31]
MAPE	1.15	1.02	0.93	0.97	0.94	1.16	1.32	1.14	0.91
RMSE	1.45	1.23	1.34	1.19	1.08	1.38	1.65	1.39	1.15
$R^2$	<b>67.1</b>	<b>76.5</b>	<b>75.1</b>	<b>78.0</b>	<b>81.6</b>	<b>73.7</b>	<b>57.3</b>	<b>69.7</b>	<b>81.7</b>
T	678	226	56	678	226	56	678	226	56

Table 2: Cross-Sectional Equity Return Analysis: Subsamples

This table reports slope and intercept estimates from the second-stage estimation of average returns on the 10 value portfolios (top panel) or the 25 size- and value portfolios (bottom panel) on the bond risk factor exposures from the first-stage estimation. The bond risk factor is the Cochrane-Piazzesi *CP* factor. In the left panel, the *CP* factor is not re-estimated (same as in the full sample), whereas in the right panel it is re-estimated on the sub-sample in question.

<b>10 Book-to-market Portfolios (Monthly Frequency)</b>						
	<b>CP from full sample</b>			<b>CP estimated on subsample</b>		
	<i>1963-2008</i>	<i>1952-1980</i>	<i>1980-2008</i>	<i>1963-2008</i>	<i>1952-1980</i>	<i>1980-2008</i>
slope	<b>1.90</b> [14.00]	<b>1.10</b> [3.00]	<b>0.93</b> [3.85]	<b>2.30</b> [14.38]	<b>0.76</b> [3.30]	<b>0.95</b> [3.64]
intercept	1.46 [3.87]	2.00 [0.97]	5.98 [15.35]	2.10 [6.50]	-3.06 [-0.91]	5.80 [13.19]
MAPE	0.26	1.29	0.75	0.27	1.21	0.79
RMSE	0.37	1.43	0.90	0.36	1.35	0.93
$R^2$	<b>96.1</b>	<b>52.8</b>	<b>64.9</b>	<b>96.3</b>	<b>57.6</b>	<b>62.3</b>
T	552	339	339	552	339	339

<b>25 Size- and Value Portfolios (Monthly Frequency)</b>						
	<b>CP from full sample</b>			<b>CP estimated on subsample</b>		
	<i>1963-2008</i>	<i>1952-1980</i>	<i>1980-2008</i>	<i>1963-2008</i>	<i>1952-1980</i>	<i>1980-2008</i>
slope	<b>2.42</b> [10.72]	<b>1.02</b> [3.81]	<b>1.90</b> [5.75]	<b>2.95</b> [9.48]	<b>0.77</b> [4.28]	<b>1.88</b> [5.08]
intercept	0.29 [0.40]	2.66 [1.48]	4.94 [8.26]	1.46 [2.09]	-3.21 [-1.08]	5.42 [9.11]
MAPE	0.85	1.64	1.58	0.98	1.57	1.74
RMSE	1.16	1.93	2.06	1.28	1.84	2.20
$R^2$	<b>83.3</b>	<b>38.7</b>	<b>58.9</b>	<b>79.6</b>	<b>44.3</b>	<b>52.9</b>
T	552	339	339	552	339	339

Table 3: Cross-Sectional Equity Return Analysis: Comparison with 3-and 4-Factor Model

This table reports slope and intercept estimates from the second-stage estimation of average returns on the 25 size- and value portfolios on the factor exposures from the first-stage estimation. The factors are the 3 Fama-French factors (first panel), the 4 Carhart factors (second panel), the yield spread and the 3 Fama-French factors (third panel), and the yield spread and the 4 Fama-French factors (fourth panel). The sample runs from June 1952 until December 2008.

25 Size- and Value Portfolios (1952-2008)								
	<i>CP</i>	<i>yspr</i>	<i>3-f</i>	<i>4-f</i>	<i>CP+3-f</i>	<i>yspr+3-f</i>	<i>CP+4-f</i>	<i>yspr+4-f</i>
CP	2.05				1.44		1.19	
	[9.02]				[3.18]		[2.67]	
yspr		1.40				1.50		1.39
		[5.56]				[7.54]		[6.60]
Mkt			-0.97	0.04	-0.55	-0.02	0.10	0.28
			[-2.60]	[0.06]	[-1.62]	[-0.10]	[0.21]	[0.89]
SMB			0.15	0.12	0.15	0.17	0.12	0.16
			[3.14]	[2.68]	[3.56]	[6.78]	[3.13]	[6.08]
HML			0.39	0.44	0.39	0.39	0.43	0.41
			[6.41]	[7.52]	[7.65]	[12.31]	[8.21]	[12.21]
UMD				3.97			2.70	1.25
				[2.42]			[1.79]	[1.24]
intercept	3.09	2.01	18.00	6.59	13.18	6.72	5.93	3.43
	[4.87]	[1.68]	[3.85]	[1.04]	[3.14]	[2.34]	[1.07]	[0.95]
MAPE	0.97	1.32	0.94	0.89	0.80	0.54	0.79	0.52
RMSE	1.19	1.65	1.27	1.11	1.03	0.65	0.95	0.61
$R^2$	<b>81.9</b>	<b>57.3</b>	<b>74.9</b>	<b>80.6</b>	<b>83.3</b>	<b>93.5</b>	<b>85.9</b>	<b>94.1</b>

Table 4: Contemporaneous Betas of Bond Portfolios in the Data

This table reports slope and intercept estimates from the second-stage estimation of average returns on the 5 CRSP bond portfolio returns on the level factor betas from a first-stage estimation. The intercept is in percent per year. The level factor is the 5-year Fama-Bliss yield in the left panel and the first principal components of the 1- through 5-year Fama-Bliss yields in the right panel. The last four rows denote the mean absolute pricing error (MAPE) and the root mean squared pricing error (RMSE) across the 25 portfolios. Both are expressed in percent per year. We also report the cross-sectional  $R^2$  from the second stage estimation, and the sample length from the first-stage estimation. The different columns denote different sampling frequencies. The sample is June 1952 until December 2008.

<b>5 CRSP Bond Portfolio Returns</b>						
predictor:	5-Year Bond Yield factor			Level Factor		
	<i>M</i>	<i>Q</i>	<i>A</i>	<i>M</i>	<i>Q</i>	<i>A</i>
slope	<b>-1.22</b>	<b>-1.07</b>	<b>-1.01</b>	<b>-2.47</b>	<b>-2.14</b>	<b>-2.01</b>
$[t - stat]$	<b>[-4.24]</b>	<b>[-4.19]</b>	<b>[-4.41]</b>	<b>[-4.11]</b>	<b>[-4.13]</b>	<b>[-4.24]</b>
intercept	1.15	0.78	0.59	1.09	0.73	0.55
$[t - stat]$	[9.92]	[5.49]	[5.02]	[8.13]	[4.71]	[4.25]
MAPE	0.08	0.08	0.08	0.09	0.09	0.08
RMSE	0.12	0.12	0.12	0.12	0.13	0.12
$R^2$	<b>85.7</b>	<b>85.4</b>	<b>86.6</b>	<b>84.9</b>	<b>85.0</b>	<b>85.7</b>
T	678	226	56	678	226	56

Table 5: Unified SDF Model for Stocks and Bonds - Pricing Errors

Panel A of this table reports pricing errors on 10 book-to-market sorted stock portfolios, the value-weighted market portfolio, and five bond portfolios of maturities 1, 2, 5, 7, and 10 years. They are expressed in percent per year. Each column corresponds to a different SDF model, as described in the text, apart from the second and third column. In the third column, we only use the bond returns to estimate the same stochastic discount factor as in the second column. The main text contains further details. The last row reports the mean absolute pricing error across all 11 securities (MAPE). Panel B reports the estimates of the prices of risk. In all specifications, we set  $\Lambda_1 = 0$ . The data are monthly for 1952.6 until 2008.12.

Panel A: Pricing Errors					
	SDF 1	SDF 2	SDF 2	SDF 3	SDF 4
	(bonds only)				
BM1	4.90	3.42	4.55	-3.28	0.01
BM2	5.79	2.14	4.83	-1.85	-0.33
BM3	6.58	2.25	5.43	-0.83	0.04
BM4	6.31	2.23	5.23	-0.99	-0.05
BM5	7.25	2.67	6.02	0.55	0.78
BM6	7.30	1.93	5.85	0.40	0.26
BM7	7.30	0.06	5.31	0.76	-1.15
BM8	9.05	1.76	7.07	2.34	0.27
BM9	9.65	2.38	7.67	2.77	0.71
BM10	10.02	2.98	8.15	2.35	0.26
Market	5.95	1.72	4.82	-1.56	-0.76
1-yr	1.12	-1.78	0.31	1.00	-0.44
2-yr	1.32	-2.97	0.13	1.15	-0.57
5-yr	1.69	-4.62	-0.06	1.43	-0.20
7-yr	2.00	-4.64	0.15	1.61	0.39
10-yr	1.63	-5.33	-0.30	1.09	0.18
MAPE	5.49	2.68	4.82	1.50	0.40
Panel B: Estimates of Prices of Risk ( $\Lambda_0$ )					
	SDF 1	SDF 2	SDF 2	SDF 3	SDF 4
	(bonds only)				
CP factor	0	0	0	0	45.90
Level factor	0	-59.22	-16.40	0	-24.00
Slope factor	0	0	0	0	0
Curvature factor	0	0	0	0	0
dp	0	0	0	-3.46	-1.94

Table 6: Non-Linear Predictability in the Value-Growth returns in the Data

This table reports slope and intercept estimates from a predictability regression of cumulative (discounted) BM10-BM1 excess returns on a non-linear function of the bond risk premium. The non-linear function of the bond risk premium is constructed as  $CP_t I_{CP_t > .0160}$  in the left panel and  $yspr_t I_{yspr_t > .0168}$  in the right panel. The first column denotes the holding period over which returns are cumulated. The first 59 months are discarded so that all returns are computed over the same sample. All cumulative results are expressed on a per annum basis so that the slope coefficients have comparable magnitudes across rows. The full sample is monthly data from June 1952 until December 2007. T-stats are computed using Newey-West standard errors with  $k - 1$  lags, where  $k$  is the horizon in months.

predictor: k	CP factor			Yield Spread		
	slope	t-stat	$R^2$	slope	t-stat	$R^2$
1-qtr	3.06	2.29	1.62	4.43	2.12	1.80
2-qtr	2.15	1.85	1.47	3.20	1.70	1.73
3-qtr	2.14	1.91	2.11	3.38	1.89	2.80
1-yr	2.38	2.09	3.43	3.62	2.02	4.22
2-yr	2.04	2.11	5.14	2.72	2.13	4.88
3-yr	1.36	2.10	3.87	1.27	1.27	1.78
4-yr	0.80	1.48	1.88	0.36	0.43	0.20
5-yr	0.32	0.63	0.40	-0.06	-0.08	0.01

Figure 1: Nominal Bond Risk Premia

The  $KLN$  factor is the fitted value of a regression of the return on the HLM stock return portfolio (long value short growth) on a constant, the one-year nominal yield and the 2- through 5-year forward rates. The  $CP$  factor is the projection of by regression of the equally-weighted average of the one-year excess return on bonds of maturities of two, three, four, and five years on a constant and the one-period lagged one-year nominal yield and the 2- through 5-year forward rates. The data run from June 1952 until December 2008.

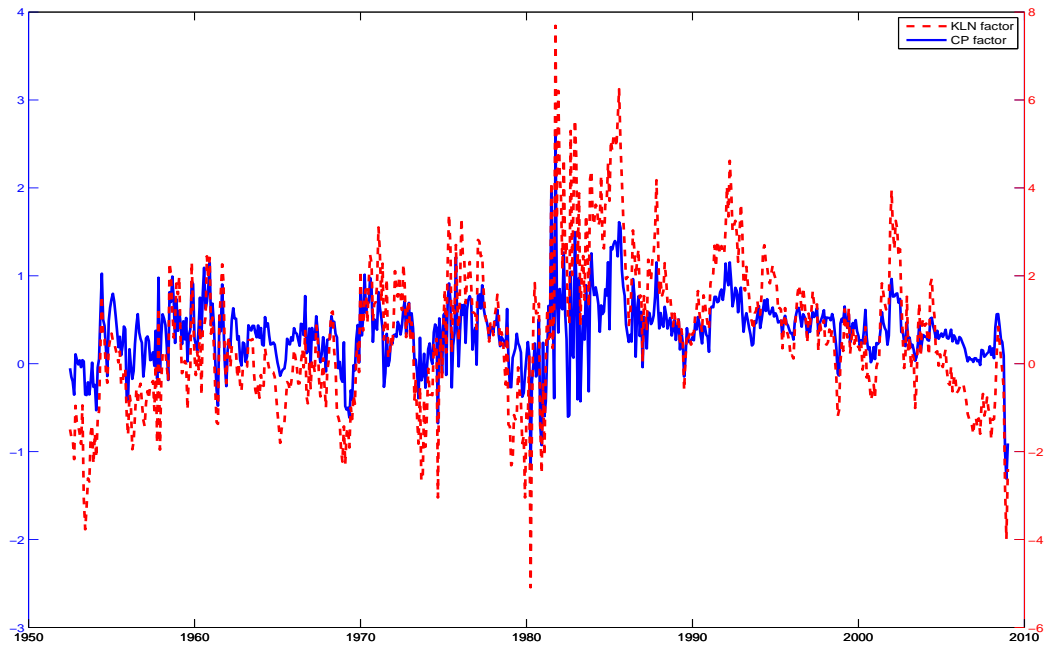


Figure 2: Exposure of 10 Book-to-Market Portfolio Excess Returns to the *KLN* and *CP* Factors

The figure plots the factor exposures (betas) of the 10 book-to-market portfolio returns  $r_{t+1}^{i,e} \equiv r_{t+1}^i - y_t^S(1) + .5 * Var[r_{t+1}^i]$  to the contemporaneous *KLN*<sub>t+1</sub> (left panels) or *CP*<sub>t+1</sub> factor (right panels). Each bar denotes the slope coefficient of a time-series regression of one portfolio return on the *KLN* or *CP* factor. The left bar is for growth stocks (log book-to-market), the right bar for value stocks (high book-to-market). The top panel uses monthly return data, the middle panel uses quarterly return data, and the bottom panel uses annual data. The data run from June 1952 until December 2008.

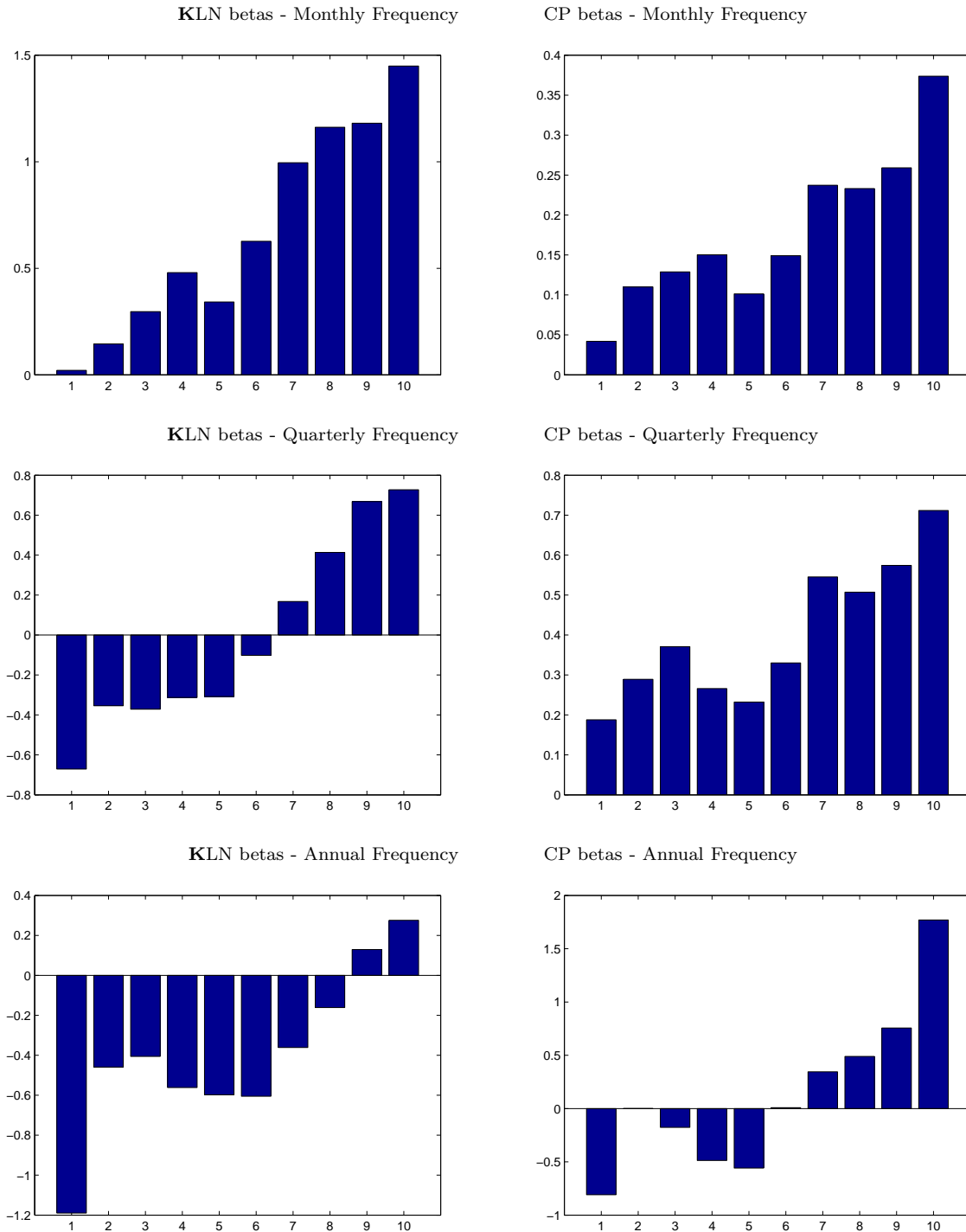




Figure 3: Exposure of 25 Size and Book-to-Market Portfolio Returns to the  $KLN$  and  $CP$  Factors

The figure plots the factor exposures (betas) of the 25 size- and value portfolio returns to the contemporaneous  $KLN_{t+1}$  (left panels) or  $CP_{t+1}$  factor (right panels). Each bar denotes the slope coefficient of a time-series regression of a portfolio return on the  $CP$  factor. The first five bars are for the smallest quintile of firms; the last five bars are for the biggest quintile of firms. Within each group of five, the book-to-market ratio increases. The top panel uses monthly return data, the middle panel uses quarterly return data, and the bottom panel uses annual data. The data run from June 1952 until December 2008.

