Valuation of Electricity Futures: Reduced-Form vs. Dynamic Equilibrium Models

Wolfgang Bühler and Jens Müller-Merbach[*]

May 2009

Abstract

Equilibrium and reduced-form models are partly competing, partly complementary approaches to value electricity futures. We present the first empirical comparison of a one-factor reduced-form model and a demand driven dynamic equilibrium model. The contribution to the literature is twofold. First, on the theoretical side we develop a dynamic generalization of the static model by Bessembinder and Lemmon. As a result we endogenously derive the term structure of futures prices and risk premia. Second, on the empirical side we test both models using price data from Nord Pool. Our main findings are as follows. (1) The equilibrium model is able to explain the increasing volatility and right-skewness of futures prices for a decreasing time to maturity. (2) The cost function in the equilibrium model has to be parameterized by the water reservoir level to obtain reasonable spot and futures prices. (3) The equilibrium model provides better out-of-sample estimates of futures prices than the reduced-form model, and it is able to capture price peaks.

JEL classification: G13
Keywords: Electricity futures, risk premia, reduced-form models, equilibrium models.

1   Introduction

The valuation of electricity futures is still a challenging topic. The non-storability of electricity results in two major consequences: First, spot prices exhibit seasonal patterns, a large volatility, and price peaks. Second, electricity futures cannot be evaluated by the cost-of-carry approach, and their prices contain risk premia on top of the expected spot price, cf. Geman (2005, p. 270). Determining these risk premia is a major theoretical and empirical challenge.

Two approaches have been proposed to price electricity futures: Reduced-form and equilibrium models. The advantages and shortcomings of these competing as well as complementary model classes are well known. Reduced-form models are flexible and parsimonious with regard to the number and the stochastic properties of the risk factors. If modeled suitably, they allow for closed-form solutions of futures prices. For typical reduced-form models we refer to Pilipović (1997), Geman and Vasicek (2001), Koekebakker and Ollmar (2005), Audet et al. (2002), Lucia and Schwartz (2002), Villaplana (2003), Geman and Roncoroni (2006), and Cartea and Figueroa (2005).

Equilibrium models focus on modeling the production and demand structure of electricity. Spot and futures prices, risk premia and their properties are determined endogenously. In their fundamental dynamic model, Routledge et al. (2001) derive their pricing results by explicitly considering also the storage possibilities of different types of fuel and its sequential conversion into electricity. In the static model by Bessembinder and Lemmon (2002) hedging activities of risk-averse electricity producers and retailers drive the futures prices. Ullrich (2007) adds capacity constraints to the Bessembinder/Lemmon-model. Benth et al. (2007) use a time-dependent allocation of market power between risk-averse producers and consumers to receive futures prices in equilibrium that may contain positive or negative risk premia.

Our model contributes to the literature in the following ways. On the theoretical side we derive a dynamic generalization of the static model by Bessembinder and Lemmon. This dynamic version has several important features. First, it endogenously generates the term structure of futures prices and its risk premia. Comparative statics show that the dynamic approach can explain increasing, decreasing, and hump-shaped term structures of risk premia

even when there is no seasonal structure in the prices. Second, it explains empirical stochastic characteristics as the increasing volatility and right-skewness of futures prices for decreasing time to maturity. Third, the dynamic setting is necessary to evaluate cascading futures as they are common at most electricity exchanges. For a more detailed derivation we refer to Bühler and Müller-Merbach (2008).

On the empirical side we present the first study that compares a reduced-form and our equilibrium pricing model for electricity futures. Both models use one exogenous factor, the spot price and the demand for electricity, respectively, which shows a strong seasonal pattern. We use the best performing one-factor reduced-form model in the study by Lucia and Schwartz (2002). In the equilibrium model, additional to the stochastic demand, the cost structure of the producers and the changing level of the water reservoir matter for the spot and futures prices.

We conduct our empirical analysis with data from the Scandinavian exchange Nord Pool. Nord Pool provides, in addition to price data, also time series of the end-user demand and of the water reservoir level. We test and compare the two models with weekly observations of futures prices during a period of 50 months. This period contains years with a rather low volatility, but also times with a record level of prices and large price jumps.

Our main empirical findings are as follows. Assuming risk-neutral agents as a benchmark we find that the equilibrium model provides better out-of-sample futures price estimates than the reduced-form model. The absolute mean estimation error, the standard deviation, and the maximum of the estimation errors are lower for this model. The result can be traced back to the following reasons. (1) The reduced-form model shows a reasonable performance during calm price periods. It is, however, not able to capture price peaks in an appropriate way. These peaks are properly reflected in the equilibrium model as the cost function depends on the water reservoir level. This variable is highly correlated with spot and futures prices. (2) The spot prices in the reduced-form model are symmetrically distributed, whereas in the equilibrium model normally distributed demand residuals are transformed by the convex cost function into skewed spot prices. (3) The seasonal component of the exogenous spot price in the reduced-form model is harder to estimate than the seasonal component of the demand. Both models, however, underestimate on average the futures prices. This result shows that there exist risk premia in futures prices. These premia are positive, i.e. the seller

of a futures contract earns on average a surplus. This finding confirms the empirical results by Longstaff and Wang (2004) and Geman and Vasicek (2001).

The implicit estimation of the market price of risk for the reduced-form model and the normalized risk aversion parameter in the equilibrium model shows the expected signs. The market price of risk is on average negative, contrary to the results reported by Lucia and Schwartz (2002), and the normalized risk aversion parameter has a positive sign.

On average, the out-of-sample pricing errors of both models are reduced if the implicitly estimated risk parameters are considered in estimating futures prices. Also, the standard deviation of the pricing errors in the equilibrium model declines, but not for the reduced-form model. Again, this result is due to the huge pricing error of this model during highly volatile spot price periods. The analysis of the estimated risk premia shows that the theoretical term structure of these premia is supported by the observed term structure for the equilibrium model, but not for the reduced-form approach.

Our paper is organized as follows. Section 2 shortly reviews the two models. Section 3 describes the spot and futures market at Nord Pool and Section 4 provides descriptive statistics for the used time series. In Section 5 we explain the estimation design and procedure. We present the empirical results in Section 6. Section 7 concludes.

2 Review of the Models

In this section we shortly review the proposed models. For a more detailed description we refer to the original papers.

2.1 The Reduced-Form Model by Lucia and Schwartz

In their paper, Lucia and Schwartz (2002) propose two different one-factor models as well as two different two-factor models. Apart from the number of factors the approaches differ whether the spot price or the log-spot price is used as an exogenous variable. In their empirical study, Lucia and Schwartz find that the spot-price model outperforms the competing model. Therefore, we restrict our analysis to their spot price model which is shortly reviewed below.

The spot price $\tilde{P}_t$ equals the sum of a deterministic (seasonal) component $f(t)$ and a stochastic component $\tilde{X}_t$ that follows an Ornstein-Uhlenbeck process

$$d\tilde{X}_t = -\kappa \tilde{X}_t dt + \sigma^X d\tilde{Z}_t, \tag{1}$$

where $\tilde{Z}_t$ is a Wiener process. $\sigma^X > 0$ denotes the constant volatility and $\kappa > 0$ the speed of adjustment.

Assuming a constant market price of risk, λ, the futures price under the risk-neutral measure (denoted by *) equals the expected spot price at the futures maturity date T. Since $\tilde{P}_t = f(t) + \tilde{X}_t$, the futures price $F_{t,T}$ at date t reads:

$$F_{t,T} = \mathrm{E}_t^*\{\tilde{P}_T\} = f(T) + (P_t - f(t))\mathrm{e}^{-\kappa(T-t)} - \lambda \frac{\sigma^X}{\kappa}(1 - \mathrm{e}^{-\kappa(T-t)}). \tag{2}$$

The futures price in (2) consists of three additive components. The first one, $f(T)$, describes the unconditionally expected future spot price. The second component, $(P_t - f(t))\mathrm{e}^{-\kappa(T-t)}$, is the difference between the current spot price and its deterministic component. This difference is discounted with the speed of adjustment κ and reflects the mean-reverting behavior: The best forecast is to expect that the actual deviation of the spot price from $f(t)$ continuously fades out. Finally, the third component corrects for the spot price risk. This component is monotonous in the time-to-maturity and proportional to the market price of risk, λ. The second and the third term in (2) can be positive or negative.

2.2 The Dynamic Equilibrium Model

In Bühler and Müller-Merbach (2008) we develop a multi-period extension of the static model by Bessembinder and Lemmon (2002). The general framework of this equilibrium model is outlined in Section 2.2.1. It has several degrees of freedom that we specify in Section 2.2.2.

2.2.1 Model Framework

The market structure is characterized by competitive, risk-averse producers G and retailers R of electricity. Both groups are aggregated to two representative agents. We assume that they

maximize their terminal wealth $\tilde{W}_\Theta$ at the planing horizon Θ in a mean-variance framework:

$$\mathrm{E}_t\{\tilde{W}_\Theta^A\} - \tfrac{1}{2}\lambda^A \mathrm{Var}_t\{\tilde{W}_\Theta^A\} , \qquad A \in \{G, R\} . \tag{3}$$

The terminal wealth is determined by the cash flows that the representative agents receive from producing or retailing electricity, respectively, as well as from their trading activities. Both agents must trade in the spot market to satisfy the exogenous end-user demand $\tilde{D}_t$ at each date t. As electricity is not storable, the demand is equal to the production volume.

When not hedged, the producer receives at date t the cash flow

$$C_t'(\tilde{D}_t)\tilde{D}_t - C_t(\tilde{D}_t). \tag{4}$$

$C_t(D)$ denotes the variable production cost as a (possibly time-varying) function of the produced quantity of electricity, D. $C_t'(D)$ denotes the marginal cost function. In a competitive equilibrium marginal costs equal the spot price. We assume that C_t' is strictly increasing in the production volume.

The retailer receives the cash flow

$$p\tilde{D}_t - C_t'(\tilde{D}_t)\tilde{D}_t. \tag{5}$$

p represents the fixed end-user price per unit of electricity which is assumed to be independent of the consumed quantity of electricity.

By trading futures, both representative agents can change the risk-return position of their cash flows in (4) and (5), and, by compounding the marking-to-market consequences to the planning horizon Θ, their terminal wealth $\tilde{W}_\Theta$. Solving (3) under the condition of market clearing determines the futures prices in equilibrium. As discussed in Basak and Chabakauri (2007), two solution concepts are available. The global solution is not time-consistent and its implementation needs a commitment device. In Bühler and Müller-Merbach (2008) we consider the time-consistent solution by dynamic programming which leads to the following

equilibrium futures prices:

$$F_{t,T} = \mathrm{E}_t\{\tilde{F}_{t+1,T}\} - \xi \sum_{\theta=t+1}^{\Theta} \mathrm{e}^{r(\Theta-\theta)}\mathrm{Cov}_t\{p\tilde{D}_\theta - C_\theta(\tilde{D}_\theta); \tilde{F}_{t+1,T}\}, \qquad t < T \le \Theta, \quad (6)$$

where $\xi = \frac{\lambda^R\lambda^G}{\lambda^R+\lambda^G}$ is the combined risk aversion factor and r the constant interest rate. In the last period before maturity, (6) can be written as

$$F_{T-1,T} = \mathrm{E}_{T-1}\{\tilde{F}_{T,T}\} - \xi \sum_{\theta=T-1}^{\Theta} \mathrm{e}^{r(\Theta-\theta)}\mathrm{Cov}_{T-1}\{p\tilde{D}_\theta - C_\theta(\tilde{D}_\theta); C'_T(\tilde{D}_T)\}, \qquad t < T \le \Theta, \ (7)$$

where $F_{T,T} = C'_T(D_T)$ is the final settlement price and equals the spot price P_T. Note that (6) represents equilibrium futures prices recursively only.

The equilibrium futures price in (6) is equal to the expected next period's futures price or spot price, respectively, minus a risk premium. This risk premium is the sum of the covariances between future cash flows and the next period's endogenous futures or spot price. The term $p\tilde{D}_\theta - C_\theta(\tilde{D}_\theta)$ denotes the aggregate net cash flow that the whole electricity sector receives in the economy. The representative retailer receives $p\tilde{D}_t$ and the producer has to pay $C_t(\tilde{D}_t)$ for the production during the time interval $[t, t+1)$. The spot and the futures market allocate the aggregate cash flow and the cash flow risk between the producer and the retailer.

2.2.2 Model Specification

Two factors in the equilibrium model are yet to be specified: The process of the state variable, i. e. the end-user demand D_t, and the cost structure. We assume that the stochastic process of $\tilde{D}_t$ is the sum of a deterministic component $\bar{D}_t$ and a stochastic component $\tilde{S}_t$. $\bar{D}_t$ reflects the characteristic seasonal patterns. The functional form of $\bar{D}_t$ that we use in the empirical analysis is described in Appendix A. The stochastic component $\tilde{S}_t$ is modeled as a first-order autoregressive process:

$$\tilde{S}_t = \rho\tilde{S}_{t-1} + \sigma^S\tilde{\epsilon}_t^S, \quad \tilde{\epsilon}_t^S \sim \mathcal{N}(0,1) \text{ i.i.d.} \tag{8}$$
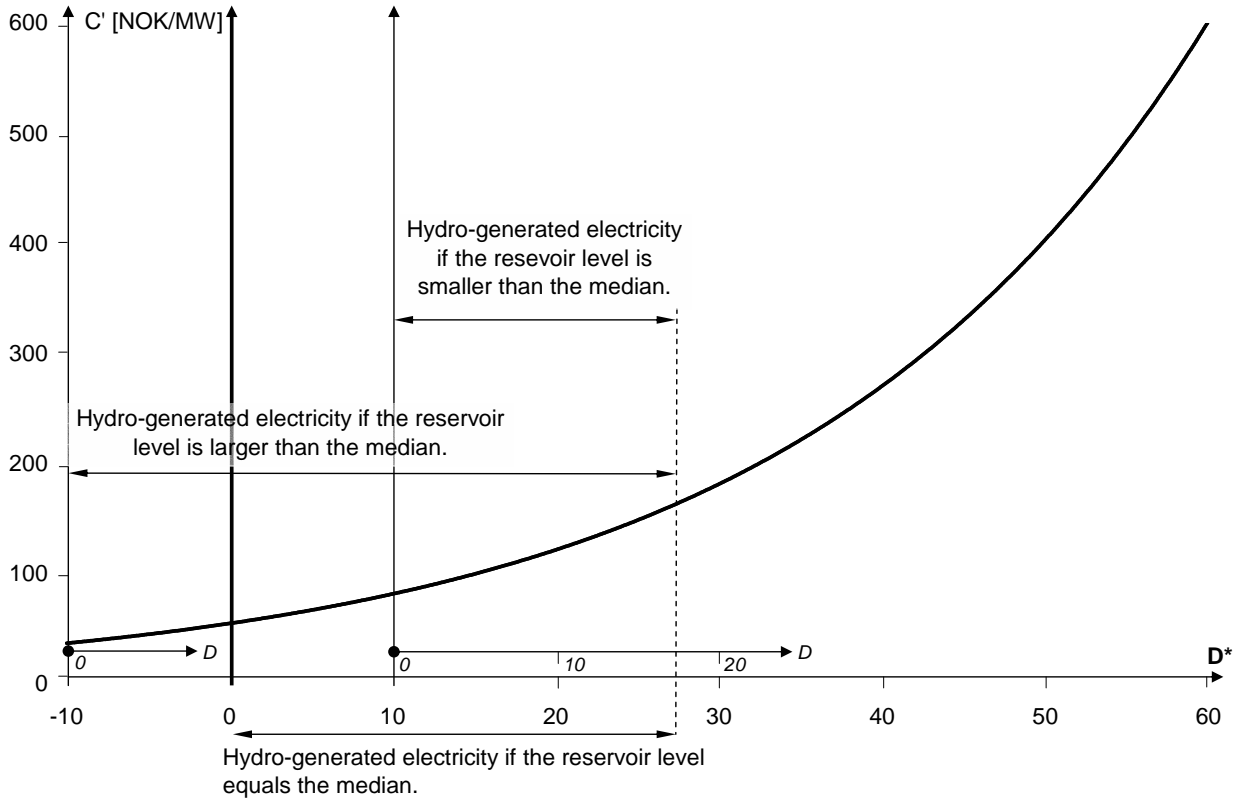
Figure 1: This figure illustrates how the marginal cost function shifts depending on the water reservoir level. D^* denotes the hypothetical production quantity if the water reservoir level was equal to the median level.

This choice is motivated by two reasons: First, on a weekly basis we found that a sophisticated estimation of the deterministic component $\bar{D}_t$ strongly reduces the higher order autocorrelations of the residuals $\tilde{S}_t = \tilde{D}_t - \bar{D}_t$, and an autoregressive process of first-order is sufficient according to the Akaike criterion. Second, the stochastic component of the spot price, X_t, in the reduced-form model by Lucia and Schwartz has to be estimated as AR(1) by construction. In order to conduct a fair comparison we adopt the same lag structure.

The cost function and the marginal cost function are defined on the end-user demand that equals the production volume. We assume an exponential function for the marginal cost. The production costs depend on the available capacity of the different power plants within the Nord Pool area. The major role in the production of electricity play hydro plants. Their current and future capacity depend on the actual and the estimated future level of the total water reservoirs. This level exhibits a deterministic pattern due to the average seasonal precipitation throughout a year and the periods of melting and freezing. The expected level of this seasonal reservoir level determines the constant amount of electricity generated by

hydro plants. Deviations from this base level are caused by not anticipated components of water reservoir levels like unusual rainy or dry periods, e. g. during summer 2002. Unexpected deviations from the expected seasonal level result in an immediate reduction or extension of the hydro production with the goal to smooth the total future production.

As hydro plants represent the cheapest technology for generating electricity an adjustment of their production reduces or extends the first and constant section of the marginal cost function. In other words, the marginal cost function shifts by an amount that depends on the deviation from the median reservoir level, cf. Figure 1. This effect is modeled by defining a reservoir-corrected end-user demand D_t^* as follows:

$$D_t^* = D_t + \gamma^* WRD_t. \tag{9}$$

WRD_t denotes the deviation from the median water reservoir level and the parameter γ^* translates a surplus (lack) of stored potential energy measured in percentage-points into additional (missing) production capacity. WRD_t is not modeled as a second stochastic state variable, but as a time-dependent model parameter. This translation of demand and production quantity results in the following modified marginal cost function

$$C'(D_t) = \exp(c_0 + c_1 D_t + \gamma WRD_t), \quad \gamma = \gamma^* c_1 \tag{10}$$

with time-invariant coefficients c_0, c_1, and γ.

2.3 Comparative Statics

From the recursive futures price formula (6) we are able to derive comparative static results for futures prices and for risk premia numerically. Figure 2 shows the term structure of risk premia for three different levels of the initial demand.

The term structure typically exhibits a local extremum. Local maxima occur for large levels of initial demand and local minima for low levels. The term premia converge to a small value for large maturities. This value can be positive or negative, depending on the parameter settings.

