Being Locked Up Hurts

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ABSTRACT

This paper examines multi-period asset allocation when portfolio rebalancing is difficult or impossible for some assets due to the existence of a lockup period. A lockup period restricts an investor's ability to rebalance his portfolio and has non-trivial effects on the allocation decision and portfolio efficiency. Our empirical analysis shows that both the unconditional strategy and conditional strategy benefit from adding hedge funds. More importantly, both the unconditional strategy and conditional strategy are hurt by the presence of a hedge fund lockup period. In an unconditional setting, we find a Sharpe ratio of 1.23 for the portfolio of stocks, bonds and hedge funds, with a three-month lockup period for hedge funds and monthly rebalancing of stocks and bonds. For the same portfolio, but without a lockup, we find a significantly higher Sharpe ratio of 1.53. The certainty equivalent is 4.2%, i.e. a three-month lockup costs the investor 4.2% per annum. Therefore, the economic significance of a lockup period is also evident. Investors compensate for the lockup period of hedge funds by making adjustments to their equity and bond holdings. Adding hedge funds to the portfolio of stocks and bonds reduces the allocation to stocks and increases the allocation to bonds in each month. Finally, the effect of a lockup period on portfolio performance is less pronounced when investing in funds of hedge funds relative to investing in individual hedge funds when the investment horizon is short, suggesting that funds of funds are able to suppress the effect of a lockup period.

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I. Introduction

An important issue for both practitioners and academics in portfolio management is to solve a multi-period investment problem. The question is how to rebalance a portfolio before the investment horizon, which is often complicated by restrictions such as the inability to go short and the fact that some positions in illiquid assets cannot be rebalanced easily. For instance, investments in hedge funds are often accompanied by a lockup period, during which investors cannot withdraw their money. There are other restrictions such as a redemption notice period and redemption frequency that make it difficult for an investor to get his money out of hedge funds. The implication of a hedge fund lockup period is best illustrated by the experience of hedge fund investors during the recent financial crisis beginning in July, 2007. Investors would like to liquidate their investments in hedge funds to avoid further losses or to meet liquidity need elsewhere. But there is no escape if the hedge fund lockup period has not yet expired.

Institutional investors from many countries show an increasing allocation (in terms of both absolute dollar amounts and portfolio weights) to hedge funds, private equity and venture capital (Source: The 2007-2008 Russell Investments Survey on Alternative Investing). In this paper, we study the asset allocation problem for a multi-period investor when some of the assets such as hedge funds have a lockup period. The framework in this paper can be modified to take into account the redemption notice period and redemption frequency constraints. The analysis extends Brandt and Santa-Clara (2006) where the multi-period investment portfolio is solved in a static Markowitz-framework. We show that a hedge fund lockup period can be incorporated into the multi-period asset allocation decision by an investor who periodically re-adjusts his portfolio. In addition, we find that the lockup constraint considered in this paper is empirically highly relevant. Our empirical analysis shows that even with a hedge fund lockup, investing in hedge funds can improve portfolio outcomes under both unconditional strategies and conditional strategies.

This paper contributes to three strands of literature. The first strand is portfolio choice and valuation of illiquid or nonmarketable assets, going back Mayer (1976) and more recently by Longstaff (2001). Mayer (1976) presents a single-period mean-variance model of capital asset pricing taking into account nonmarketable assets. Our paper naturally extends the single-period framework to a multi-period setting. Longstaff (2001) develops a continuous-time model with thin-trading interpretation of illiquidity and shows the shadow cost of illiquidity for investors.

In this model, investors can trade only for a limited quantity of a security. The illiquidity facing hedge fund investors, however, is different from the thin-trading type. When investors start to invest in hedge funds with a lockup period, they can't sell any quantity of stakes until the lockup restriction expires. In addition, since investors often hold a portfolio of liquid and illiquid assets, it is interesting to investigate how they rebalance portfolio holding of liquid assets in each period, taking into account illiquid holdings, to maximize the multi-period utility.

This paper also contributes to the hedge fund literature by evaluating the economic value of hedge fund investments from a portfolio perspective. The evaluation of hedge fund performance has been studied in several papers, including Agarwal and Naik (2004), Fung, Hsieh, Naik and Ramadorai (2008), Kosowski, Naik, and Teo (2005), Malkiel and Saha (2005). In these studies, the performance of individual hedge funds or groups of hedge funds is typically evaluated on the basis of a factor model or a benchmark model. We take a portfolio perspective and compute optimal allocations to different asset classes in a portfolio. The benefits of adding hedge funds to a portfolio are evaluated by incremental Sharpe ratios or certainty equivalents.

Finally, our paper contributes to the literature of share restrictions of hedge funds. Asset classes such as hedge funds are often considered attractive investments because of their superior risk-return profile and low correlations to stocks and bonds. However, investments in hedge funds often face more restrictions than investments in stocks and bonds. For instance, many hedge funds impose a lockup period, ranging from a few months up to several years. Ang and Bollen (2008) model hedge fund lockups and notice periods as a real option. Aragon (2007), Derman (2007), Liang and Park (2008) examine the liquidity premium that a hedge fund investor is expected to earn from investing in hedge funds with a lockup period. In our paper, we derive the cost of a lockup period, taking into account the possibility of rebalancing of stocks and bonds in a portfolio of stocks, bonds and hedge funds. In other words, investors can at least partially hedge unexpected changes in hedge fund returns by changing holdings in stocks and bonds.

As in Brandt and Santa-Clara (2006), we solve a multi-period portfolio problem that consists of a set of timing portfolios and conditional portfolios. In a multi-period setting, a timing portfolio for a risky asset is a strategy that invests in only risky assets in one period and in only the risk-free asset in all remaining periods. Therefore, a multi-period asset allocation can be derived by solving a static Markowitz problem on the basis of timing portfolios and scaled returns or conditional portfolios. We incorporate the constraint of a lockup period for hedge funds to the asset allocation problem. If we assume that the investment horizon is equal to the length of the lockup period, there are no timing portfolios for hedge funds, because once an investment in hedge funds is made, an investor has to hold on to it until the lockup restriction expires. A portfolio of stocks, bonds and hedge funds with a lockup will certainly behave differently from a portfolio of the same assets without a hedge fund lockup period, in terms of allocations to different assets over time, as well as portfolio performance.