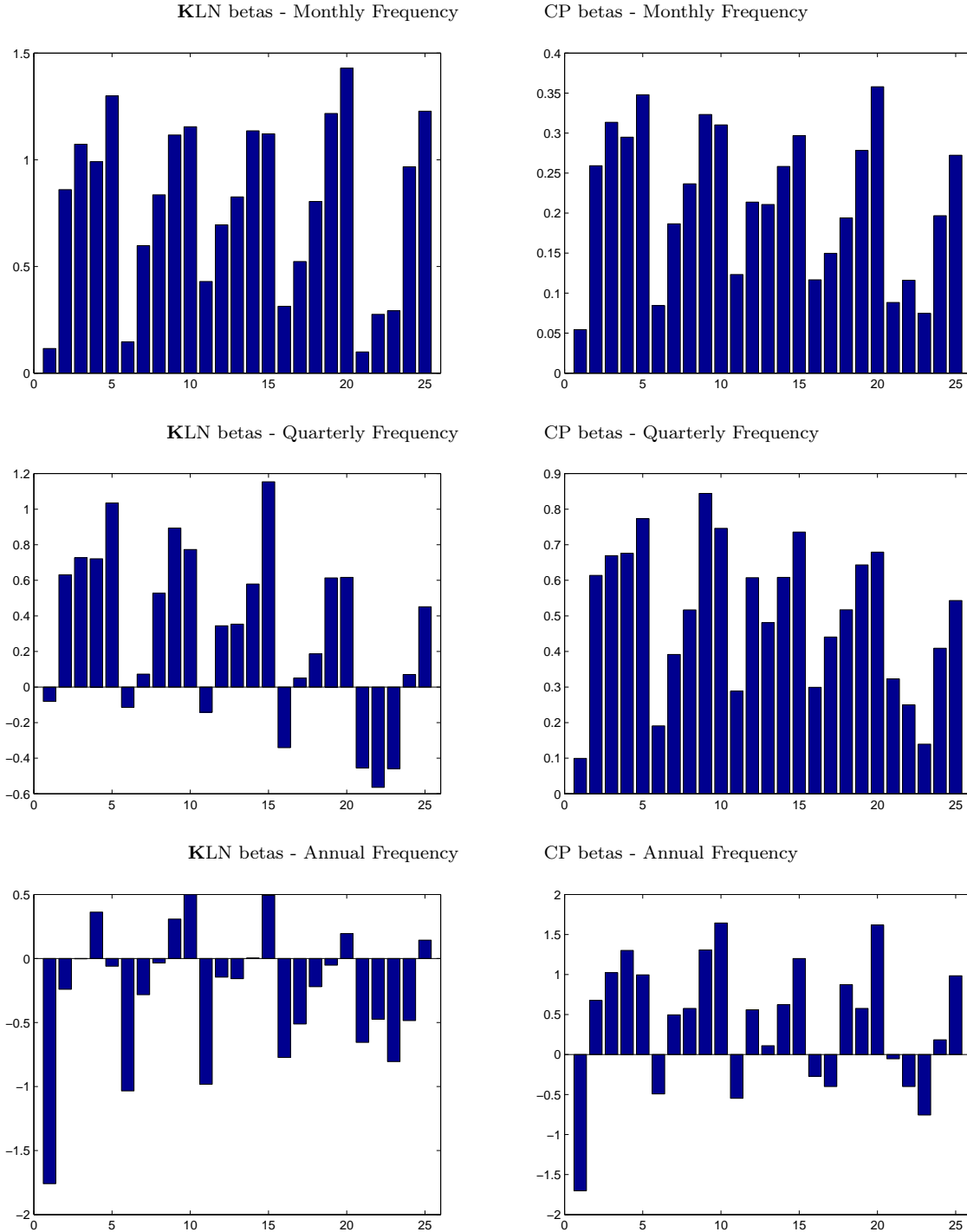


Figure 4: Exposure to the Yield Spread

The figure plots the factor exposures (betas) of the 10 book-to-market portfolio returns (left panels) and the 25 size- and value portfolio returns (right panel) to the yield spread. The data are monthly for June 1952 until December 2008 (top row), quarterly (middle row), and annual (bottom row).

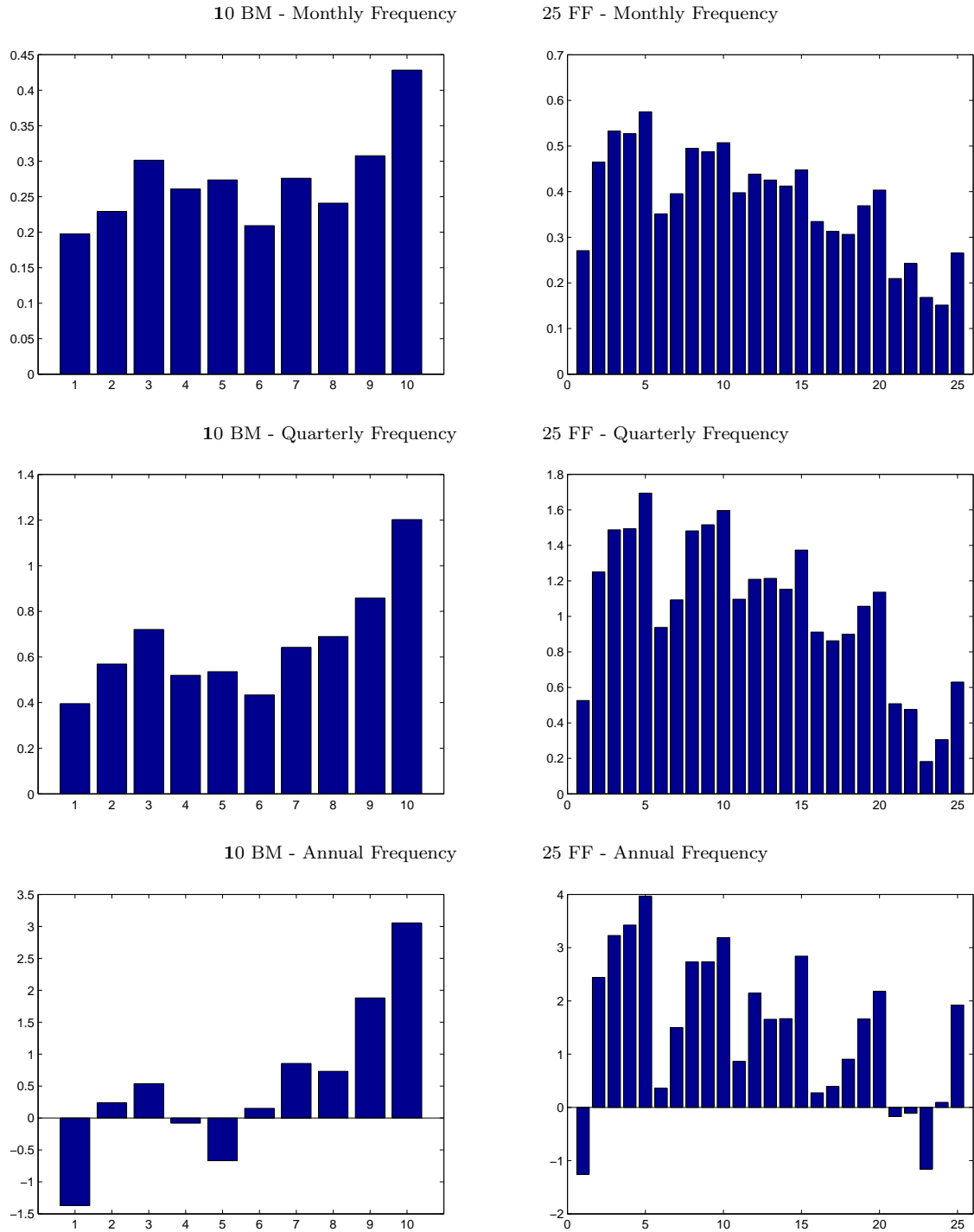


Figure 5: Average Realized vs Predicted Returns: *CP* Factor

Scatter diagram of average realized versus predicted excess returns. The predicted excess return is generated by the OLS estimation with the *CP* factor as only explanatory variable. The test assets are the 10 book-to-market portfolio returns (top panel) and the 25 size- and value portfolio returns (bottom panel). The solid line is the 45 degree line. The data run from June 1952 until December 2008.

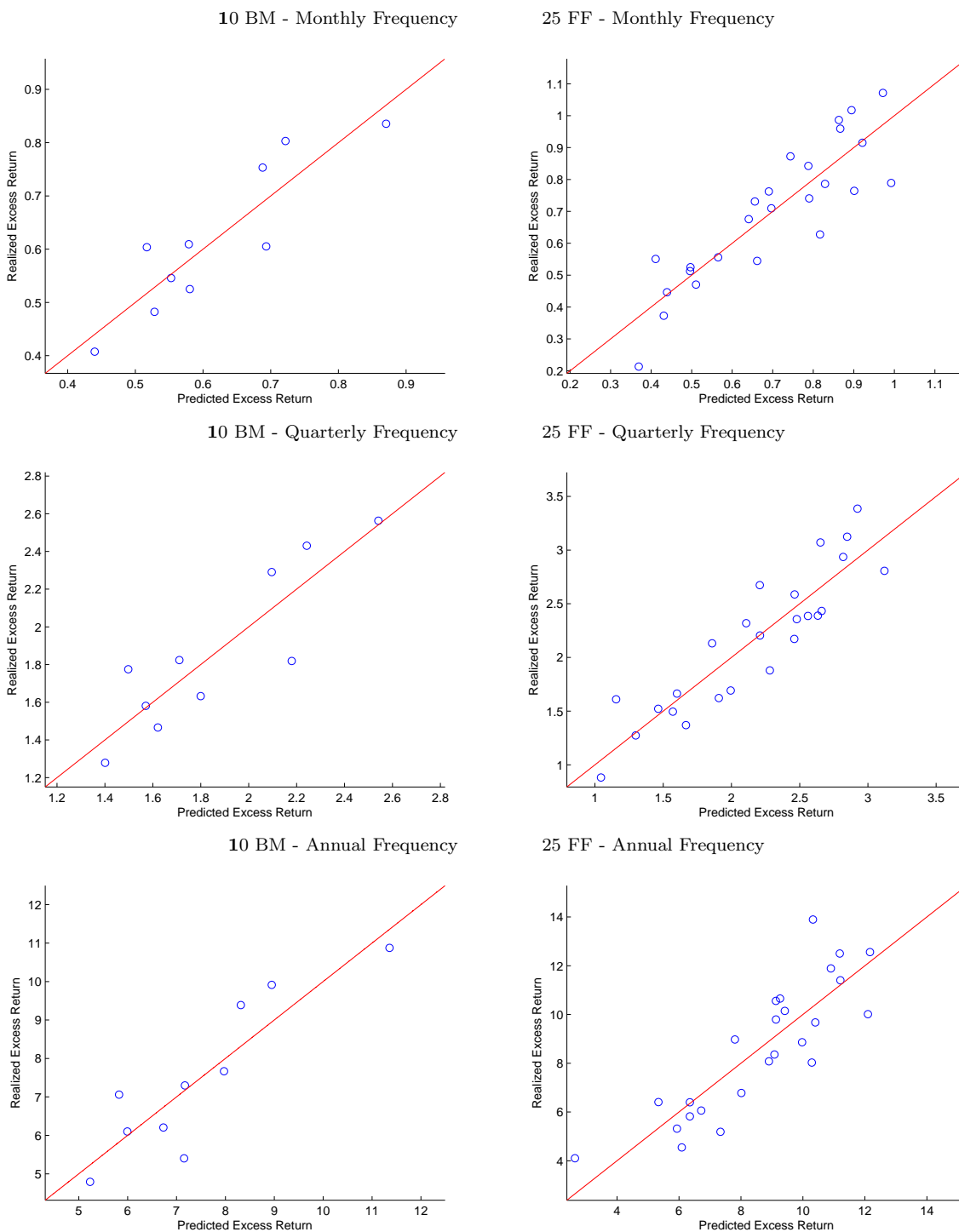


Figure 6: Exposure of 5 Bond Portfolio Excess Returns to the Level Factor in Data

The figure plots the factor exposures (betas) of five CRSP bond portfolio returns (1-yr, 2-yr, 5-yr, 7-yr, and 10-yr maturities) to the contemporaneous level factor, measured as the 5-year nominal bond yield  $y^s(20)$ . Each bar denotes the slope coefficient of a time-series regression of one portfolio return on the level factor. The left bar is for the shortest maturity bond, the right bar for longest maturity bonds. The top panel uses monthly return data, the middle panel uses quarterly return data, and the bottom panel uses annual data. The data run from June 1952 until December 2008.

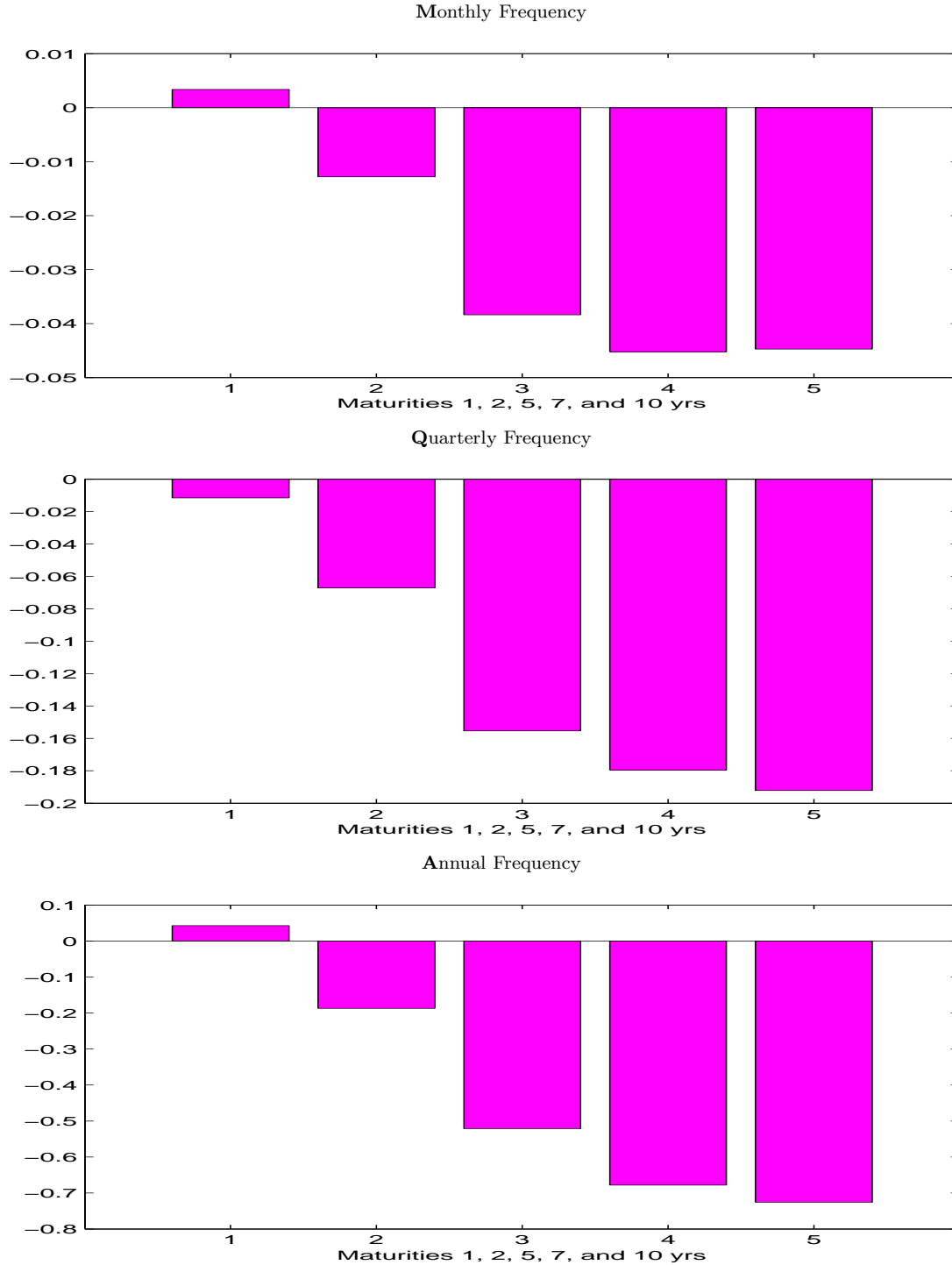


Figure 7: SDF Model-Implied Yields

This figure plots annual yields on nominal bonds of maturities 1- through 5 years as implied by stochastic discount factor model estimated in Section 3.3. The data are Fama-Bliss yields on nominal bonds of maturities 1- through 5 years.

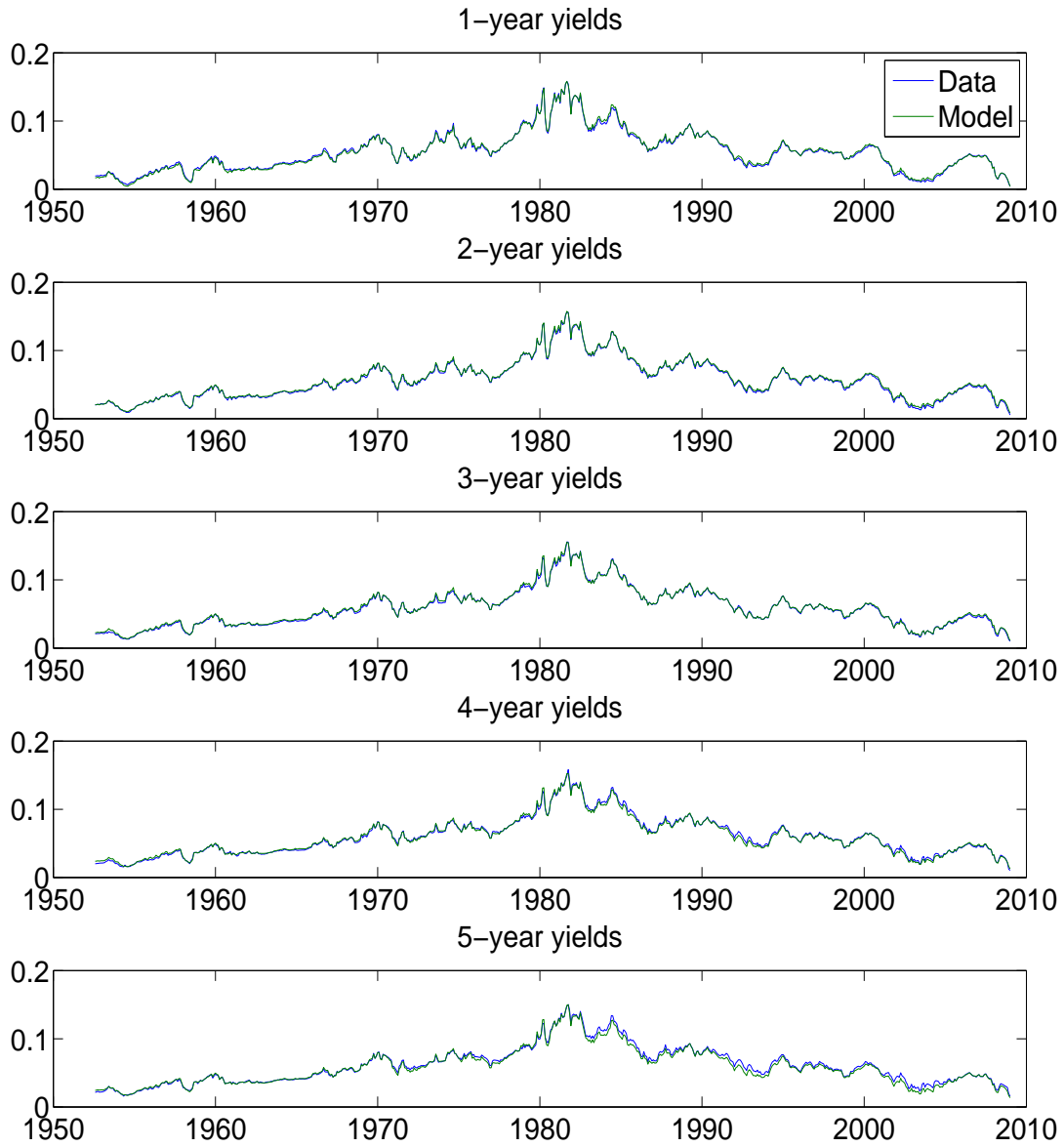


Figure 8: NBER Recessions, the CP Factor and the Yield Spread

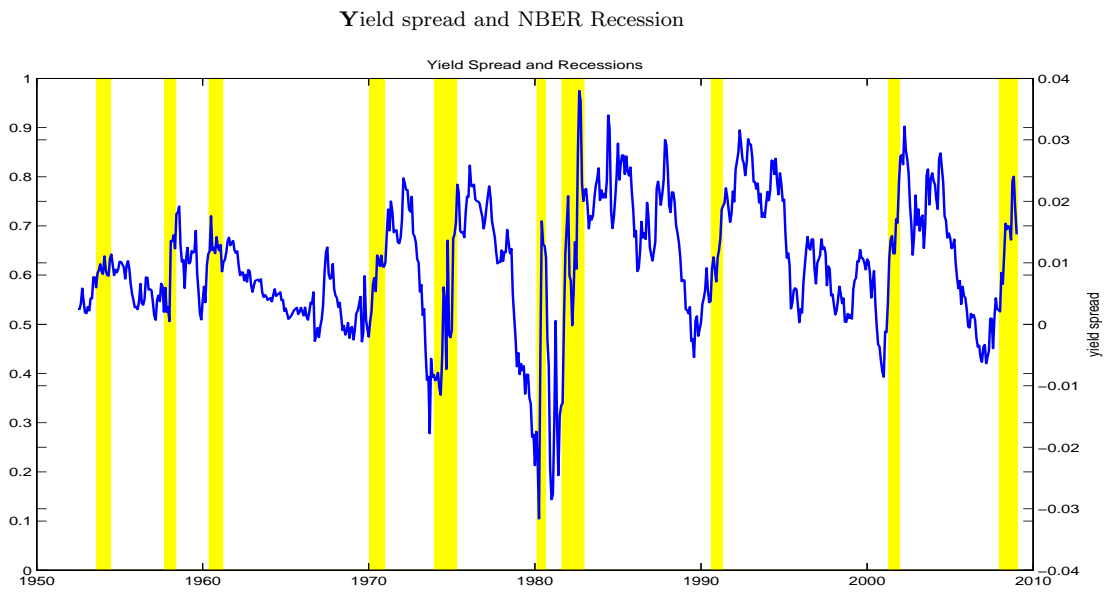
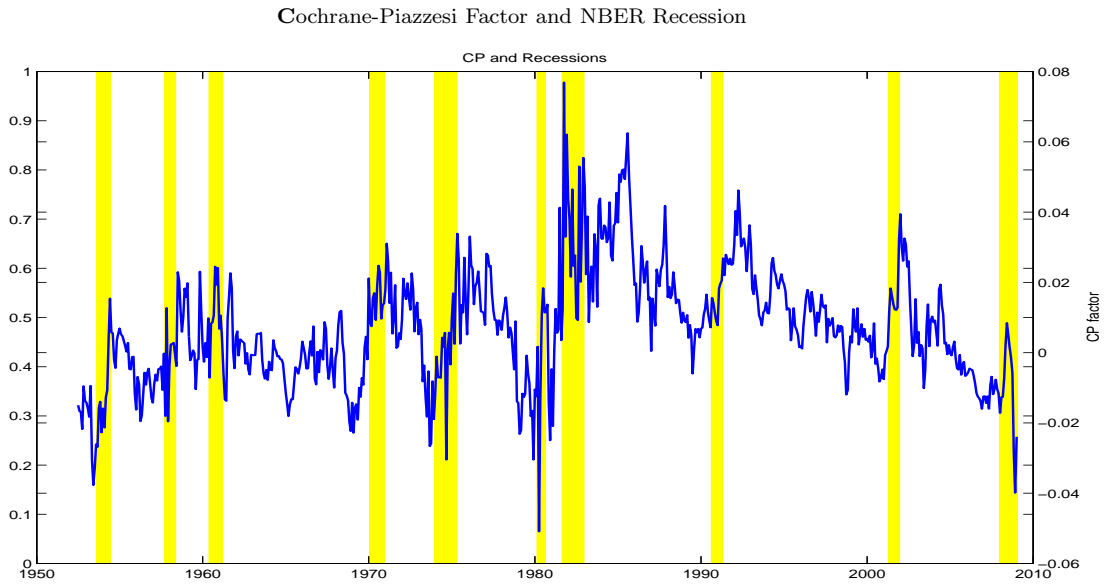


Figure 9: Value versus Growth Returns By CP Factor and Yield Spread Quartiles

The left panels denote average monthly returns on the 10 book-to-market portfolios, multiplied by 1200, by CP quartile. The right panels denote average monthly returns on the 10 book-to-market portfolios, multiplied by 1200, by yield spread quartile. The top row is for the top quartile, the second row for the 50-75 percentile, the third row for the 12-50 percentile, and the bottom row for the lowest 25 percent CP or yield spread realizations.