For the reduced-form model, the risk premium $-\lambda \frac{\sigma^X}{\kappa}(1 - e^{-\kappa T})$ can be represented ana-
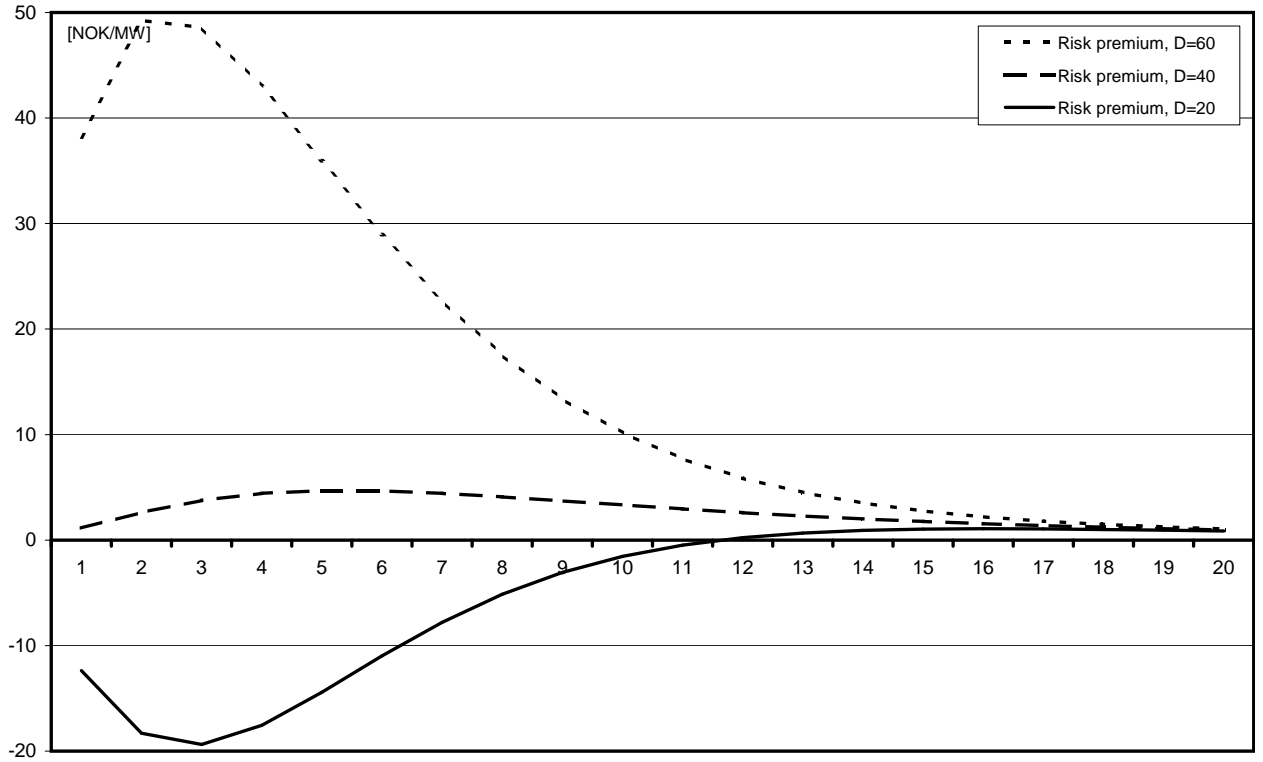
Figure 2: This figure shows the term structure of the risk premium in the equilibrium model with an exponential function of marginal cost for three states of demand. For $D = 60$ GW, the spot price is equal to $270.43 NOK$, for the median case of $D = 40$ it is 148.41, and for $D = 20$ the spot price is equal to 81.45.

lytically. It is monotone and asymptotically approaches $-\lambda \frac{\sigma^X}{\kappa}$. As σ^X and κ are strictly positive, the sign of λ determines whether the risk premium is positive or negative.

3 Contracts at Nord Pool

For our empirical study we choose the Scandinavian electricity market because it is one of the most early liberalized electricity markets in Europe. Trading at the Scandinavian electricity exchange Nord Pool began in 1992. Furthermore, Nord Pool not only provides price data, but also time series on electricity consumption and production volumes and on the water reservoir level of the Scandinavian hydro plants. These plants account for about 55 % of the production.

For our purposes, two market segments at Nord Pool are interesting: The spot market and the so-called financial market. At the latter one, futures, forwards, and options are traded.

All prices used in our study are given in NOK/MW.[1] In the spot market electricity is traded for physical delivery during each single hour of the subsequent day (or days, if weekends or exchange holidays follow). This market is organized as a single auction market. All traders have to provide their buy or sell offers for each hour of the subsequent day. Nord Pool then calculates 24 hourly market clearing prices. The equally weighted average of those prices is called the system price which we refer to as the daily spot price hereafter.

In the financial market forwards and futures are traded continuously. The underlying of all contracts is the 24-hour-delivery of electricity per day at a constant rate during a specified delivery period. All contracts are cash settled. The delivery periods of the listed contracts range from one day to one year. Futures are used for shorter delivery periods, forward contracts are listed for delivery periods of one month and longer. The time-to-maturity of the listed contracts, i. e. the time until the delivery period begins, ranges from two days up to three years. In detail, the following contracts are listed or were listed during our observation period:

- *Day futures* with a delivery period of one day are listed with a time-to-maturity between two and a maximum of ten days. These contracts show a rather low liquidity.

- *Week futures* have a delivery period from Monday through Sunday and a time-to-maturity up to eight weeks. They are the most actively traded contracts at Nord Pool, however, liquidity decreases with increasing time-to-maturity.

- *Block futures* cover a delivery period of four weeks. The listed maturities comprise the time interval from eight up to 48 weeks. Block futures were actively traded but were successively replaced by moth forwards in 2003.

- *Month forwards* were introduced in 2003. Their delivery periods equal the calendar months, i. e. 28 to 31 days. The listed maturities cover the subsequent six months.

- For longer delivery periods, *quarter*, *season*, and *year forwards* are or were traded at Nord Pool. These contracts are the least liquid ones, sometimes only one trade per week is documented.

---

[1]Between 2003 and 2006, Nord Pool subsequently switched from NOK to Euro.

Some of these contracts have a special cascade feature that is not found in other commodity markets. A certain time before maturity they are split up into into a volume-equivalent bundle of contracts with shorter maturities. In detail, block futures are split into the corresponding four week futures contracts eight weeks before its delivery period begins. For the longer maturities, year forwards were split into season forwards and are nowadays split into quarter forwards that in turn are split into month forwards.[2] This cascade structure allows the market participants to hedge their exposures more precisely for closer maturities. Note that forward contracts are not split into futures.

The financial market Nord Pool only provides so-called closing prices that are used for the daily settlement of futures. The closing price is the last registered trading price at a randomly chosen point in time within the last ten minutes of trading.[3] If there are no trades at a certain exchange day, Nord Pool uses several procedures to estimate a closing price. As Nord Pool also provides the daily trading volumes for each contract, we can identify closing prices that are based on trading.

4   Descriptive Statistics

Nord Pool published daily spot prices since 1992. However, we do not use spot prices before 11/01/1996, however, because Finish electricity firms gained access to Nord Pool in October 1996.[4] The available times series of daily production, consumption, import, and export volumes start at 03/29/1999.[5] The water reservoir level and its deviation from the historical median are measured once a week and are available since 1996, daily futures prices and trading volumes since 1995. We only consider transaction prices and not prices that were set by Nord Pool for settlement purposes only.

For the fixed tariff rate p in the equilibrium model we use the average price of a "1-year/new fixed-price contracts" for households and industrial customers. This price is provided as a quarterly time series by Statistics Norway. For the interest rate r we use the 3-month-NIBOR (Norwegian Interbank Offered Rate) provided by the central bank of Norway.

---

[2]The types and the cascade structure of the listed derivatives at Nord Pool are subject to current changes.
[3]See Nord Pool ASA (ed.) (2004) for details.
[4]Nord Pool ASA (ed.) (1998), p. 9.
[5]Prior to that date, the referred time series were available for only some of the countries.

All time series end on 08/04/2004. This provides us with a period of 64 months for the shortest time series, the production and consumption volumes. As we need a period of over one year to estimate the parameters of the models, we test both models for a period of 50 months from 05/24/2000 to 08/04/2004 (cf. Section 5 for details).

4.1 Daily Spot Prices

In this section we present descriptive statistics of daily spot prices, daily demand, and the weekly measured water reservoir level for the 64-months period from 03/29/1999 until 08/04/2004. Quantities will be given in GW, prices in NOK/MW if not denoted otherwise.

Figure 5 in the Appendix plots the level of the daily spot price and its first difference during the five-year period and Table 1 shows the corresponding descriptive statistics. As mentioned before, the spot price exhibits some seasonality in that the prices during the winter time are usually higher than in the preceding summer. However, this seasonality is superimposed by strong changes in the mean level of the spot price. Table 1 shows that in the last two 12-month periods of our observation period the yearly mean as well as the median were about twice as high as in earlier periods. We also observe a fluctuating standard deviation.

The rather high values for the kurtosis in combination with the mostly positive skewness reflect the occasional spikes in the spot price process. The price spikes usually occur in the winter time. In Figure 5 we marked two examples of single spikes, A and B, and a period of high prices and frequent spikes, C. Spikes A (387.78 NOK/MW) and B (633.36 NOK/MW) happened on 01/24/2000 and 02/05/2001, respectively, and disappeared one day later. They are caused by an exceptionally large electricity demand on those two days. This can be seen in Figure 6 that plots the daily demand volumes.

The third example, C, covers a period of about four months of record high prices that cannot be explained by demand alone. We argue that these large spot prices are caused by unusually low water reservoir levels. The period of low precipitation began in summer 2002. Figure 8 shows that during this period the water reservoir deviation from the long term median is negative and decreasing. As a consequence the production capacity of hydro plants was reduced and producers were forced to run plants with higher variable costs.[6] This period

---

[6]Cf. Bye (2003) for a detailed analysis of this period in Norway.

	01.04.97– 31.03.98	01.04.98– 31.03.99	01.04.99– 31.03.00	01.04.00– 31.03.01	01.04.01– 31.03.02	01.04.02– 31.03.03	01.04.03– 31.03.04	whole sample
Daily Spot Price, P_t [NOK/MW]								
Obs.	365	365	366	365	365	365	366	2841
Mean	130.27	110.52	108.98	125.33	178.08	258.84	252.70	171.39
Std. dev.	29.14	31.69	30.37	60.11	27.15	162.99	33.23	90.45
Skewness	−0.018	−0.202	2.381	2.511	0.710	1.399	−0.388	2.378
Kurtosis	−0.29	1.24	19.12	15.22	2.40	1.65	1.20	10.47
Minimum	58.21	21.27	50.43	31.85	119.07	80.65	128.91	21.27
Maximum	234.25	266.47	387.78	633.36	335.80	831.41	343.25	831.41
Daily Spot Price Differences, $P_t - P_{t-1}$ [NOK/MW]								
Obs.	365	365	366	365	365	365	366	2840
Mean	0.04	−0.07	0.03	0.26	−0.22	0.33	0.02	0.00
Std. dev.	10.09	11.23	20.02	37.50	17.40	19.66	16.42	20.09
Skewness	0.750	−0.599	3.920	3.156	1.722	−0.223	0.964	3.358
Kurtosis	3.54	21.12	111.60	66.41	18.82	6.71	4.77	121.88
Minimum	−42.02	−96.44	−197.32	−264.29	−98.79	−95.92	−55.65	−264.29
Maximum	45.58	76.81	265.49	440.59	151.74	94.00	89.03	440.59

Table 1: Descriptive statistics for the daily spot price P_t and its first differences at Nord Pool in period between 11/01/1996 and 08/10/2004 and for subperiods of 12 months each.

will be a challenge for both models.

The high absolute values of the minimum and maximum in Table 1 are an obvious consequence of the price spikes. The high kurtosis and the mostly positive skewness do not support the assumption of normal distribution. However, note that the values in Table 1 reflect also the seasonal patterns in the spot prices. We will discuss the distribution of spot prices without the deterministic (seasonal) component in Section 6.

We tested the spot price series for unit roots with the Augmented Dickey-Fuller (ADF) test, including a constant. With a test value of −3.007 the hypothesis of a unit root is rejected on the 5%-level if we include up to 35 lags. However, in four out of 217 estimation periods we found a unit root in the spot price series after having extracted the deterministic components on a weekly basis (cf. Section 6.2).

	all futures	week futures	block futures	W1	W2	W3	W4	W5	W6	W7	W8
Obs.	5121	2258	2863	406	406	405	399	300	208	114	20
Mean	3.70	8.84	−0.36	4.26	7.04	8.50	9.06	10.61	13.10	14.87	36.05
Std. dev.	67.38	48.47	78.95	24.21	39.98	47.20	53.86	59.20	60.93	64.02	51.96
Skewness	−1.416	3.462	−2.111	5.168	4.333	4.114	3.610	2.260	2.704	2.186	0.450
Kurtosis	15.27	35.44	9.43	54.61	45.27	42.30	36.09	24.92	19.64	16.16	−1.52

	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	B11
Obs.	352	348	340	327	318	301	271	238	191	120	57
Mean	6.55	5.26	2.27	0.27	−1.03	−3.62	−5.28	−4.55	−2.37	−5.67	−16.62
Std. dev.	65.19	74.88	78.06	79.87	76.22	76.89	83.63	86.44	82.86	91.54	104.91
Skewness	−0.084	−0.996	−1.857	−2.299	−2.744	−2.757	−2.610	−2.409	−2.162	−2.147	−2.324
Kurtosis	11.48	11.73	10.20	9.38	10.56	9.98	8.54	7.55	6.99	6.28	5.27

Table 2: This table shows the ex-post calculated differences between observed futures prices before maturity and realized spot prices during the delivery period. We used weekly average futures prices during the period 11/01/1996 until 08/10/2004.

4.2 Futures Prices

Futures prices also exhibit seasonal patterns as well as an increasing trend during the observation period. We therefore abstain from presenting descriptive statistics on futures prices, but focus on the differences between futures and spot prices. Following Longstaff and Wang (2004), we calculate ex-post differences between futures prices $F_{t,T}$ observed at time t and realized spot prices P_T during the delivery periods of the futures contracts. These differences provide a first empirical insight into the term structure of the risk premia. To avoid a possible bias in our estimates which results from the larger liquidity of short-term contracts, we take weekly averages of traded prices for each contract instead of daily prices. As a consequence, a contract with only one transaction during a certain week has the same weight as more actively traded contracts.

Table 2 shows the results. Wx denotes week futures with x weeks until maturity, Bx denotes block futures with x 4-week periods until decomposition in four equivalent week futures. We find positive ex-post differences on average over all contracts. This result is remarkable as the spot prices on average showed an increasing trend during the observation period. If that trend was not anticipated by the market participants, one would expect negative ex-post differences. Furthermore, we also find positive ex-post differences for week futures, but

differences close to zero for block futures. For week futures the difference increases with the maturity.

We observe positive, but decreasing ex-post differences for the first four block futures and slightly negative ones for the block futures with longer times to maturity. Note that there are only few observations for the W8 and the B11 contract. These differences can be understood as estimates of the risk premium. Therefore, our results support, basically, the term structure of risk premia as derived in the equilibrium model.

The standard deviation of the ex-post differences increases with maturity. This observation reflects the increasing forecasting uncertainty and results in insignificant ex-post differences for the block futures. Applying a t-test with Newey/West correction shows that only the ex-post differences for the W1, W2, W3, W6, and W7 contracts are significant at the 5%-level.

4.3   Daily Electricity Production and Demand

For the equilibrium model we need the total electricity consumption or production in the Nord Pool area. If this was a closed market, the end-user consumption of electricity had to equal the production (after transmission losses) at each point in time. However, a part of the average daily used electricity can be exported from or imported into the Nord Pool area. Since the spot price results from market clearing of total demand and supply, we define the electricity supply as the production in the Nord Pool area plus the electricity import. Analogously, we refer to the demand as the consumption in the Nord Pool area plus the export into countries outside of this area. According to the non-storability of electricity, these two quantities must be equal at each date. On a daily basis, we find that those two quantities differ by less than 1% in 99% of all days. The correlation between them is 99.96 % and 99.77 % for the first differences in the observation period. We attribute these minor differences to measurement errors.

Figure 6 shows the daily demand as defined above. This time series exhibits a very strong seasonal component. The differences between demand volumes in the same period of different years are much smaller in absolute and percentage terms than for the time series of spot prices. Table 3 underlines this observation. Compared to Table 1 for the spot prices, the characteristics of the sample distribution are much more stable for the yearly subperiods.

	01.04.99– 31.03.00	01.04.00– 31.03.01	01.04.01– 31.03.02	01.04.02– 31.03.03	01.04.03– 31.03.04	whole sample
Daily Electricity Demand, D_t [GW]						
Obs.	366	365	365	365	366	1961
Mean	38.58	40.28	40.14	40.67	39.41	39.44
Std. dev.	6.69	6.98	7.15	6.95	7.05	6.96
Skewness	0.104	0.351	0.002	−0.017	0.053	0.181
Kurtosis	−1.13	−0.93	−1.21	−1.13	−1.10	−1.04
Minimum	25.58	26.38	25.19	25.65	25.14	25.14
Maximum	52.74	58.19	54.18	54.97	56.30	58.19
First Differences of Daily Electricity Demand, D_t [GW]						
Obs.	366	365	365	365	366	1960
Mean	0.01	−0.00	−0.02	0.02	−0.00	−0.00
Std. dev.	2.30	2.28	2.41	2.34	2.24	2.30
Skewness	0.530	0.647	0.539	0.549	0.365	0.531
Kurtosis	0.68	0.50	0.49	0.43	0.64	0.54
Minimum	−6.42	−4.70	−6.34	−5.71	−7.47	−7.47
Maximum	7.06	6.63	6.92	6.44	5.98	7.06

Table 3: Descriptive statistics for the daily electricity demand in the Nord Pool area, measured in GW. Statistics are given for the whole sample that comprises the 64-month period between 29/03/1999 and 08/10/2004 (1961 observations) and for subperiods of 12 months.

Like the spot prices the demand shows downward spikes for the week-ends. The strongest downward spike can be observed at the Scandinavian holiday midsummer (end of June). By visual comparison of the two time series we find that the demand volume not only explains the two price spikes A and B, but also part of the general behavior of the spot price as both rise in the autumn and winter time and decrease in the spring time until summer. The correlation between daily demand and the spot price is 43.7 % and 48.8 % for their first differences.

The relation between the spot price and the demand volume is non-linear. Figure 7 shows the scatter plot of the daily spot price and the demand volume. We draw two conclusions from this plot. First, for the same demand volume the spot price varies considerably. An electricity demand of 40 GW, e.g., is related with spot prices between 80 and 350 NOK/MW. Therefore, there exists at least one more factor that has an important impact on the spot prices. In the next subsection we will identify the deviation of the water reservoir level from its median as an additional determinant. Second, under the assumption of a competitive

market among producers the spot price equals the marginal cost of the last produced unit of electricity. Given this assumption, Figure 7 indicates qualitatively that the marginal costs are slightly, progressively increasing in the daily production volume.

There are at least three possible reasons why the spot prices could deviate from the unobservable marginal cost function:

1. The system prices at Nord Pool are usually determined one day, sometimes several days before delivery. Therefore, the two quantities are determined asynchronously.

2. By using daily data we average across 24 hourly data of demand volumes and spot prices, i. e. points of the marginal cost function. As the marginal cost function is presumably convex, the average spot price is an upward biased estimator of the spot price for the average demand volume. The absolute amount of this bias depends on the production level.

3. The marginal cost function may vary over time. Apart from temporary plant outages and from building new or breaking down old power plants, the level of the water reservoirs and thus the production capacity of hydro plants has a major impact.

In our analysis we will not model the first two of the above mentioned effects, but take into account the third one as we consider it to be the most crucial one.

4.4   Water Reservoir

The water reservoir level for Norway, Sweden, and Finland is published once a week in percentage points of the total reservoir capacity in the Nord Pool area. Figure 8 in the Appendix shows the time series of percentage levels and the deviation from its median in percentage points of the total reservoir capacity. Table 4 presents the the key sample statistics of the deviation. Note that the seasonal pattern in the level series is caused by the melting period starting in April and the freezing period starting around November. The first one increases the inflow into the water reservoirs while in the freezing period the inflows are reduced. The seasonal component of the production volumes, i. e. the water outflow, has only a minor impact on the reservoir level.