The paper uses broad market indices as the proxy for stocks, bonds and hedge funds/funds of funds in the empirical analysis. We rescale optimal portfolio weights such that the first period portfolio is the tangency portfolio. The (first period) tangency portfolio investor can move to the right of the tangency portfolio in the following periods, investing more in risky assets funded by a risk-free loan, or move to the left of the tangency portfolio, investing in the risk-free asset as well as risky assets. A lockup period has two important implications for investors. First of all, when there is no lockup period, an investor is relatively more aggressive in the first period in that he reduces total investments in risky assets in the second and third period. With a portfolio of stocks, bonds and hedge funds (using the HFRI composite as the proxy), the investor is going to invest about 19% and 24% in the risk-free asset in the second and third month, respectively. Keeping the mix of risky assets constant, this implies that the portfolio is less risky over time. Nevertheless, the investor also adjusts the proportions of stocks, bonds and hedge funds in the mix of risky assets. Within the mix of risky assets, the same investor will increase allocations to stocks and bonds, and substantially reduce allocations to hedge funds in the second and third month. Since bonds are relatively less risky than stocks and hedge funds, intertemporal adjustment of the mix of risky assets also favors a less risky strategy. Hence, without a lockup period, the investor becomes more conservative as the investment horizon approaches. When there is a three-month lockup period, however, the investor behavior is quite different. Allocations to stocks and bonds tend to increase over time, while allocations to hedge funds are stable because of a lockup period. This implies that after the first period, the investor will invest more than 100% in risky assets funded by a risk-free loan. To buy additional risky assets, the investor shorts the risk-free asset by about 8% and 28% in the second and third month, respectively. In this sense, the investor is more conservative in the beginning and becomes more aggressive over time, which is the opposite of the behavior when there is no lockup period. In addition, within the mix of risky assets, the proportion of stocks increases a bit in the third month, offset by the decrease in the proportion of hedge funds. The proportion of bonds in the mix of risky assets are relatively stable over time. Therefore, even though the investor has short positions in the risk-free asset in the second and third period, bond investments in the mix of risky assets are more or less the same as before. Overall, the investor seems to be more aggressive over time due to additional purchase of risky assets financed by borrowing. In contrast, the investor is less aggressive over time when there is no lockup; both investments in the risk-free assets and the proportion of bonds in the mix of risky assets increase in the following periods. Such difference in portfolio strategies stresses the importance of taking into account a lockup period in the investment decision.

In addition, this paper investigates hedge demands arising from the inclusion of hedge funds with a lockup period. Indeed, we find that a lockup period induces large, negative hedge demands for stocks in order to obtain the desired intertemporal equity exposure that cannot be obtained by hedge funds due to the lockup constraint. For a portfolio of stocks, bonds and hedge funds (using the HFRI composite as the proxy) with a three-month lockup period under the unconditional strategy, the Markowitz demands for stocks are 24%, 23% and 17% in Month 1, Month 2 and Month 3, decreasing over time. For bonds, the Markowitz demands increase from 22% in Month 1, to 30% in Month 2 and 39% in Month 3. The inclusion of the HFRI composite generates hedge demands of –47% , –41%, and –30% for stocks, and 9%, 5%, and 11% for bonds over the three months. As a result, the total demands for stocks in the three-asset portfolio are lower than the Markowitz demands over the three months, and display an upward trend over time. The hedge demands for bonds are positive in all three months, so the total demands for bonds are higher than the Markowitz demands for bonds and increase over time.

Our empirical analysis shows that both the unconditional strategy and the conditional strategy can be improved upon when adding hedge funds to the stock/bond portfolio, but portfolio performance is hurt when there is a hedge fund lockup period. For instance, the annualized Sharpe ratio for the unconditional strategy with stocks, bonds and funds of hedge funds (HFRIFOF composite) with a three-month lockup period is 1.23, which is significantly higher, both economically and statistically, than the Sharpe ratio of 0.91 for the unconditional strategy of stocks and bonds only. But if there is no hedge fund lockup period, the portfolio Sharpe ratio with the three asset classes is 1.53, which is significantly higher from the reported Sharpe ratio of 1.23 for the unconditional strategy with stocks, bonds and hedge funds with a three-month lockup period. The effect of a lockup period is stronger when the HFRI composite index and HFRI strategy indices are considered relative to fund of funds indices (i.e. HFRIFOF composite and strategy indices), especially at a short investment horizon. This suggests that fund of funds managers are able to structure their funds in such a way that their clients are hurt less by a lockup period.

In terms of certainty equivalents, an investor is willing to pay as much as 3.4% per year in order to move from the portfolio of stocks and bonds to the portfolio of stocks, bonds and funds of funds (HFRIFOF composite), even if there is a three-month lockup period. Hence, the economic value of hedge fund investments is large to an investor. In addition, an investor with the portfolo of stocks, bonds and funds of funds (HFRIFOF composite) is willing to pay 4.2% per year in order to make the lockup period 'disappear'. In other words, without the lockup period, the certainty equivalent of adding hedge funds to the portfolio of stocks and bonds will be much larger at 7.6%. It is in this sense that a lockup period hurts. A lockup period takes away some utility gains, but overall gains from inclusion of hedge funds are still positive and large.

The rest of the paper is organized as follows. Section II explains the methodology to derive optimal asset allocations for a quadratic utility investor facing a lockup period for hedge funds. The asset allocations for a generalized utility function are discussed in the Appendix to this paper. Section III describes the data. Section IV shows empirical results of a lockup period under the unconditional strategy, while Section V presents empirical results in the conditional framework. The bootstrap results are shown in Section VI. Finally, Section VII concludes.

II. Asset Allocation with a Lockup Period

We consider the allocation problem for a risk-averse investor. The investor's portfolio consists of liquid assets and illiquid assets. Liquid assets include stocks, bonds, money market instruments, etc., while illiquid assets can be hedge funds, private equity and venture capital investment. We restrict our attention to stocks, bonds, Treasury bills and hedge funds in our empirical analysis, but the same method is applicable when a portfolio includes other asset classes with similar liquidity features. The investor can change allocations to liquid assets every period, but adjusting allocations to illiquid assets is difficult if not impossible. The form of illiquidity in this paper is restricted to the situation in which a lockup period is imposed for investments in hedge funds.

A. Multi-period Asset Allocation with Lockup Constraints

We first illustrate the two-period asset allocation problem with lockup constraints, and generalize the method to the longer period setting. There are K_1 liquid risky assets and K_2 illiquid risky assets with a lockup period equal to L. For simplicity, the investment horizon has the same length as the lockup period. Consider the two-period quadratic utility optimization problem for an investor:

$$\max \ E_t\left[r^p_{t\to t+2} - \frac{\gamma}{2}\left(r^p_{t\to t+2}\right)^2 \right], \tag{1}$$

where $r^p_{t\to t+2}$ is the excess portfolio return over two periods and γ is the coefficient of risk aversion. Denote portfolio weights on liquid assets and illiquid assets at time t by $w_{z,t}$ and $w_{x,t}$, respectively. In addition, denote the one-period gross return on the risk-free asset at time t by R^f_t, and gross returns of illiquid assets by R^x_{t+1}. The vector r_{t+1} contains one-period excess returns of liquid risky assets. The two-period excess return of the portfolio with only liquid assets is:

$$\begin{aligned} r^p_{t\to t+2} &= \left(R^f_t + w'_t r_{t+1}\right)\left(R^f_{t+1} + w'_{t+1} r_{t+2}\right) - R^f_t R^f_{t+1} \\ &= w'_t\left(R^f_{t+1} r_{t+1}\right) + w'_{t+1}\left(R^f_t r_{t+2}\right) + \left(w'_t r_{t+1}\right)\left(w'_{t+1} r_{t+2}\right) \\ &\approx w'_t\left(R^f_{t+1} r_{t+1}\right) + w'_{t+1}\left(R^f_t r_{t+2}\right). \end{aligned} \tag{2}$$

Because r_{t+1} and r_{t+2} are excess returns, the product $\left(w'_t r_{t+1}\right)\left(w'_{t+1} r_{t+2}\right)$ is very small at short horizons, so the excess portfolio return over two periods is approximately the sum of $w'_t\left(R^f_{t+1} r_{t+1}\right)$ and $w'_{t+1}\left(R^f_t r_{t+2}\right)$.