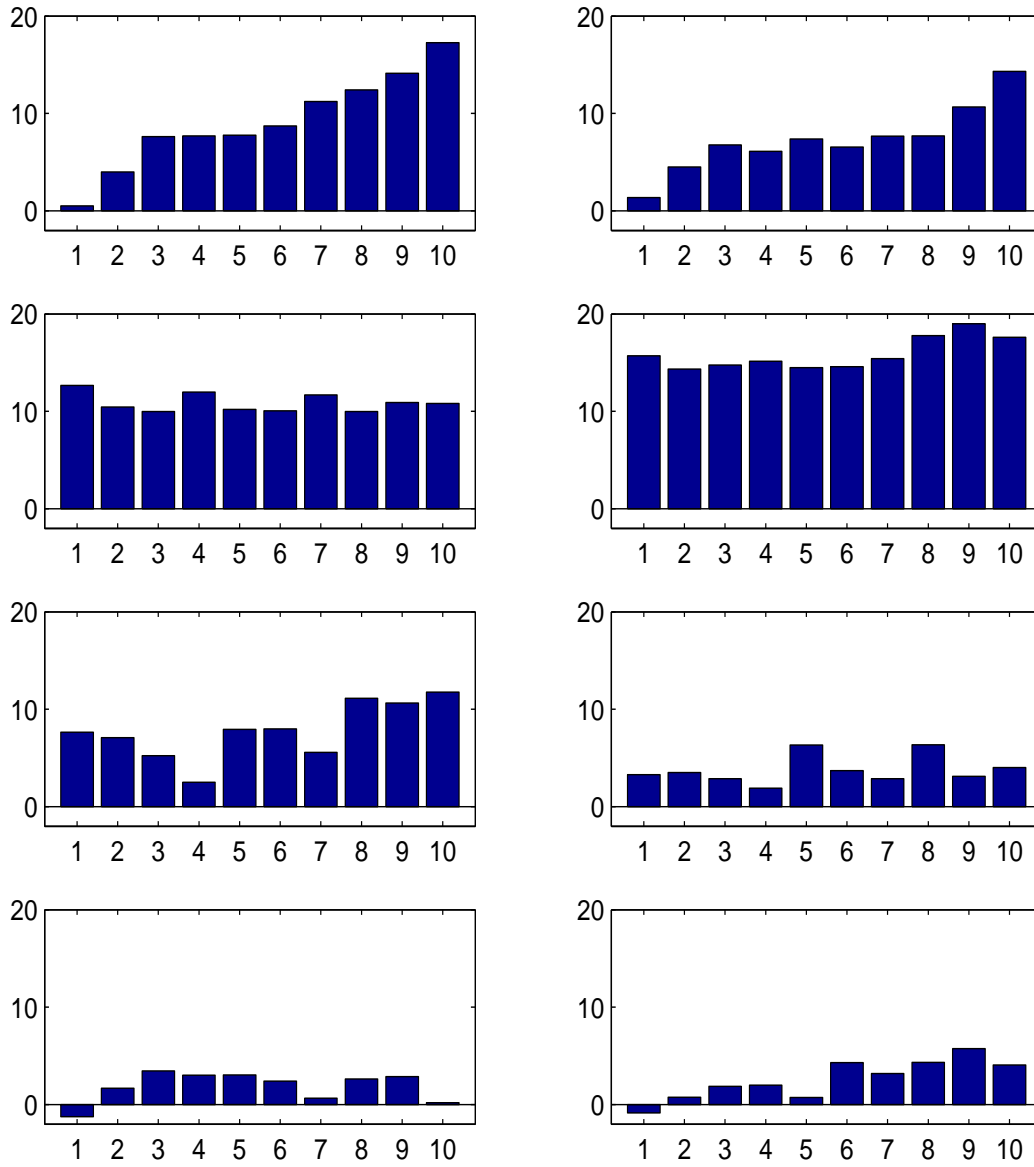


Figure 10: Exposure of 10 Book-to-Market Portfolio Dividend Growth rates to the  $CP$  and Yield Spread Factor in Data

The figure plots the factor exposures (betas) of the 10 book-to-market portfolio dividend growth rates  $\delta d_{t+1}$  to the contemporaneous  $CP_{t+1}$  factor (left panels) and yield spread (right panels). Each bar denotes the slope coefficient of a time-series regression of one portfolio dividend growth rate on the  $CP$  factor or the yield spread. The left bar is for growth stocks (log book-to-market), the right bar for value stocks (high book-to-market). The top panel uses monthly return data, the middle panel uses quarterly return data, and the bottom panel uses annual data. The data run from June 1952 until December 2008.

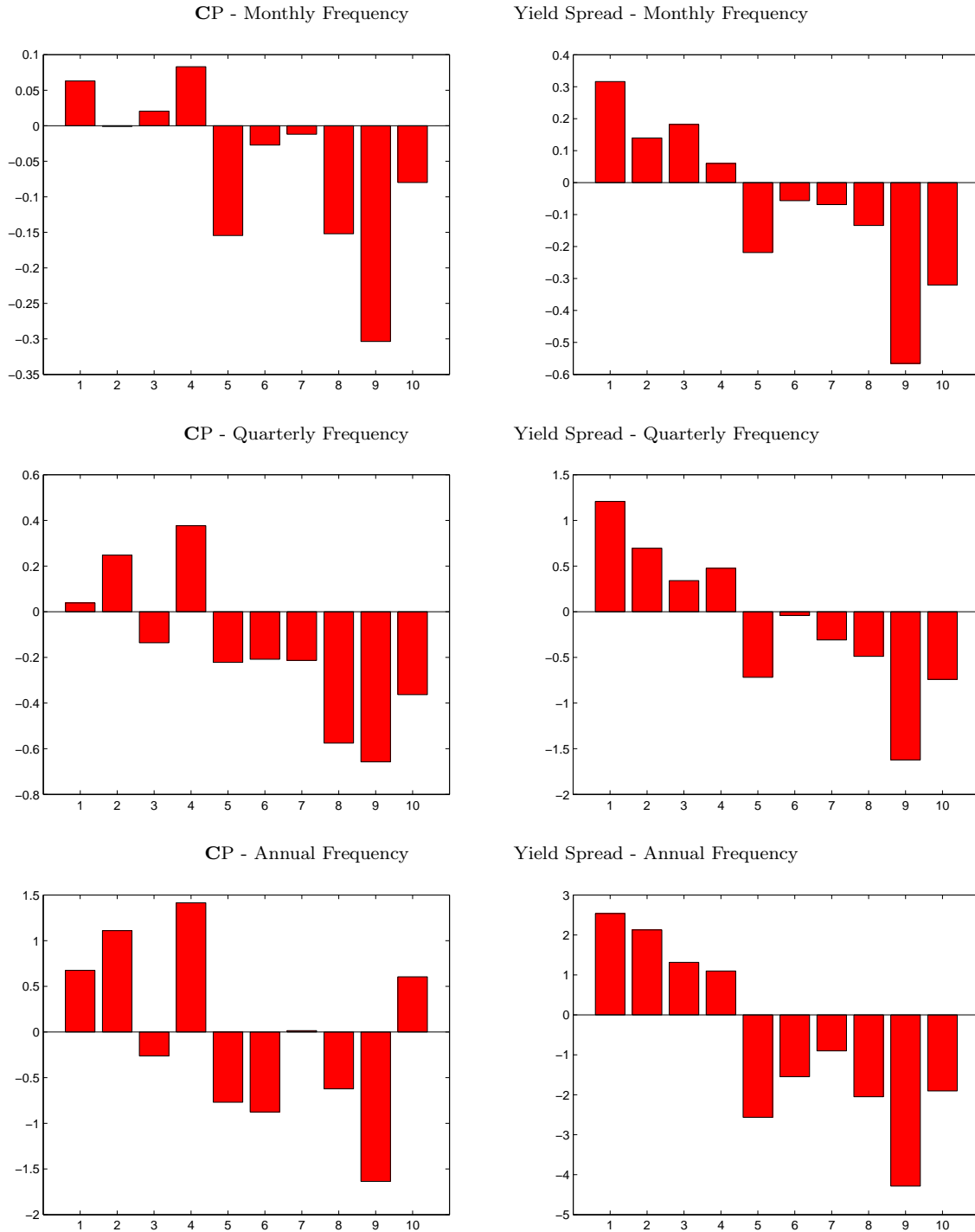
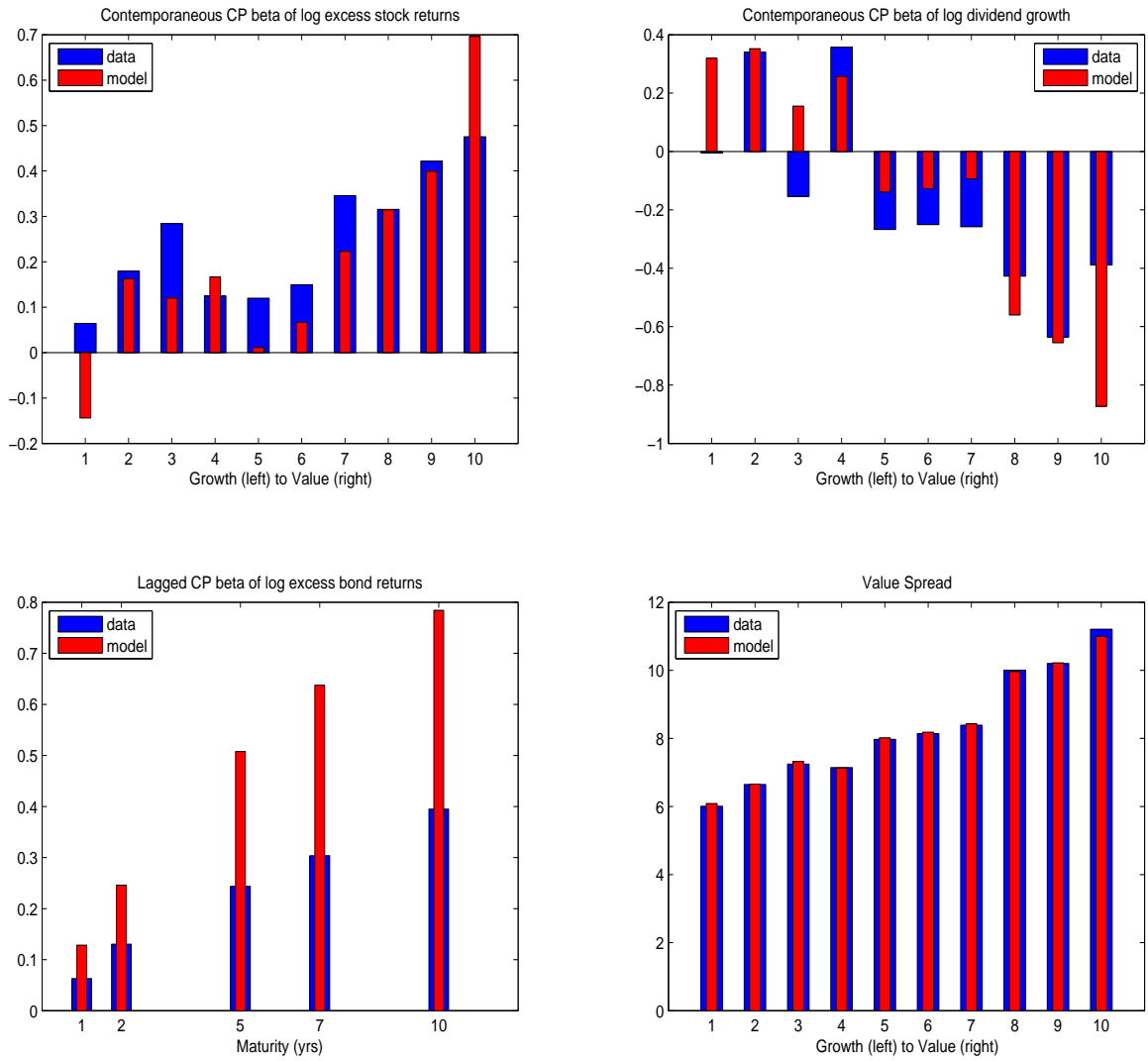




Figure 11: Risk Premia and CP Betas in Equilibrium Model



# LONG-RUN RISK MODEL IN DETAIL: TECHNICAL APPENDIX

February 25, 2009

This appendix provides a complete derivation of our modified long-run risk model in section A. It calibrates the model in Section B and discusses the results in Section C.

## A The Model

### A.1 Setup

**Preferences** The long-run risk literature works off the class of preferences due to Kreps and Porteus (1978), Epstein and Zin (1989), and Duffie and Epstein (1992). These preferences impute a concern for the timing of the resolution of uncertainty. A first parameter  $\alpha$  governs risk aversion and a second parameter  $\rho$  governs the willingness to substitute consumption inter-temporally. In particular,  $\rho$  is the inverse of the inter-temporal elasticity of substitution (EIS). Bansal and Yaron (2004) show that the stochastic discount factor can be written as a function of consumption growth and the return to the wealth portfolio:

$$M_{t+1} = \beta^{\frac{1-\alpha}{1-\rho}} \left( \frac{C_{t+1}}{C_t} \right)^{-\rho \frac{1-\alpha}{1-\rho}} (R_{t+1}^c)^{\frac{\rho-\alpha}{1-\rho}}. \quad (1)$$

The return on a claim to aggregate consumption, the *total wealth return*, can be written as

$$R_{t+1}^c = \frac{W_{t+1}}{W_t - C_t} = \frac{C_{t+1}}{C_t} \frac{WC_{t+1}}{WC_t - 1}.$$

We start by using the Campbell (1991) approximation of the log total wealth return  $r_t^c = \log(R_t^c)$  around the long-run average log wealth-consumption ratio  $A_0^c \equiv E[w_t - c_t]$ .<sup>1</sup>

$$r_{t+1}^c = \kappa_0^c + \Delta c_{t+1} + wc_{t+1} - \kappa_1^c wc_t, \quad (2)$$

where we define the log *wealth-consumption* ratio  $wc$  as

$$wc_t \equiv \log \left( \frac{W_t}{C_t} \right) = w_t - c_t.$$

The linearization constants  $\kappa_0^c$  and  $\kappa_1^c$  are non-linear functions of the unconditional mean wealth-consumption ratio  $A_0^c$ :

$$\kappa_1^c = \frac{e^{A_0^c}}{e^{A_0^c} - 1} > 1 \quad \text{and} \quad \kappa_0^c = -\log(e^{A_0^c} - 1) + \frac{e^{A_0^c}}{e^{A_0^c} - 1} A_0^c. \quad (3)$$

---

<sup>1</sup>Throughout, variables with a subscript zero denote unconditional averages.

Using the definition of the total wealth return, one can then show that the log SDF becomes:

$$m_{t+1} = \frac{1-\alpha}{1-\rho} [\log \beta + \kappa_0^c] - \kappa_0^c - \alpha \Delta c_{t+1} - \frac{\alpha-\rho}{1-\rho} (wc_{t+1} - \kappa_1^c wc_t) \quad (4)$$

**Technology** We mostly adopt the consumption growth specification of Bansal and Yaron (2004):

$$\Delta c_{t+1} = \mu_c + x_t + \sigma_t \eta_{t+1}, \quad (5)$$

$$x_{t+1} = \rho_x x_t + \varphi_e \sigma_t e_{t+1}, \quad (6)$$

$$\sigma_{t+1}^2 = \bar{\sigma}^2 + \nu_1 (\sigma_t^2 - \bar{\sigma}^2) + \sigma_w \sigma_t w_{t+1}, \quad (7)$$

where  $(\eta_t, e_t, w_t)$  are i.i.d. standard normal innovations. Consumption growth contains a low-frequency component  $x_t$  and is heteroscedastic, with conditional variance  $\sigma_t^2$ . The two state variables  $x_t$  and  $\sigma_t^2 - \bar{\sigma}^2$  capture time-varying growth rates and time-varying economic uncertainty.