	01.04.99– 31.03.00	01.04.00– 31.03.01	01.04.01– 31.03.02	01.04.02– 31.03.03	01.04.03– 31.03.04	whole sample
Obs.	52	52	52	52	53	282
Mean	3.82	4.96	−0.69	−8.38	−14.54	−3.66
Std. dev.	3.75	2.52	5.92	9.48	3.72	9.39
Skewness	−0.190	0.483	−0.325	0.007	−0.393	−0.397
Kurtosis	−1.51	−0.12	−1.28	−1.58	−0.95	−1.24
Minimum	−2.11	0.20	−11.63	−21.99	−22.05	−22.05
Maximum	9.17	10.94	6.79	5.94	−9.61	10.94

Table 4: Descriptive statistics of the weekly water reservoir deviation from its median in the Nord Pool area, measured in percentage-points of the total reservoir capacity. Statistics are given for the whole sample that comprises the 64-months period between 04/01/1999 and 08/10/2004 (280 weekly observations) and for subperiods of 12 months.

Market participants know and anticipate the seasonal fluctuation of the reservoir level. Therefore, as the scatter plot in Figure 9 shows, the absolute reservoir level contributes only little to the explanation of spot prices. The correlation between the first differences of the spot price and the reservoir level is only −3.7%.

The picture changes strongly if we replace the absolute reservoir level by its weekly deviation in percentage-points from the observed reservoir level and its long-term median. We choose the median instead of the mean because this value is published by Nord Pool and serves as a reference for all market participants. Figure 10 shows a clear negative relation between the reservoir deviation and the spot prices. The reason for this relationship is fairly obvious. If a relatively high electricity demand coincides with below median water reservoir levels, this demand has partly to be served by expensive plants like gas-fired turbines, i. e. the marginal costs and, therefore, the spot prices are large. Vice versa, if there is an unexpected reservoir surplus, a larger proportion of demand can be served by the cheap hydro plants.

5 Estimation Procedure

The basic structure of our estimation procedure is described in Figure 3. The total test period runs from 05/24/2000 to 08/04/2004, i. e. 220 Wednesdays. We observe futures prices on 216 of these Wednesdays, four of them are holidays. We take futures prices from the following Thursday if that is not a holiday. Otherwise we skipped a week. This leaves
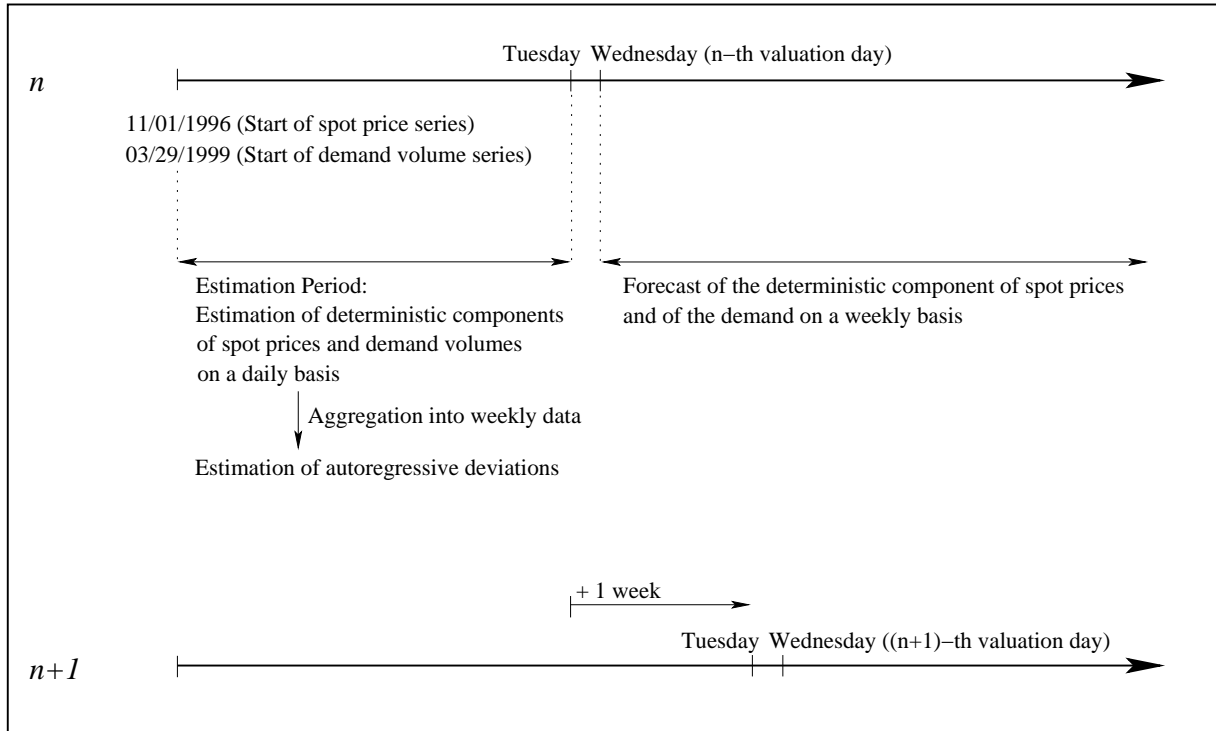
Figure 3: This figure shows the moving time windows for estimating the deterministic and the stochastic part of the spot prices and for valuation of futures prices. We evaluate futures prices on 220 Wednesdays, i. e. $n = 1, \ldots, 220$, whereof we skip three that were holidays. The first valuation day is Wednesday, 05/24/2000, the last one is Wednesday, 08/04/2004.

us with 217 observations. For each of these valuation days we determine theoretical futures prices using data up to the preceding Tuesday. These theoretical futures prices are compared with observed prices.

We choose Wednesdays as valuation dates for two reasons: First, Wednesdays provide us with the largest sample of trading days as very few holidays happen to be on Wednesdays. Second, the weekly reservoir data are published on Wednesdays at 1 p. m. These data refer to the reservoir level of the preceding Monday. Therefore, futures prices of Wednesdays are the earliest ones that incorporate the new information on the water reservoir level.

For each of the 217 valuation days we perform the following estimation steps. First, we estimate the deterministic component of the spot price in the reduced-form model, $f(t)$, and of the end-user demand in the equilibrium model, $\bar{D}(t)$, using daily data. This data frequency is used to capture as much information as possible. We apply dummy variables for day-of-week and holiday effects and a continuous, piecewise linear function to represent the

seasonal patterns.[7] These estimated daily demands and spot prices are averaged to weekly deterministic time series. Second, we average the observed daily spot prices and demand volumes to obtain weekly data. These weekly data are used to determine the parameters of the stochastic components of the spot price and the demand volume. By two reasons we consider only weekly data in our estimation procedure: First, the computation time for the equilibrium model increases with the order $O(N^2)$ in the number of time steps. Using daily data would lead to unacceptable long computation times. Second, we only consider week and block futures as those are the most liquid ones. Thus, daily calculations are not necessary.

As explained in Section 2.2.2, we also consider the deviation from the median water reservoir level as an additional determinant for the marginal costs in the equilibrium model. This determinant is not modeled as a second stochastic factor, but as a time-dependent parameter. This simplification implies that the current water reservoir deviations is used as a predictor for future water reservoir deviations.

The weekly differences between the observed and the deterministic values of the spot price and the demand volume, respectively, define the stochastic deviations X_t and S_t in (1) and (8).

5.1 Estimating the Reduced-form Model

Our procedure to estimate the parameters of the reduced-form model consists of the following seven steps:

1. We estimate the deterministic components of the spot price process with daily data. The seasonality within a year is modeled by 12 overlapping triangular functions. To account for the week-end effects we introduce dummy variables for Fridays, Saturdays, and Sundays. We furthermore add dummy variables for a specified set of 24 holidays days. For each of the 217 Wednesdays (valuation days), we use an estimation window from 11/01/1996 until the preceding Tuesday. A more detailed description of this step is given in Section A of the Appendix.

---

[7]Lucia and Schwartz (2002) use two alternative methods: a cosine-function and a piecewise constant function.

2. Based on the 217 regression results of Step 1 we calculate time series of deterministic spot prices, $f(T)$, for the future maturity dates T of futures contracts up to 08/04/2005.

3. We aggregate the forecasted deterministic spot prices as well as the observed spot prices to obtain weekly mean prices. The difference of these prices yields the weekly time series of the spot price deviation, X_t.

4. We model the time series X_t as a first order autoregressive process

$$X_t = \phi X_{t-1} + \epsilon_t^X \tag{11}$$

and use the estimate $\hat{\phi}$ as a proxy for the speed of adjustment, $\hat{\kappa} = 1 - \hat{\phi}$, and the standard deviation of the regression as a proxy for σ^X.

5. Finally we calculate theoretical prices for week futures. If the agents are assumed to be risk-neutral, the futures prices are obtained from (2) by setting λ to zero. Otherwise, λ is obtained implicitly as described in Section 5.3. The prices of block futures are computed as the average of the prices of the underlying week futures.

Our estimation design differs in two aspects from the estimation design by Lucia and Schwartz (2002): First, we use a different approach for extracting the deterministic components. Second, we aggregate daily data into weekly data.

5.2 Estimating the Equilibrium Model

We analyze the equilibrium by the following seven steps:

1. The seasonal component of the daily end-user demand in the Nord Pool area is estimated analogously to the seasonal component of the spot price in the reduced-form model. The estimation window begins on 03/29/1999 because Nord Pool does not provide a longer times series for demand or production data. The estimation window ends at the Tuesday of each particular valuation week.

2. We sum up the estimated daily seasonal volumes to obtain weekly seasonal demand data. Analogously, we obtain the observed demand volumes per week. The differences

between these two time series provides us with the stochastic component $\tilde{S}_t$ on a weekly basis.

3. We estimate the parameters $\hat{\rho}$ and $\hat{\sigma}^S$ of the AR(1)-process $\tilde{S}_t$ as defined in (8).

4. For the estimation of the marginal cost function $C_t'(D_t^*)$ we simultaneously estimate the slope coefficient c_1 and the shift variable γ of the water reservoir deviation. Taking the logarithm on (10) leads to the linear estimation equation

$$\ln(P_t) = c_0 + c_1 D_t + \gamma WRD_t + \epsilon_t^c \tag{12}$$

where autocorrelation is considered in ϵ_t^c.

5. We receive the cost function by integrating over the marginal cost function. We cannot observe the fixed costs of production, however, as those are independent from the demand, they are not needed for evaluating the risk premia.

6. The last necessary parameters are the tariff rate p that retailers charge their customers and the interest rate. We linearly interpolate between the quarterly published tariff data in order to achieve a weekly time series. The calculated weekly tariff prices vary between 130 and 227 NOK/MW. For the interest rate we use daily observations.

7. Finally, we calculate theoretical prices for week futures. Under risk neutrality ($\xi = 0$) they are easily obtained from (6). If agents are not risk-neutral, ξ is determined implicitly as described in the next section. The theoretical futures prices are determined recursively by dynamic programming.

5.3 Evaluation of Futures Prices

Our valuation analysis consists of three steps: First, we evaluate both models out-of-sample as if market participants were risk-neutral, i. e. we assume $\lambda = 0$ for the reduced-form model and $\xi = 0$ for the equilibrium model.

In the second step, we introduce risk aversion. For both models we estimate the implicit risk aversion parameters by minimizing the root mean squared error (RMSE) between the observed and the model prices by using observed futures prices of four subsequent Wednesdays.

	Average Results				Full Sample
	Mean	Std. dev.	Minimum	Maximum	
	Reduced-form Model				
Estimation window					11/01/1996 − 08/03/2004
Observations		217 estimations			2833 daily obs.
$\bar{R}^2$	0.917	0.041	0.848	0.965	0.965
S. E.	16.23	2.30	11.12	18.03	17.02
	Equilibrium Model				
Estimation window					03/29/1999 − 08/03/2004
Observations		217 estimations			1952 daily obs.
$\bar{R}^2$	0.986	0.001	0.984	0.987	0.986
S. E.	0.800	0.027	0.720	0.837	0.810

Table 5: Key results of the deterministic components for daily data on average and for the last estimation, i. e. the full sample.

We use only traded week- and block-futures, but no forward contracts. For calculating the RMSE and other deviations, we weight each observation by the volume of the underlying. A block futures price thus has four times the weight of a week futures price.

For the equilibrium model we implemented the state space of the end-user demand as a grid with 21 points for each time step. The reservoir deviation is set to its most recent value. We identify the planning horizon Θ with the longest futures' maturity plus four more time steps. An earlier analysis revealed that this amount suffices in order to suppress finite horizon effects.

In the third step, we conduct a second out-of-sample test by incorporating the previously estimated parameters λ and ξ. The comparison of the out-of-sample results assuming risk-neutral or risk-averse agents allows us to draw conclusions about the importance of the risk premia in electricity futures prices.

6 Results

6.1 Deterministic Components

In Table 5 we present some basic regression results for the deterministic components of the demand and the spot prices. Detailed results for the full observation period are given in Table 12 in the Appendix. We find that the holiday effects are more pronounced in the demand volume than in the spot price. 22 out of 24 holidays have a significant effect on the demand on the 1%-level. On the 5%-level, all 24 days significantly effect the demand and only 14 holidays effect the spot price. Furthermore, all coefficients of the triangular basis functions are significantly different from zero for the demand, but only four are significant for the spot price on the 5%-level. The dummy variables for Fridays, Saturdays, and Sundays are all highly significant for the spot price as well as for the demand volume.

Table 5 shows that the R^2 is higher for the demand volume than for the spot price. The standard error of the regression, standardized on the mean of spot prices or daily demand, respectively, is larger for the spot price than for the demand. These results reflect the intuition from Figures 5 and 6 that the seasonal patterns are much stronger in the demand series than in the spot price series.

Figures 11 and 12 in the Appendix exemplarily plot the deterministic components of both models for the estimation period. Visual inspection affirms that the deterministic component of the demand explains total demand much better than in the case of the spot price. The residuals of the spot price may even exceed the deterministic component. The regular downward spikes in the deterministic components as well as in the observed data are due to the end-of-week effects. We like to point out that the use of monthly dummy variables instead of triangular functions would result in a rather irregular behavior of the stochastic component for both exogenous variables.

6.2 Stochastic Processes

As described in Section 5 we estimate the AR(1)-processes of the residuals X_t and S_t, respectively, from weekly data by non-linear least squares. The upper part of Table 6 shows

Parameter	Average Results				Full Sample		
	Mean	Std. dev.	Minimum	Maximum	Estimation	t-stat.	p-value
			Reduced-form Model				
ϕ^X	0.906	0.041	0.806	1.040	0.951	60.05	< 0.001
$\kappa = 1 - \phi^X$	0.094	0.041	-0.040	0.195	0.050		
$\sigma^X = $ S. E.	19.74	5.94	12.24	28.02	26.28		
R^2	0.824	0.059	0.660	0.899	0.899		
Jarque-Bera					17346		
Durbin-Watson					1.80		
			Equilibrium Model				
ρ	0.526	0.095	0.182	0.632	0.580	12.09	< 0.001
$\sigma^S = $ S. E.	1.306	0.096	1.065	1.428	1.321		
R^2	0.3025	0.080	0.042	0.412	0.345		
Jarque-Bera					21.79		
Durbin-Watson					1.95		

Table 6: Descriptive statistics and estimation results of the weekly AR(1)-processes of the stochastic component of the spot price, X_t, and of the demand, S_t, respectively. The left part shows the average of all estimates, the right part makes use of the full observation period.

descriptive statistics of X_t, the lower part the main estimation results for S_t.

The higher moments and the Jarque-Bera statistic in Table 6 show that the assumption of normal distribution is acceptable for the stochastic demand S_t, but not for the stochastic spot price X_t. Especially the large kurtosis of X_t reflects fat tails in the distribution of spot prices. The Durbin-Watson statistic supports the modeling of autoregressive processes for both series.

For the 217 time series of the stochastic component of the spot price, X_t, we find a mean autocorrelation coefficient $\hat{\phi} = 0.906$. For the whole estimation window $\hat{\phi}$ is equal to 0.951. $\hat{\phi}$ is larger than 1 in four out of the 217 estimation windows. A Dickey-Fuller test (four lags, without a constant) does not reject the hypothesis of a unit root in five cases (significance level of 5%-level). The estimation periods in these five cases end between 12/04/2002 and 01/15/2003. This is the period when daily spot prices at Nord Pool reached new record levels, cf. to period C in Figure 5. These cases are not excluded from the subsequent analysis.

For the stochastic component of the demand, S_t, the mean autocorrelation coefficient, $\hat{\rho}$, equals 0.526, and 0.580 for the full observation period. With a maximum across all estimation windows equal to 0.632, the process S_t is always stationary. We find rather low values of

Parameter	Average Results				Full Sample		
	Mean	Std. dev.	Minimum	Maximum	Estimate	t-stat.	p-value
c_0	0.377	0.47	3.18	4.72	4.64	35.43	< 0.001
c_1	0.0322	0.0065	0.0203	0.0410	0.0209	21.67	< 0.001
γ	−0.0135	0.0062	−0.0320	−0.0019	−0.0094	−2.19	0.029
R^2			0.944			0.963	

Table 7: This table shows key statistics for the estimated coefficients of the marginal cost function on average over all estimation periods and for the full sample, i. e. the last period. The estimates for the full sample are based on 281 observations between 03/31/1999 and 08/10/2004.

$\hat{\rho}$ around 0.20 for the first few estimation intervals which are artefacts due to the short observation periods. We did not exclude those values neither.

The autocorrelation coefficient and the R^2 for the stochastic spot price X_t are much larger than for the stochastic demand S_t. This result corresponds to the findings in Section 6.1 that a great amount of demand variation can be explained by seasonal patterns, while the spot price fluctuates more randomly. Figures 11 and 12 show the residuals X_t and S_t, respectively, on a daily basis. The former exhibits only a weak mean-reversion characteristic, the latter, however, quickly oscillates around zero.

6.3   Marginal Cost Function

Table 7 shows the average parameter values of the marginal cost function and the estimates for the full sample. The parameter values of c_0 and c_1 are positive and affirm the assumption of an increasing marginal cost function. As expected, the estimates of γ are negative. The mean of $−0.0135$ shows a reduction of the aggregate hydro plant production by $0.0135/0.0322 = 0.42$ GW if the water reservoir level is one percentage point below its median for a particular day. Since we determine γ on a weekly basis, a production of 0.42 GW during one week is equivalent to 70.6 GWh. The total reservoir capacity of the hydro plants in the Nord Pool area amounts to approximately 123,500 GWh, i. e. one percentage-point is equal to 1,235 GWh. This figure implies that in case of a negative water reservoir deviation hydro plant operators reduce their production such that approximately 6% of the deviation is recovered after one week.

Figure 13 in the Appendix shows the relation between D_t and the weekly spot price, and

	Reduced-form Model		Equilibrium Model	
	$f_{t,T}^{\text{A,RF}}$	$|f_{t,T}^{\text{A,RF}}|$	$f_{t,T}^{\text{A,EQ}}$	$|f_{t,T}^{\text{A,EQ}}|$
Mean	−25.12	44.54	−37.03	43.62
Median	−24.23	25.65	−25.60	28.87
Minimum	−527.59	0.0039	−561.38	0.0006
Maximum	1988.76	1988.76	75.17	561.38
Std. dev.	105.55	98.94	59.70	55.07
Skewness	9.400	10.669	−3.703	4.578

Table 8: This table shows descriptive statistics of the valuation errors $f_{t,T}^{\text{A,RF/EQ}} = \hat{F}_{t,T}^{\text{A,RF/EQ}} - F_{t,T}$ and of the absolute valuation errors $|f_{t,T}^{\text{A,RF/EQ}}|$. Differences are given in NOK. The valuation period covers the interval from 05/03/2000 to 08/10/2004. The study is based on 2,046 futures prices.

Figure 14 plots D_t^* versus the weekly spot price and the estimated marginal cost functions for the last valuation day.