Brandt and Santa-Clara (2006) interpret $w'_{z,t}\left(R^f_{t+1} r_{t+1}\right)$ and $w'_{z,t+1}\left(R^f_t r_{t+2}\right)$ as "timing portfolios". First, $w'_{z,t}\left(R^f_{t+1} r_{t+1}\right)$ is the two-period excess return from investing in risky assets at time t and then investing in the risk-free asset. Second, $w'_{z,t+1}\left(R^f_t r_{t+2}\right)$ is the two-period excess return from investing in the risk-free asset at time t and then investing in risky assets.

When the portfolio includes assets with a two-period lockup, the two-period portfolio excess return takes the form of the following:

$$r^{p}_{t\to t+2} = \left(R^{f}_{t} + w'_{z,t} r_{t+1}\right)\left(R^{f}_{t+1} + w'_{z,t+1} r_{t+2}\right) - R^{f}_{t} R^{f}_{t+1} + w'_{x,t} r^{x}_{t\to t+2}$$
$$\approx w'_{z,t}\left(R^{f}_{t+1} r_{t+1}\right) + w'_{z,t+1}\left(R^{f}_{t} r_{t+2}\right) + w'_{x,t} r^{x}_{t\to t+2}. \qquad (3)$$

where $r^{x}_{t\to t+2}$ is the K_2 dimensional vector of excess returns of illiquid assets, and for each illiquid asset, $r^{x}_{i,t\to t+2} = R^{x}_{i,t+1} R^{x}_{i,t+2} - R^{f}_{t} R^{f}_{t+1}$ for $i = 1,2,...,K_2$. For the two-period investment in illiquid assets, one dollar will grow by $R^{x}_{i,t+1} R^{x}_{i,t+2}$ and after paying back the risk-free loan, the two-period excess return on illiquid assets is $R^{x}_{i,t+1} R^{x}_{i,t+2} - R^{f}_{t} R^{f}_{t+1}$. There is no "timing" portfolio for illiquid assets since they are locked up over two periods.

The S dimensional vector of z_t is a set of state variables at time t. The portfolio weights are assumed to be linear in state variables. For liquid risky assets,

$$w_{z,t} = \beta_1 z_t \text{ and } w_{z,t+1} = \beta_2 z_{t+1}, \qquad (4)$$

where the matrices β_1 and β_2 both have a dimension of $K_1 \times S$. For illiquid assets, we have

$$w_{x,t} = \beta_x z_t, \qquad (5)$$

where β_x is a $K_2 \times S$ matrix. Throughout this paper, the portfolio strategy using the constant as the only state variable is defined as the "unconditional strategy". If state variables include time-varying instruments, then the portfolio strategy is called the "conditional strategy". For the optimal portfolio under the conditional strategy, an investor can simply maximize the utility (1) after inserting (3) into the utility function. It is obvious that the unconditional strategy is the special case of the conditional strategy with constants being the only state variable.

The equations (4) and (5) express portfolio weights as linear combinations of state variables, and the two-period portfolio excess return in (3) becomes

$$r^{p}_{t\to t+2} = (\beta_1 z_t)'\left(R^{f}_{t+1} r_{t+1}\right) + (\beta_2 z_{t+1})'\left(R^{f}_{t} r_{t+2}\right) + (\beta_x z_t)' r^{x}_{t\to t+2}. \qquad (6)$$

Using some linear algebra, we find

$$(\beta_1 z_t)'\left(R^{f}_{t+1} r_{t+1}\right) = vec(\beta_1)' R^{f}_{t+1}\left(z_t \otimes r_{t+1}\right), \qquad (7)$$

$$(\beta_2 z_{t+1})'\left(R^{f}_{t} r_{t+2}\right) = vec(\beta_2)' R^{f}_{t}\left(z_{t+1} \otimes r_{t+2}\right), \qquad (8)$$

$$(\beta_x z_t)' r^{x}_{t\to t+2} = vec(\beta_x)'\left(z_t \otimes r^{x}_{t\to t+2}\right). \qquad (9)$$

where $vec(\beta_j)$ is a vector that stacks the columns of the matrix β_j, $j = 1,2,x$, and $\otimes$ is the Kronecker product. The investment menu becomes a set of scaled returns or expanded asset return space, $\tilde{r}_{t+1} = z_t \otimes r_{t+1}$, $\tilde{r}_{t+2} = z_{t+1} \otimes r_{t+2}$ and $\tilde{r}^x_{t\to t+2} = z_t \otimes r^x_{t\to t+2}$. The investor's problem is to choose a set of parameters to maximize the multi-period quadratic utility:

$$\max_{\tilde{w}} \; E_t\left[\tilde{w}'\tilde{r}_{t\to t+2} - \frac{\gamma}{2}\tilde{w}'\tilde{r}_{t\to t+2}\tilde{r}'_{t\to t+2}\tilde{w}\right], \tag{10}$$

where parameters $\tilde{w}' = \left(vec(\beta_1)' \quad vec(\beta_2)' \quad vec(\beta_x)'\right)$ can be considered as unconditional weights in the portfolio of expanded assets, and scaled returns of those expanded assets are defined as $\tilde{r}'_{t\to t+2} = \left(\left(R^f_{t+1}\tilde{r}_{t+1}\right)' \quad \left(R^f_t\tilde{r}_{t+2}\right)' \quad \left(\tilde{r}^x_{t\to t+2}\right)'\right)$. Once unconditional weights $\tilde{w}$ are derived, they can be inserted into equations (4) and (5) to compute the portfolio weights of liquid and illiquid assets. The unconditional weights $\tilde{w}$ that maximize the conditional expected utility at all dates t should also maximize the unconditional expected utility. The optimization still makes use of the static Markowitz approach on the basis of the unconditional moments of scaled returns. The optimal static or unconditional weights are:

$$\tilde{w} = \frac{1}{\gamma}E[\tilde{r}_{t\to t+2}\tilde{r}'_{t\to t+2}]^{-1}E[\tilde{r}_{t\to t+2}]. \tag{11}$$