The two changes we make relative to Bansal and Yaron (2004) are that (1) the innovation in equation (7) is not  $\sigma_w w_{t+1}$  but  $\sigma_w \sigma_t w_{t+1}$ , and (2) we assume that  $x_t$  and to  $\sigma_t^2 - \bar{\sigma}^2$  have a non-zero conditional correlation  $\chi$ . We engineer this through a non-zero unconditional correlation  $\chi$  between the contemporaneous shocks  $e_{t+1}$  and  $w_{t+1}$ :

$$\text{Corr}[e_{t+1}, w_{t+1}] = \text{Cov}[e_{t+1}, w_{t+1}] = E[e_{t+1} w_{t+1}] = \chi.$$

All other shocks have zero correlation with each other at all leads and lags. To find the correlation between  $x_t$  and to  $\sigma_t^2 - \bar{\sigma}^2$ , start by working out

$$\begin{aligned} E_t[x_{t+1}(\sigma_{t+1}^2 - \bar{\sigma}^2)] &= E_t[(\rho_x x_t + \varphi_e \sigma_t e_{t+1})(\nu_1 (\sigma_t^2 - \bar{\sigma}^2) + \sigma_w \sigma_t w_{t+1})], \\ &= \rho_x \nu_1 x_t (\sigma_t^2 - \bar{\sigma}^2) + \rho_x x_t \sigma_w \sigma_t E_t[w_{t+1}], \\ &\quad + \varphi_e \sigma_t \nu_1 (\sigma_t^2 - \bar{\sigma}^2) E_t[e_{t+1}] + \varphi_e \sigma_t^2 \sigma_w E_t[e_{t+1} w_{t+1}], \\ &= \rho_x \nu_1 x_t (\sigma_t^2 - \bar{\sigma}^2) + \varphi_e \sigma_w \chi \sigma_t^2. \end{aligned}$$

The conditional covariance between  $x_{t+1}$  and  $\sigma_{t+1}^2 - \bar{\sigma}^2$  equals:

$$\begin{aligned} \text{Cov}_t[x_{t+1}, \sigma_{t+1}^2 - \bar{\sigma}^2] &= E_t[x_{t+1}(\sigma_{t+1}^2 - \bar{\sigma}^2)] - E_t[x_{t+1}]E_t[(\sigma_{t+1}^2 - \bar{\sigma}^2)], \\ &= \rho_x \nu_1 x_t (\sigma_t^2 - \bar{\sigma}^2) + \varphi_e \sigma_w \chi \sigma_t^2 - \rho_x \nu_1 x_t (\sigma_t^2 - \bar{\sigma}^2), \\ &= \varphi_e \sigma_w \chi \sigma_t^2. \end{aligned}$$

It follows that the conditional correlation between  $x_{t+1}$  and  $\sigma_{t+1}^2 - \bar{\sigma}^2$  is also  $\chi$ :

$$\begin{aligned} \text{Corr}_t[x_{t+1}, \sigma_{t+1}^2 - \bar{\sigma}^2] &= \frac{\text{Cov}_t[x_{t+1}, \sigma_{t+1}^2 - \bar{\sigma}^2]}{\text{Std}_t[x_{t+1}] \text{Std}_t[\sigma_{t+1}^2 - \bar{\sigma}^2]}, \\ &= \frac{\chi \varphi_e \sigma_w \sigma_t^2}{\varphi_e \sigma_w \sigma_t^2} = \chi. \end{aligned}$$

The *unconditional* variance of  $x_{t+1}$  and  $\sigma_{t+1}^2 - \bar{\sigma}^2$  is:

$$\begin{aligned} V[x_{t+1}] &= E[x_{t+1}^2] = \frac{\varphi_e^2 \bar{\sigma}^2}{1 - \rho_x^2}, \\ V[\sigma_{t+1}^2 - \bar{\sigma}^2] &= E[(\sigma_{t+1}^2 - \bar{\sigma}^2)^2] = \frac{\sigma_w^2 \bar{\sigma}^2}{1 - \nu_1^2}. \end{aligned}$$

The *unconditional* covariance between  $x_{t+1}$  and  $\sigma_t^2 - \bar{\sigma}^2$  equals:

$$Cov[x_{t+1}, \sigma_{t+1}^2 - \bar{\sigma}^2] = \frac{\chi\varphi_e\sigma_w\bar{\sigma}^2}{1 - \rho_x\nu_1}$$

The *unconditional* correlation between  $x_{t+1}$  and  $\sigma_t^2 - \bar{\sigma}^2$  equals:

$$\begin{aligned} Corr[x_{t+1}, \sigma_{t+1}^2 - \bar{\sigma}^2] &= \frac{Cov[x_{t+1}, \sigma_{t+1}^2 - \bar{\sigma}^2]}{Std[x_{t+1}]Std[\sigma_{t+1}^2 - \bar{\sigma}^2]}, \\ &= \frac{\frac{\chi\varphi_e\sigma_w\bar{\sigma}^2}{1 - \rho_x\nu_1}}{\frac{\varphi_e\bar{\sigma}}{\sqrt{1 - \rho_x^2}} \frac{\sigma_w\bar{\sigma}}{\sqrt{1 - \nu_1^2}}} = \chi \frac{\sqrt{1 - \rho_x^2}\sqrt{1 - \nu_1^2}}{1 - \rho_x\nu_1}. \end{aligned}$$

The unconditional correlation between these two state variables is zero at all other leads and lags.

**Dividend Growth** We specify dividend growth processes for various assets or portfolios  $i$ , making two important modifications to the specification in Bansal and Yaron (2004):

$$\Delta d_{t+1}^i = \mu_d^i + \phi_{d,x}^i x_t + \phi_{d,\sigma}^i (\sigma_t^2 - \bar{\sigma}^2) + \varphi_d^i \sigma_t u_{t+1}^i \quad (8)$$

In Bansal and Yaron (2004), the shock  $u_t$  is assumed orthogonal to  $(\eta, e, w)$  and  $\phi_{d,\sigma}^i = 0$ . Then, correlation between consumption and dividend growth comes through the two state variables  $x_t$  and  $\sigma_t$ . Instead, we allow for a correlation  $\iota^i$  between  $\eta_{t+1}$  and  $u_{t+1}^i$ . All other correlations are kept at zero. This opens up an additional channel of correlation and is a modification also made by (Bansal and Shaliastovich 2007). It turns out to be immaterial for our purposes. Much more important is the non-zero loading of dividend growth on economic uncertainty  $\phi_{d,\sigma}^i \neq 0$ . Its role will become clear below.

Defining returns ex-dividend and using the Campbell (1991) linearization, the log return on a claim to the dividend of portfolio  $i$  can be written as:

$$r_{t+1}^i = \Delta d_{t+1}^i + pd_{t+1}^i + \kappa_0^i - \kappa_1^i pd_t^i,$$

with coefficients

$$\kappa_1^i = \frac{e^{A_0^i}}{e^{A_0^i} - 1} > 1, \quad \text{and} \quad \kappa_0^i = -\log(e^{A_0^i} - 1) + \frac{e^{A_0^i}}{e^{A_0^i} - 1} A_0^i,$$

which depend on the long-run log price-dividend ratio  $A_0^i$ .

**Inflation** We follow Wachter (2006), Piazzesi and Schneider (2006), and Bansal and Shaliastovich (2007), and model inflation as

$$\pi_{t+1} = \bar{\pi}_t + \varphi_{\pi,\eta}\sigma_t\eta_{t+1} + \varphi_{\pi,e}\varphi_e\sigma_t e_{t+1} + \sigma_\pi \xi_{t+1}, \quad (9)$$

where expected inflation  $\bar{\pi}_t = E_t[\pi_{t+1}]$  is given by

$$\bar{\pi}_{t+1} = \mu_\pi + \rho_\pi(\bar{\pi}_t - \mu_\pi) + \phi_\pi x_t + \zeta_{\pi,\eta}\sigma_t\eta_{t+1} + \zeta_{\pi,e}\varphi_e\sigma_t e_{t+1} + \sigma_z \xi_{t+1}, \quad (10)$$

The innovation  $\xi_t$  is orthogonal to all other innovations  $(\eta_t, e_t, w_t, u_t^i)$ . Expected inflation mean-reverts, it carries long-run risk with loading  $\phi_\pi$ , and its innovations are correlated with unexpected inflation, and with the temporary and persistent component of consumption growth.

The nominal SDF is given by:

$$m_{t+1}^{\$} = m_{t+1} - \pi_{t+1}.$$

Expected inflation (demeaned),  $\bar{\pi}_t - \mu_\pi$ , is a third state variable that drives the nominal yield curve. We are interested in understanding its variance and covariance with the other two state variables  $x_{t+1}$  and  $\sigma_t^2 - \bar{\sigma}^2$ . First, we calculate

$$\begin{aligned} Cov[x_{t+1}, \bar{\pi}_{t+1} - \mu_\pi] &= \rho_x \rho_\pi Cov[x_t, \bar{\pi}_t - \mu_\pi] + \rho_x \phi_\pi E[x_t^2] + \zeta_{\pi,e} \varphi_e^2 \bar{\sigma}^2, \\ &= \frac{\rho_x \phi_\pi}{1 - \rho_x^2} + \zeta_{\pi,e} \varphi_e^2 \bar{\sigma}^2. \end{aligned}$$

The covariance with economic uncertainty is

$$\begin{aligned} Cov[\sigma_{t+1}^2 - \bar{\sigma}^2, \bar{\pi}_{t+1} - \mu_\pi] &= \nu_1 \rho_\pi Cov[\sigma_t^2 - \bar{\sigma}^2, \bar{\pi}_t - \mu_\pi] + \nu_1 \phi_\pi Cov[x_t, \sigma_t^2 - \bar{\sigma}^2] + \chi \zeta_{\pi,e} \varphi_e \sigma_w \bar{\sigma}^2, \\ &= \chi \frac{\frac{\nu_1 \phi_\pi}{1 - \rho_x \nu_1} + \zeta_{\pi,e}}{1 - \nu_1 \rho_\pi} \varphi_e \sigma_w \bar{\sigma}^2. \end{aligned}$$

We can now calculate the unconditional variance of expected inflation

$$\begin{aligned} E[(\bar{\pi}_{t+1} - \mu_\pi)^2] &= \rho_\pi^2 E[(\bar{\pi}_t - \mu_\pi)^2] + \phi_\pi^2 E[x_{t+1}^2] + \zeta_{\pi,\eta}^2 \bar{\sigma}^2 + \zeta_{\pi,e}^2 \varphi_e^2 \bar{\sigma}^2 + \sigma_z^2 + 2\phi_\pi \rho_\pi E[x_t, \bar{\pi}_t - \mu_\pi], \\ &= \frac{\zeta_{\pi,\eta}^2 \bar{\sigma}^2 + \left[ \frac{\phi_\pi^2}{1 - \rho_x^2} + \zeta_{\pi,e}^2 + 2\phi_\pi \rho_\pi \frac{\rho_x \phi_\pi + \zeta_{\pi,e}}{1 - \rho_x \rho_\pi} \right] \varphi_e^2 \bar{\sigma}^2 + \sigma_z^2}{1 - \rho_\pi^2}. \end{aligned}$$

## A.2 Equity pricing

**Euler Equation** The starting point of the analysis is the Euler equation  $E_t[M_{t+1} R_{t+1}^i] = 1$ , where  $R_{t+1}^i$  denotes a gross return between dates  $t$  and  $t+1$  on some asset  $i$  and  $M_{t+1}$  is the SDF. In logs:

$$E_t[m_{t+1}] + E_t[r_{t+1}^i] + \frac{1}{2} Var_t[m_{t+1}] + \frac{1}{2} Var_t[r_{t+1}^i] + Cov_t[m_{t+1}, r_{t+1}^i] = 0. \quad (11)$$

The same equation holds for the real risk-free rate  $y_t(1)$ , so that

$$y_t(1) = -E_t[m_{t+1}] - \frac{1}{2} Var_t[m_{t+1}]. \quad (12)$$

The expected excess return becomes:

$$E_t[r_{t+1}^{e,i}] = E_t[r_{t+1}^i - y_t(1)] + \frac{1}{2} Var_t[r_{t+1}^i] = -Cov_t[m_{t+1}, r_{t+1}^i] = -Cov_t[m_{t+1}, r_{t+1}^{e,i}], \quad (13)$$

where  $r_{t+1}^{e,i}$  denotes the excess return on asset  $i$  corrected for the Jensen term.

**The Consumption Claim** In what follows we focus on the return on a claim to aggregate consumption, denoted  $r^c$ , where

$$r_{t+1}^c = \kappa_0^c + \Delta c_{t+1} + w c_{t+1} - \kappa_1^c w c_t,$$

and derive the five terms in equation (11) for this asset.

Taking logs on both sides of the non-linear SDF expression in equation (1) delivers an expression of the log SDF as a function of log consumption changes and the log total wealth return

$$m_{t+1} = \frac{1-\alpha}{1-\rho} \log \beta - \frac{1-\alpha}{1-\rho} \rho \Delta c_{t+1} + \left( \frac{1-\alpha}{1-\rho} - 1 \right) r_{t+1}^c. \quad (14)$$

We conjecture that the log wealth-consumption ratio is linear in the two states  $x_t$  and  $\sigma_t^2 - \bar{\sigma}^2$ ,

$$w c_t = A_0^c + A_1^c x_t + A_2^c (\sigma_t^2 - \bar{\sigma}^2).$$

As BY, we assume joint conditional normality of consumption growth,  $x$ , and the variance of consumption growth. We verify this conjecture from the Euler equation (11).

Using the conjecture for the wealth-consumption ratio, we compute innovations in the total wealth return, and its conditional mean and variance:

$$\begin{aligned} r_{t+1}^c - E_t [r_{t+1}^c] &= \sigma_t \eta_{t+1} + A_1^c \varphi_e \sigma_t e_{t+1} + A_2^c \sigma_w \sigma_t w_{t+1} \\ E_t [r_{t+1}^c] &= r_0 + (1 - (\kappa_1^c - \rho_x) A_1^c) x_t - A_2^c (\kappa_1^c - \nu_1) (\sigma_t^2 - \bar{\sigma}^2) \\ V_t [r_{t+1}^c] &= [1 + (A_1^c \varphi_e)^2 + (A_2^c)^2 \sigma_w^2 + 2 A_1^c \varphi_e A_2^c \sigma_w \chi] \sigma_t^2 \\ r_0 &= \kappa_0^c + A_0^c (1 - \kappa_1^c) + \mu_c \end{aligned}$$

Substituting in the expression for the log total wealth return  $r^c$  into the log SDF, we compute innovations, and the conditional mean and variance of the log SDF:

$$\begin{aligned} m_{t+1} - E_t [m_{t+1}] &= -\alpha \sigma_t \eta_{t+1} - \frac{\alpha - \rho}{1 - \rho} A_1^c \varphi_e \sigma_t e_{t+1} - \frac{\alpha - \rho}{1 - \rho} A_2^c \sigma_w \sigma_t w_{t+1}, \\ E_t [m_{t+1}] &= m_0 + \left( -\alpha + \frac{\alpha - \rho}{1 - \rho} A_1^c (\kappa_1^c - \rho_x) \right) x_t + \frac{\alpha - \rho}{1 - \rho} (\kappa_1^c - \nu_1) A_2^c (\sigma_t^2 - \bar{\sigma}^2) \\ V_t [m_{t+1}] &= \left[ \alpha^2 + \left( \frac{\alpha - \rho}{1 - \rho} \right)^2 \{ (A_1^c \varphi_e)^2 + (A_2^c \sigma_w)^2 + 2 \chi A_1^c \varphi_e A_2^c \sigma_w \} \right] \sigma_t^2 \\ m_0 &= \frac{1-\alpha}{1-\rho} \log \beta - \frac{\alpha - \rho}{1 - \rho} [\kappa_0^c + A_0^c (1 - \kappa_1^c)] - \alpha \mu_c \end{aligned} \quad (15)$$

The conditional covariance between the log consumption return and the log SDF is given by the conditional expectation of the product of their innovations

$$\begin{aligned} Cov_t [r_{t+1}^c, m_{t+1}] &= E_t [r_{t+1}^c - E_t [r_{t+1}^c], m_{t+1} - E_t [m_{t+1}]] \\ &= \left[ -\alpha - \frac{\alpha - \rho}{1 - \rho} \{ (A_1^c \varphi_e)^2 + (A_2^c \sigma_w)^2 + 2 \chi A_1^c \varphi_e A_2^c \sigma_w \} \right] \sigma_t^2 \end{aligned}$$

Using the method of undetermined coefficients and the five components of equation (11), we can solve for the constants  $A_0^c$ ,  $A_1^c$ , and  $A_2^c$ :

$$A_1^c = \frac{1 - \rho}{\kappa_1^c - \rho_x}, \quad (16)$$

$$A_2^c = .5 \frac{(1 - \rho)(1 - \alpha)}{\kappa_1^c - \nu_1} \left[ 1 + \frac{(A_1^c \varphi_e)^2 + (A_2^c \sigma_w)^2 + 2\chi A_1^c \varphi_e A_2^c \sigma_w}{(1 - \rho)^2} \right], \quad (17)$$

$$0 = \frac{1 - \alpha}{1 - \rho} [\log \beta + \kappa_0^c + (1 - \kappa_1^c) A_0^c] + (1 - \alpha) \mu_c + \frac{1}{2} (1 - \alpha)^2 \left[ 1 + \frac{(A_1^c \varphi_e)^2 + (A_2^c \sigma_w)^2 + 2\chi A_1^c \varphi_e A_2^c \sigma_w}{(1 - \rho)^2} \right] \bar{\sigma}^2 \quad (18)$$

The first equation is the same as in the case of no correlation between the state variables. The second and third equations are implicit functions of  $A_2^c$  and  $A_0^c$ . Because  $\kappa_0^c$  and  $\kappa_1^c$  are non-linear functions of  $A_0^c$ , this system of three equations needs to be solved simultaneously and numerically. Our computations indicate that the system has a unique solution. This verifies the conjecture that the log wealth-consumption ratio is linear in the two state variables.

**The Consumption Risk Premium** According to (13), the risk premium (expected excess return corrected for a Jensen term) on the consumption claim is given by

$$\begin{aligned} E_t [r_{t+1}^{c,e}] &= -Cov_t [r_{t+1}^c, m_{t+1}] \\ &= \left[ \alpha + \frac{\alpha - \rho}{1 - \rho} \{ (A_1^c \varphi_e)^2 + (A_2^c \sigma_w)^2 + 2\chi A_1^c \varphi_e A_2^c \sigma_w \} \right] \sigma_t^2 \\ &= (\lambda_{m,\eta} + \lambda_{m,e} A_1^c \varphi_e + \lambda_{m,w} A_2^c \sigma_w + 2\chi \lambda_{m,e} \lambda_{m,w}) \sigma_t^2, \end{aligned} \quad (19)$$

with the market price of risk vector  $\Lambda = [\lambda_{m,\eta}, \lambda_{m,e}, \lambda_{m,w}]$  given by

$$\begin{aligned} \lambda_{m,\eta} &= \alpha, \\ \lambda_{m,e} &= \frac{\alpha - \rho}{1 - \rho} A_1^c \varphi_e = \frac{\alpha - \rho}{\kappa_1^c - \rho_x} \varphi_e, \\ \lambda_{m,w} &= \frac{\alpha - \rho}{1 - \rho} A_2^c \sigma_w. \end{aligned}$$

Note that the market price of temporary consumption growth shocks  $\eta_{t+1}$  is positive and equal to the risk aversion coefficient  $\alpha$ , the market price of long-run consumption growth shocks  $e_{t+1}$  is positive, and the market price of economic uncertainty (innovations  $w_{t+1}$  in  $\sigma^2 - \bar{\sigma}^2$ ) is *negative* because  $A_2^c$  will turn out to be negative.

**Correlations expected consumption growth and expected total wealth return** Expected total wealth returns and expected consumption growth rates are equal to:

$$E_t [r_{t+1}^c] = r_0^c + \rho x_t - A_2^c (\kappa_1^c - \nu_1) (\sigma_t^2 - \bar{\sigma}^2), \quad (20)$$

$$E_t [\Delta c_{t+1}] = \mu_c + x_t. \quad (21)$$

The unconditional variance of these quantities is:

$$\begin{aligned} V [E_t [r_{t+1}^c]] &= \rho^2 V [x_t] + [A_2^c (\kappa_1^c - \nu_1)]^2 V [\sigma_t^2 - \bar{\sigma}^2] - 2\rho A_2^c (\kappa_1^c - \nu_1) Cov [x_t, \sigma_t^2 - \bar{\sigma}^2], \\ V [E_t [\Delta c_{t+1}]] &= V [x_t]. \end{aligned}$$

The unconditional covariance between expected returns and expected consumption growth rates is

$$Cov(E_t[r_{t+1}^c], E_t[\Delta c_{t+1}]) = \rho V[x] - A_2^c(\kappa_1^c - \nu_1)Cov[x_t, \sigma_t^2 - \bar{\sigma}^2]$$

This then leads to the unconditional correlation, which is straightforward to compute.