6.4 Futures Prices under Risk-neutrality

We first determine theoretical futures prices under the assumption of risk-neutrality. This case is considered as a benchmark for the identification of the seasonal components and the stochastic parameters of the processes.

In Table 8 we report descriptive statistics of the valuation errors $f_{t,T}^{\text{A,RF/EQ}} = \hat{F}_{t,T}^{\text{A,RF/EQ}} - F_{t,T}$ and of the absolute valuation errors $|f_{t,T}^{\text{A,RF/EQ}}|$ between estimated futures prices $\hat{F}_{t,T}^{\text{A,RF/EQ}}$ and observed futures prices $F_{t,T}$. The indices EQ and RF denote the equilibrium model and the reduced-form model, respectively. We find that the equilibrium model explains futures prices slightly better than the reduced-form model in terms of the mean absolute error, but not in terms of the median or the mean error. Both models clearly underestimate futures prices. Even though the mean and median results do not clearly favor one specific model, the variation does. The error range for the reduced-form model is roughly three times as large as for the equilibrium model. Moreover, the standard deviations show that the equilibrium model is more robust.

In order to gain an impression of the results, we calculated the mean deviation (MD) and the mean absolute deviation (MAD) across all observations for rolling intervals of eight weeks. Figure 17 in the Appendix depicts the results. We find that the reduced-form model provides good estimates in periods when the spot price does not fluctuate very much, e. g.

during summer 2000. Contrary, the equilibrium model provides better results when the spot price exhibits its characteristic spikes. For example, the extreme weather conditions in combination with a very low reservoir level in winter 2002/03 result in extremely high prices. During this period the equilibrium model explains futures prices considerably better than the reduced-form model.

We argue that there are two main reasons for the better performance of the production-based equilibrium approach during volatile markets: First, the reduced-form model cannot reflect the distribution of the spot price deviations X_t. As shown in Table 6, the Jarque-Bera statistic indicates that residuals from the AR(1)-estimation of the stochastic spot price are not normally distributed. The production-based approach in the equilibrium model introduces the skewness of spot prices by transforming normally distributed demand deviations, S_t, into right-skewed spot prices due to the increasing marginal cost function.

Second, the stochastic deviation of the spot price, X_t, represents a larger part of spot and futures prices in the reduced-form model, cf. Figure 11. A large value of X_t can be due to an extreme event (e. g., a week of cold weather) that does not affect futures prices or it can be caused by lasting changes in the determinants of electricity prices (e. g., a change in the water reservoir deviation) that does affect futures prices. The former case would require a small value of κ, the latter a large value of κ. The estimated $\hat{\kappa}$ is supposed to be somewhere in between.

For the stochastic part of the demand the described problem is of less importance: First, changes in the stochastic demand residual, S_t, are mostly caused by weather conditions and, thus, will in general fade out quickly. This is reflected by the low value of $\hat{\rho}$, cf. Table 6. Second, the deterministic component of the demand provides good estimates of future demand in comparison to the deterministic component of the spot price, cf. Figure 12. Therefore, S_t has a smaller impact on forecasted demands and marginal costs than X_t. We conclude that a large absolute value of X_t may lead to large deviations between the theoretical and the observed futures prices.

Figure 17 shows that the value of the mean meviation is mostly negative for both models, restating that both models tend to underestimate the observed futures prices. For the time period between Dec. 2002 and Jan. 2003, however, futures prices are strongly overestimated

in the reduced-form model. This period comprise those valuation days with estimates $\hat{\kappa} < 0$ (non-stationary processes of X_t).

A further analysis of the prices generated by the reduced-form model reveals that the futures prices of shorter maturities are typically evaluated quite accurately and sometimes overestimated. Futures prices of longer maturities, however, are clearly underestimated. We find a significant relationship between time to maturity and the valuation error if we exclude the observations during winter 2002/03 (Dec. 2002 until Feb. 2003). We do not find a corresponding relationship for the equilibrium model.

We conclude that the assumption of risk-neutrality results in futures prices that are on average lower than the observed ones, i. e. there exist implicit premia in futures prices. These premia are positive on average, i. e. the seller of a futures contract can earn a surplus. This conclusion confirms the empirical findings of Longstaff and Wang (2004) who empirically analyze forward prices at the Pennsylvania-New Jersey-Maryland market.

6.5   Estimation of Risk Parameters

We implicitly estimate the market price of risk, λ, and the risk parameter of the equilibrium model, ξ, by minimizing the root mean squared error between the theoretical futures prices and the traded prices of all block- and week-futures observed during a particular valuation interval of eight weeks.

In the case of the reduced-form model, we expect the market price of risk, λ, to be negative on average according to (2) and our considerations in Section 6.4. For the equilibrium model, the risk parameter has to be positive by construction if market participants are risk-averse. In order to compensate the underestimation of the futures prices in this model by a positive ξ, the sum of the covariances in (6) has to be negative on average.

Table 9 shows the aggregate results for the estimated risk parameters. The market price of risk, λ, is negative as expected. Note that the values for λ are much larger in absolute terms than those that Lucia and Schwartz (2002) report. Furthermore, Lucia and Schwartz (2002) report, surprisingly, mostly positive λ. Wilkens and Wimschulte (2007) also find positive values when evaluating the Lucia/Schwartz-Model with data from the German electricity market.

	Reduced-form Model	Equilibrium Model
	λ	ξ
Mean	-0.2389	0.1569
Median	-0.2632	0.0270
Minimum	-0.7998	-0.6566
Maximum	1.0204	1.9497
Std. dev.	0.3175	0.4092

Table 9: This table shows the results of the implicitly estimated parameters of risk aversion.

The larger absolute values of λ in our study are due to our approach of estimating the autocorrelation on a weekly basis. Recall that the risk premium in the reduced-form model is equal to $-\lambda \frac{\sigma^X}{\kappa}(1 - e^{-\kappa(T-t)})$. In order to achieve a certain risk premium, λ must increase if the fraction $\frac{\sigma^X}{\kappa}$ decreases, c. p. From our weekly approach we receive larger estimates for the speed of adjustment, $\hat{\kappa}$, and for the spot price volatility, $\hat{\sigma}^X$, than Lucia and Schwartz (2002) do. Our estimates for $\hat{\kappa}$ are on average around 0.09 per week, and for $\hat{\sigma}^X$ around 20 NOK per week (cf. Table 6), i. e. $\frac{\hat{\sigma}^X}{\hat{\kappa}} = 222$. Lucia and Schwartz (2002) report a $\hat{\kappa}$ of 0.01 per day, and a value of 9 for $\hat{\sigma}^X$, i. e. $\frac{\hat{\sigma}^X}{\hat{\kappa}} = 900$. This larger value results from not considering autocorrelation of higher lags in the daily residuals, leading to an overestimation of $\hat{\sigma}^X$.

The different test periods also account for part of the difference in absolute values and especially for negative instead of positive values of λ. Lucia and Schwartz (2002) test their model during a twelve-month period in 1998/1999 when spot prices were lower on average than in the years before. Therefore, their estimated deterministic component $f(t)$ of the spot price overestimates the spot prices in the test period, leading to mostly positive values of λ. We encountered the opposite situation. Table 1 shows that the twelve-month average spot price increased until 2003 and stayed on a high level until the end of our available time series. From an ex-post perspective, the deterministic component of the spot price then inevitably underestimates the observed and future spot prices. If market participants can partly predict such increasing spot prices – e. g. by considering water reservoir levels – and evaluate futures accordingly, the reduced-form model must underestimate futures prices when risk-neutrality is assumed. A negative λ that is appropriately large in absolute terms can compensate such estimation errors.

As an exception in our sample, λ adopts large positive values in the intervals covering December 2002 and January 2003 when spot prices were extremely high. This effect is

caused by negative estimates of $\hat{\kappa}$ in those particular intervals.

Depending on the sign of λ, the term $-\lambda\frac{\sigma^X}{\kappa}(1 - e^{-\kappa(T-t)}))$ is either strictly positive or strictly negative for all maturities, it increases and asymptotically converges to a constant with increasing time to maturity $(T-t)$. Thus, it matches the curve of valuation errors that basically increases with time-to-maturity as described in the previous section. Figure 18 in the Appendix shows a scatter plot of the in-sample estimated risk premia in the reduced-form model. In the majority of observations, those are clearly positive and adopt values up to 180 NOK. Especially during the winter 2002/03, the risk premia reach enormous negative values that do not seem economically meaningful.

The time series of λ features a level-autocorrelation of 0.98. We therefore conclude that λ contains valuable information and contributes to forecasting futures prices even when applied out-of-sample as we test in the next section even though we observe eight changes of sign in the time series. The means between λ in warm and cold periods (April until September vs. October until March) show little difference. We conclude that λ does not incorporate seasonal effects that discriminate between summer and winter.

The estimated values of ξ in the equilibrium model are mostly and on average positive as expected. However, in 36 out of 217 cases ξ is negative. The negative values mostly occur in subsequent valuation intervals. The level-autocorrelation of ξ amounts to 0.95, i. e. ξ is supposed to improve futures pricing out-of-sample, too. However, the time series of ξ exhibits six changes of sign. For those valuation dates we expect the results from the out-of-sample test in Section 6.6 to be worse than those from the risk-neutral evaluation.

The technical reason for negative ξ lies in the covariance term of (6). In the majority of cases, the expected future spot price is smaller than the observed futures price as shown in the previous section. In order to generate a positive risk premium with a positive value of ξ, the sum of covariances in (6) must be negative. To meet this requirement, the covariances between the future demands D_t times the end-user price and the next period's futures price have to be smaller than the covariances between the future production costs $C(D_t)$ and the futures price.

For low demand volumes the slope of the cost function (i. e. the marginal cost function) is smaller than the average end-user price. These low demand volumes would therefore

contribute positively to the covariance term. If the probability of high demand volumes is small, the whole covariance term can be positive.

According to the reasoning above, negative values of ξ should be possible when only futures maturing in the summer time are evaluated, but not necessarily when the valuation day lies in the summer time as it is the case in our results.

However, the estimation of the marginal cost function used at those particular valuation dates is dominated by observations at summer dates. This effect arises from the beginning of each estimation period at the end of March 1999. Therefore, each estimation of the marginal cost function is based on more summer than winter observations. This effect especially holds for the earlier valuation dates. Furthermore, the estimation of the marginal cost functions before winter 2002/03 in general lacks observations of extremely large spot prices. Both facts lead to estimates of the marginal cost function with a slope that is too low.

Our analysis of the risk premia at these particular days has shown that the underestimated marginal cost function forces the parameter ξ to be negative in such cases. Still, for higher production levels or low reservoir levels, the cost function is steep enough to support positive values of ξ.

Generally, in the equilibrium model a non-zero value of ξ does not shift the whole futures curve into one direction as the reduced-form model does, but adds large premia for short maturities and lower premia for long maturities. Also a combination of positive premia at the short end and negative premia for longer maturities may occur. Figure 19 in the Appendix shows estimated risk premia along time to maturity. The risk premium in the equilibrium model adopts its extreme values for some short maturity. It approaches zero for long maturities and therefore cannot compensate theoretical miss-pricing of long-term futures. It indirectly depends on seasonal effects as it considers covariances between futures prices on the one hand side and production costs, marginal production costs, and demand volumes on the other hand side.

Thus, if the calculation with $\xi = 0$ shows a large difference between calculated and observed prices along the whole futures curve, even a large absolute value of ξ contributes only little to the reduction of the estimation errors for long maturities. On the other hand side, if the estimation of ξ is based on few futures contracts with only short maturities, i. e. after August

	Reduced-form Model		Equilibrium Model	
	$f_{t,T}^{\mathrm{C,RF}}$	$|f_{t,T}^{\mathrm{C,RF}}|$	$f_{t,T}^{\mathrm{C,EQ}}$	$|f_{t,T}^{\mathrm{C,EQ}}|$
Mean	-8.02	35.24	-19.32	36.01
Median	-3.44	12.11	-12.02	24.58
Minimum	-508.67	0.0195	-523.13	0.0172
Maximum	2683.42	2683.42	128.19	523.13
Std. dev.	124.58	119.76	54.51	45.25
Skewness	10.985	12.841	-3.002	4.687

Table 10: This table shows descriptive statistics of the valuation errors $f_{t,T}^{\mathrm{C,RF/EQ}} = \hat{F}_{t,T}^{\mathrm{C,RF/EQ}} - F_{t,T}$ and of the absolute valuation errors $|f_{t,T}^{\mathrm{C,RF/EQ}}|$. Differences are given in NOK. The valuation period covers the interval from 06/28/2000 to 08/10/2004. The study is based on 1,971 futures prices.

2003 when block futures were delisted and only week futures are available, the compensation of estimation errors for only those shorter futures is feasible. From a technical point of view, the compensation then works in a similar way as in the reduced-form model. As Figure 16 exemplary shows, the risk premium in both models increases with time-to-maturity for the first few weeks.

However, the estimates of ξ become less reliable if fewer futures prices enter the estimation procedure. Figure 20 shows that the implicitly estimated ξ varies stronger after August 2003. Also the variation of λ increases, but to a much less extent.

6.6 Risk-adjusted Evaluation

We finally conduct a second out-of-sample test of futures prices by incorporating the risk parameters λ and ξ. We apply the values of λ and ξ that were implicitly estimated in the previous valuation interval. The results are presented in Table 10.

We find that incorporating risk premia improves on average the forecasting quality of the reduced-form model only in the mean and the median of the valuation errors. The standard deviation and the error range, defined as the difference between maximum and minimum error, decreases for the equilibrium model, but increases for the reduced-form model, i. e. it even looses robustness.

In comparison to each other, the models show almost equal values of the absolute valuation error, but we find a lower median for the reduced-form model. As can be seen from Figure 21, the reduced-form model gains forecasting ability for calm market situations relative to

the equilibrium model, but fails during volatile periods. This leads to a more than twice as high standard deviation of the errors of the reduced-form model than of the equilibrium model. Specifically, this result is driven by some extreme forecasting errors that derive from large prices in the winter 2002/03.

Both models continue to underestimate the futures prices on average, even though to a much lesser extent. We conjectured in the previous sections that the different functional forms of the risk premium term structures will allow the reduced-form model to better compensate for mispricing of long-term futures. Therefore, we regress the valuation error on the time to maturity. We do not find a significant relationship for the equilibrium model. However, we find a significant positive relationship for the reduced-form model, i. e. the underpricing decreases on average with increasing time to maturity.

We again examine the correlations between the error measures of both models. For the valuation error we find a correlation of 26.6%, for the absolute valuation error it is 31.9%. Both dropped slightly compared to our first study. This confirms that the adjustment by incorporating risk premia works differently in both models.

As discussed the risk premia necessarily contain a fraction due to the misestimation of expected futures prices and another one for actual, but unobservable risk premia. These two cannot be separated. However, if we assume that the errors of the estimation of the expected spot prices are symmetrically distributed with a constant – presumably negative – mean, than the estimated risk premia should represent fraction of the actual risk premia plus a constant.

In order to evaluate the estimated risk premia, $\hat{\Lambda}_{t,T}^{\text{C, RF/EQ}} = \hat{F}_{t,T}^{\text{C, RF/EQ}} - \hat{F}_{t,T}^{\text{A, RF/EQ}}$, we regress them on the valuation errors $f_{t,T}^{\text{A,RF/EQ}}$ from our first study:

$$\hat{\Lambda}_{t,T}^{\text{C, RF/EQ}} = \hat{\zeta}_0^{\text{RF/EQ}} + \hat{\zeta}_A^{\text{RF/EQ}} f_{t,T}^{\text{A,RF/EQ}} + u_{t,T}^{\text{RF/EQ}} \tag{13}$$

with $u_{t,T}^{\text{RF/EQ}}$ i. i. d. distributed. The results are given in table 11.

As expected, the estimated constants are positive, indicating that the risk premia cover up part of the undervaluation. Surprisingly, the slope $\hat{\zeta}_A^{\text{RF}}$ for the reduced-form model does not significantly differ from zero. Thus, on average the risk premium in the reduced-form model does not capture any structure of the errors in our first study. However, we did the

Regression	Reduced-form Model				Equilibrium Model			
	$\hat{\zeta}_0^{\mathrm{RF}}$	$\hat{\zeta}_A^{\mathrm{RF}}$	t-Stat.	R^2	$\hat{\zeta}_0^{\mathrm{GG}}$	$\hat{\zeta}_A^{\mathrm{GG}}$	t-Stat.	R^2
$f^{\mathrm{A,\ RF/GG}}$	17.91	-0.004	-0.04	< 0.01	9.61	-0.241	-11.81	0.20

Table 11: This table shows the results of regressing the estimated risk premia of the study in Section 6.6 on the valuation errors of the study in Section 6.4.

same regression for subsamples and found a significant negative slope if we leave out all observations of the winter 2002/03. For the equilibrium model, we find a significant negative slope for the full sample as well as for subsamples. We conclude that the risk premium given by the equilibrium model is able to explain partly the structure of the valuation errors.

7 Conclusion

We empirically compare the performance of a one-factor reduced-form model by Lucia and Schwartz (2002) and the one-factor equilibrium model by Bühler and Müller-Merbach (2008). Our analysis is based on 217 daily observations of traded week and block futures prices at the Scandinavian electricity exchange Nord Pool.

The main difference between the two models is the choice of the exogenous variable: The spot price in the reduced-form model versus the end-user demand in the equilibrium model. The end-user demand and, to a lower extent the spot price, exhibit seasonal patterns which possibly dominate risk premia in futures prices. Therefore, we carefully estimate the seasonal components. In addition, the equilibrium model requires a carefully estimated marginal cost function. This cost function translates the symmetrically distributed demand into right-skewed spot prices. In contrast, the reduced-form model assumes a symmetric distribution of spot prices.

In case of risk-neutral agents, our results show that the production-based equilibrium model explains futures prices slightly better than the competing reduced-form model. The main reason for this result is the better performance of the equilibrium model in periods of highly volatile spot prices.

To consider risk aspects in futures prices, we implicitly estimate the risk parameter of each model by minimizing the RMSE between observed and theoretical futures prices. An out-of-

sample test shows that both models result in lower average errors with a slight advantage for the reduced-form model. However, the reduced-form model exhibits a much larger standard deviation of the valuation errors, as it still fails to capture price spikes appropriately.

Finally, we compare the model-free estimates of risk premia in futures prices with the model dependent premia. Our results support the term structure of risk premia as derived form the equilibrium model. The reduced-form model has not enough flexibility to explain the observed premia.

The implementation of the equilibrium model is much more challenging compared to the reduced-from model. It requires more input data and considerably higher computational effort. This advantage of the reduced-form model is diminished if the spot price is modeled by jump-diffusions in order to cope with price peaks. Villaplana (2003) and Geman and Roncoroni (2006) provide examples in this direction.

References

Audet, N., Heiskanen, P., Keppo, J., and Vehviläinen, I. (2002). Modelling of Electricity Forward Curve Dynamics. Working Paper, University of Michigan, USA.

Basak, S. and Chabakauri, G. (2007). Dynamic Mean-Variance Asset Allocation. Working Paper, London Business School.

Benth, F. E., Cartea, A., and Kiesel, R. (2007). Pricing Forward Contracts in Power Markets by the Certainty Equivalence Principle: Explaining the Sign of the Risk Premium. Working Paper.

Bessembinder, H. and Lemmon, M. (2002). Equilibrium Pricing and Optimal Hedging in Electricity Forward Markets. *Journal of Finance*, 57(3), 1347–1382.

Bühler, W. and Müller-Merbach, J. (2008). Risk Premia of Electricity Futures: A Dynamic Equilibrium Model. In Geman, H. (Ed.), *Risk Management in Commodity Markets*, chapter 5, 61–80. Chichester: Wiley.

Bye, T. (2003). A Nordic Energy Market under Stress. *Economic Survey*, (4), 26–37.