The sample analogue of the population moments in equation (11) leads to a consistent estimate of the unconditional weights $\tilde{w}$, which is a vector of length $(2K_1S + K_2S)$. The optimal weights $\tilde{w}$ is with respect to scaled returns or expanded asset returns, but we can recover the optimal portfolio weights on K_1 risky assets at time t and $t+1$, $w_{z,t}$ and $w_{z,t+1}$ as

$$w^i_{z,t} = \left(\tilde{w}_{(i)} \quad \tilde{w}_{(i+K_1)} \quad \cdots \quad \tilde{w}_{(i+(S-1)K_1)}\right)z_t,\; i = 1,2,\cdots,K_1. \tag{12}$$

$$w^i_{z,t+1} = \left(\tilde{w}_{(i+K_1S)} \quad \tilde{w}_{(i+K_1+K_1S)} \quad \cdots \quad \tilde{w}_{(i+(S-1)K_1+K_1S)}\right)z_{t+1},\; i = 1,2,\cdots,K_1. \tag{13}$$

For illiquid assets, the portfolio weights at time t can be derived in the same way as those of liquid risky assets.

$$w^i_{x,t} = \left(\tilde{w}_{(i+2K_1S)} \quad \tilde{w}_{(i+K_2+2K_1S)} \quad \cdots \quad \tilde{w}_{(i+(S-1)K_2+2K_1S)}\right)z_t,\; i = 1,2,\cdots,K_2. \tag{14}$$

However, the static optimal portfolio weights in (11) do not give direct solutions to the portfolio weights of illiquid assets at time $t+1$. We can normalize the initial portfolio value to one and the portfolio weight of illiquid asset i is the ratio of its value to the portfolio value at the beginning of time $t+1$.

We can generalize the method above to the L-period asset allocation problem with lockup constraints on certain risky assets. The optimal static portfolio weights are

$$\tilde{w} = \frac{1}{\gamma} E[\tilde{r}_{t \to t+L} \tilde{r}'_{t \to t+L}]^{-1} E[\tilde{r}_{t \to t+L}], \tag{15}$$

where $\tilde{r}_{t \to t+L}$ is a set of timing portfolios with scaled returns of liquid assets and L-period excess returns of illiquid assets scaled by the information set z_t.

The solution in (15) may produce negative weights for illiquid assets with a lockup period. In reality, while shorting stocks and bonds is relatively easy, shorting illiquid assets is either too costly or impossible. For instance, investors cannot short hedge funds or transfer their stakes in hedge funds to other investors. In this case, investors should add a nonnegative constraint on portfolio weights of illiquid assets to the analysis.

B. Econometric Issues

We estimate the set of portfolio weights in (15) by sample analogue. In addition, we can test whether the portfolio weights of each asset are equal across the investment horizon by a Wald test or F test. The construction of the estimated covariance matrix of $\tilde{w}$ and the test procedure follow the method by Britten-Jones (1999).

Given a time-series sample of asset returns, the estimation of $\tilde{w}$ can be sensitive to the choice of starting dates of the sample. Specifically, for a lockup period of L, we have L choices of starting dates, and the resulting L sets of the estimated $\tilde{w}$ are all consistent asymptotically. Following Jegadeesh and Titman (1993), and Rouwenhorst (1998), we consider L strategies that contribute equally to a composite portfolio. Specifically, at the start of each period, the composite portfolio consists of L sub-portfolios. Each sub-portfolio invests optimally according to one set of estimated $\tilde{w}$ on the basis of an estimation window. For example, suppose the lockup period is two months and the sample data consists of ten-year monthly asset returns. We can estimate $\tilde{w}$ using two different windows: one starting one month later than another in the data. The composite portfolio invests one half according to the first set of estimated $\tilde{w}$ and one half according to the second set of estimated $\tilde{w}$. The method is comparable to that in Jegadeesh and Titman (1993), and Rouwenhorst (1998). In those two papers, they report the monthly average return of K strategies for a K-month holding period in order to evaluate the relative strength portfolios.

C. Generalized Utility Function and GMM Estimation

The quadratic utility in expression (1) can be considered as a second-order approximation of power utility. The approximation is not a serious concern if asset returns are normally distributed. However, asset classes such as hedge funds may exhibit some non-normality such as large, negative skewness or excess kurtosis. This is especially true for hedge funds pursuing a relative value strategy such as merger arbitrage. Extreme gains or losses are more likely for such hedge funds than what normality implies. Therefore, the quadratic utility maximization may result in asset allocations undesirable from the perspective of a power utility investor. To measure the effect of including the third and fourth moments on the asset allocation decision, portfolio characteristics and economic values of hedge fund investments, this paper also considers a higher order approximation of power utility similar to the fourth-order approximation scheme in Brandt and Santa-Clara (2006) and Brandt et al. (2009). The optimal weights are implied in the first order conditions of the expected utility maximization based on the fourth-order approximation. Brandt and Santa-Clara (2006) suggest a guessing method to obtain optimal weights. Nevertheless, optimal weights and covariance matrix can be derived by the generalized method of moments (GMM, Hansen and Singleton (1982)). On the basis of the fourth-order approximation, we find that results of asset allocations and portfolio characteristics are quantitative similar to those under the second-order approximation[1].

III. Data

For hedge funds, we obtain various hedge fund indices and fund of funds indices from Hedge Fund Research, Inc. (HFR, Inc.). A fund of funds or fund of hedge funds is a hedge fund that invests with multiple managers of hedge funds or managed accounts. Since a fund of funds holds a diversified portfolio of hedge funds, it lowers the risk of investing with an individual hedge fund manager and gives access to hedge funds that are closed to new money (Nicholas (2004)). The length of the lockup period depends on liquidity of underlying individual hedge funds in fund of funds portfolios. Some funds of funds require no lockup periods, but a lockup

[1] The derivation and empirical results of optimal asset allocations under the fourth-order approximation of power utility are available upon request.