**The Dividend Claims** We conjecture, as we did for the wealth-consumption ratio, that the log price dividend ratio on a stock  $i$ , where the notation  $i$  is suppressed, is linear in the two state variables:

$$pd_t^i = A_0^i + A_1^i x_t + A_2^i(\sigma_t^2 - \bar{\sigma}^2).$$

As we did for the return on the consumption claim, we compute innovations in the dividend claim return, and its conditional mean and variance:

$$\begin{aligned} r_{t+1}^i - E_t[r_{t+1}^i] &= \varphi_d^i \sigma_t u_{t+1}^i + A_1^i \varphi_e \sigma_t e_{t+1} + A_2^i \sigma_w \sigma_t w_{t+1} \\ E_t[r_{t+1}^i] &= r_0^i + [\phi_{d,x}^i - A_1^i(\kappa_1^i - \rho_x)]x_t + [\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)](\sigma_t^2 - \bar{\sigma}^2) \\ V_t[r_{t+1}^i] &= [(\varphi_d^i)^2 + (A_1^i \varphi_e)^2 + (A_2^i \sigma_w)^2 + 2\chi A_1^i \varphi_e A_2^i \sigma_w] \sigma_t^2, \\ r_0^i &= \kappa_0^i + A_0^i(1 - \kappa_1^i) + \mu_d^i \end{aligned}$$

Finally, the conditional covariance between the log SDF and the log dividend claim return is

$$Cov_t[m_{t+1}, r_{t+1}^i] = - \left[ i^i \alpha \varphi_d^i + \frac{\alpha - \rho}{1 - \rho} \{ A_1^c A_1^i \varphi_e^2 + \chi A_1^c A_2^i \varphi_e \sigma_w + \chi A_2^c A_1^i \varphi_e \sigma_w + A_2^c A_2^i \sigma_w^2 \} \right] \sigma_t^2.$$

From the Euler equation for this return  $E_t[m_{t+1}] + E_t[r_{t+1}^i] + \frac{1}{2}V_t[m_{t+1}] + \frac{1}{2}V_t[r_{t+1}^i] + Cov_t[m_{t+1}, r_{t+1}^i] = 0$  and the method of undetermined coefficients, we can use the same procedure as above, and solve for the constants  $A_0^i$ ,  $A_1^i$ , and  $A_2^i$ :

$$\begin{aligned} A_1^i &= \frac{\phi_{d,x}^i - \rho}{\kappa_1^i - \rho_x}, \\ A_2^i &= \frac{\left[ \frac{\alpha - \rho}{1 - \rho} A_2^c (\kappa_1^c - \nu_1) + \phi_{d,\sigma}^i + .5H^i \right]}{\kappa_1^i - \nu_1}, \\ 0 &= m_0 + \kappa_0^i + A_0^i(1 - \kappa_1^i) + \mu_d^i + .5H^i \bar{\sigma}^2 \end{aligned}$$

where

$$\begin{aligned} H^i &= \{ \alpha^2 + (\varphi_d^i)^2 - 2i^i \alpha \varphi_d^i \} + \{ (A_1^i \varphi_e)^2 + (A_2^i \sigma_w)^2 + 2\chi A_1^i \varphi_e A_2^i \sigma_w \} \\ &+ \left( \frac{\alpha - \rho}{1 - \rho} \right)^2 \{ (A_1^c \varphi_e)^2 + (A_2^c \sigma_w)^2 + 2\chi A_1^c \varphi_e A_2^c \sigma_w \} \\ &- 2 \frac{\alpha - \rho}{1 - \rho} \{ A_1^c A_1^i \varphi_e^2 + \chi A_1^c A_2^i \varphi_e \sigma_w + \chi A_2^c A_1^i \varphi_e \sigma_w + A_2^c A_2^i \sigma_w^2 \} \end{aligned} \quad (22)$$

Again, this is a non-linear system in three equations and three unknowns, which we solve numerically.

The equity risk premium on dividend claim  $i$  is:

$$E_t[r_{t+1}^{e,i}] = \left[ i^i \alpha \varphi_d^i + \frac{\alpha - \rho}{1 - \rho} \{ A_1^c A_1^i \varphi_e^2 + \chi A_1^c A_2^i \varphi_e \sigma_w + \chi A_2^c A_1^i \varphi_e \sigma_w + A_2^c A_2^i \sigma_w^2 \} \right] \sigma_t^2. \quad (23)$$



Equation (23) shows that the equity risk premium consists of a compensation for bearing temporary shock risk (first term), long-run risk (second term) and uncertainty risk (third term). The fourth term is new and only arises when there is non-zero correlation  $\chi$  between the two state variables. The market price of temporary cash-flow risk is  $\alpha > 0$ , the market price of long-run risk is  $A_1^c \varphi_e > 0$ , and the market price of uncertainty risk is  $A_2^c \sigma_w < 0$ . These signs make sense: when long-run risk is low (economic uncertainty is high), the marginal utility growth for the representative investor is high, and she is willing to pay a high price for a security that has high dividend growth in such states of the world. The new, fourth term in (23) lowers the equity risk premium when the state variables are positively correlated because  $A_1^c A_2^i + A_2^c A_1^i < 0$ . Indeed, when high uncertainty goes hand in hand with high future growth, the world is a less risky place. Finally, we note that the only source of variation in equity risk premia over time is economic uncertainty  $\sigma_t^2$ .

**Correlations expected dividend growth and expected return** Expected stock returns and expected dividend growth rates on any portfolio  $i$  are equal to:

$$E_t [r_{t+1}^i] = r_0^i + \rho x_t + [\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)](\sigma_t^2 - \bar{\sigma}^2), \quad (24)$$

$$E_t [\Delta d_{t+1}^i] = \mu_d^i + \phi_{d,x}^i x_t + \phi_{d,\sigma}^i (\sigma_t^2 - \bar{\sigma}^2). \quad (25)$$

The unconditional variance of these quantities is:

$$\begin{aligned} V [E_t [r_{t+1}^i]] &= \rho^2 V[x_t] + [\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)]^2 V[\sigma_t^2 - \bar{\sigma}^2] + 2\rho[\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)] Cov[x_t, \sigma_t^2 - \bar{\sigma}^2], \\ V [E_t [\Delta d_{t+1}^i]] &= (\phi_{d,x}^i)^2 V[x_t] + (\phi_{d,\sigma}^i)^2 V[\sigma_t^2 - \bar{\sigma}^2] + 2\phi_{d,\sigma}^i \phi_{d,x}^i Cov[x_t, \sigma_t^2 - \bar{\sigma}^2]. \end{aligned}$$

The unconditional covariance between expected stock returns and expected dividend growth rates is

$$\begin{aligned} Cov (E_t [r_{t+1}^i], E_t [\Delta d_{t+1}^i]) &= \phi_{d,x}^i \rho V[x_t] + \phi_{d,\sigma}^i [\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)] V[\sigma_t^2 - \bar{\sigma}^2] \\ &\quad + (\rho \phi_{d,\sigma}^i + \phi_{d,x}^i [\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)]) Cov[x_t, \sigma_t^2 - \bar{\sigma}^2] \end{aligned}$$

This then leads to the unconditional correlation, which is straightforward to compute.

The important thing to note is that this covariance is (1) asset-specific because it depends on both  $\phi_{d,x}^i$  and  $\phi_{d,\sigma}^i$ , and (2) it can be positive or negative depending on  $\phi_{d,x}^i$  and  $\phi_{d,\sigma}^i$ , holding fixed  $\chi$ . In the standard LRR model, the covariance between the state variables is zero and  $\phi_{d,\sigma}^i = 0$ . In that case, the sign of the covariance between expected returns and expected dividend growth depends on the sign of  $\phi_{d,x}^i \rho$ , which is positive.

The unconditional correlation between expected and unexpected dividend growth is zero for all portfolios by the law of iterated expectations:

$$Corr (E_t [\Delta d_{t+1}^i], \Delta d_{t+1}^i - E_t [\Delta d_{t+1}^i]) = Corr(\phi_{d,x}^i x_t + \phi_{d,\sigma}^i (\sigma_t^2 - \bar{\sigma}^2), \varphi_d^i \sigma_t u_{t+1}) = 0.$$

Likewise, the correlation between expected and unexpected returns is zero. This is the same as in the standard BY model.

### A.3 Bond Pricing

**The Risk-free Rate** According to equation (12), the expression for the risk-free rate is given by

$$\begin{aligned} y_t(1) &= -A(1) - B(1)x_t - C(1)(\sigma_t^2 - \bar{\sigma}^2) \\ A(1) &= m_0 + .5\Gamma(0)\bar{\sigma}^2, \\ B(1) &= -\rho, \\ C(1) &= \frac{\alpha - \rho}{1 - \rho}(\kappa_1^c - \nu_1)A_2^c + .5\Gamma(0), \end{aligned} \quad (26)$$

where the notation  $\Gamma(0)$  is defined as:

$$\Gamma(0) \equiv \alpha^2 + \left(\frac{\alpha - \rho}{1 - \rho}\right)^2 \{(A_1^c \varphi_e)^2 + (A_2^c \sigma_w)^2 + 2\chi A_1^c \varphi_e A_2^c \sigma_w\} \quad (27)$$

**The Real Yield Curve** The price of a  $\tau$ -period real zero-coupon bond satisfies:

$$P_t(\tau) = E_t \left[ e^{m_{t+1} + \log P_{t+1}(\tau-1)} \right].$$

This defines a recursion with  $P_t(0) = 1$ . The corresponding bond yield is  $y_t(\tau) = -\log(P_t(\tau))/\tau$ . It is easy to show that the LRR model gives rise to a affine real term structure (Bansal and Shaliastovich 2007):

$$y_t(\tau) = -\frac{A(\tau)}{\tau} - \frac{B(\tau)}{\tau}x_t - \frac{C(\tau)}{\tau}(\sigma_t^2 - \bar{\sigma}^2),$$

The coefficients  $A(\tau)$ ,  $B(\tau)$ , and  $C(\tau)$  satisfy the following recursions:

$$A(\tau + 1) = m_0 + A(\tau) + .5\Gamma(\tau)\bar{\sigma}^2, \quad (28)$$

$$B(\tau + 1) = \rho_x B(\tau) - \rho, \quad (29)$$

$$C(\tau + 1) = \nu_1 C(\tau) + \frac{\alpha - \rho}{1 - \rho}(\kappa_1^c - \nu_1)A_2^c + .5\Gamma(\tau) \quad (30)$$

where

$$\begin{aligned} \Gamma(\tau) &\equiv \alpha^2 + \left[ B(\tau) - \frac{\alpha - \rho}{\kappa_1^c - \rho_x} \right]^2 \varphi_e^2 + \left[ C(\tau) - \frac{\alpha - \rho}{1 - \rho} A_2^c \right]^2 \sigma_w^2 \\ &\quad + 2\chi \left[ B(\tau) - \frac{\alpha - \rho}{\kappa_1^c - \rho_x} \right] \varphi_e \left[ C(\tau) - \frac{\alpha - \rho}{1 - \rho} A_2^c \right] \sigma_w, \end{aligned} \quad (31)$$

and where the recursion is initialized at  $A(0) = 0$ ,  $B(0) = 0$ , and  $C(0) = 0$ .

*Proof.* Guess and verify

$$\begin{aligned} P_t(\tau + 1) &= E_t[\exp\{m_{t+1} + \log(P_{t+1}(\tau))\}] \\ &= E_t[\exp\{m_{t+1} + A(\tau) + B(\tau)x_{t+1} + C(\tau)(\sigma_{t+1}^2 - \bar{\sigma}^2)\}] \\ &= \exp\{m_0 + A(\tau) + (B(\tau)\rho_x - \rho)x_t + \left[ \nu_1 C(\tau) + \frac{\alpha - \rho}{1 - \rho}(\kappa_1^c - \nu_1)A_2^c \right] (\sigma_t^2 - \bar{\sigma}^2)\} \times \\ &\quad E_t \left[ \exp \left\{ -\alpha\sigma_t\eta_{t+1} + \left[ B(\tau) - \frac{\alpha - \rho}{\kappa_1^c - \rho_x} \right] \varphi_e \sigma_t \epsilon_{t+1} + \left[ C(\tau) - \frac{\alpha - \rho}{1 - \rho} A_2^c \right] \sigma_w \sigma_t w_{t+1} \right\} \right] \end{aligned}$$

Note that the conditional expectation term is equal to  $\exp\{.5\Gamma(\tau)\sigma_t^2\}$ , where  $\Gamma(\tau)$  is defined in (31). The rest follows from recognizing that

$$\log P_t(\tau + 1) = A(\tau + 1) + B(\tau + 1)x_t + C(\tau + 1)(\sigma_t^2 - \bar{\sigma}^2)$$

and matching up coefficients. □

**Real Bond Returns** Define the 1-period log return on a real bond of maturity  $\tau$  as:

$$r_{t+1}^b(\tau) = \tau y_t(\tau) - (\tau - 1)y_{t+1}(\tau - 1).$$

The one-period excess return subtracts the real short rate:

$$r_{t+1}^b(\tau) - y_t(1) = \tau y_t(\tau) - (\tau - 1)y_{t+1}(\tau - 1) - y_t(1).$$

In general, the excess log return on buying an  $\tau$ -period real bond at time  $t$  and selling it at time  $t + m$  as an  $\tau - m$  period bond is:

$$\tau y_t(\tau) - (\tau - m)y_{t+m}(\tau - m) - m y_t(1).$$

The same expression holds for nominal excess bond returns, which we denote with a \$ superscript. Innovations in one-period real bond returns are given by

$$\begin{aligned} r_{t+1}^b(\tau) - E_t[r_{t+1}^b(\tau)] &= (\tau - 1)(-y_{t+1}(\tau - 1) + E_t[y_{t+1}(\tau - 1)]), \\ &= B(\tau - 1)\varphi_e\sigma_t e_{t+1} + C(\tau - 1)\sigma_w\sigma_t w_{t+1}. \end{aligned}$$

The conditional variance of the bond return therefore equals:

$$V_t[r_{t+1}^b(\tau)] = \left\{ [B(\tau - 1)\varphi_e]^2 + [C(\tau - 1)\sigma_w]^2 + 2\chi B(\tau - 1)\varphi_e C(\tau - 1)\sigma_w \right\} \sigma_t^2.$$

The real bond risk premium (the expected excess return with Jensen adjustment) is:

$$\begin{aligned} E_t[r_{t+1}^{b,e}(\tau)] &= \tau y_t(\tau) - (\tau - 1)E_t[y_{t+1}(\tau - 1)] - y_t(1) + .5V_t[r_{t+1}^b(\tau)], \\ &= [.5\Gamma(0) - .5\Gamma(\tau - 1) + .5\left\{ [B(\tau - 1)\varphi_e]^2 + [C(\tau - 1)\sigma_w]^2 + 2\chi B(\tau - 1)\varphi_e C(\tau - 1)\sigma_w \right\}] \sigma_t^2 \\ &= \left\{ B(\tau - 1)\frac{\alpha - \rho}{\kappa_1^c - \rho_x}\varphi_e^2 + C(\tau - 1)\frac{\alpha - \rho}{1 - \rho}A_2^c\sigma_w^2 \right. \\ &\quad \left. + \chi B(\tau - 1)\varphi_e\frac{\alpha - \rho}{1 - \rho}A_2^c\sigma_w + \chi C(\tau - 1)\varphi_e\frac{\alpha - \rho}{\kappa_1^c - \rho_x}\sigma_w \right\} \sigma_t^2 \end{aligned}$$

We have verified that this risk premium equals  $-Cov_t[m_{t+1}, r_{t+1}^b(\tau) - E_t[r_{t+1}^b(\tau)]]$ .

**The Nominal Yield Curve** The price of a  $\tau$ -period nominal zero-coupon bond satisfies:

$$P_t^{\$}(\tau) = E_t\left[e^{m_{t+1}^{\$} + \log P_{t+1}^{\$}(\tau - 1)}\right].$$

This defines a recursion with  $P_t^{\$}(0) = 1$ . The corresponding bond yield is  $y_t^{\$}(\tau) = -\log(P_t^{\$}(\tau))/\tau$ . Nominal bond yields are also affine in the state vector (Bansal and Shaliastovich 2007):

$$y_t^{\$}(\tau) = -\frac{A^{\$}(\tau)}{\tau} - \frac{B^{\$}(\tau)}{\tau}x_t - \frac{C^{\$}(\tau)}{\tau}(\sigma_t^2 - \bar{\sigma}^2) - \frac{D^{\$}(\tau)}{\tau}(\bar{\pi}_t - \mu_{\pi}),$$

The coefficients  $A(\tau)$ ,  $B^{\$}(\tau)$ , and  $C^{\$}(\tau)$  satisfy the following recursions

$$A^{\$}(\tau + 1) = m_0 - \mu_{\pi} + A^{\$}(\tau) + .5 \left[ -\sigma_{\pi} + D^{\$}(\tau)\sigma_z \right]^2 + .5\Gamma^{\$}(\tau)\bar{\sigma}^2, \quad (32)$$

$$B^{\$}(\tau + 1) = B^{\$}(\tau)\rho_x - \rho + D^{\$}(\tau)\phi_{\pi}, \quad (33)$$

$$C^{\$}(\tau + 1) = C^{\$}(\tau)\nu_1 + \frac{\alpha - \rho}{1 - \rho}(\kappa_1^c - \nu_1)A_2^c + .5\Gamma^{\$}(\tau), \quad (34)$$

$$D^{\$}(\tau + 1) = D^{\$}(\tau)\rho_{\pi} - 1, \quad (35)$$

initialized at  $A^{\$}(0) = 0$ ,  $B^{\$}(0) = 0$ ,  $C^{\$}(0) = 0$ , and  $D^{\$}(0) = 0$ , and where

$$\begin{aligned} \Gamma^{\$}(\tau) \equiv & \left[ -\alpha - \varphi_{\pi,\eta} + D^{\$}(\tau)\zeta_{\pi,\eta} \right]^2 + \left[ B^{\$}(\tau) - \frac{\alpha - \rho}{\kappa_1^c - \rho_x} - \varphi_{\pi,e} + D^{\$}(\tau)\zeta_{\pi,e} \right]^2 \varphi_e^2 \\ & + \left[ C^{\$}(\tau) - \frac{\alpha - \rho}{1 - \rho}A_2^c \right]^2 \sigma_w^2 + 2\chi \left[ B^{\$}(\tau) - \frac{\alpha - \rho}{\kappa_1^c - \rho_x} - \varphi_{\pi,e} + D^{\$}(\tau)\zeta_{\pi,e} \right] \varphi_e \left[ C^{\$}(\tau) - \frac{\alpha - \rho}{1 - \rho}A_2^c \right] \sigma_w. \end{aligned}$$

*Proof.* We conjecture that the  $t+1$ -price of a  $\tau$ -period bond is exponentially affine in the state and solve for the coefficients in the process of verifying this conjecture using the Euler equation:

$$\begin{aligned} P_t^{\$}(\tau + 1) &= E_t[\exp\{m_{t+1}^{\$} + \log(P_{t+1}^{\$}(\tau))\}] \\ &= E_t[\exp\{m_{t+1} - \pi_{t+1} + A^{\$}(\tau) + B^{\$}(\tau)x_{t+1} + C^{\$}(\tau)(\sigma_{t+1}^2 - \bar{\sigma}^2) + D^{\$}(\tau)(\bar{\pi}_{t+1} - \mu_{\pi})\}] \\ &= \exp\{m_0 - \mu_{\pi} + A^{\$}(\tau) + [B^{\$}(\tau)\rho_x - \rho + D^{\$}(\tau)\phi_{\pi}]x_t \\ &\quad + [C^{\$}(\tau)\nu_1 + \frac{\alpha - \rho}{1 - \rho}(\kappa_1^c - \nu_1)A_2^c](\sigma_t^2 - \bar{\sigma}^2) + [D^{\$}(\tau)\rho_{\pi} - 1](\bar{\pi}_t - \mu_{\pi})\} \times \\ &\quad E_t \left[ \exp\{[-\alpha - \varphi_{\pi,\eta} + D^{\$}(\tau)\zeta_{\pi,\eta}]\sigma_t\eta_{t+1}\} \times \right. \\ &\quad \left. \{ + \left[ -\frac{\alpha - \rho}{\kappa_1^c - \rho_x} - \varphi_{\pi,e} + B^{\$}(\tau) + D^{\$}(\tau)\zeta_{\pi,e} \right] \varphi_e\sigma_t e_{t+1} \} \times \right. \\ &\quad \left. \{ + \left[ -\frac{\alpha - \rho}{1 - \rho}A_2^c + C^{\$}(\tau) \right] \sigma_w\sigma_t w_{t+1} + [-\sigma_{\pi} + D^{\$}(\tau)\sigma_z] \xi_{t+1} \} \right] \end{aligned}$$

We use the joint standard normality of the four mutually-orthogonal shocks. Taking logs and collecting terms, we obtain a linear equation for  $\log(P_t^{\$}(\tau + 1))$  where the coefficients  $A^{\$}(\tau + 1)$  through  $D^{\$}(\tau + 1)$  satisfy (32)- (35).  $\square$

We have verified that the one-period nominal short rate satisfies:

$$y_t^{\$}(1) = -E_t[m_{t+1}^{\$}] - .5V_t[m_{t+1}^{\$}] = -E_t[m_{t+1} - \pi_{t+1}] - .5V_t[m_{t+1} - \pi_{t+1}]$$

for the coefficients  $A^{\$}(1)$  through  $D^{\$}(1)$  in (32)- (35).