Cartea, A. and Figueroa, M. G. (2005). Pricing in Electricity Markets: A Mean Reverting Jump Diffusion Model with Seasonality. *Applied Mathematical Finance*, 12(4), 313–335.

Geman, H. (2005). *Commodities and Commodity Derivatives*. Chichester: Wiley.

Geman, H. and Roncoroni, A. (2006). Understanding the Fine Structure of Electricity Prices. *Journal of Business*, 79(3), 7–14.

Geman, H. and Vasicek, O. (2001). Plugging into Electricity. *Risk*, 93–97.

Koekebakker, S. and Ollmar, F. (2005). Forward Curve Dynamics in the Nordic Electricity Market. *Managerial Finance*, 31(6), 73–94.

Longstaff, F. A. and Wang, A. W. (2004). Electricity Forward Prices: A High-frequency Empirical Analysis. *Journal of Finance*, 59(4), 1877–1900.

Lucia, J. J. and Schwartz, E. S. (2002). Electricity Prices and Power Derivatives: Evidence from the Nordic Power Exchange. *Review of Derivatives Research*, 5(1), 5–50.

Nord Pool ASA (ed.) (1998). Elbørsen – the nordic power exchange's 5th anniversary issue.

Nord Pool ASA (ed.) (2004). Trading rules for financial electricity contracts and certificate contracts. Manual.

Pilipović, D. (1997). *Energy Risk*. New York et al.: McGraw-Hill.

Routledge, B. R., Seppi, D. J., and Spatt, C. S. (2001). The Spark Spread: Cross-Commodity Equilibrium Restrictions and Electricity. Working Paper, Carnegie Mellon University.

Ullrich, C. J. (2007). Constrained Capacity and Equilibrium Forward Premia in Electricity Markets. Working Paper, Pennsylvania State University.

Villaplana, P. (2003). Pricing Power Derivatives: A Two-factor Jump-diffusion Approach. Working Paper.

Wilkens, S. and Wimschulte, J. (2007). The Pricing of Electricity Futures: Evidence from the European Energy Exchange. *The Journal of Futures Markets*, 27(4), 387–410.

A   Estimation of the Deterministic Components

To model the piecewise linear function of the seasonal pattern, we define 12 overlapping triangular functions $M_{i,t}$ around certain equidistant calendar days $\bar{t}_i$.

The selection of triangular functions $M_{i,t}$ as regressors yields a piecewise linear and continuous function for the seasonal component, cf. Figure 4. Note that this procedure does not require more coefficients than conventional dummy variables.

The effects of specific days on the spot price or the daily demand are captured by dummy variables FRI_t, SAT_t, and SUN_t for week-end days, and $H_{h,t}$ for holidays and for additional days with a significant impact on the endogenous variable. The residual in the regression is assumed to follow an autoregressive process whose lag is determined by the Akaike criterion. The triangular functions are defined as follows:

$$M_{i,t} = \frac{1}{28} \cdot \begin{cases} \overline{\Delta t} - (\bar{t}_i - t), & 0 \leq \bar{t}_i - t < \overline{\Delta t} \\ \overline{\Delta t} - (t - \bar{t}_i), & 0 < t - \bar{t}_i < \overline{\Delta t} \\ 0, & \text{elsewhere} \end{cases} \tag{14}$$

with $\overline{\Delta t} = 28$ days and the following calendar days $\bar{t}_i$:

$\bar{t}_1$	01/01	$\bar{t}_2$	01/29	$\bar{t}_3$	02/26	$\bar{t}_4$	03/26	$\bar{t}_5$	04/23	$\bar{t}_6$	05/21
$\bar{t}_7$	06/18	$\bar{t}_8$	08/14	$\bar{t}_9$	09/11	$\bar{t}_{10}$	10/09	$\bar{t}_{11}$	11/06	$\bar{t}_{12}$	12/04

We use this procedure for both models. We choose 13 calendar days (instead of 12 corresponding to the calendar months) for the simple reason that $365 \mod 13 = 1$, i. e. we can cover one year more regularly when applying 13 instead of 12 intervals. July 16 and 17 are used as benchmark days, i. e. not covered by any of the triangular functions. We choose these days as they are the ones with the lowest production in the long-term average.

The regression equation for the reduced-form model is given by:

$$P_t = \beta_0 + \sum_{i=1}^{12} \beta_i M_{i,t} + \beta_{FRI} FRI_t + \beta_{SAT} SAT_t + \beta_{SUN} SUN_t + \sum_h \beta_h H_{h,t} + \epsilon_t^P \tag{15}$$

$$\text{with} \quad \epsilon_t^P = \phi_1 \epsilon_{t-1}^P + \phi_2 \epsilon_{t-2}^P + \phi_3 \epsilon_{t-3}^P + \ldots + u_t^P \quad , u_t^P \sim \mathcal{N}(0, \sigma^P) \tag{16}$$
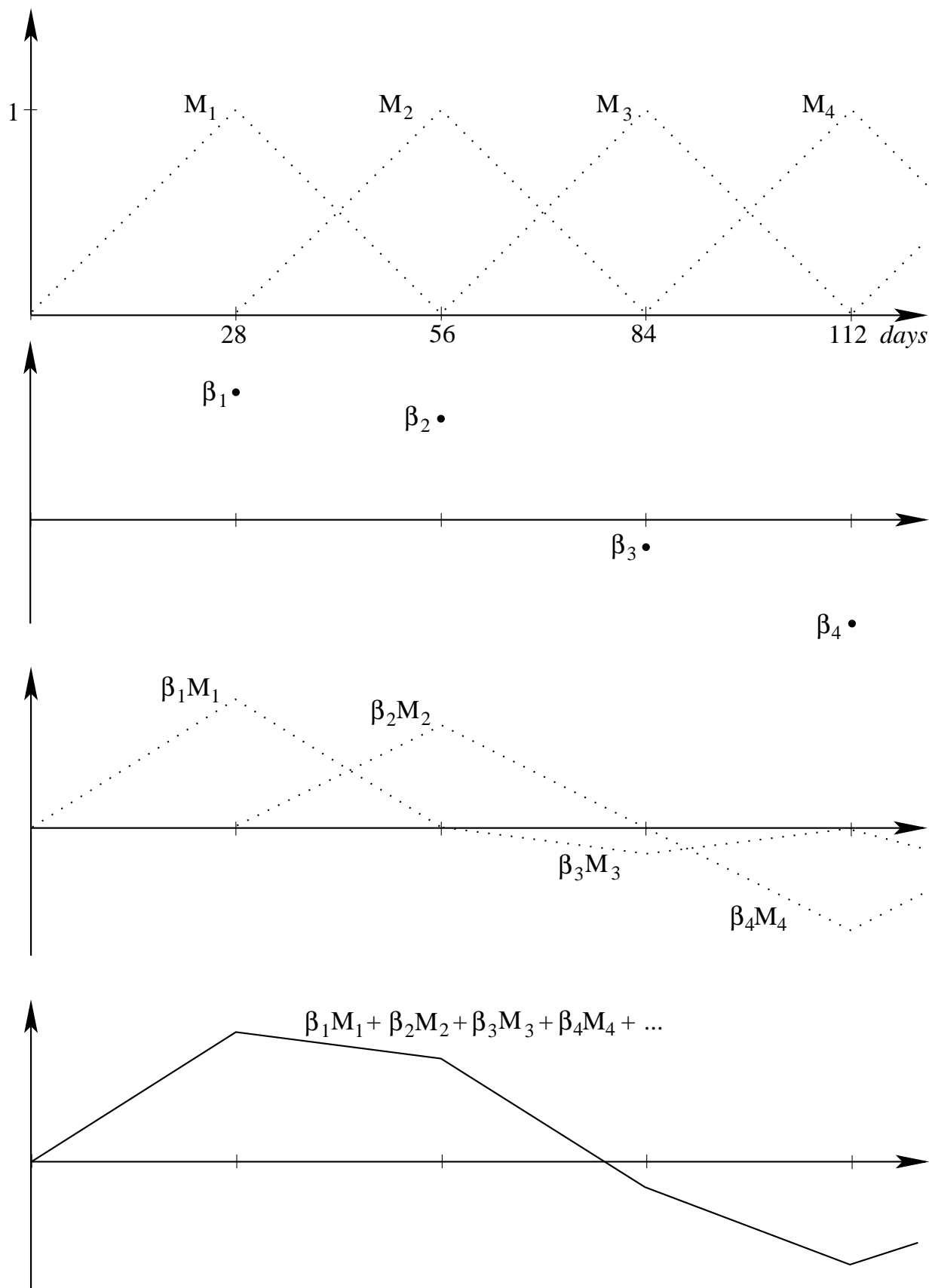
Figure 4: This figure illustrates the principle of describing the deterministic components in the spot price and in the demand series.

The time index t in (15) and (16) runs from 11/01/1996 until the Tuesday preceding the valuation day (Wednesday). For the equilibrium model the endogenous variable P_t in (15) is replaced by the daily demand D_t and the autoregressive residuals have three lags only. The estimation results for the total observation period of spot prices and daily demand volumes are given in Table 12.

	Spot Prices (Reduced-form Model) 11/01/1996–08/03/2004		Electricity Demand (Equilibrium Model) 03/29/1999–08/03/2004	
Estimation Window No. of obs.	2833		1955	
	Estimate [NOK]	t-Statistic	Estimate [GW]	t-Statistic
β_0 (Const.)	125.00	3.64	30.83	49.11
β_1 (01/01)	137.08	3.63	19.93	21.10
β_2 (01/29)	74.33	1.98	18.91	20.09
β_3 (02/26)	69.97	1.90	18.26	19.52
β_4 (03/26)	42.88	1.21	13.40	14.33
β_5 (04/23)	34.72	1.05	8.22	9.16
β_6 (05/21)	25.40	0.87	4.96	5.57
β_7 (06/18)	34.40	1.53	2.99	3.13
β_8 (08/14)	31.30	1.36	3.30	3.34
β_9 (09/11)	38.53	1.27	4.96	5.32
β_{10} (10/09)	42.21	1.23	9.60	10.20
β_{11} (11/06)	64.78	1.77	14.00	14.89
β_{12} (12/04)	102.59	2.75	16.76	17.82
FRI	−3.40	−4.45	−0.38	−8.19
SAT	−14.40	−17.06	−3.56	−60.33
SUN	−19.01	−25.12	−4.32	−94.13
No. of significant holidays ($\alpha = 5\%$)	14		24	
ϕ_1 (AR(1))	0.77	40.67	1.13	49.67
ϕ_2 (AR(2))	0.02	0.95	−0.36	−10.74
ϕ_3 (AR(3))	0.27	11.43	0.12	5.20
ϕ_4 (AR(4))	−0.08	−3.95	–	–
Adj. R^2	0.965		0.986	
S. E.	17.04		0.81	
Log likelihood	−12034.7		−2337.6	
	Residuals u_t^P, u_t^D			
Min.	−162.61		−3.27	
Max.	367.36		3.45	
Std. Dev.	16.91		0.80	
Skewness	4.55		0.05	
Kurtosis	124.46		4.56	
Jarque-Bera-Stat.	$1.75 \cdot 10^7$		199.52	

Table 12: This table shows the results of the daily estimation of both models, cf. (15) and (16). The estimation windows comprises the whole available time series.
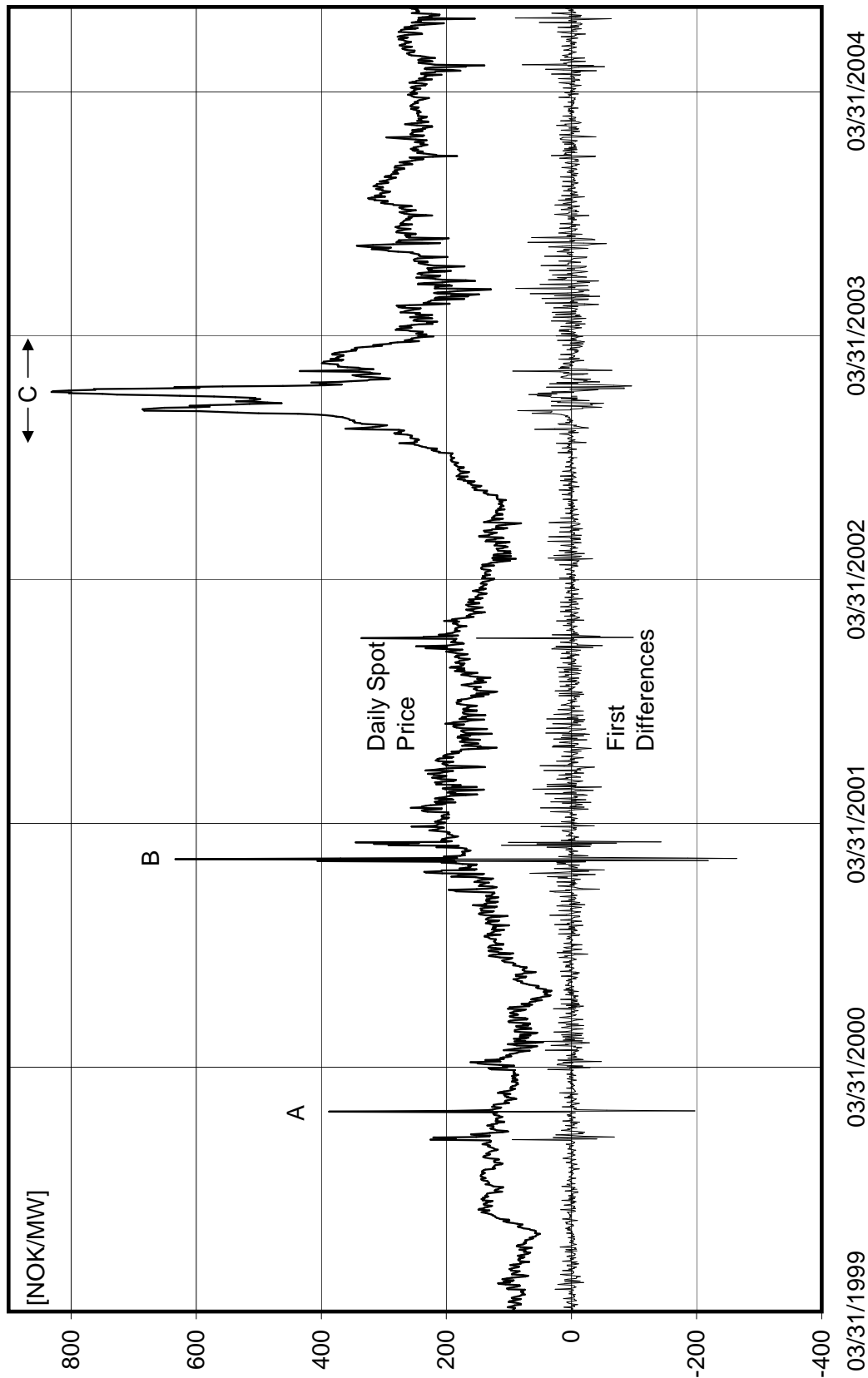
Figure 5: This figure shows the daily spot price ("system price") and its first differences at Nord Pool for the period from 03/31/1999 until 08/04/2004. The price spikes A and B and the period of high prices, C, correspond to those marked in Figures 6, 7, and 8.
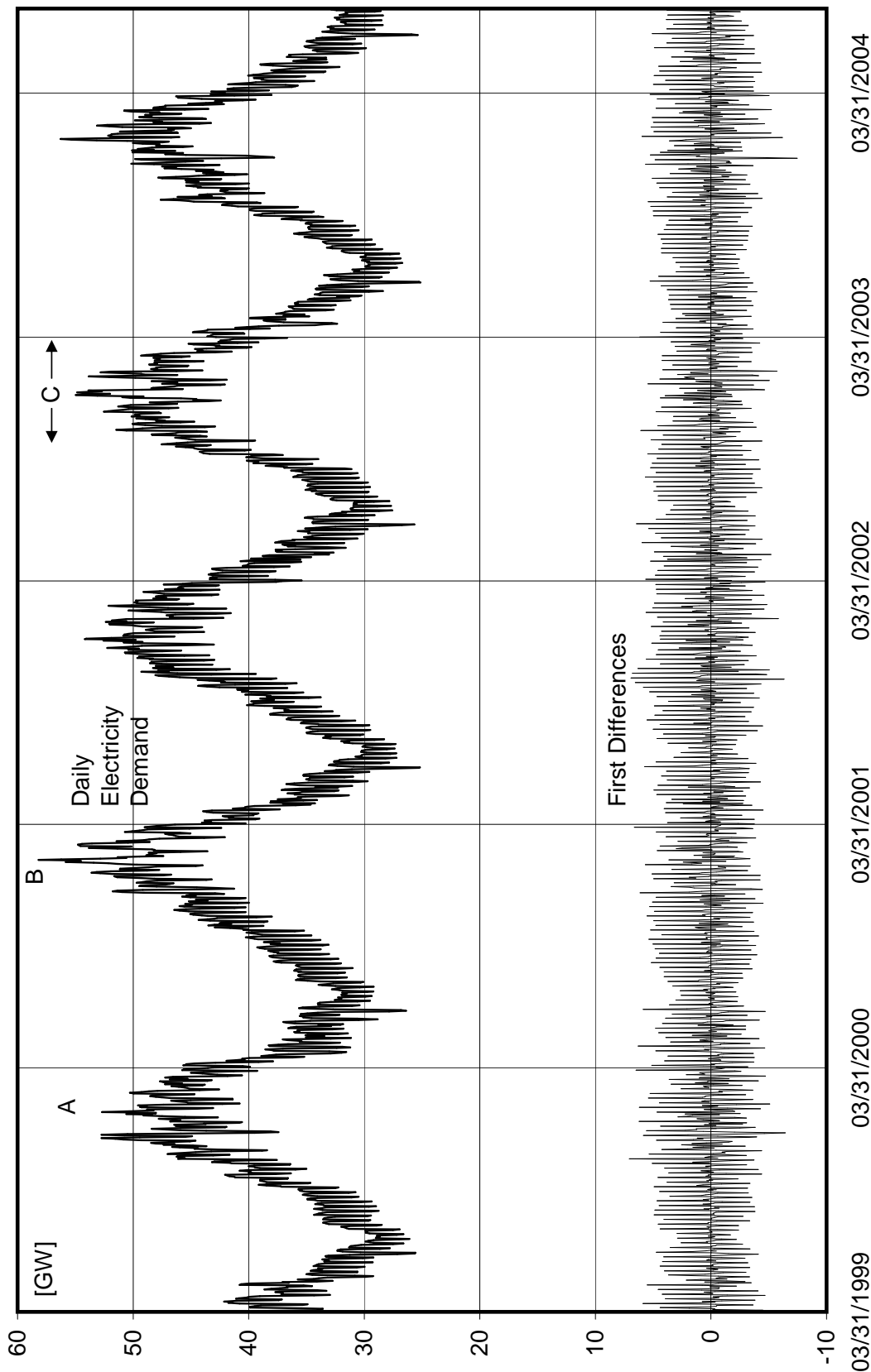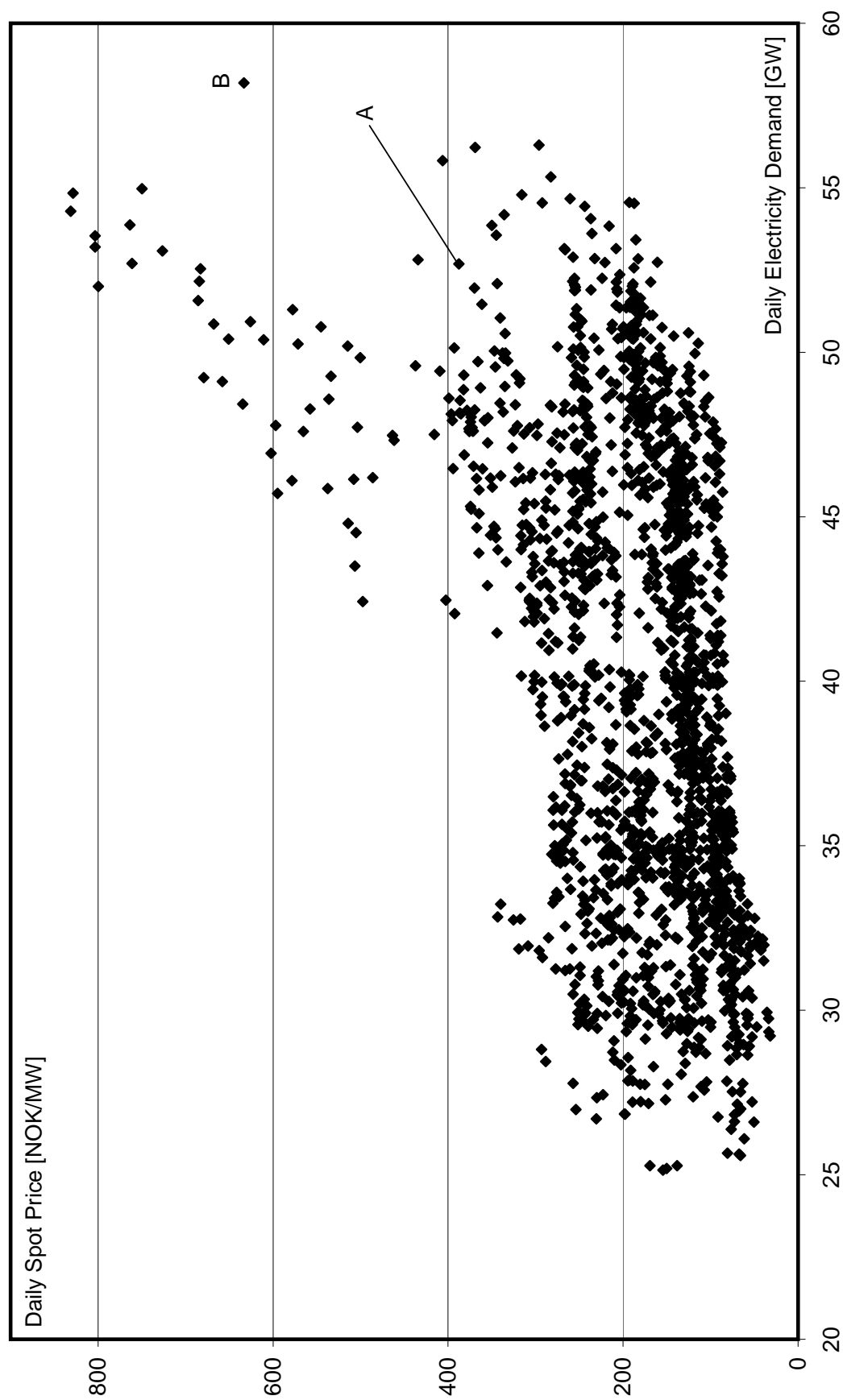
Figure 6: This figure shows the daily electricity demand and its first differences in the Nord Pool area (Norway, Sweden, Finland) for the period from 03/31/1999 until 08/04/2004. The price spikes A and B and the period of high prices, C, correspond to those marked in Figures 5, 7, and 8.