period of three months up to two years is not uncommon. An individual U.S. hedge fund typically requires a one-year lockup period plus a notice period ranging from one month to three months. In contrast, less than 40 percent of funds of funds require a lockup period, and among those funds of funds that do, about two third of them set a lockup period of six months or longer (Nicholas (2004)). The HFRI Fund of Funds Composite Index (HFRIFOF) is an equal weighted index that includes over 800 funds of hedge funds with at least USD 50 Million under management. Monthly returns are net of all fees. HFR, Inc. also provides four equal weighted sub-indices according to the classification of fund of funds strategies: Conservative, Diversified, Market Defensive, and Strategic. A fund of funds is classified as "Conservative" if it tends to invest in funds with conservative strategies such as Equity Market Neutral, Fixed Income Arbitrage, etc. that exhibit low historical volatilities. A fund of funds is "Diversified" if it invests with various strategies/managers and exhibits performance close to that of the HFRIFOF composite index. A "Market Defensive" fund of funds invests in hedge funds with short-biased strategies and exhibits a low or negative correlation with the equity market benchmark. Finally, a "Strategic" fund of funds tends to invest in hedge funds with more opportunistic strategies and exhibits greater volatility relative to the HFRIFOF composite index. For the composite index based on individual or single-strategy hedge funds, we use the HFRI Fund Weighted Composite Index (HFRI), which is an equal weighted index based on more than 2000 single-strategy hedge funds. The HFRI index excludes funds of funds to prevent double counting of performance figures. In addition, HFR, Inc. classifies single-strategy hedge funds into four primary strategies: Equity Hedge, Event-Driven, Macro, and Relative Value. Each primary strategy includes several sub-strategies. HFR, Inc. provides detailed descriptions of primary and sub-strategies in its products and website. From CRSP, we obtain the value-weighted NYSE index as the proxy for stocks, the 1-month Treasury bill as the proxy for the risk-free asset, and the Fama Bond Portfolio (Treasuries) with maturities greater than 10 years as the proxy for bonds. We construct quarterly returns from monthly index returns of stocks, bonds, and hedge funds. The relatively short sample period for the hedge fund data limits the empirical analysis to the sample period from December 1989 through December 2007. Table 1 gives summary statistics of stocks, bonds and hedge funds.

Over the sample period, the average return and volatility of stocks are 11.4% and 12.6%, respectively. Bonds have an average return of 8.5% and volatility of 7.9%, but the Sharpe ratio of bonds is only slightly lower than that of stocks. The HFRIFOF composite index has a lower average return (9.7%) and volatility (5.5%) compared to stocks, and a Sharpe ratio of 1.03, which is almost twice as large as the Sharpe ratio of stocks or bonds. The HFRIFOF Conservative index has the lowest volatility among all fund of funds indices, consistent with the style classification. The HFRIFOF Diversified index shows a similar average return and volatility compared to the composite index. Although average returns and volatilities differ among four HFRIFOF strategy indices, their Sharpe ratios are not too far away from each other. In contrast, the HFRI Relative Value shows a Sharpe ratio that is higher than the other three HFRI strategy indices and the HFRI composite index, mainly due to its low volatility. But given the nature of the Relative Value strategy, the returns of this strategy show fat tails. The excess kurtosis of the Relative Value returns is 10.43, much larger than what it would be under the normal distribution. In this case, comparisons on the basis of means and volatilities should be made with caution. The average returns of the HFRI composite index and the HFRI strategy indices are quite high compared to stocks, bonds and fund of funds indices. The average return of the HFRI composite index is 13.2%, which is 3.5% higher than the average return of the HFRIFOF composite index, while the volatility of the HFRI composite index is about 6.6%, only 1.1% higher than that of the HFRIFOF composite index. The difference in Sharpe ratios of the two composite indices is 0.35, so it seems that funds of funds offer lower risk-adjusted returns relative to the aggregate individual hedge funds. The double fee structure of fund of funds investments may account for some of the difference in risk-adjusted returns, but some studies argue that the greater survivorship bias underlying single-strategy hedge funds may cause reported under-performance of funds of funds (e.g. Fung and Hsieh (2000)).

We obtain the state variables from CRSP[2]. We include the market dividend-price ratio that is known to predict asset returns.[3] The market dividend-price ratio is based on the value-weighted NYSE equity index, calculated as the ratio of sum of dividends over past twelve months to the NYSE index level. Figure 1 plots the time series of the market dividend-price ratio from December 1989 to December 2007. The market dividend-price ratio is closely linked to the ups and downs of the U.S. stock market, so the long bull market in 1990s results in a downward trend of the market dividend-price ratio during this period. Table 2 gives the

[2] This paper includes the market dividend-price ratio as the state variable. Results based on other state variables such as the short-term interest rate, term spread and default spread are available on request.

[3] See Campbell (1987), Campbell and Shiller (1988a), (1988b), Campbell and Viceira, (1999), (2002), Cochrane (2007), Fama and French (1988), (1989), Keim and Stambaugh (1986), Hodrick (1992), and Lettau and Ludvigson (2005). Goyal and Welch (2007) and Campbell and Thompson (2007) include a comprehensive list of these variables along with some others as predictors used in predictability studies.

correlation matrix for risky asset returns and the market dividend-price ratio. For most hedge fund indices, their correlations to the market dividend-price ratio are stronger than the correlations of stocks and bonds to the market dividend-price ratio. The correlations of hedge fund returns to stock returns are moderate for most hedge fund indices, except for the HFRIFOF Market Defensive. Stock returns and bond returns are weakly correlated as expected. Most hedge fund index returns have a low correlation to bond returns, with the exception of the HFRI Macro. Notice that the correlation of stock returns to the HFRIFOF composite index returns is high (0.43), but lower than the correlation of stock returns to the HFRI composite index returns (0.69). This implies that the HFRIFOF is a better diversifier than the HFRI does. On the other hand, a high correlation of stock returns to hedge fund returns indicates that hedge funds and funds of funds have large equity exposures.

IV. Unconditional Strategy with a Three-Month Lockup

Section A starts by reporting optimal portfolio weights of the unconditional strategy with a three-month hedge fund lockup period. We are interested in the difference in allocations to stocks and bonds when hedge funds are added to the portfolio, as well as changes in investment patterns over the three-month investment horizon. We rescale optimal total demands for stocks, bonds and hedge funds such that the first-period portfolios are tangency portfolios, i.e. the sum of portfolio weights of stocks and bonds (and hedge funds in a three-asset portfolio) in the first period is equal to 1. Hence, after the first period, an investor makes two allocation decisions: how to allocate between risky assets and the risk-free asset, and how to allocate across different risky assets within the mix of risky assets[4]. We decompose the total demand for stocks and bonds in a portfolio of stocks, bonds and hedge funds into the Markowitz or speculative demand and the hedge demand. Section B compares the performance of the unconditional strategy with a lockup period and without a lockup period. In addition, we test whether adding hedge funds improves the Sharpe ratio of the stock/bond portfolio.

[4] Portfolio weights of stocks, bonds and hedge funds in Table 3, Table 4, Table 6 and Table 7 are proportions of stocks, bonds and hedge funds to the total portfolio value of risky assets and the risk-free asset. Whenever the mix of risky assets is mentioned in the paper, it refers to a portfolio of risky assets, excluding the risk-free asset. The proportion of each risky asset to the mix of risky assets is not equivalent to the portfolio weight of the asset in the portfolio of all assets, whenever the investment in the risk-free asset is nonzero.

A. Optimal Demands, Markowitz Demands and Hedge Demands

Table 3 reports results for the unconditional strategy with a three-month hedge fund lockup period. The estimated parameters, portfolio performance and test statistics are the averages of three-month rolling windows. We can think of this as the result of a strategy that always invests 1/L of wealth for three months (i.e. $L = 3$ in Section IV), starting every month, just as in Jegadeesh and Titman (1993) and Rouwenhorst (1998) (see Section II.B. Econometric Issues). The t-statistics for portfolio weights are based on the covariance matrix that is estimated following the approach by Britten-Jones (1999).