**Inflation Risk Premium** The expected excess return on a one-period nominal bond over a one-period real bond measures the inflation risk premium at the short end of the term structure. It is given by  $-cov_t[m_{t+1}, \pi_{t+1}]$ , where

$$-cov_t[m_{t+1}, \pi_{t+1}] = \left( \alpha\varphi_{\pi,\eta} + \frac{\alpha - \rho}{\kappa_1^c - \rho_x} \varphi_{\pi,e} \varphi_e^2 + \chi \frac{\alpha - \rho}{1 - \rho} A_2^c \sigma_w \varphi_{\pi,e} \varphi_e \right) \sigma_t^2.$$

In principle, the inflation risk premium can be either negative or positive depending on the sign of  $\chi$ . However, it takes the same sign for all periods. When  $\varphi_{\pi,\eta} = 0$  and  $\chi = 0$ , which is the case in (Bansal and Shaliastovich 2007), the inflation risk premium equals the second term.

**Nominal Bond Returns** Define the 1-period log return on a nominal bond of maturity  $\tau$  as:

$$\begin{aligned} r_{t+1}^{b,\$}(\tau) &= \tau y_t^\$(\tau) - (\tau - 1) y_{t+1}^\$(\tau - 1), \\ &= -A^\$(\tau) - B^\$(\tau) x_t - C^\$(\tau) (\sigma_t^2 - \bar{\sigma}^2) - D^\$(\tau) (\bar{\pi}_t - \mu_\pi) \\ &\quad + A^\$(\tau - 1) + B^\$(\tau - 1) x_{t+1} + C^\$(\tau - 1) (\sigma_{t+1}^2 - \bar{\sigma}^2) + D^\$(\tau - 1) (\bar{\pi}_{t+1} - \mu_\pi) \end{aligned}$$

Expected nominal bond returns are:

$$\begin{aligned} E_t \left[ r_{t+1}^{b,\$}(\tau) \right] &= A^\$(\tau - 1) - A^\$(\tau) + \left[ B^\$(\tau - 1) \rho_x + D^\$(\tau - 1) \phi_\pi - B^\$(\tau) \right] x_t \\ &\quad + \left[ C^\$(\tau - 1) \nu_1 - C^\$(\tau) \right] (\sigma_t^2 - \bar{\sigma}^2) + \left[ D^\$(\tau - 1) \rho_\pi - D^\$(\tau) \right] (\bar{\pi}_t - \mu_\pi). \end{aligned}$$

Innovation in nominal bond returns are:

$$\begin{aligned} r_{t+1}^{b,\$}(\tau) - E_t \left[ r_{t+1}^{b,\$}(\tau) \right] &= (\tau - 1) \left( -y_{t+1}^\$(\tau - 1) + E_t \left[ y_{t+1}^\$(\tau - 1) \right] \right), \\ &= D^\$(\tau - 1) \zeta_{\pi,\eta} \sigma_t \eta_{t+1} + \left[ B^\$(\tau - 1) + D^\$(\tau - 1) \zeta_{\pi,e} \right] \varphi_e \sigma_t \epsilon_{t+1} \\ &\quad + C^\$(\tau - 1) \sigma_w \sigma_t w_{t+1} + D^\$(\tau - 1) \sigma_z \xi_{t+1}. \end{aligned}$$

Expected excess bond returns adjusted for a Jensen term, or nominal bond risk premia, are given by:

$$\begin{aligned} E_t \left[ r_{t+1}^{b,e,\$}(\tau) \right] &\equiv -Cov_t \left[ m_{t+1}^\$, r_{t+1}^{b,e,\$}(\tau) \right] \\ &= Cov_t \left[ \pi_{t+1}, r_{t+1}^{b,e,\$}(\tau) \right] - Cov_t \left[ m_{t+1}, r_{t+1}^{b,e,\$}(\tau) \right] \\ &= r_0^{b,e,\$}(\tau) + F^\$(\tau) (\sigma_t^2 - \bar{\sigma}^2) \end{aligned}$$

where

$$\begin{aligned} r_0^{b,e,\$}(\tau) &= D^\$(\tau - 1) \sigma_z \sigma_\pi + F^\$(\tau) \bar{\sigma}^2 \\ F^\$(\tau) &= (\varphi_{\pi,\eta} + \alpha) D^\$(\tau - 1) \zeta_{\pi,\eta} + \\ &\quad \left( \varphi_{\pi,e} + \frac{\alpha - \rho}{\kappa_1^c - \rho_x} \right) \left[ B^\$(\tau - 1) + D^\$(\tau - 1) \zeta_{\pi,e} \right] \varphi_e^2 + \frac{\alpha - \rho}{1 - \rho} A_2^c C^\$(\tau - 1) \sigma_w^2 + \\ &\quad \chi \frac{\alpha - \rho}{1 - \rho} A_2^c \sigma_w \left[ B^\$(\tau - 1) + D^\$(\tau - 1) \zeta_{\pi,e} \right] \varphi_e + \chi \frac{\alpha - \rho}{\kappa_1^c - \rho_x} \varphi_e C^\$(\tau - 1) \sigma_w \end{aligned}$$

We have used the fact that:

$$\begin{aligned}
Cov_t \left[ \pi_{t+1}, rx_{t+1}^{\$}(\tau) \right] &= \left\{ \varphi_{\pi,\eta} D^{\$}(\tau-1) \zeta_{\pi,\eta} + \varphi_{\pi,e} \left[ B^{\$}(\tau-1) + D^{\$}(\tau-1) \zeta_{\pi,e} \right] \varphi_e^2 \right. \\
&\quad \left. + \chi \varphi_{\pi,e} \varphi_e C^{\$}(\tau-1) \sigma_w \right\} \sigma_t^2 + D^{\$}(\tau-1) \sigma_z \sigma_{\pi} \\
Cov_t \left[ m_{t+1}, rx_{t+1}^{\$}(\tau) \right] &= \left\{ -\alpha D^{\$}(\tau-1) \zeta_{\pi,\eta} - \frac{\alpha - \rho}{\kappa_1^c - \rho_x} \left[ B^{\$}(\tau-1) + D^{\$}(\tau-1) \zeta_{\pi,e} \right] \varphi_e^2 \right. \\
&\quad - \frac{\alpha - \rho}{1 - \rho} A_2^c C^{\$}(\tau-1) \sigma_w^2 - \chi \frac{\alpha - \rho}{1 - \rho} A_2^c \sigma_w \left[ B^{\$}(\tau-1) + D^{\$}(\tau-1) \zeta_{\pi,e} \right] \varphi_e \\
&\quad \left. - \chi \frac{\alpha - \rho}{\kappa_1^c - \rho_x} \varphi_e C^{\$}(\tau-1) \sigma_w \right\} \sigma_t^2.
\end{aligned}$$

In sum, nominal expected excess bond returns (nominal bond risk premia) only vary over time because economic uncertainty ( $\sigma_t^2 - \bar{\sigma}^2$ ) varies. The loading  $F(\tau)$  measures their sensitivity to this factor; it depends on the maturity of the bond.

**The Cochrane-Piazzesi Factor** Define the nominal forward rate for a loan between period  $t + \tau$  and  $t + \tau + 1$  as usual from nominal bond prices:

$$f_t^{\$}(\tau) = \log P_t^{\$}(\tau-1) - \log P_t^{\$}(\tau).$$

In the model, the forward rate is

$$\begin{aligned}
f_t^{\$}(\tau) &= \left( A^{\$}(\tau-1) - A^{\$}(\tau) \right) + \left( B^{\$}(\tau-1) - B^{\$}(\tau) \right) x_t + \\
&\quad \left( C^{\$}(\tau-1) - C^{\$}(\tau) \right) (\sigma_t^2 - \bar{\sigma}^2) + \left( D^{\$}(\tau-1) - D^{\$}(\tau) \right) (\bar{\pi}_t - \mu_{\pi})
\end{aligned}$$

Given that the nominal model has three state variables, we can invert the state variables from any three forward rates, say  $f_t^{\$}(1)$ ,  $f_t^{\$}(3)$ , and  $f_t^{\$}(5)$ . Put differently, one linear combination of forward rates is perfectly correlated with  $(\sigma_t^2 - \bar{\sigma}^2)$ . This is the linear combination of interest, because this state variable is the only source of time variation in bond and stock risk premia. Hence, this linear combination is the best predictor of future bond returns. It is perfectly correlated with expected bond returns. We call in the Cochrane-Piazzesi factor because the weights  $(c_1, c_3, c_5)$  display a tent-shaped function as in Cochrane and Piazzesi (2005):

$$CP_t = c_1 f_t^{\$}(1) + c_3 f_t^{\$}(3) + c_5 f_t^{\$}(5) = b_{CP} (\sigma_t^2 - \bar{\sigma}^2). \quad (36)$$

In order to pin down the ratio of the standard deviation of  $CP_t$  relative to that of  $\sigma_t^2$ ,  $b_{CP}$ , we form annual expected excess bond returns on nominal bonds of horizons 2-, 3-, 4-, and 5-years in the model. They only depend on  $\sigma_t^2$ . We set the  $CP$  factor equal to the equally-weighted average of these four returns, as we did in the data. Hence  $b_{CP}$  is not a free parameter but rather a function of the structural parameters of the model.

#### A.4 CP Betas

The question we want to ask of the long-run risk model is whether it is able to generate the cross-sectional variation in the exposures of excess returns of book-to-market-sorted stock portfolios to

the  $CP$  factor. We showed in the main text that there is a clearly increasing pattern from growth to value. Since this is an equilibrium model, stock returns are endogenous, and only cash-flow growth can be specified exogenously. This leads us to impose the additional discipline that the model must match also the cross-sectional variation in the exposures of dividend growth rates of book-to-market-sorted stock portfolios to the  $CP$  factor. We also showed dividend growth betas that were decreasing from growth (mildly positive) to value (strongly negative). Third, the  $CP$  factor is a strong predictor of bond returns in the data; we want to capture this predictability quantitatively. We now provide closed-form expressions for the contemporaneous return and dividend growth betas and for lagged- $CP$  betas for excess bond returns. We start from the expressions for expected stock returns, expected excess stock returns, and expected dividend growth rates on stock portfolios in equation (24)-(25).

**Stock Returns and Excess Returns** The unconditional, contemporaneous,  $CP$  beta for *excess* stock returns is equal to:

$$\beta^{res} \equiv \beta \left( r_{t+1}^{e,i}, CP_{t+1} \right) = \nu_1 \beta_{Lagged}^{res} + \beta_{Innov}^{res}, \quad (37)$$

$$\beta_{Lagged}^{res} = \frac{\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1) + \frac{\alpha-\rho}{1-\rho} A_2^c(\kappa_1^c - \nu_1) + .5\Gamma(0)}{b_{CP}}, \quad (38)$$

$$\beta_{Innov}^{res} = \left( \chi A_1^i \frac{\varphi_e}{\sigma_w} + A_2^i \right) \frac{1 - \nu_1^2}{b_{CP}}, \quad (39)$$

where the expression for  $\Gamma(0)$  was given earlier in this appendix. The contemporaneous  $CP$ -beta of excess returns is the sum of the lagged beta, multiplied by the persistence of  $CP$ ,  $\nu_1$ , plus the innovation beta. The latter arises from the fact that  $pd$  innovations and  $CP$  innovations are both driven by the shock  $w_{t+1}$  and from the fact that innovations to  $w_{t+1}$  and  $e_{t+1}$  may be correlated ( $\chi \neq 0$ ).

Likewise, the unconditional, contemporaneous,  $CP$  beta for stock returns is equal to:

$$\beta^{rs} \equiv \beta \left( r_{t+1}^i, CP_{t+1} \right) = \nu_1 \beta_{Lagged}^{rs} + \beta_{Innov}^{rs}, \quad (40)$$

$$\beta_{Lagged}^{rs} = \frac{\rho}{b_{CP}} \frac{Cov[x_t, \sigma_t^2 - \bar{\sigma}^2]}{V[\sigma_t^2 - \bar{\sigma}^2]} + \frac{\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)}{b_{CP}}, \quad (41)$$

$$\beta_{Innov}^{rs} = \left( \chi A_1^i \frac{\varphi_e}{\sigma_w} + A_2^i \right) \frac{1 - \nu_1^2}{b_{CP}}. \quad (42)$$

The unconditional lagged  $CP$ -beta for *realized* returns is the same as for expected returns:  $\beta \left( r_{t+1}^i, CP_t \right) = \beta \left( E_t \left[ r_{t+1}^i \right], CP_t \right)$ . The reason is that unexpected returns have a zero unconditional correlation with  $CP_t$  by the law of iterated expectations.

**Dividend Growth Rates** The contemporaneous  $CP$  beta for dividend growth rates, denoted with a superscript  $\Delta d$ , can be shown to equal:

$$\beta^{\Delta d} \equiv \beta \left( \Delta d_{t+1}^i, CP_{t+1} \right) = \nu_1 \beta_{Lagged}^{\Delta d}, \quad (43)$$

$$\beta_{Lagged}^{\Delta d} = \left( \chi \frac{\phi_{d,x}^i}{1 - \rho_x \nu_1} \frac{\varphi_e}{\sigma_w} \frac{1 - \nu_1^2}{b_{CP}} + \frac{\phi_{d,\sigma}^i}{b_{CP}} \right). \quad (44)$$

The contemporaneous CP-beta for dividend growth is proportional to the lagged CP-beta, with a constant of proportionality of  $\nu_1$ . The reason is that dividend growth innovations are orthogonal to  $CP$  innovations because  $u_{t+1}$  and  $w_{t+1}$  are uncorrelated. The unconditional covariance between *realized* dividend growth rates  $\Delta d_{t+1}$  and  $CP_t$  is the same as the covariance with expected growth rates because unexpected dividend growth rates have a zero correlation with  $CP_t$  because  $Cov(\varphi_d^2 \sigma_t u_{t+1}^i, \sigma_t^2 - \bar{\sigma}^2) = 0$ , again because of the law of iterated expectations.

**Bond Returns and Excess Returns** For comparison, we also work out the betas of nominal *excess* bond returns with respect to CP:

$$\beta^{reb}(\tau) \equiv \beta\left(r_{t+1}^{b,e,\$}(\tau), CP_{t+1}\right) = \nu_1 \beta_{Lagged}^{reb}(\tau) + \beta_{Innov}^{reb}(\tau), \quad (45)$$

$$\beta_{Lagged}^{reb}(\tau) = \frac{.5\Gamma^{\$}(0) - .5\Gamma^{\$}(\tau - 1)}{b_{CP}}, \quad (46)$$

$$\beta_{Innov}^{reb}(\tau) = \left(\chi \left[B^{\$}(\tau - 1) + D^{\$}(\tau - 1)\zeta_{\pi,e}\right] \frac{\varphi_e}{\sigma_w} + C^{\$}(\tau - 1)\right) \frac{(1 - \nu_1^2)}{b_{CP}}, \quad (47)$$

where the expression for  $\Gamma^{\$}(\tau)$ , for  $\tau \geq 0$ , is given above. We will focus on the excess bond return predictability by CP, by matching lagged excess return bond betas (equation 46) in model and data.

Finally, the contemporaneous CP beta of nominal bond returns is given by:

$$\beta^{rb}(\tau) \equiv \beta\left(r_{t+1}^{b,\$}(\tau), CP_{t+1}\right) = \nu_1 \beta_{Lagged}^{rb}(\tau) + \beta_{Innov}^{rb}(\tau), \quad (48)$$

$$\begin{aligned} \beta_{Lagged}^{rb}(\tau) &= \frac{\rho}{b_{CP}} \frac{Cov[x_t, \sigma_t^2 - \bar{\sigma}^2]}{V[\sigma_t^2 - \bar{\sigma}^2]} + \frac{1}{b_{CP}} \frac{Cov[\bar{\pi}_t - \mu_{\pi}, \sigma_t^2 - \bar{\sigma}^2]}{V[\sigma_t^2 - \bar{\sigma}^2]} \\ &\quad + \frac{-\frac{\alpha-\rho}{1-\rho}(\kappa_1^c - \nu_1)A_2^e - .5\Gamma^{\$}(\tau - 1)}{b_{CP}}, \end{aligned} \quad (49)$$

$$\beta_{Innov}^{rb}(\tau) = \left(\chi \left[B^{\$}(\tau - 1) + D^{\$}(\tau - 1)\zeta_{\pi,e}\right] \frac{\varphi_e}{\sigma_w} + C^{\$}(\tau - 1)\right) \frac{(1 - \nu_1^2)}{b_{CP}}. \quad (50)$$

$$(51)$$

Note that the first term of  $\beta_{Lagged}^{rb}(\tau)$  is common with the beta of stock returns; it is a yield curve effect. The second term is also a yield curve effect. Both disappear if we look at excess returns.

**Beta decomposition** Recall the definition of a return

$$r_{t+1}^i = \kappa_0^i + \Delta d_{t+1}^i + pd_{t+1}^i - \kappa_1^i pd_t^i$$

and the definition of the  $pd$  ratio:

$$pd_t^i = A_0^i + A_1^i x_t + A_2^i (\sigma_t^2 - \bar{\sigma}^2)$$

Therefore, the contemporaneous return beta is:

$$\beta^{rs} - \beta^{\Delta d} = -\frac{A_1^i(\kappa_1^i \nu_1 - 1)}{b_{CP}} \frac{Cov[x_t, \sigma_t^2 - \bar{\sigma}^2]}{V[\sigma_t^2 - \bar{\sigma}^2]} - \frac{A_2^i(\kappa_1^i \nu_1 - 1)}{b_{CP}}$$



If  $\kappa_1^i \nu_1 - 1 > 0$ , which is a statement about the relative persistence of  $x$  and  $\sigma_t^2 - \bar{\sigma}^2$ , and  $\chi < 0$ , then both terms on the RHS are positive because  $A_1^i > 0$  and  $A_2^i < 0$ . Since value stocks have higher  $A_1^i$  and more negative  $A_2^i$ , this explains the increasing pattern in the LHS from growth to value.

Another way to understand this is by using

$$pd_t^i = \frac{\kappa_0^i}{\kappa_1^i - 1} + \Delta d_t^{i,H} - r_t^{i,H},$$

Therefore, the contemporaneous return beta is:

$$\beta_{CP}^{rs} - \beta^{\Delta d} = \beta \left[ CP_{t+1}, \Delta d_{t+1}^{i,H} - \kappa_1^i \Delta d_t^{i,H} \right] - \beta \left[ CP_{t+1}, r_{t+1}^{i,H} - \kappa_1^i r_t^{i,H} \right] \quad (52)$$

The first beta on the RHS measures the covariance of innovations of  $CP_{t+1}$  with *news* about future dividend growth rates. It equals:

$$\beta \left[ CP_{t+1}, \Delta d_{t+1}^{i,H} - \kappa_1^i \Delta d_t^{i,H} \right] = -\frac{\phi_{d,x}^i \nu_1 \text{Cov}[x_t, \sigma_t^2 - \bar{\sigma}^2]}{b_{CP} V[\sigma_t^2 - \bar{\sigma}^2]} - \frac{\phi_{d,\sigma}^i \nu_1}{b_{CP}} \quad (53)$$

$$+ \left\{ \chi \frac{\phi_{d,x}^i}{(\kappa_1^i - \rho_x)} \frac{\varphi_e}{\sigma_w} + \frac{\phi_{d,\sigma}^i}{(\kappa_1^i - \nu_1)} \right\} \frac{(1 - \nu_1^2)}{b_{CP}} \quad (54)$$

The second beta on the RHS measures the covariance of innovations of  $CP_{t+1}$  with *news* about future stock returns. It equals:

$$\beta \left[ CP_{t+1}, r_{t+1}^{i,H} - \kappa_1^i r_t^{i,H} \right] = \frac{-\rho \nu_1 \text{Cov}[x_t, \sigma_t^2 - \bar{\sigma}^2]}{b_{CP} V[\sigma_t^2 - \bar{\sigma}^2]} - \frac{[\phi_{d,\sigma}^i - A_2^i (\kappa_1^i - \nu_1)] \nu_1}{b_{CP}} \quad (55)$$

$$+ \left\{ \chi \frac{\rho}{(\kappa_1^i - \rho_x)} \frac{\varphi_e}{\sigma_w} + \frac{[\phi_{d,\sigma}^i - A_2^i (\kappa_1^i - \nu_1)]}{\kappa_1^i - \nu_1} \right\} \frac{(1 - \nu_1^2)}{b_{CP}} \quad (56)$$

It is easy to verify that the difference between these two betas delivers the first expression we derived for  $\hat{\beta}_{CP}^r - \hat{\beta}_{CP}^{\Delta d}$ .