Figure 7: This figure shows the daily spot price and the daily demand in a scatter plot for the period from 03/31/1999 until 08/10/2004. The correlation between the spot price and the production is 39.2%, for their first differences it is 49.7%. The points A and B correspond to those marked in Figure 5 and 6. The data points in the upper left area mostly correspond to the period of high prices, C, which is not marked in this figure.
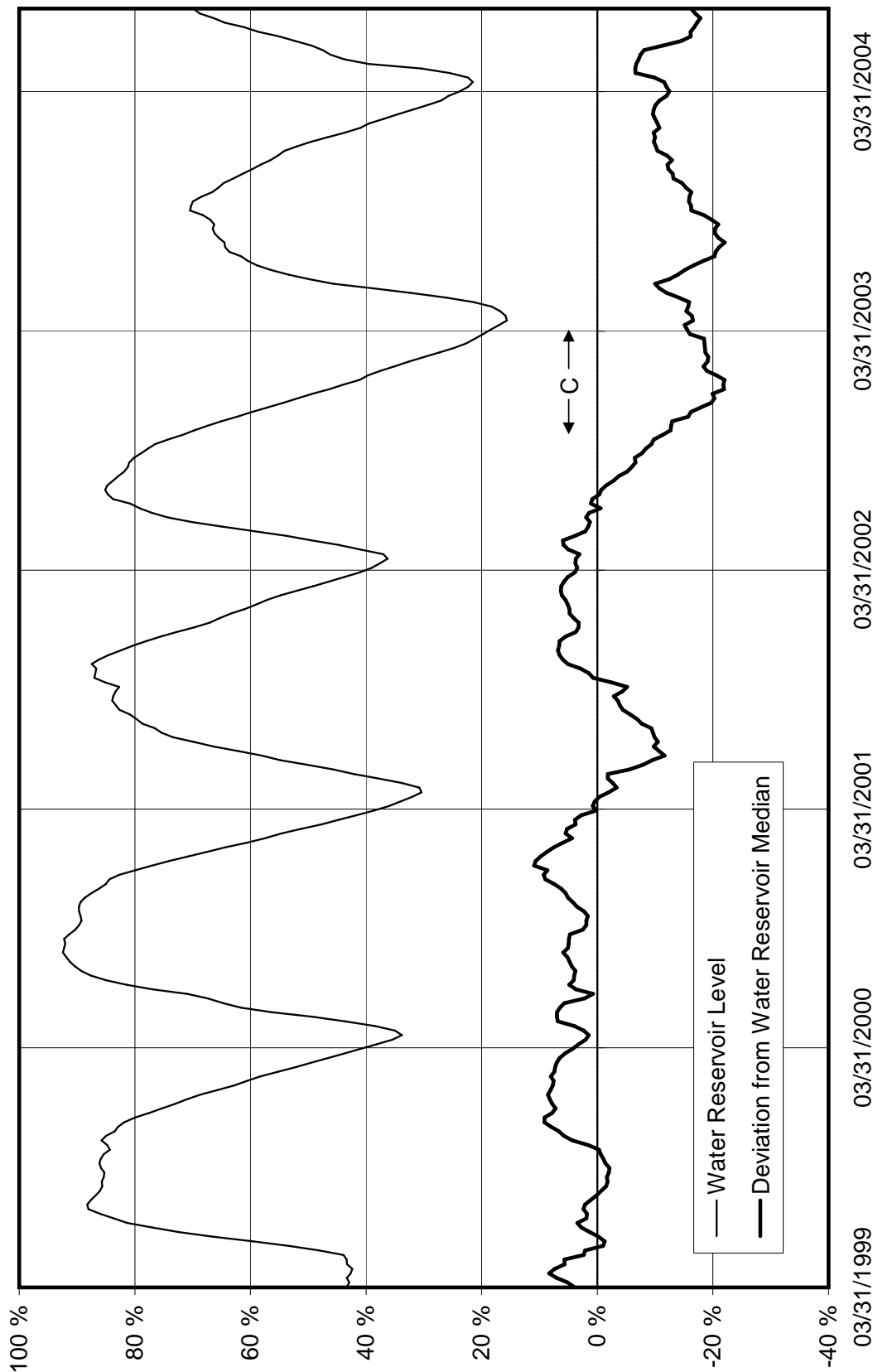
Figure 8: This figure shows the water reservoir level and its deviation from the historical median (280 data points from 03/31/1999 until 08/10/2004). It is measured in percentage points of the total reservoir capacity. The correlation between the spot price and the deviation from the median reservoir level amounts to -75.9%. The time period C corresponds to those in the previous Figures.
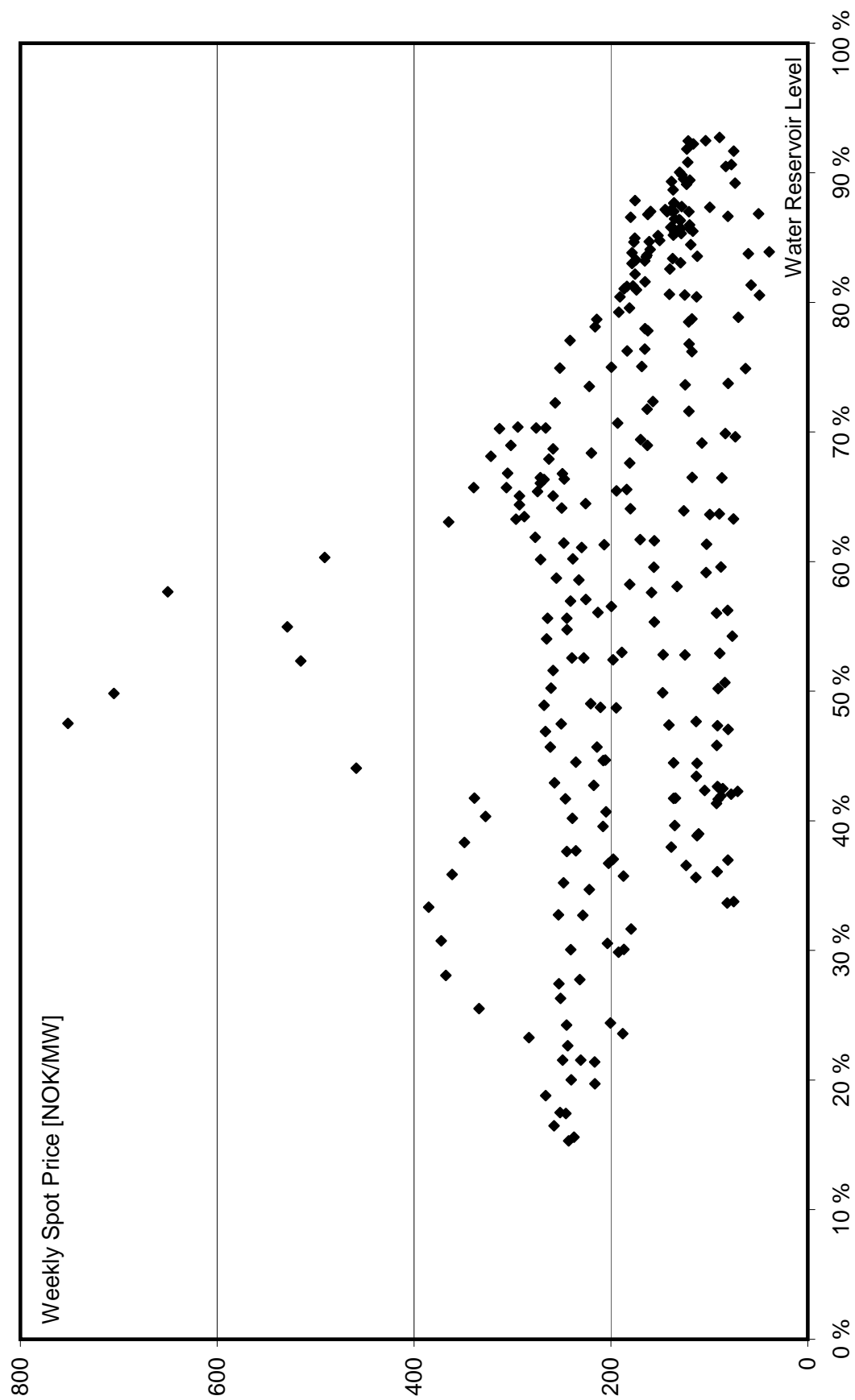
Figure 9: The weekly spot price and the water reservoir level in a scatter plot (280 data points from 03/31/1999 until 08/10/2004). The correlation between the spot price and the reservoir level is -37.1%, but only -3.7% for the first differences.
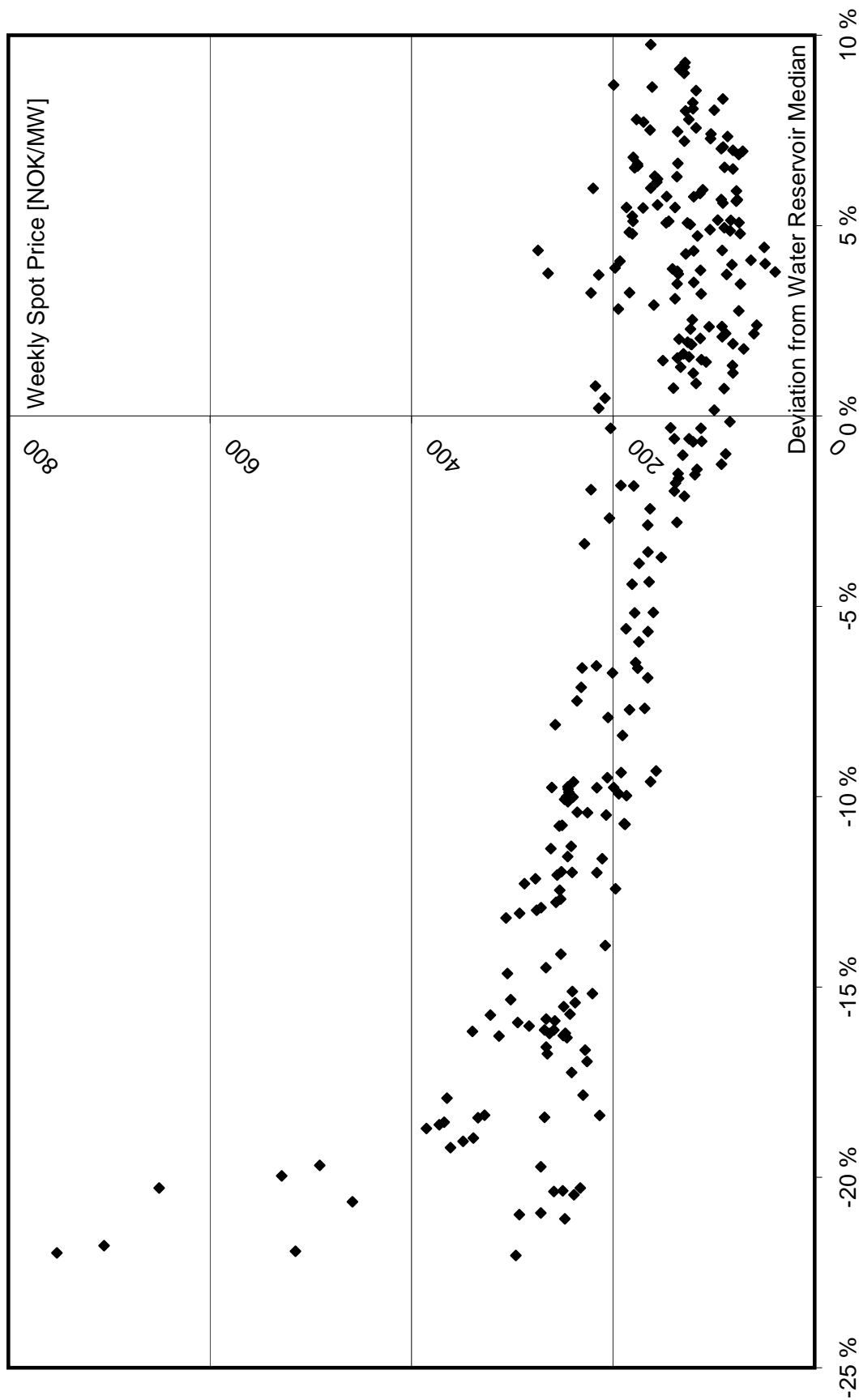
Figure 10: This figure shows a scatter plot of the weekly spot price and the water reservoir deviation from its median in %-points (280 data points from 03/31/1999 until 08/10/2004). The correlation between the spot price and the deviation from the reservoir median is −75.9%, for the first differences it is −13.8%.