Results for the unconditional strategy in Table 3 show that portfolio weights vary in a systematic way over the investment horizon. The variation in portfolio weights is caused by the presence of timing portfolios. To start out, in the portfolio of stocks and bonds only, allocations to stocks and bonds display distinct patterns over the investment horizon. Over the three months, allocations to stocks decrease monotonically from 52% to 37% while allocations to bonds increase monotonically from 48% to 85%. Thus, an investor starts with a relatively risky portfolio and gradually adjusts his portfolio holdings in order to obtain a less risky portfolio by the end of the investment horizon. However, since the increase in allocations to bonds is greater than the decrease in allocations to stocks in the second and third month, the investor will invest more than 100% in stocks and bonds after the first month. Such a portfolio is considered to be more risky, but the increase in risk due to leverage in the second and third month is probably offset somewhat by the increasing allocations to bonds in the mix of risky assets. We test the restriction that allocations to stocks or bonds are equal across three months, and the p-values indicate that the null hypothesis cannot be rejected for both stocks and bonds.

Adding hedge funds to a portfolio of stocks and bonds changes the pattern of portfolio weights of stocks over the investment horizon, while the pattern of portfolio weights of bonds remains monotonically increasing. For example, inclusion of the HFRI composite with a three-month lockup period will reverse the pattern of investments in stocks from being monotonically decreasing to be monotonically increasing from –23% in Month 1 to –12% in Month 3. However, inclusion of the HFRIFOF composite with a three-month lockup period to the portfolio of stocks and bonds will change the pattern of investments in stocks over the three-month period from being monotonically decreasing to being an inverted U-shape, as allocations to stocks increase from 9% in Month 1 to 14% in Month 2, and decrease to 12% in Month 3.

A three-month lockup period has significant impact on the investor behavior over the investment horizon. When there is no lockup period, an investor is relatively more conservative over time as he increases allocations to the risk-free asset in the second and third month. For the portfolio of stocks, bonds and hedge funds (using the HFRI composite as the proxy), investments in the risk-free asset are about 19% and 24% in the second and third month, respectively. Hence, if the investor keeps investing in stocks, bonds and hedge funds in the same proportions as in the first period, the portfolio is going to be less risky. Interestingly, the investor also adusts the proportion of stocks, bonds and hedge funds in the mix of risky assets over time. Within the mix of risky assets, the investor will increase allocations to stocks and bonds, and reduce allocations to hedge funds in the second and third month. For instance, in the third month, allocations to stocks, bonds and hedge funds are 7%, 38% and 32%. This implies that the proportion of bond investments in the mix of risky assets is about 50% in the third month, which is sharpely higher than the proportion of bond investments in the mix in the first period, 22%. Compared to a proportion of more than 78% in the first month, stocks plus hedge funds only account for about 50% in the mix of risky assets in the third month. Since bonds are relatively less risky than stocks and hedge funds, adjustment of the mix of risky assets leads to a less risky portfolio. Together with the increasing allocations to the risk-free assets, the portfolio strategy is less aggressive over time. To sum it up, without a lockup period, an investor becomes more conservative as the investment horizon approaches.

When there is a three-month lockup period, allocations to stocks and bonds increase over time. Stocks account for –23%, –18%, and –12% and bonds have weights of 31%, 35% and 50% over the three-month period. In the first month, hedge funds have a weight of 92%. The number of shares invested in hedge funds is the same for remaining periods and portfolio weights of hedge funds will be around 92% in the second and third month, unless there are sharpe month-to-month changes in portfolio values. Average allocations to hedge funds are 91% and 90% in the second and third month (not included in Table 3 to stress that they are results of growth in hedge fund and portfolio values over time), only slightly lower than the portfolio weight of hedge funds in the first month. This implies that after the first period, the investor will invest more than 100% in risky assets funded by a risk-free loan. The investor purchases additional risky assets by shorting the risk-free asset of 8% and 28% in the second and third month, respectively. In this sense, the investor becomes more aggressive over time, as he has shorting positions in the risk-free asset after the first month. In addition, within the mix of risky assets, the proportion of stocks increases a bit in the second and third month, while there is an decrease in the proportion of hedge funds over the same periods. The proportion of bonds in the mix are relatively stable over time. Therefore, even though the investor has some a short position in the risk-free asset to purchase additional risky assets in the second and third period, bond investments in the mix of risky assets are more or less the same as before. Overall, the investor seems to be more aggressive over time as the portfolio is tilted to additional amount of risky assets financed by shorting the risk-free asset. In contrast, the investor is more conservative over the investment horizon when there is no lockup; both investments in the risk-free assets and the proportion of bonds in the mix of risky assets increase as the investment horizon approaches. Such constrasting investor behaviors underscore the importance of taking into consideration of a lockup period in the multi-period asset allocation decision.

To further investigate these changes in the patterns of portfolio weights of stocks and bonds, we calculate Markowitz (or pure speculative) demands and hedge demands for stocks and bonds in the three-asset portfolio with a hedge fund lockup period. Table 4 shows the optimal demand for stocks and bonds as the combination of the Markowitz demand and the hedge demand, using either the HFRI or the HFRIFOF composite index as the proxy for hedge funds in the three-asset portfolio with a lockup restriction. Specifically, the optimal demand for stocks and bonds in the three-asset portfolio takes the following form[5]:

$$w_z = \frac{1}{\hat{\gamma}} \Sigma_{zz}^{-1} \mu_z + \left(- \Sigma_{zz}^{-1} \Sigma_{z,x} w_{x,t}^* \right). \tag{16}$$

The w_z is the optimal demand for stocks and bonds over the investment horizon, Σ_{zz}^{-1} is the covariance matrix of timing portfolio of stocks and bonds, μ_z is the vector of expected returns of timing portfolios of stocks and bonds, $\Sigma_{z,x}$ is the covariance matrix of three-period excess returns of hedge funds and timing portfolio of stocks and bonds, while $w_{x,t}^*$ is the optimal demand for hedge funds in the three-asset portfolio. The Markowitz demand for stocks and

[5] Note that the optimal weights in equation (11) are based quadratic utility or mean-second moment utility. However, from the optimal weights derived from mean-second moment utility to those derived from mean-variance utility is only a matter of rescaling the risk-aversion. Hence, we can get a mean-variance version of equation (11). See Britten-Jones (1999) for a description on the conversion of mean-second moment portfolios to mean-variance portfolios.