## A.5 The Relationship Between Stocks and Bonds

**The Equity Risk Premium and the Campbell-Shiller Decomposition** Expected discounted future equity returns and dividend growth rates are given by:

$$r_t^{i,H} \equiv E_t \left[ \sum_{j=1}^{\infty} (\kappa_1^i)^{-j} r_{t+j}^i \right] = \frac{r_0^i}{\kappa_1^i - 1} + \frac{\rho}{\kappa_1^i - \rho_x} x_t + \frac{[\phi_{d,\sigma}^i - A_2^i (\kappa_1^i - \nu_1)]}{\kappa_1^i - \nu_1} (\sigma_t^2 - \bar{\sigma}^2) \quad (57)$$

$$\Delta d_t^{i,H} \equiv E_t \left[ \sum_{j=1}^{\infty} (\kappa_1^i)^{-j} \Delta d_{t+j}^i \right] = \frac{\mu_d^i}{\kappa_1^i - 1} + \frac{\phi_{d,x}^i}{\kappa_1^i - \rho_x} x_t + \frac{\phi_{d,\sigma}^i}{\kappa_1^i - \nu_1} (\sigma_t^2 - \bar{\sigma}^2) \quad (58)$$

From these expressions, it is easy to see that

$$pd_t^i = \frac{\kappa_0^i}{\kappa_1^i - 1} + \Delta d_t^{i,H} - r_t^{i,H},$$

and to compute the elements of the variance-decomposition:

$$V[pd_t^i] = Cov[pd_t^i, \Delta d_t^{i,H}] + Cov[pd_t^i, -r_t^{i,H}] = V[\Delta d_t^{i,H}] + V[r_t^{i,H}] - 2Cov[\Delta d_t^{i,H}, r_t^{i,H}].$$

We can also derive an expression for the expected future real bond yield (the risk-free rate):

$$y_t^H(1) \equiv E_t \left[ \sum_{j=1}^{\infty} (\kappa_1^i)^{-j} y_{t+j-1}(1) \right] = \frac{-A(1)}{\kappa_1^i - 1} + \frac{\rho}{\kappa_1^i - \rho_x} x_t + \frac{-C(1)}{\kappa_1^i - \nu_1} (\sigma_t^2 - \bar{\sigma}^2) \quad (59)$$

$$(60)$$

This makes it clear that the first term in the expected future stock return expression is a risk-free rate effect. It drops out once we look at expected excess returns:

$$r_t^{i,e,H} = r_t^{i,H} - y_t^H(1) = \frac{r_0^i + A(1)}{\kappa_1^i - 1} + \frac{[\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1) + C(1)]}{\kappa_1^i - \nu_1} (\sigma_t^2 - \bar{\sigma}^2)$$

The covariance between expected future dividend growth and expected future returns:

$$\begin{aligned} Cov(r_t^{i,H}, \Delta d_t^{i,H}) &= \frac{\phi_{d,x}^i \rho}{(\kappa_1^i - \rho_x)^2} V[x_t] + \frac{\phi_{d,\sigma}^i [\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)]}{(\kappa_1^i - \nu_1)^2} V[\sigma_t^2 - \bar{\sigma}^2] \\ &\quad + \frac{\rho \phi_{d,\sigma}^i + \phi_{d,x}^i [\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1)]}{(\kappa_1^i - \rho_x)(\kappa_1^i - \nu_1)} Cov[x_t, \sigma_t^2 - \bar{\sigma}^2]. \end{aligned}$$

It is similar to that of expected one-period ahead dividend growth and expected returns in equation (57). Indeed, if  $\rho_x = \nu_1$ , then it equals the latter covariance divided by  $(\kappa_1^i - \rho_x)^2 = (\kappa_1^i - \nu_1)^2$ . Note that when  $\chi = 0$  and  $\phi_{d,\sigma}^i = 0$ , only the first term survives in the covariance between expected returns and expected growth rates. Because  $\rho > 0$  and  $\phi_{d,x}^i > 0$ , it is always positive. Furthermore, this is a pure risk-free rate effect: the correlation between the expected *excess* return and expected dividend growth rate is exactly zero when  $\chi = 0$  and  $\phi_{d,\sigma}^i = 0$ . In our model, this is no longer necessarily the case because of the additional cash-flow effect coming through  $\phi_{d,\sigma}^i \neq 0$ :

$$\begin{aligned} Cov(r_t^{i,e,H}, \Delta d_t^{i,H}) &= \frac{\phi_{d,\sigma}^i [\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1) + C(1)]}{(\kappa_1^i - \nu_1)^2} V[\sigma_t^2 - \bar{\sigma}^2] \\ &\quad + \frac{\rho \phi_{d,\sigma}^i + \phi_{d,x}^i [\phi_{d,\sigma}^i - A_2^i(\kappa_1^i - \nu_1) + C(1)]}{(\kappa_1^i - \rho_x)(\kappa_1^i - \nu_1)} Cov[x_t, \sigma_t^2 - \bar{\sigma}^2]. \end{aligned}$$

Finally, for the consumption claim, we get the following expected future return, excess return, and growth expressions:

$$r_t^{c,H} = \frac{r_0^c}{\kappa_1^c - 1} + \frac{\rho}{\kappa_1^c - \rho_x} x_t - A_2^c (\sigma_t^2 - \bar{\sigma}^2) \quad (61)$$

$$\Delta c_t^H = \frac{\mu_c}{\kappa_1^c - 1} + \frac{1}{\kappa_1^c - \rho_x} x_t \quad (62)$$

$$r_t^{c,e,H} = \frac{r_0^c - A(1)}{\kappa_1^c - 1} + \frac{\frac{\alpha-1}{1-\rho} A_2^c (\kappa_1^c - \nu - 1) + .5\Gamma(0)}{(\kappa_1^c - \nu - 1)} (\sigma_t^2 - \bar{\sigma}^2) \quad (63)$$

**The Term Structure of Equity** We derive the price of dividend strips (or zero-coupon equity), the price of a claim to unit of dividends of firm  $i$  at some future date  $t$ . See also Lettau and Wachter (2007). Recall the log-linearized stock return expression for security or portfolio  $i$ :

$$r_{t+1}^i = \kappa_0^i + \Delta d_{t+1}^i + p d_{t+1}^i - \kappa_1^i p d_t^i. \quad (64)$$

Log price-dividend ratios on dividend strips of horizon  $\tau$ ,  $\widetilde{p d}_t^i(\tau)$ , are affine in the state vector:

$$\widetilde{p d}_t^i(\tau) = A^m(\tau) + B^m(\tau)x_t + C^m(\tau)(\sigma_t^2 - \bar{\sigma}^2),$$

where the coefficients  $A^m(\tau)$ ,  $B^m(\tau)$ , and  $C^m(\tau)$  follow recursions

$$\begin{aligned} A^m(\tau+1) &= A^m(\tau) + m_0 + \mu_d^i + \frac{1}{2}\Gamma^m(\tau)\bar{\sigma}^2, \\ B^m(\tau+1)' &= \rho_x B^m(\tau) + \phi_{d,x}^i - \rho, \\ C^m(\tau+1)' &= \nu_1 C^m(\tau) + \phi_{d,\sigma}^i + \frac{\alpha - \rho}{1 - \rho}(\kappa_1^c - \nu_1)A_2^c + \frac{1}{2}\Gamma^m(\tau), \\ \Gamma^m(\tau) &= \alpha^2 + \varphi_d^2 - 2\alpha\varphi_{dt} + \left[ B^m(\tau) - \frac{\alpha - \rho}{1 - \rho}A_1^c \right]^2 \varphi_e^2 + \left[ C^m(\tau) - \frac{\alpha - \rho}{1 - \rho}A_2^c \right]^2 \sigma_w^2 \\ &\quad + 2\chi \left[ B^m(\tau) - \frac{\alpha - \rho}{1 - \rho}A_1^c \right] \left[ C^m(\tau) - \frac{\alpha - \rho}{1 - \rho}A_2^c \right] \varphi_e \sigma_w \end{aligned}$$

initialized at  $A^m(0) = 0$ ,  $B^m(0) = 0$ , and  $C^m(0) = 0$ .

*Proof.* We conjecture that the log  $t+1$ -price of a  $\tau$ -period strip, scaled by the dividend in period  $t+1$ , is affine in the state

$$\widetilde{p d}_{t+1}^i(\tau) = A^m(\tau) + B^m(\tau)'x_{t+1} + C^m(\tau)(\sigma_{t+1}^2 - \bar{\sigma}^2)$$

and solve for the coefficients  $A^m(\tau+1)$ ,  $B^m(\tau+1)$ , and  $C^m(\tau+1)$  in the process of verifying this conjecture using the Euler equation:

$$\begin{aligned} \exp \widetilde{p d}_t^i(\tau+1) &= E_t \left[ \exp \{ m_{t+1} + \Delta d_{t+1}^i + \widetilde{p d}_{t+1}^i(\tau) \} \right] \\ &= \exp \{ m_0 + \mu_d^i + A^m(\tau) + (\phi_{d,x}^i - \rho + B^m(\tau)\rho_x)x_t + \left[ \phi_{d,\sigma}^i + \frac{\alpha - \rho}{1 - \rho}(\kappa_1^c - \nu_1)A_2^c + C^m(\tau)\nu_1 \right] (\sigma_t^2 - \bar{\sigma}^2) \} \\ &\quad E_t \left[ \exp \{ -\alpha\sigma_t\eta_{t+1} + \left[ B^m(\tau) - \frac{\alpha - \rho}{1 - \rho}A_1^c \right] \varphi_e\sigma_t e_{t+1} + \left[ C^m(\tau) - \frac{\alpha - \rho}{1 - \rho}A_2^c \right] \sigma_w\sigma_t w_{t+1} + \varphi_d\sigma_t u_{t+1} \} \right] \end{aligned}$$

We use the log-normality of the shocks to work out the expectation. Taking logs and collecting terms, we obtain the conjectured linear expression for  $\widetilde{p d}_t^i(\tau+1)$  where the recursions are as given above.  $\square$

## B Calibration

Our goal is to ask whether the model can quantitatively account for the patterns in CP betas and returns we document in the main text. Therefore, we refrain from a wholesale re-calibration or

estimation of the long-run risk model and make minimal deviations from the literature. Table 1 compares our calibration in the first column to the calibration in Bansal and Shaliastovich (2007) (Column 2) and that of the original Bansal and Yaron (2004) (Column 3). All parameters are quarterly since we solve the model at quarterly frequency and compare it to quarterly data. This avoids complications arising from time aggregation. Our parameters are identical to those in Bansal and Shaliastovich (2007), with the following exceptions. First, we introduce heteroscedasticity in  $\sigma_t^2 - \bar{\sigma}^2$ . That is, we replace  $\sigma_w w_{t+1}$  by  $\sigma_w \sigma_t w_{t+1}$  in equation (7), and multiply our value of  $\sigma_w$  by  $\bar{\sigma}$  to keep the unconditional variance of economic uncertainty the same. This change is inconsequential. Second, we estimate  $\nu_1$ , the autocorrelation coefficient of economic uncertainty  $\sigma_t^2 - \bar{\sigma}^2$ . Third, we estimate the parameter  $\chi$ , which governs correlation between shocks to  $x_t$  and  $\sigma_t^2$ . Fourth, we estimate the loadings of dividend growth  $\phi_{d,\sigma}^i$  on  $\sigma_t^2 - \bar{\sigma}^2$  and  $\phi_{d,x}^i$  on  $x_t$  for each portfolio. We use 10 “book-to-market” portfolios and 1 “market” portfolio. Prior to the estimation, we set the unconditional mean log real dividend growth rate,  $\mu_d^i$ , equal to its observed value in the data for each of the 11 portfolios. We then estimate  $2 + 2 \times 11 = 24$  parameters in order to minimize the distance between model and data along the following 38 dimensions: 11 contemporaneous log excess stock return CP betas, 11 contemporaneous log real dividend growth CP betas, 11 equity risk premia, and 5 lagged bond return CP betas for maturities 1-, 2-, 5-, 7-, and 10-years. The estimation is by non-linear least squares.

## B.1 Mapping from Monthly to Quarterly Parameters

The Bansal-Yaron model is calibrated and parameterized to monthly data. Since we want to use data on quarterly consumption and dividend growth, and a quarterly series for the wealth-consumption ratio, we recast the model at quarterly frequencies. We assume that the quarterly process for consumption growth, dividend growth, the low frequency component and the variance has the exact same structure than at the monthly frequency, with mean zero, standard deviation 1 innovations, but with different parameters. This appendix explains how the monthly parameters map into quarterly parameters. We denote all variables, shocks, and all parameters of the quarterly system with a tilde superscript. Our parameter values are listed at the end of this section, together with details on the simulation approach.

Obviously, the preference parameters do not depend on the horizon ( $\tilde{\alpha} = \alpha$  and  $\tilde{\rho} = \rho$ ), except for the time discount factor  $\tilde{\beta} = \beta^3$ . Also, the long-run average log wealth-consumption ratio at the quarterly frequency is lower than at the monthly frequency by approximately  $\log(3)$ , because log of quarterly consumption is the log of three times monthly consumption.

We accomplish this by matching the conditional and unconditional mean and variance of log consumption and dividend growth. Log quarterly consumption growth is the sum of log consumption growth of three consecutive months. We obtain  $\Delta \tilde{c}_{t+1} \equiv \Delta c_{t+3} + \Delta c_{t+2} + \Delta c_{t+1}$

$$\Delta \tilde{c}_{t+1} = 3\mu_c + (1 + \rho_x + \rho_x^2)x_t + \sigma_t \eta_{t+1} + \sigma_{t+1} \eta_{t+2} + \sigma_{t+2} \eta_{t+3} + (1 + \rho_x)\varphi_e \sigma_t e_{t+1} + \varphi_e \sigma_{t+1} e_{t+2} \quad (65)$$

Log quarterly dividend growth looks similar:

$$\Delta \tilde{d}_{t+1} = 3\mu_d + \phi(1 + \rho_x + \rho_x^2)x_t + \varphi_d \sigma_t u_{t+1} + \varphi_d \sigma_{t+1} u_{t+2} + \varphi_d \sigma_{t+2} u_{t+3} + \phi(1 + \rho_x)\varphi_e \sigma_t e_{t+1} + \phi \varphi_e \sigma_{t+1} e_{t+2} \quad (66)$$

First, we rescale the long-run component in the quarterly system, so that the coefficient on it in the consumption growth equation is still 1:

$$\tilde{x}_t = (1 + \rho_x + \rho_x^2)x_t.$$

Table 1: Quarterly Calibration

<i>Parameter</i>	<i>Our model</i>	<i>BS 2008</i>	<i>BY 2004</i>
Preferences			
$\alpha$	8	8	10
$\rho$	2/3	2/3	2/3
$\beta$	.9961	.9961	.9970
Consumption Process			
$\mu$	.0048	.0048	.0045
$\bar{\sigma}$	.0056	.0056	.0135
$\rho_x$	.9732	.9732	.9383
$\varphi_e$	.1617	.1617	.1264
$\nu_1$	EST	.9880	.9615
$\sigma_w$	.1984e-5/ $\bar{\sigma}$	.1984e-5	.3932e-5
$\chi$	EST	NA	NA
Dividend Process			
$\mu_d^m$	.0045	.0048	.0045
$\varphi_d^m$	5.9860	5.9860	4.4960
$\phi_{d,x}^m$	EST	1.5	3
$\iota^m$	.10	.10	NA
$\phi_{d,\sigma}^m$	EST	NA	NA
Inflation Process			
$\mu_\pi$	.0096	.0096	NA
$\varphi_{\pi,\eta}$	0	0	NA
$\varphi_{\pi,e}$	-2	-2	NA
$\sigma_\pi$	.0061	.0061	NA
$\rho_\pi$	.5718	.5718	NA
$\phi_\pi$	-.35	-.35	NA
$\zeta_{\pi,\eta}$	0	0	NA
$\zeta_{\pi,e}$	-1	-1	NA
$\sigma_z$	6.9282e-6	6.9282e-6	NA

*Notes:* This Table lists our benchmark parameter choices. All parameters are expressed in quarterly units. See Appendix B.1 on how we go from the monthly parameters of Bansal and Shaliastovich (2007) to the quarterly values in the second column and from the monthly values of Bansal and Yaron (2004) to the quarterly values in the third column.

Second, we equate the unconditional mean of consumption and dividend growth :

$$\tilde{\mu} = 3\mu, \tilde{\mu}_d = 3\mu_d.$$

These imply that we also match the the conditional mean of consumption growth:

$$E_t[\Delta c_{t+3} + \Delta c_{t+2} + \Delta c_{t+1}] = 3\mu + (1 + \rho_x + \rho_x^2)x_t = \tilde{\mu} + \tilde{x}_t = E_t[\Delta \tilde{c}_{t+1}]$$

Third, we also match the conditional mean of dividend growth by setting the quarterly leverage parameter  $\tilde{\phi} = \phi$ .

Fourth, we match the unconditional variance of quarterly consumption growth:

$$\begin{aligned}
V[\Delta\tilde{c}_{t+1}] &= (1 + \rho_x + \rho_x^2)^2 V[x_t] + \sigma^2 [3 + (1 + \rho_x)^2 \varphi_e^2 + \varphi_e^2] \\
&= (1 + \rho_x + \rho_x^2)^2 \frac{\varphi_e^2 \sigma^2}{1 - \rho_x^2} + \sigma^2 [3 + (1 + \rho_x)^2 \varphi_e^2 + \varphi_e^2] \\
&= \frac{\tilde{\varphi}_e^2 \tilde{\sigma}^2}{1 - \tilde{\rho}_x^2} + \tilde{\sigma}^2
\end{aligned}$$

The first and second equalities use the law of iterated expectations to show that

$$V[\sigma_{t+j}\eta_{t+j+1}] \equiv E[E_{t+j}\{\sigma_{t+j}^2\eta_{t+j+1}^2\}] - (E[E_{t+j}\{\sigma_{t+j}\eta_{t+j+1}\}])^2 = E[\sigma_{t+j}^2] - 0 = \sigma^2$$

and the same argument applies to terms of type  $V[\sigma_{t+j}e_{t+j+1}]$ . Coefficient matching on the variance of consumption expression delivers expressions for  $\tilde{\sigma}^2$  and  $\tilde{\varphi}_e$ :

$$\begin{aligned}
\tilde{\sigma}^2 &= \sigma^2 [3 + (1 + \rho_x)^2 \varphi_e^2 + \varphi_e^2] \\
\tilde{\varphi}_e^2 &= \varphi_e^2 \frac{(1 - \tilde{\rho}_x^2)(1 + \rho_x + \rho_x^2)^2 \sigma^2}{1 - \rho_x^2} \frac{\sigma^2}{\tilde{\sigma}^2} \\
&= \frac{(1 - \rho_x^6)(1 + \rho_x + \rho_x^2)^2}{1 - \rho_x^2} \frac{\varphi_e^2}{3 + (1 + \rho_x)^2 \varphi_e^2 + \varphi_e^2},
\end{aligned}$$

where the third equality uses the first equality. Note that we imposed  $\tilde{\rho}_x = \rho_x^3$ , which follows from a desire to match the persistence of the long-run cash-flow component. Recursively substituting, we find that the three-month ahead  $x$  process has the following relationship to the current value:

$$x_{t+3} = \rho_x^3 x_t + \varphi_e \sigma_{t+2} e_{t+3} + \rho_x \varphi_e \sigma_{t+1} e_{t+2} + \rho_x^2 \varphi_e \sigma_t e_{t+1}$$

which compares to the quarterly equation

$$\tilde{x}_{t+1} = \tilde{\rho}_x \tilde{x}_t + \tilde{\varphi}_e \tilde{\sigma}_t \tilde{e}_{t+1}$$

The two processes now have the same auto-correlation and unconditional variance.