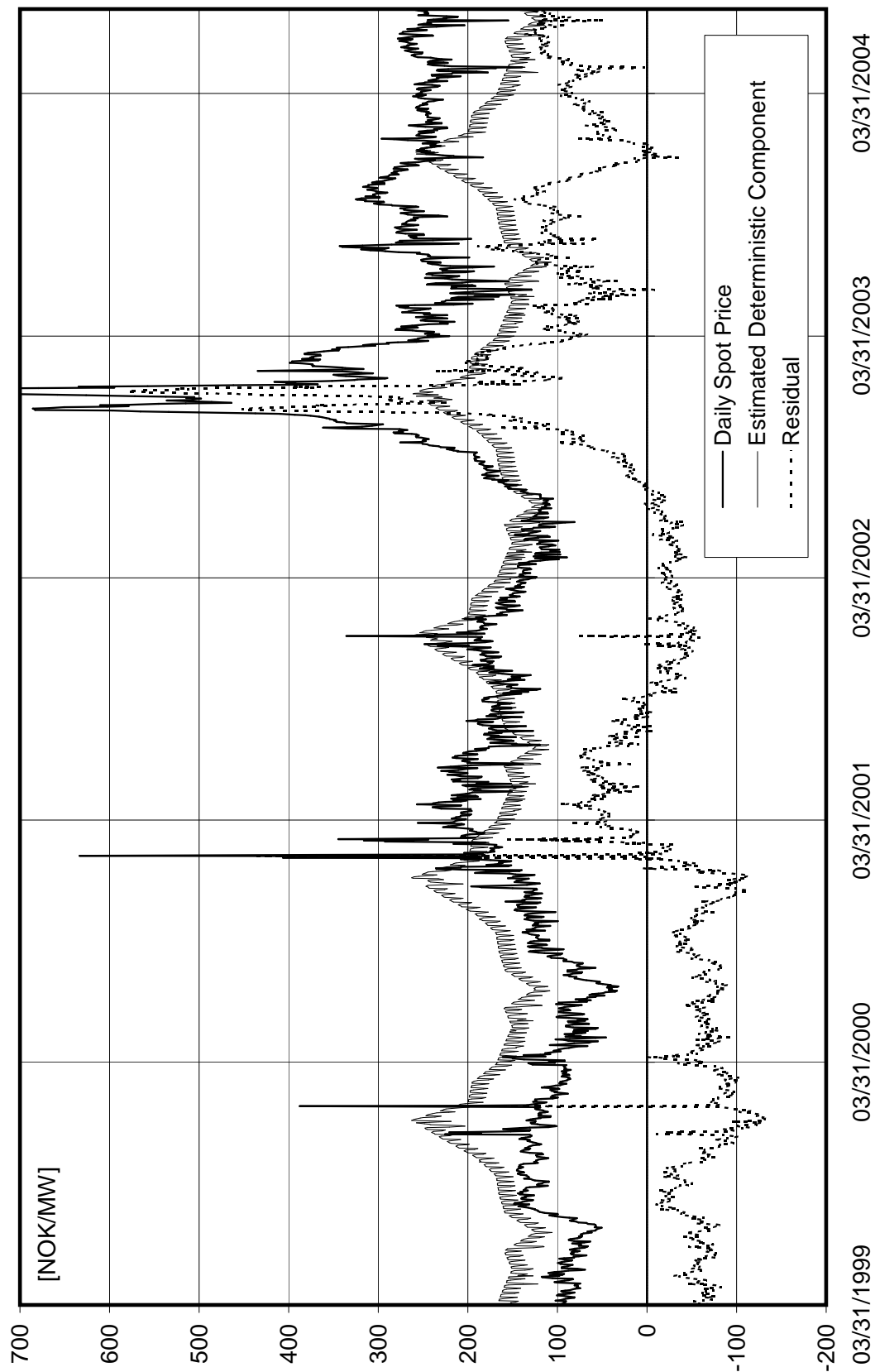
Figure 11: This figure shows the daily spot price, P_t, its estimated deterministic component, $f(t)$, for the reduced-form model, and the daily residual (estimation window: 11/01/1996–08/04/2004).
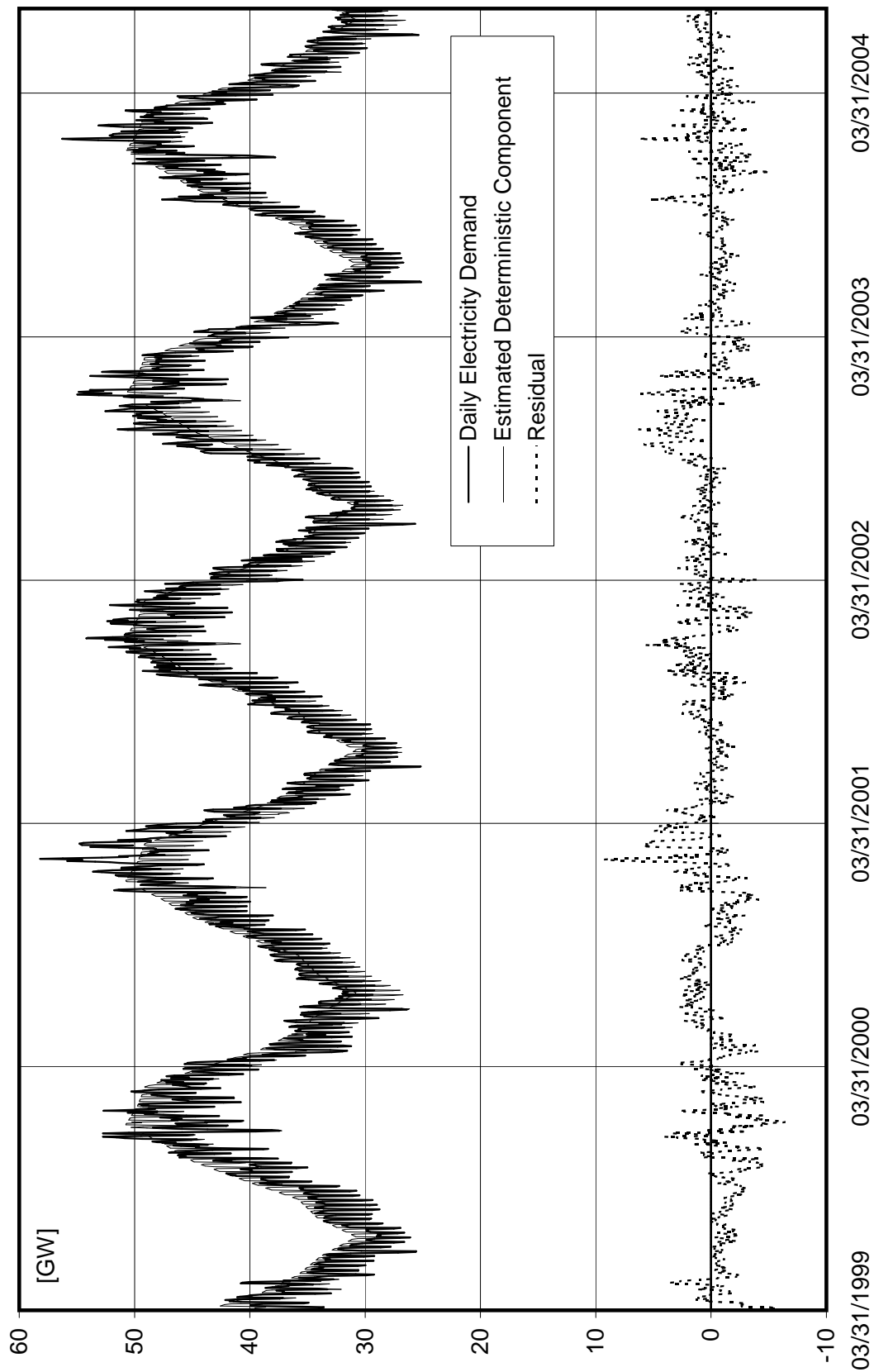
Figure 12: This figure shows the daily electricity demand D_t, its estimated deterministic component $\bar{D}_t$ used in the equilibrium model, and the residual S_t (estimation window: 03/29/1999–08/04/2004).
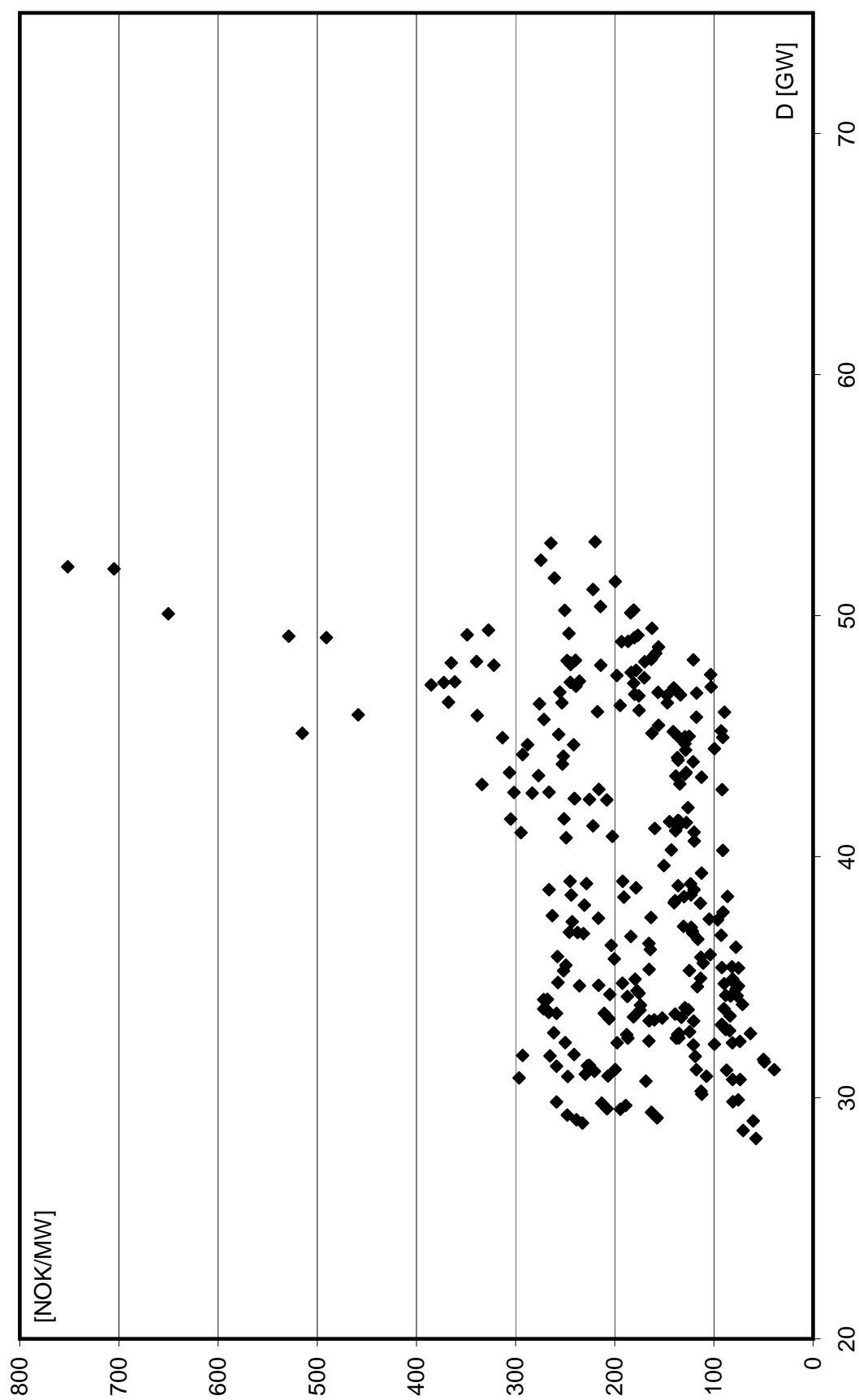
Figure 13: This figure shows the scatter plot of weekly average demand D_t and the weekly average spot prices P_t during the observation period.
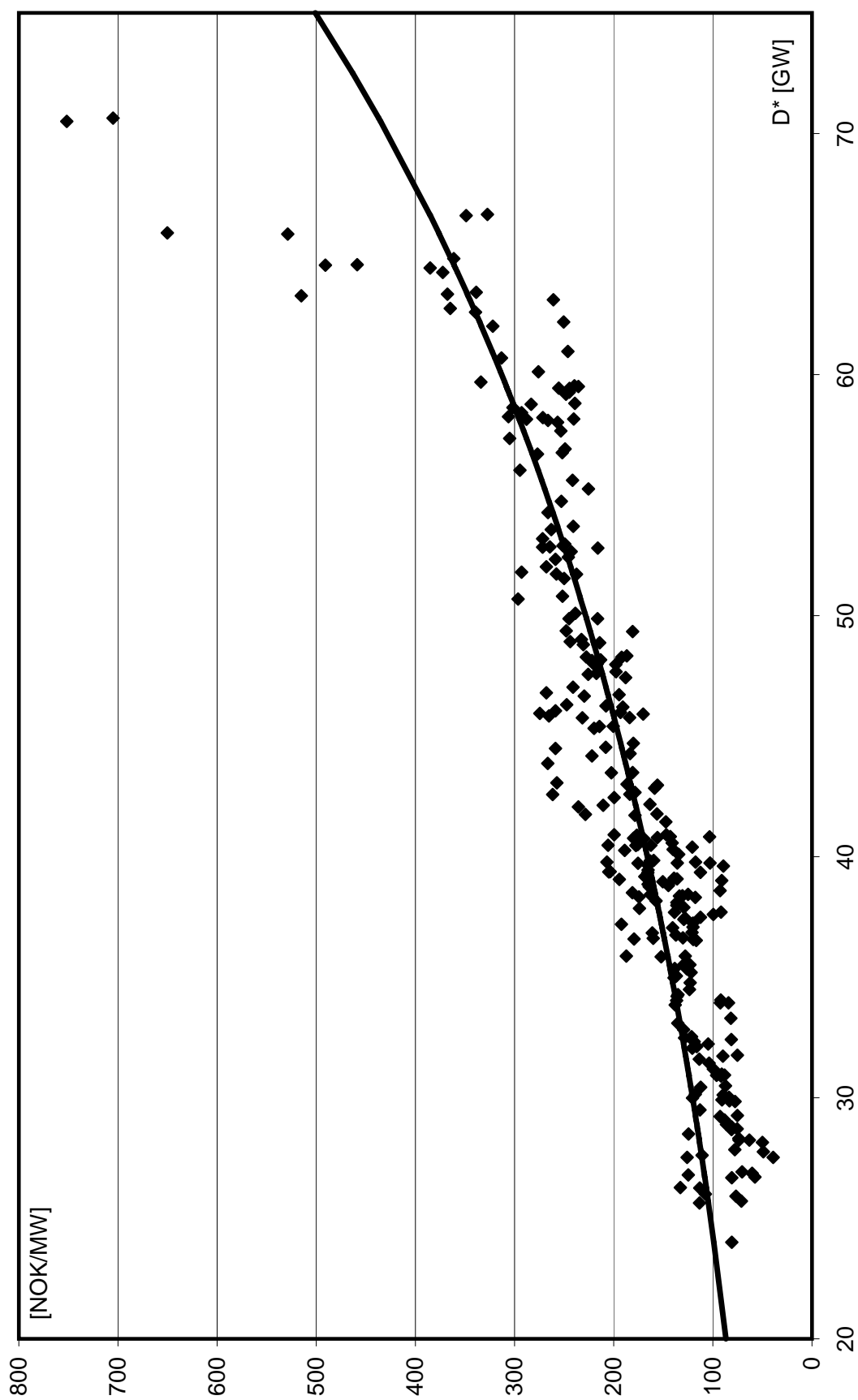
Figure 14: This figure shows the estimated marginal cost function $C'(D^*) = \exp(4.64 + 0.0209D^*)$ where $D^* = D - 0.4498WRD$) and the scatter plot of the adjusted demand D^* and the weekly average spot prices. The parameters of the marginal cost function are estimated on the full observation period, i. e. the figure shows an in-sample comparison.
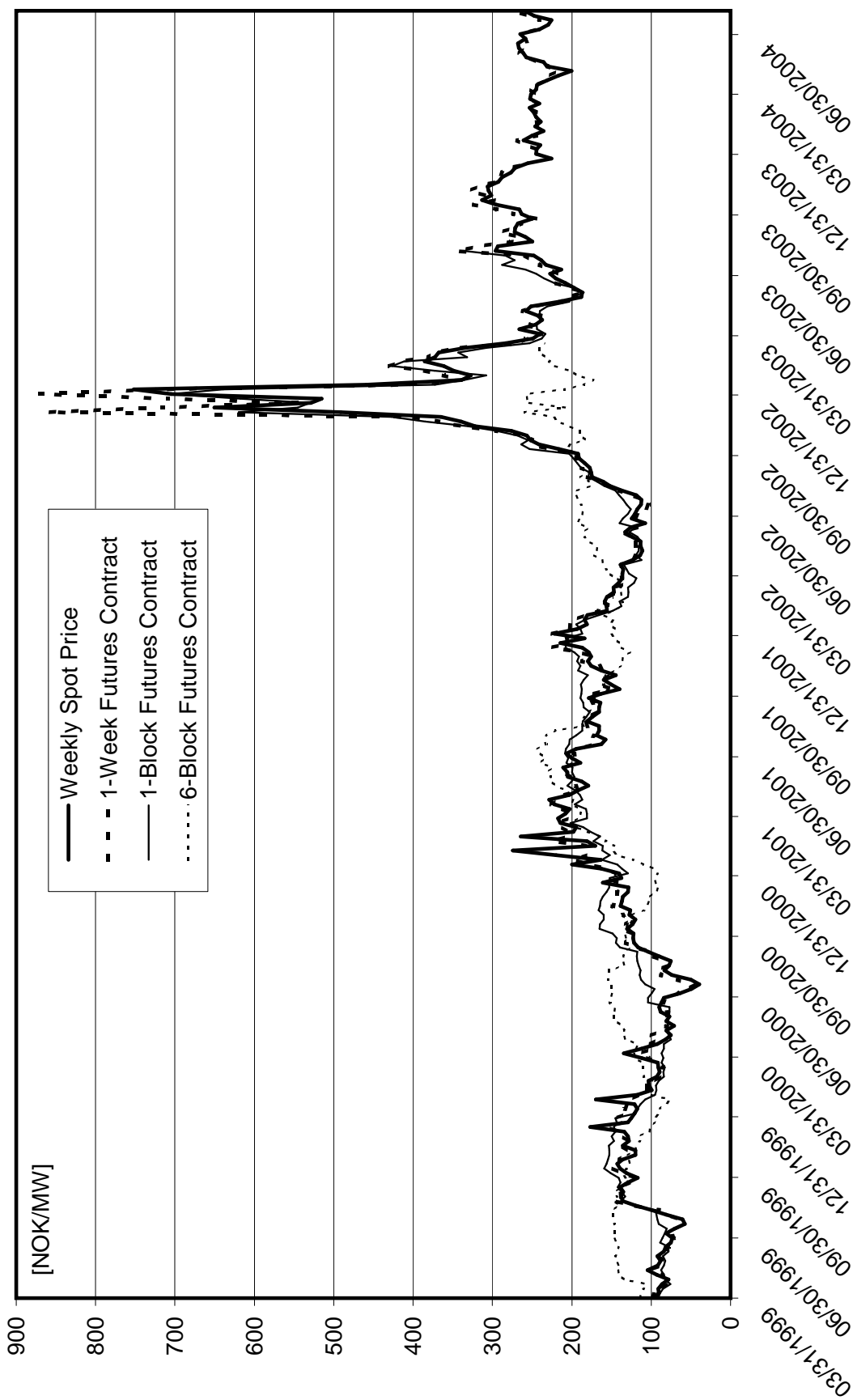
Figure 15: This figure shows the weekly average spot price, the price of the week futures contract with the shortest available maturity (1-Week Futures Contract), of the block futures contract with the shortest available maturity (1-Block Futures Contract), and of the block futures contract that matures in $6 \cdot 4 = 24$ weeks (6-Block Futures Contract). Note that block futures were delisted in August 2003.