bonds in the three-asset portfolio, $\frac{1}{\hat{\gamma}}\Sigma_{zz}^{-1}\mu_z$, is simply the optimal demand for stocks and bonds in a portfolio of stocks and bonds only. The hedge demand, $-\Sigma_{zz}^{-1}\Sigma_{z,x}w_{x,t}^{*}$, is the product of two determinants: the optimal demand for hedge funds at time t, denoted by $w_{x,t}^{*}$, and $\Sigma_{zz}^{-1}\Sigma_{z,x}$, which are slope coefficients from the regression of three-month excess returns of hedge funds on a constant and returns of timing portfolios of stocks and bonds:

$$r_{t\to t+3}^{x} = \alpha + b_{s,1}'\left(R_{t+1}^{f}R_{t+2}^{f}r_{t+1}^{s}\right) + b_{s,2}'\left(R_{t}^{f}R_{t+2}^{f}r_{t+2}^{s}\right) + b_{s,3}'\left(R_{t}^{f}R_{t+1}^{f}r_{t+3}^{s}\right)$$
$$+ b_{b,1}'\left(R_{t+1}^{f}R_{t+2}^{f}r_{t+1}^{b}\right) + b_{b,2}'\left(R_{t}^{f}R_{t+2}^{f}r_{t+2}^{b}\right) + b_{b,3}'\left(R_{t}^{f}R_{t+1}^{f}r_{t+3}^{b}\right) + \varepsilon_t, \qquad (17)$$

For instance, the hedge demand for stocks in the first period is $-(w_{x,t}^{*})'\cdot b_{s,1}$. The hedge demands for stocks in other periods and those for bonds follow the same logic.

From Table 4, the restriction that hedge demands (as well as optimal demands) are equal across three months cannot be rejected by the Wald test for all cases. We find that for each month, the hedge demand is negative for stocks and positive for bonds. Furthermore, the hedge demand for stocks is most negative in the beginning and increases over time, which results in a pattern of optimal demands different from that of Markowitz demands for stocks. For instance, adding the HFRIFOF composite to the portfolio of stocks and bonds gives rise to a small allocation to stocks relative to the Markowitz demand in the first month (9% vs. 27%). The Markowitz demand decreases to 26% in the second month, while the total demand increases to 14% due to an increase in the hedge demand. In the third month, the total demand for stocks decreases to 12%, as the increase in the hedge demand is more than offset by the decrease in the Markowitz demand. This is the reason that the total demands for stocks exhibit an inverted U-shape. For bond investments in the three-asset portfolio, changes in portfolio weights are dominated by changes in the Markowitz demands. The hedge demands for bonds are relatively small; changes in the hedge demands over the three-month horizon are not large enough to reverse the pattern of total investments in bonds.

We can explain the difference in the patterns and magnitudes of the hedge demands for stocks and those for bonds by examining the second determinant of the hedge demands, which is the set of slope coefficients from the regression (17). We look at the correlations between hedge funds and stocks or bonds in Table 2 to get a rough estimation of the magnitude and direction of the hedge demand[6]. From Table 2, the correlation between stocks and the HFRI composite is 0.69, while the correlation between bonds and the HFRI composite is near zero. In other words, hedge funds look more like stocks. To hedge the changes in the value of hedge funds, an investor can simply go short on stocks, and the hedge demand for stocks is $-(w^*_{x,t})' \cdot b_{s,1}$ for the first month. As bonds and hedge funds are weakly correlated, the hedge demand for bonds is relatively small.

Investing in hedge funds when there is a lockup period, basically leads to an exogenously given exposure to hedge funds after the first period, which induces additional hedge demand for stocks and bonds. The optimal investment in stocks and bonds in the three-asset portfolio is the sum of the Markowitz demands and the hedge demand. The Markowitz demand is the optimal portfolio weights of stocks and bonds when the investment menu includes stocks and bonds only. The hedge demand arises because the investor wants to hedge the changes in the value of hedge fund investment, which is locked up for three months. A negative hedge demand for stocks implies that the overall allocation to stocks will be lower than it would be in the portfolio consisting of only stocks and bonds.

The patterns of investments in stocks differ when different hedge fund indices are used as a proxy. We can explain the difference by examining the difference in the two determinants of hedge demands for stocks. The first determinant $w^*_{x,t}$ is larger when the HFRI composite is included in the portfolio, relative to the allocation to hedge funds in the portfolio of stocks, bonds and the HFRIFOF composite. In addition, the correlation between stock returns and the HFRI composite returns is 0.69, higher than the correlation between stock returns and the HFRIFOF composite, 0.43. In other words, the second determinant, $b_{s,1}$, is more likely to be larger when the HFRI composite is the hedge fund proxy. Both determinants work in the same direction such that the hedge demands are larger (in absolute value or magnitude) in the portfolio of stocks, bonds and HFRI than those in the portfolio of stocks, bonds and the HFRIFOF composite. In fact, the hedge demands are so much larger than the Markowitz demands for stocks when the HFRI composite is included in the portfolio that they lead to negative total demand for stocks.

[6] The estimation is not precise because the dependent variable is the three-month excess return of the hedge fund and independent variables are returns on timing portfolios of stocks and bonds. The correlation matrix in Table 2, however, is based on monthly series of excess returns on hedge funds, stocks and bonds.

The total allocations to bonds in the three-asset portfolios are similar to the Markowitz demands when either the HFRI or the HFRIFOF composite is the proxy. Adding either hedge fund composite would not change the trend of investments in bonds. In all cases, total allocations to bonds increases monotonically over the three-month period. The hedge demands for bonds are larger in the portfolio of stocks, bonds and the HFRI composite than those in the portfolio of stocks, bonds and the HFRIFOF composite, mostly due to a large portfolio weight of the HFRI composite (i.e. a large $w^*_{x,t}$, the first determinant of the hedge demand for bonds). Nevertheless, the hedge demands for bonds are small relative to the Markowitz demands, and the variation (month-to-month difference) in the hedge demands is not large enough to make a difference in the trend of total investments in bonds. For instance, in the portfolio of stocks, bonds and HFRI composite, the Markowitz demand for bonds is 22%, 30% and 39% in the first, second and third month. The corresponding hedge demands for bonds are 9%, 5% and 11% (5%, 3% and 4% in the portfolio of stocks, bonds and the HFRIFOF composite). The variation in the Markowitz demands is 8% from month 1 to month 2, and 9% from month 2 to month 3. In contrast, the month-to-month variation in the hedge demands is less than 6% (1% in the portfolio of stocks, bonds and the HFRIFOF composite).

B. Portfolio Efficiency and Certainty Equivalents

The above analysis has shown that taking into account lockup periods for hedge funds has important portfolio implications. A question of considerable importance is now whether hedge funds offer diversification benefits when they are added to the portfolio of stocks and bonds only. Table 5 reports performance of portfolios of stocks, bonds and hedge funds under the unconditional strategy. The p-values, as they appear in the table, are calculated based on the averaged test statistics over the three overlapping samples. In each case, a different hedge fund index is used as the proxy. The mean excess return and volatility of the two-asset portfolio are 7.1% and 7.8%, respectively. The Sharpe ratios of three-asset portfolios are much higher than the two-asset portfolio. The difference in mean returns, volatilities and Sharpe ratios of three-asset portfolios is large for all hedge fund indices. For instance, the portfolio of stocks, bonds and the HFRIFOF composite with a lockup has a mean excess return of 6.6% with a volatility of 5.4%, compared to a mean excess return of 9.5% and a volatility of 6.1% for the portfolio of stocks, bonds and the HFRI composite. The Sharpe ratio of the first portfolio above is 1.23, lower than the Sharpe ratio of 1.55 of the second portfolio.