Fifth, we match the unconditional variance of dividend growth. Given the assumptions we have made so far, this pins down  $\varphi_d$ :

$$\tilde{\varphi}_d^2 = \frac{3\varphi_d^2 + \phi^2(1 + \rho_x)^2 \varphi_e^2 + \phi^2 \varphi_e^2}{3 + (1 + \rho_x)^2 \varphi_e^2 + \varphi_e^2}$$

Sixth, we match the autocorrelation and the unconditional variance of economic uncertainty  $\sigma_t^2$ . Iterating forward, we obtain an expression that relates variance in month  $t$  to the one in month  $t + 3$ :

$$\sigma_{t+3}^2 - \sigma^2 = \nu_1^3 (\sigma_t^2 - \sigma^2) + \sigma_w \nu_1^2 w_{t+1} + \sigma_w \nu_1 w_{t+2} + \sigma_w w_{t+3}$$

By setting  $\tilde{\nu}_1 = \nu_1^3$  and  $\tilde{\sigma}_w^2 = \sigma_w^2(1 + \nu_1^2 + \nu_1^4)$ , we match the autocorrelation and variance of the quarterly equation

$$\tilde{\sigma}_{t+1}^2 - \tilde{\sigma}^2 = \tilde{\nu}_1 (\tilde{\sigma}_t^2 - \tilde{\sigma}^2) + \tilde{\sigma}_w \tilde{w}_{t+1}$$

## C Results

We now discuss the estimation results in detail. The main text of the paper summarizes the findings, focussing on the intuition.

### C.1 Standard LRR Model

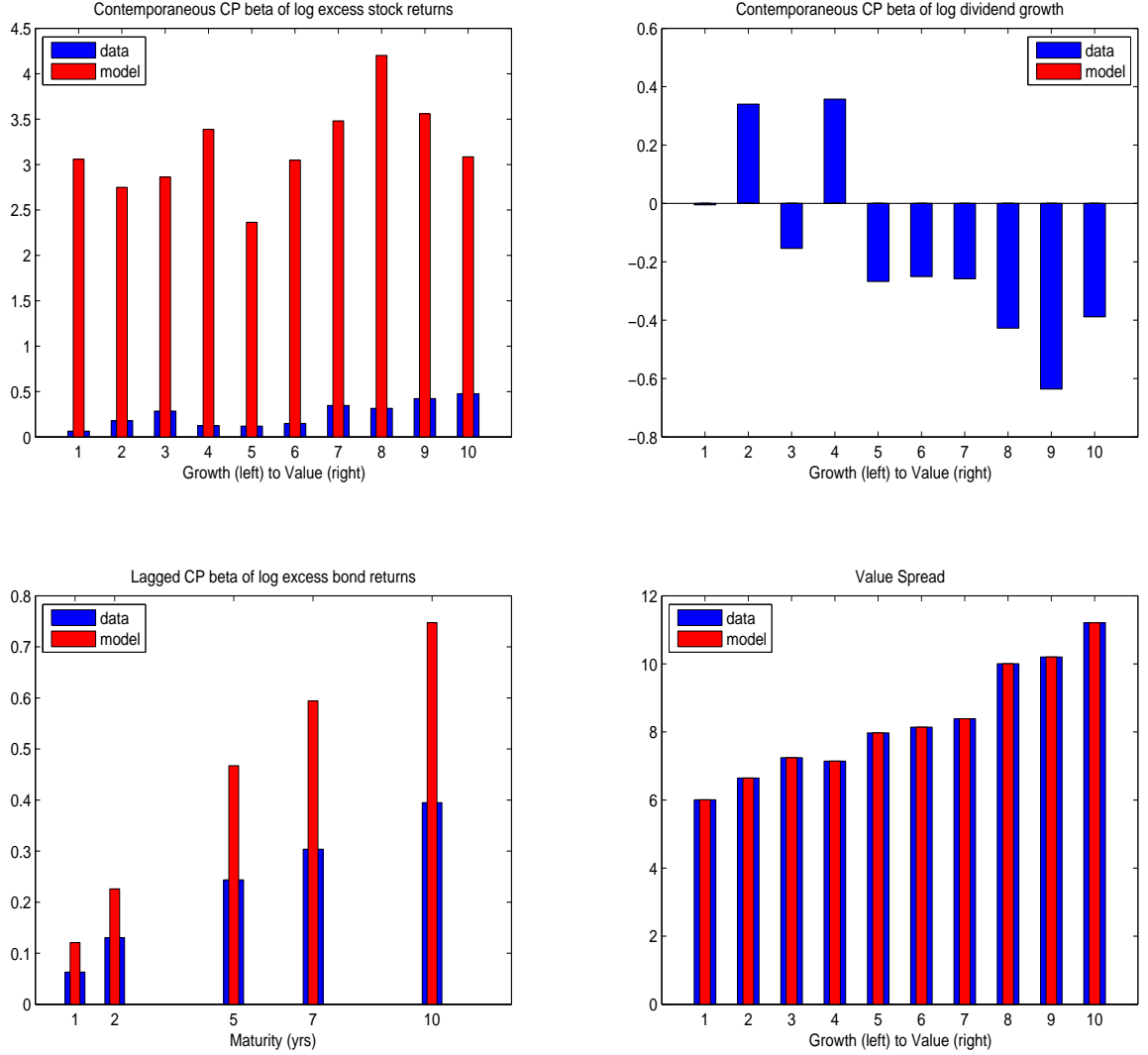
As a benchmark, Figure 1 shows these 38 moments for the Bansal and Shaliastovich (2007) calibration, henceforth B-S calibration. This calibration sets  $\chi = 0$  and  $\phi_{d,\sigma}^i = 0$ . It also features a high value for  $\nu_1$ , so that economic uncertainty is very persistent, in fact more persistent than long-run growth ( $\nu_1 > \rho_x$ ). To give this model its best chance at matching the value premium, we estimate  $\phi_{d,x}^i$  in order to match risk premia on each portfolio, following Bansal, Dittmar, and Lundblad (2005), Bansal, Kiku, and Yaron (2006), and Hansen, Heaton, and Li (2008). The bottom right panel shows that the model is flexible enough to exactly match not only the equity risk premium on the market portfolio but also the risk premia on each of the 10 book-to-market portfolios. The calibration accomplishes this by having value stocks carry more long-run risk:  $\phi_{d,x}^i$  is higher for value than for growth. In other words, value stocks are risky because their dividend growth is high in states of the world where long-run consumption growth  $x_t$  is high. These are states of low marginal utility growth for the representative investor. Panel A of 2 shows that  $\phi_{d,x}^i$  ranges from 3.3 for BM1 to 5.3 for BM10. It is 3.5 for the market portfolio. A higher  $\phi_{d,x}^i$  translates in a higher  $A_1^i$ , which increases the equity risk premium through the second term in equation (??).

However, inspection of the two top panels of Figure 1 shows that this calibration fails to generate the observed relationship between the  $CP$  factor and excess stock returns and dividend growth rates. First, equation (43) shows that when both  $\phi_{d,\sigma}^i = 0$  and  $\chi = 0$ , the contemporaneous dividend growth betas are zero for all portfolios. Because expected dividend growth does not depend on economic uncertainty  $\sigma_t^2$ , it is uncorrelated with  $CP$  (which perfectly correlates with economic uncertainty). Second, the excess stock return betas do not show the increasing pattern we found in the data, and they are at least a factor of five too big. Equation (37) helps us understand why. The innovation betas are all negative because  $\chi = 0$  and  $A_2^i < 0$ . They become increasingly negative from growth to value: from -8 to -16. The lagged excess return betas in (38) are the sum of  $\phi_{d,\sigma}^i$ ,  $-A_2^i(\kappa_1^i - \nu_1)$ , which increases from 360 to 630 from growth to value, and  $\frac{\alpha - \rho}{1 - \rho} A_2^c(\kappa_1^c - \nu_1) + .5\Gamma(0) = -993 + 1,046 = 53$ . Given  $b_{CP} = 35$ , when  $\phi_{d,\sigma}^i = 0$ , these lagged excess stock return betas range from 11 to 19. Combining the two results in contemporaneous betas that are too high without a pronounced pattern from growth to value. Finally, the first beta on the right hand side of equation (??) is zero when  $\chi = 0$  and  $\phi_{d,\sigma}^i = 0$ . The second term is negative: high bond risk premia (or positive innovations in  $CP$ ) coincide with negative innovations in expected future stock returns, which seems counter-intuitive. In addition, there is no clear pattern in this beta difference as opposed to the strongly increasing pattern in the data.

### C.2 Results for Our Calibration

Our estimation sets  $\chi = .2048 > 0$  and  $\phi_{d,\sigma}^i < 0$  in order to match the 38 moments. It also substantially lowers the value for  $\nu_1$  from .9880 to .9296 ( $\nu_1 < \rho_x$ ). The lower persistence we estimate in economic uncertainty facilitates the interpretation of  $\sigma_t^2$  (or  $CP_t$ ) as a business cycle variable. Figure 2 shows that our calibration does a good job matching the patterns and magnitudes of the  $CP$  exposures of excess stock returns, dividend growth rates, while maintaining the good

Figure 1: Risk Premia and Betas for  $\chi = 0$  and  $\phi_{d,\sigma}^i = 0$



fit for the value premium. What drives these results? First, a sufficiently negative  $\phi_{d,\sigma}^i$  directly generates a negative dividend growth beta. Panel B of Table 2 shows that the estimation chooses negative loadings  $\phi_{d,\sigma}^i$  for all portfolios. They range from -537 for growth (BM1) and decrease monotonically to -883 for value (BM10). According to this channel, stocks, and in particular value stocks, have lower expected dividend growth when economic uncertainty, and therefore  $CP$ , is high. Second, this makes them risky because they have low cash flow growth exactly when marginal utility



Table 2: Cash-Flow Parameters and the Value Premium

In Panel A, we use the benchmark calibration of Bansal and Shaliastovich (2007). In panel B, we use our own calibration. The first column reports the unconditional mean dividend growth rate. The second column reports the loading of dividend growth on the long-run risk state variable  $x_t$ . The third column reports the loading of dividend growth on the economic uncertainty state variable  $\sigma_t^2$ . The fourth column reports the equity risk premium, the expected excess log stock return (including a Jensen adjustment). The last column reports the mean price-dividend ratio.

<b>Panel A: B-S Calibration</b>					
	$\mu_d^i$	$\phi_{d,x}^i$	$\phi_{d,\sigma}^i$	Risk premium	PD
market	1.79	3.52	0	6.82	16.56
BM1	0.60	3.30	0	6.01	15.13
BM2	1.65	3.45	0	6.65	16.53
BM3	2.08	3.70	0	7.24	16.54
BM4	1.41	3.82	0	7.14	15.07
BM5	3.16	3.85	0	7.97	18.21
BM6	2.69	4.09	0	8.14	16.48
BM7	2.49	4.31	0	8.39	15.57
BM8	3.17	5.17	0	10.01	14.83
BM9	3.86	5.05	0	10.20	16.16
BM10	4.89	5.33	0	11.21	17.39

<b>Panel B: Own Calibration</b>					
	$\mu_d^i$	$\phi_{d,x}^i$	$\phi_{d,\sigma}^i$	Risk premium	PD
market	1.79	4.42	-575.17	6.81	17.17
BM1	0.60	4.17	-536.69	6.09	15.30
BM2	1.65	4.35	-558.70	6.66	17.05
BM3	2.08	4.71	-615.94	7.32	17.11
BM4	1.41	4.74	-614.86	7.13	15.64
BM5	3.16	4.92	-655.90	8.02	19.31
BM6	2.69	5.16	-687.54	8.18	17.43
BM7	2.49	5.39	-716.69	8.43	16.49
BM8	3.17	6.28	-854.99	9.97	16.30
BM9	3.86	6.22	-851.68	10.22	17.96
BM10	4.89	6.39	-883.53	11.00	20.48

growth is high. Hence the value premium. This  $\phi_{d,\sigma}^i$  effect reinforces the  $\phi_{d,x}^i$  effect. Panel B shows that value stocks continue to have higher loadings on long run growth  $x_t$  than growth stocks, just as in Panel A. The dividend growth properties of the market portfolio resemble those of the third growth decile portfolio, consistent with the finding in Jurek and Viceira (2006) that growth stocks make up about 70% of the U.S. market capitalization.

Because  $\chi$  is estimated to be .20,  $x_t$  and  $\sigma_t^2$  have a conditional correlation of 20%. Positive correlation between the two state variables implies that times with higher growth are also times with more economic uncertainty. Such an economy is less risky relative to the one with  $\chi = 0$  because the shocks to uncertainty (which causes prices to fall) are hedged by a higher long-term growth rate. The interest rate increases (slightly) because the demand for bonds is lower because of reduced precautionary savings motives. Because the economy is less risky, the price dividend ratios

on all assets are higher and the risk premia lower. This reduction in risk is why all  $\phi_{d,x}^i$  values are somewhat higher in Panel B ( $\chi = 0.2$ ) than in Panel A ( $\chi = 0$ ). Third, the positive  $\chi$  helps to increase the excess stock return innovation betas, while the negative  $\phi_{d,\sigma}^i$  helps to lower the lagged excess stock return betas. Both help to match the observed magnitude of the contemporaneous excess stock return betas. A positive  $\chi$  suggests that economic uncertainty is highest at the end of a recession, when growth is about to resume. Such a positive correlation arises endogenously in models of information production (Van Nieuwerburgh and Veldkamp 2006). It also increases the correlation between expected returns and expected dividend growth. (Binsbergen and Koijen 2007) estimate this correlation for the aggregate stock market and conclude it is positive. In both our model and the B-S calibration, the lagged excess bond return betas are somewhat too high. This is not surprising, in the model bond returns and  $CP$  are both driven by  $\sigma_t^2$ , with no offsetting effect from cash-flow growth as in the case of stocks. Finally, our calibration predicts a strongly increasing pattern in the beta difference of equation (??), just as in the data. The first beta on the right hand side is positive and increasing from growth to value. A high bond risk premium (at the end of a recession or the beginning of a boom) implies good news about future dividend growth, and more so for value stocks than for growth stocks. The second term is also positive and increasing. A high bond risk premium (at the end of a recession) coincides with a high equity risk premium, and more so for value than for growth stocks. The cash-flow effect dominates the discount rate effect.

Matching these betas does not come at the expense of matching the moments of consumption, interest rates, or inflation, for the most part because we kept these parameters at their standard values. Table 3 plots unconditional means, standard deviations (both expressed in percent per year), and quarterly autocorrelation in the benchmark B-S calibration (first 3 columns) and in our calibration (last 3 columns). The moments in our model are both close to the ones in the B-S calibration and close to the data.

Figure 2: Risk Premia and Betas for  $\chi > 0$  and  $\phi_{d,\sigma}^i < 0$

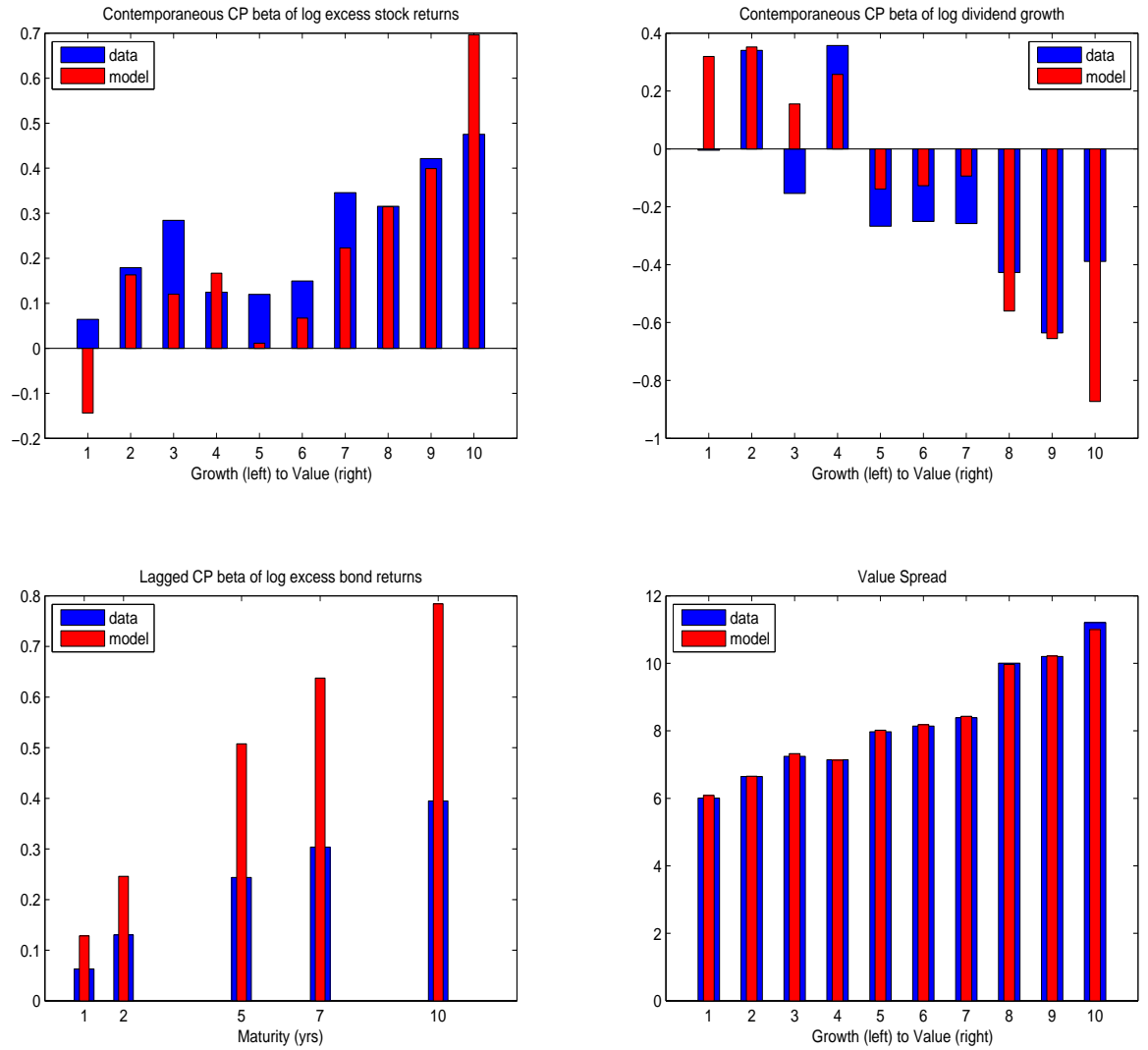


Table 3: Other Moments of Interest

This table reports unconditional means (expressed in percent per year), standard deviations (expressed in percent per year), and the quarterly first-order autocorrelation in the benchmark B-S calibration (first 3 columns) and in our calibration (last 3 columns). The moments reported for stocks are for the market portfolio, and are denoted with a <sup>m</sup> superscript. The notation follows the main text.

	Panel A: B-S Calibration			Panel B: Our Calibration		
	mean	stdev	ac(1)	mean	stdev	ac(1)
$x$	0.00	0.78	0.97	0.00	0.78	0.97
$\sigma^2$	0.0123	0.0026	0.99	0.0123	0.0011	0.93
$\bar{\pi} - \mu_\pi$	3.84	0.84	0.84	3.84	0.84	0.84
$\Delta c$	1.92	1.36	0.32	1.92	1.36	0.32
$\Delta d^m$	1.79	7.20	0.14	1.79	7.50	0.21
$\pi$	3.84	1.52	0.29	3.84	1.52	0.29
$y(1)$	2.19	0.54	0.97	2.34	0.52	0.97
$y^s(1)$	6.18	0.52	0.63	6.33	0.51	0.62
$y^s(20)$	6.30	0.15	0.92	6.48	0.14	0.91
$y^s(20) - y^s(1)$	0.12	0.73	0.11	0.15	0.73	0.11
$E_t \left[ r_{t+1}^{b,e,s}(20) \right]$	-0.94	0.20	0.99	-0.76	0.07	0.93
$E_t \left[ r_{t+1}^{b,e,s}(20) \right]$	0.21	0.04	0.99	0.26	0.02	0.93
$CP$	0.45	0.09	0.99	0.57	0.04	0.93
$WC$	67.75	5.66	0.98	74.83	4.29	0.97
$PD^m$	16.56	32.77	0.98	16.89	34.77	0.97
$r_{t+1}^m - y_t(1)$	5.68	15.15	0.01	5.42	17.23	0.00
$E_t[r_{t+1}^m - y_t(1)]$	5.68	1.17	0.99	5.42	0.73	0.95
$E_t[\Delta d_{t+1}^m]$	1.79	2.75	0.97	1.79	3.46	0.97
$E_t[r_{t+1}^{e,m}]$	6.82	1.42	0.99	6.90	0.60	0.93

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