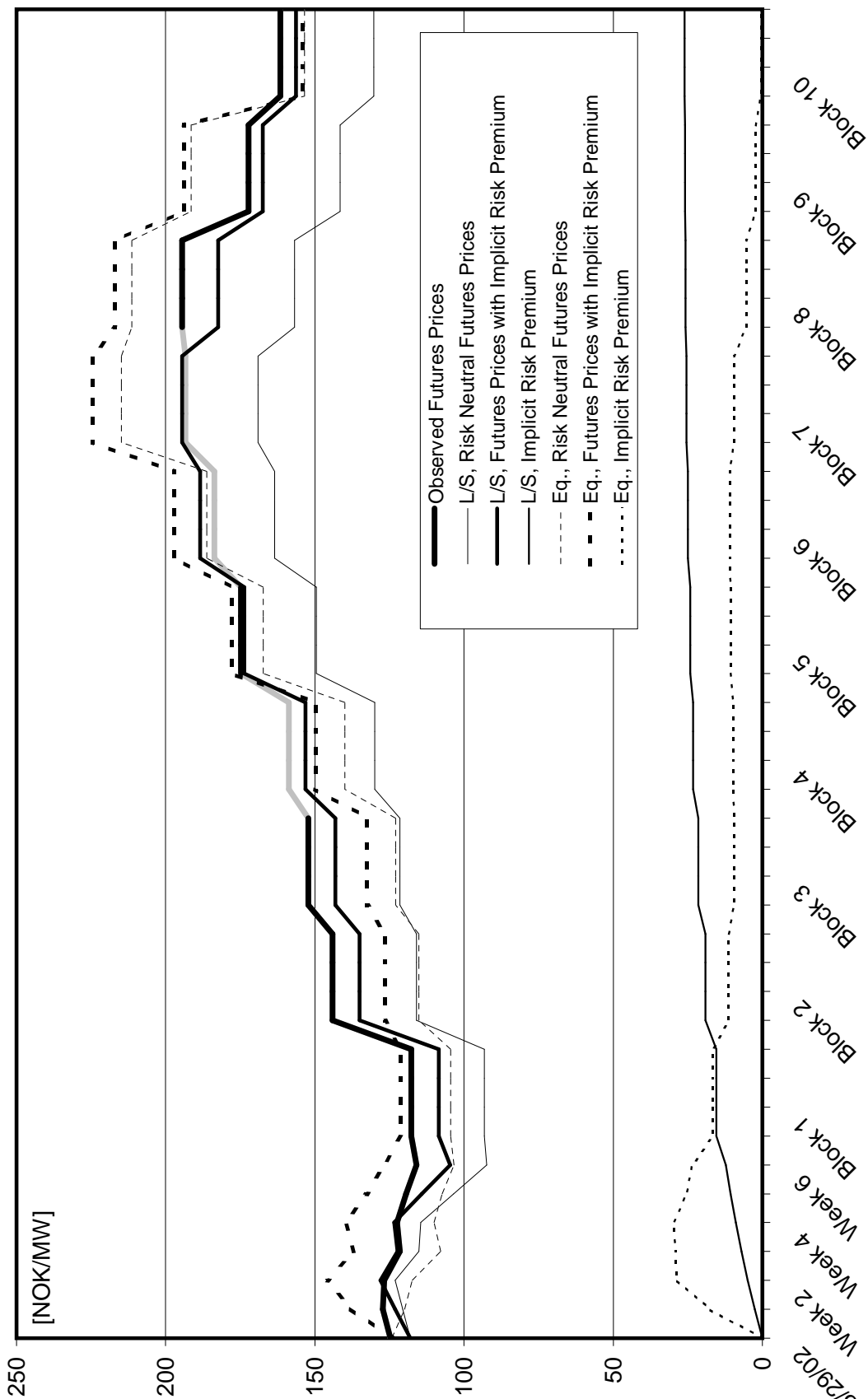
Figure 16: This figure shows the observed futures curve, the theoretical futures curves and the theoretical futures premia at 05/29/2002. The grey sections of the observed futures curve denote settlement prices that were not traded and thus not considered in the implicit estimation of risk parameters.
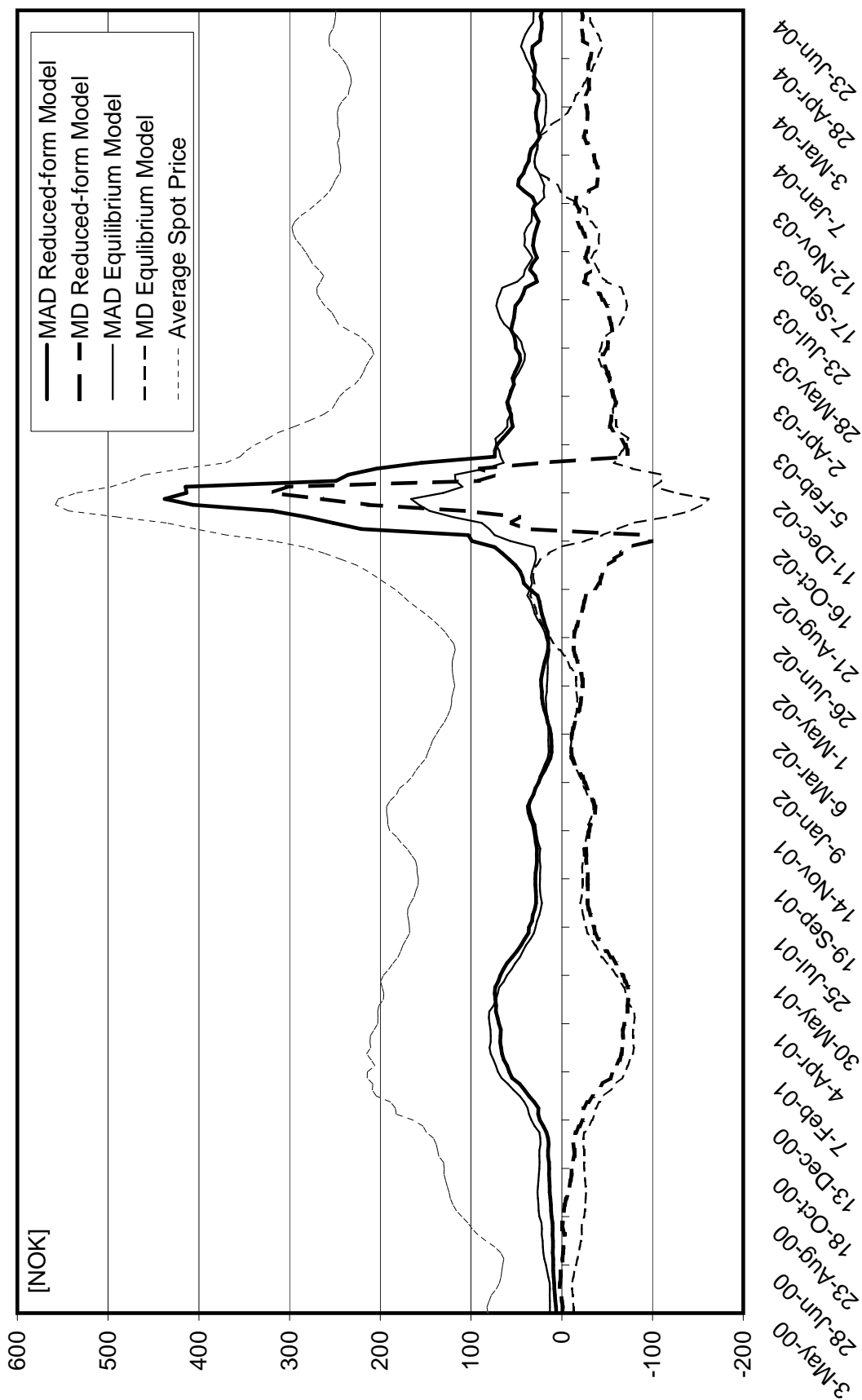
Figure 17: This figure shows the MAD and the MD for both models in the case of risk neutrality ($\lambda = 0$, $\xi = 0$) and the 8-week average spot price.
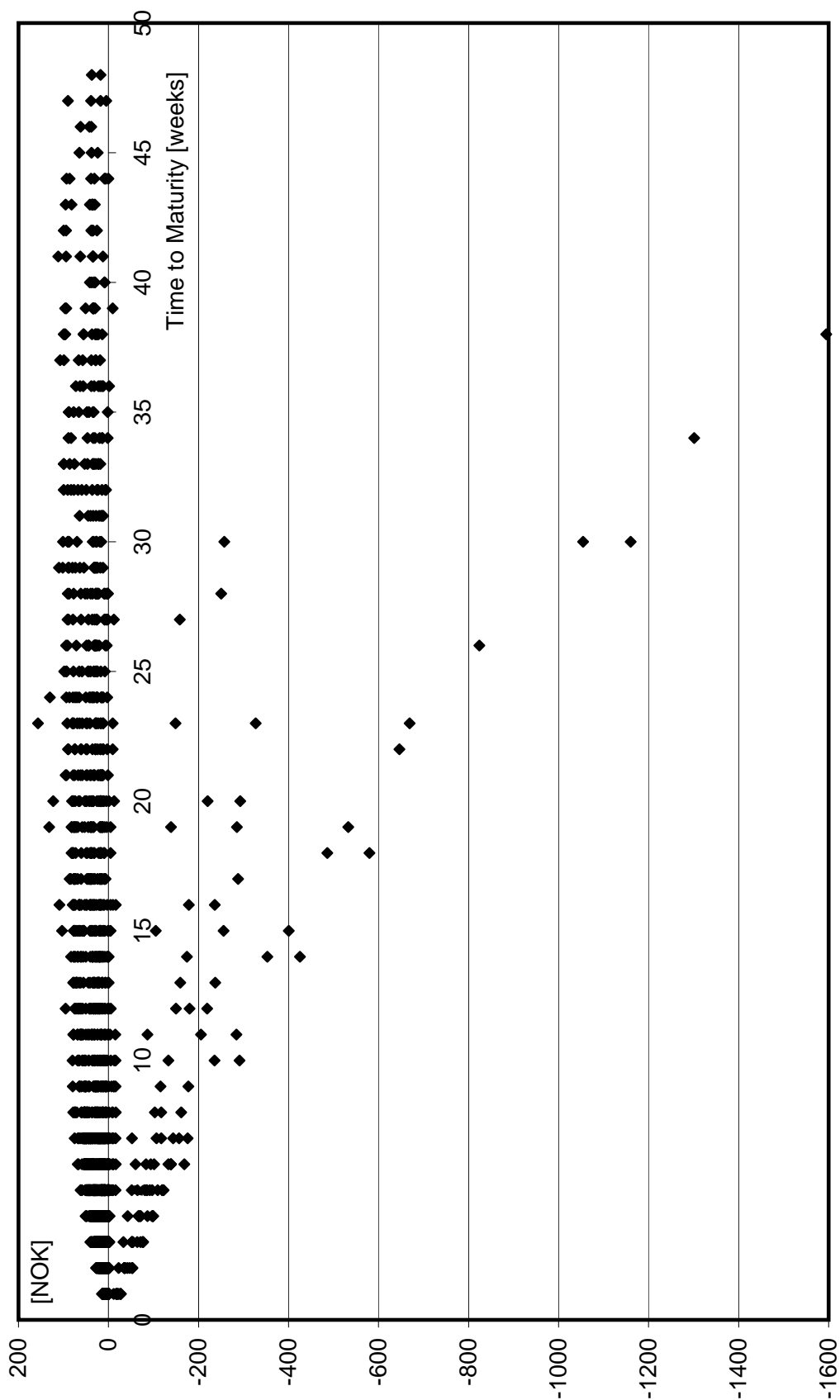
Figure 18: This Figure shows a scatter plot of the in-sample estimated risk premia of the reduced-form model along the time to maturity.
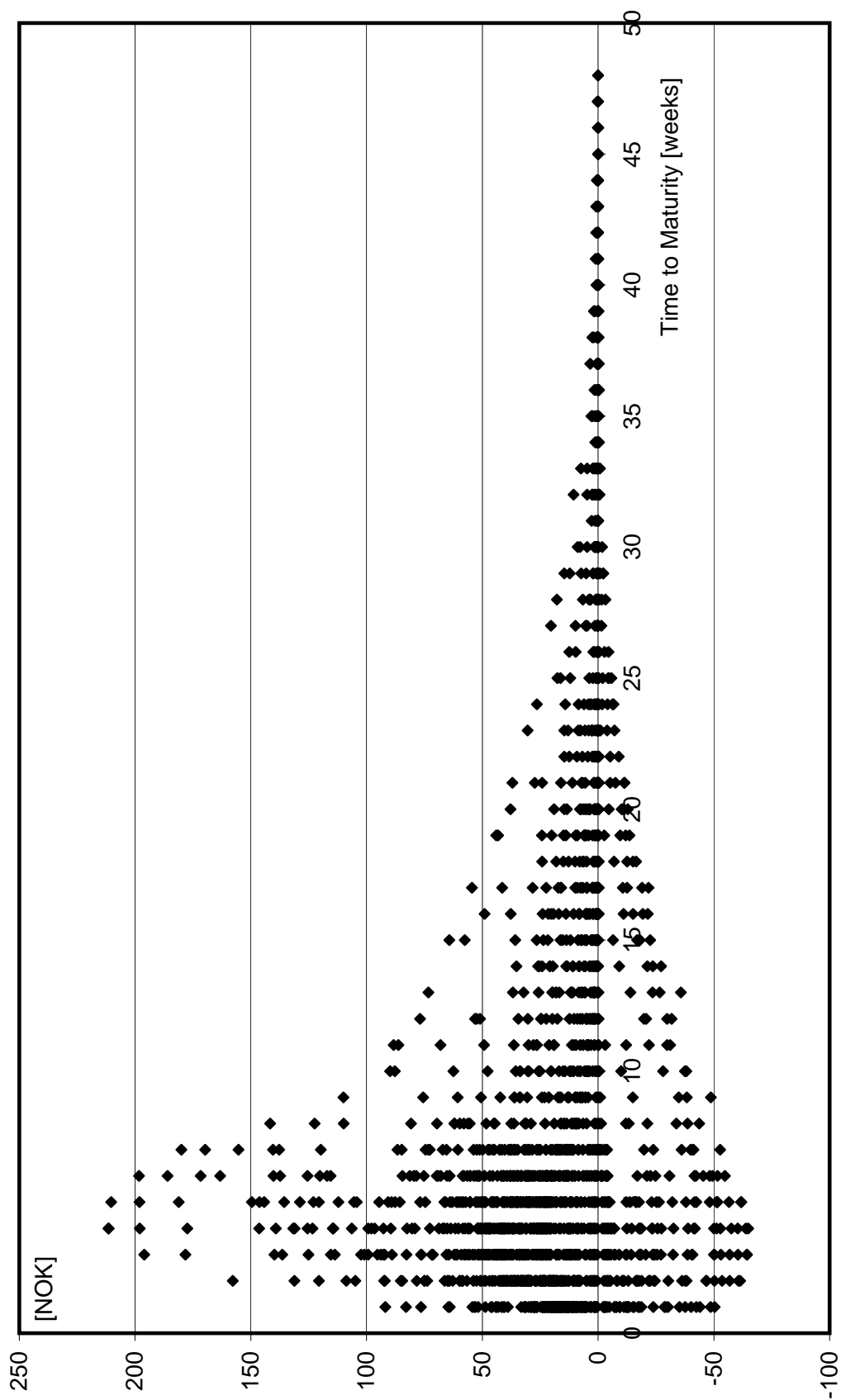
Figure 19: This Figure shows a scatter plot of the in-sample estimated risk premia of the equilibrium model along the time to maturity.
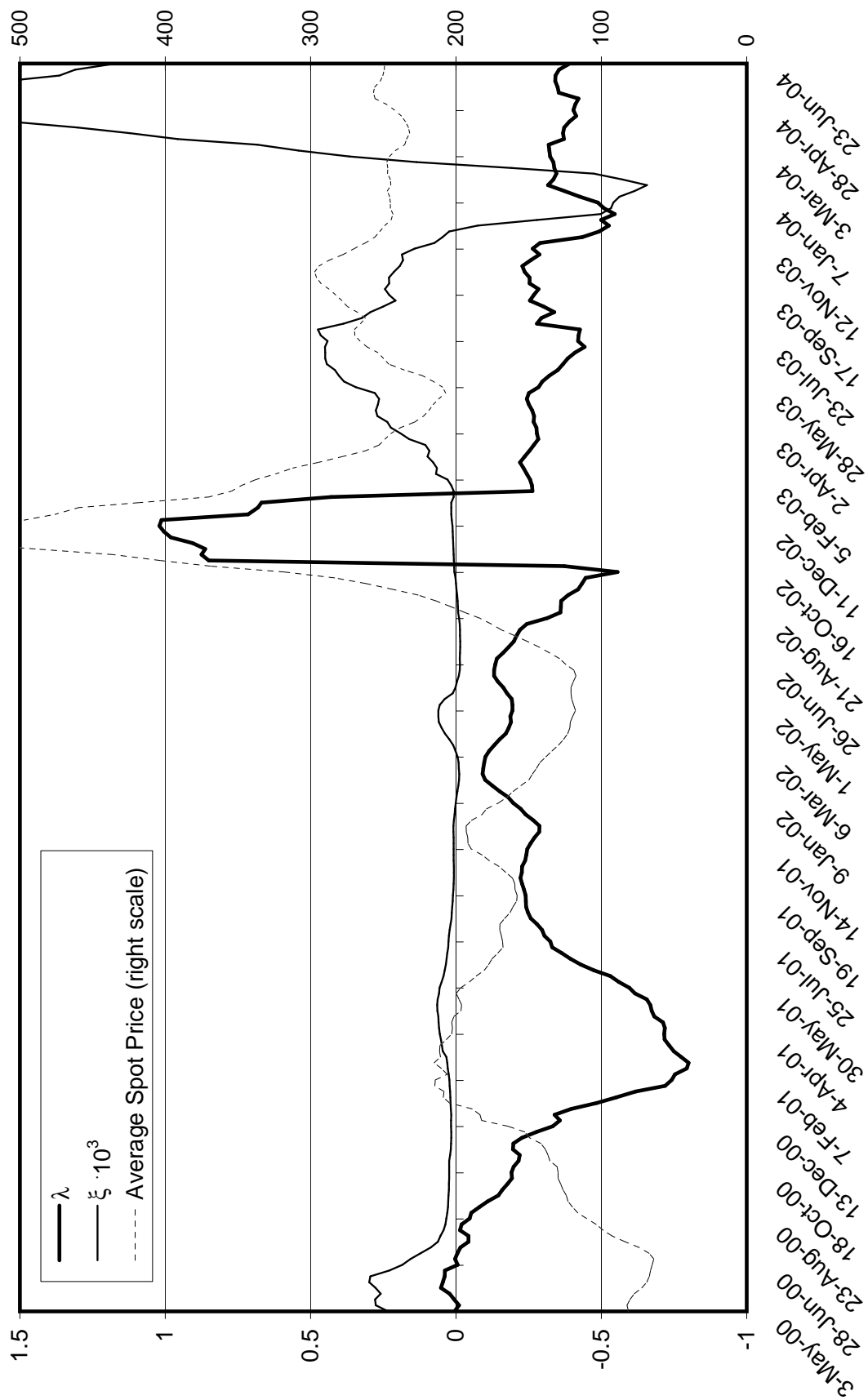
Figure 20: This figure shows the implicitly estimated λ and ξ of the reduced-form model and the equilibrium model, respectively.
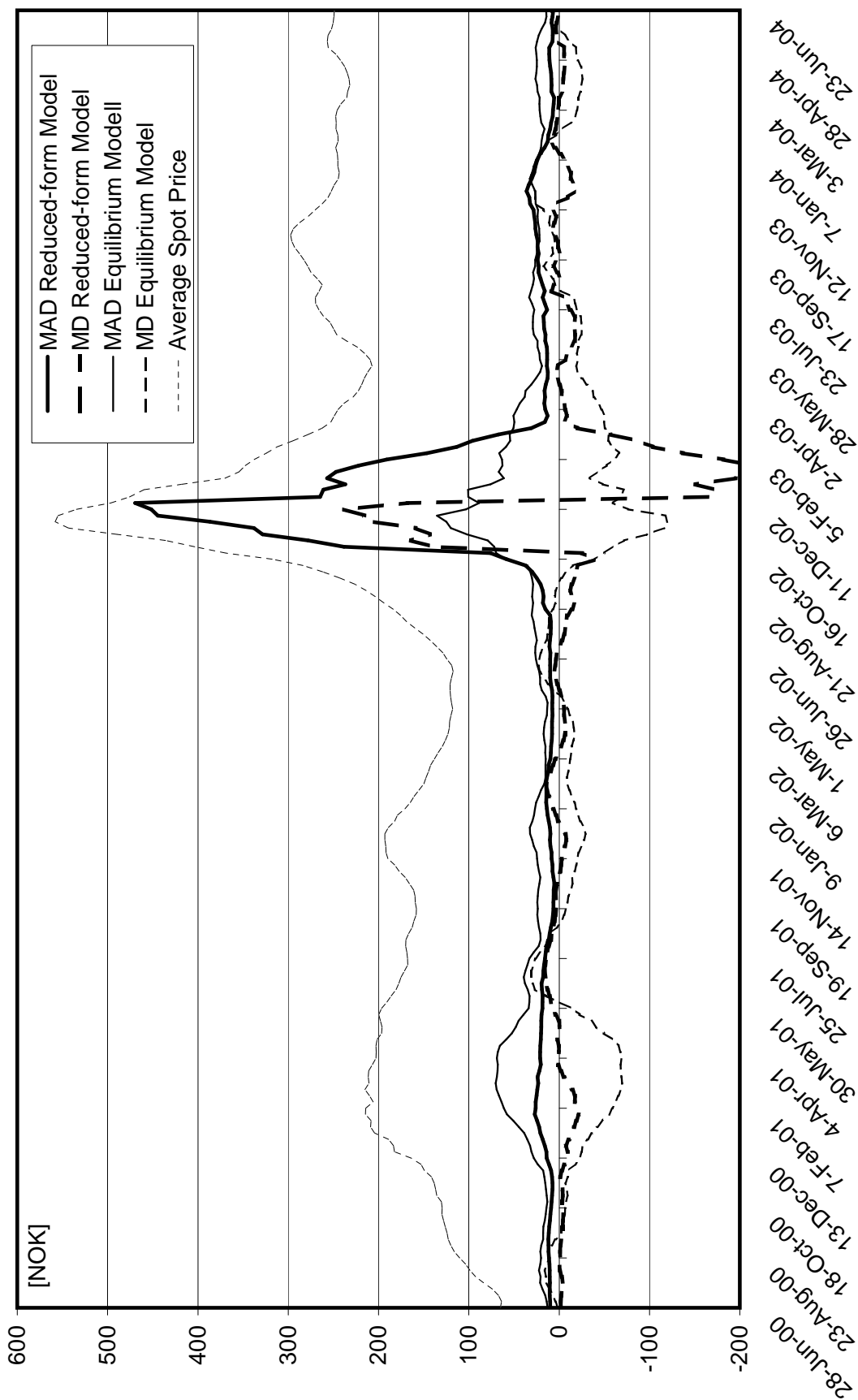
Figure 21: This figure shows the MAD and the MD for both models with the estimated λ and ξ applied out-of-sample, and the 8-week average spot price.