The test of portfolio efficiency follows Jobson and Korkie (1982) and De Roon and Nijman (2001). Denote the sample Sharpe ratio for the benchmark portfolio r^p by $\hat{\theta}_p$, and the sample Sharpe ratio for the portfolio of test assets r and benchmark assets r^p, by $\hat{\theta}$. The Wald statistic of the Sharpe ratio test is:

$$\xi_W = T\left(\frac{\hat{\theta}^2 - \hat{\theta}_p^2}{1+\hat{\theta}_p^2}\right) \sim \chi_K^2 \tag{17}$$

where T is the sample size and K is the degrees of freedom. The degrees of freedom are the difference in the number of parameters between the two portfolios. A Sharpe ratio takes into account only the mean and standard deviation of portfolio returns and it is a proper measure of performance only if portfolio returns are normally distributed. Monthly returns of some hedge fund indices seem to have excess kurtosis, especially for HFRI Relative Value index. Surprisingly, for a portfolio of stocks, bonds and any hedge fund index, even though the skewness and excess kurtosis of portfolio excess returns are not zero, they are not too different from those implied by a normal distribution. Jacque-Bera test does not reject the null hypothesis that three-period portfolio excess returns are normally distributed, even if the portfolio invests in the HFRI Relative Value. One possible explanation is that lower frequency asset returns are more likely to be normally distributed. Even though monthly returns have excess kurtosis for some hedge fund indices, quarterly returns of those hedge fund indices appear to have much smaller excess kurtosis. For instance, the excess kurtosis for HFRI Relative Value quarterly returns is 3.05, much lower than the excess kurtosis of 10.43 on the basis of HFRI Relative Value monthly returns. Another reason is that three-asset portfolios also have long positions in bonds that have negative excess kurtosis for quarterly returns, and sometimes short positions in stocks that have positive excess kurtosis. The bottom line is that the first two moments of three-period portfolio excess returns are sufficient to describe portfolio characteristics, and the Sharpe ratio test can be justified.

From the p-values of the Sharpe ratio test in Table 5, the difference in Sharpe ratios between the two-asset portfolio and every three-asset portfolio is statistically significant at the 1% significance level, suggesting that the two-asset portfolio can be significantly improved upon by adding hedge funds.

An investor who ignores the existence of a hedge fund lockup period will get a wrong estimate of portfolio performance. From Table 3 and Table 4, we know that the existence of a three-month lockup period for hedge funds makes a difference in the allocations to stocks, bonds and hedge funds over the investment horizon. If there would be no hedge fund lockup period, a portfolio of stocks, bonds and hedge funds would have a higher Sharpe ratio, relative to a portfolio of stocks, bonds and hedge funds with a lockup period of three months, regardless of the choice of the hedge fund proxy. As shown in Table 5, the difference in Sharpe ratios between the three-asset portfolio with a hedge fund lockup period and the three-asset portfolio without a hedge fund lockup is large and statistically significant (except for the case when the HFRI Relative Value as the hedge fund proxy). For instance, the portfolio of stocks, bonds and the HFRIFOF composite with a lockup has the Sharpe ratio of 1.23, but the Sharpe ratio is 1.53 if there is no lockup period. The difference is statistically significant at the 1% significance level. Similarly, for the portfolio of stocks, bonds and the HFRI composite, the difference in Sharpe ratios is 0.23 (1.55 vs. 1.78). Hence, overlooking the existence of a hedge fund lockup period may overstate the performance of three-asset portfolios.

We calculate certainty equivalents for an investor with a mean-variance utility function and the relative risk aversion of 10, as the difference in utilities. We report two certainty equivalents. The first certainty equivalent is the difference in utilities derived from a portfolio of stocks, bonds and hedge funds with a lockup period, and a portfolio of stocks and bonds. That is, the certainty equivalent for a three-asset portfolio with a lockup period can be considered as the fee an investor is willing to pay in order to move from a two-asset portfolio to a portfolio of stocks, bonds and hedge funds. The portfolio of stocks, bonds and the HFRI composite has a certainty equivalent of 7.9% with a three-month lockup period, while the certainty equivalent of the portfolio of stocks, bonds and the HFRIFOF composite is 3.4% per year with a three-moth lockup. The second certainty equivalent is the utility cost of having a lockup for an investor, calculated as the utility derived from a portfolio of stocks, bonds and hedge funds without a lockup minus the utility derived from a portfolio of stocks, bonds and hedge funds with a lockup. For instance, a three-month lockup period costs an investor 3.9% (4.2%) per annum, when considering the portfolio of stocks, bonds and the HFRI composite (HFRIFOF composite). An alternative interpretation is that an investor with the portfolio of stocks, bonds and the HFRI composite is willing to pay 3.9% per annum in order to get rid of the lockup restriction.

V. Conditional Strategy with a Three-Month Lockup

This section reports the portfolio weights and performance of various portfolios under the conditional strategy. We consider asset allocations conditional on one state variable, i.e. the market dividend-price ratio. We analyze the (average) total demand for stocks and bonds in the conditional portfolio of stocks, bonds and hedge funds, as a combination of the speculative demand (Markowitz demand) and the hedge demand due to investments in hedge funds with a three-month lockup period, similar to the previous section. We test the difference in Sharpe ratios of the three-asset portfolio with a lockup period and the portfolio without a lockup period. Furthermore, we test whether using the conditional strategy improves the efficiency of the unconditional strategy. As before, we rescale optimal portfolio weights such that portfolios in the first month are tangency portfolios.

A. Portfolio Decision Conditional on the Market Dividend-Price ratio

Table 6 reports the results of asset allocations under the conditional strategy. The state variable is standardized to have a zero mean and a volatility of one, so the intercepts or constant terms are average allocations over the sample period. Average allocations to stocks and bonds change with the passage of time. For the two-asset allocation, we cannot reject the null hypothesis that average allocations to stocks as well as to bonds are equal across time. The average allocations to stocks are not too different across the three sub-periods (53%, 59% and 50%), while the average allocation to bonds is 47% in the first month and increases from 53% in month 2 to 65% in month 3. This implies that bonds become relatively important in the portfolio as the investment horizon approaches. In addition, for all three-asset portfolios, average portfolio weights of bonds are positive and appear to be increasing over time, a similar pattern to what we found for the two-asset portfolio.

Changes in the state variable lead to changes in portfolio weights under the conditional strategy. The sign of coefficients on the market dividend-price ratio in determining portfolio weights of stocks changes over time. The investor's responses to changes in the state variable will depend on whether hedge funds are added to the portfolio, which hedge fund index is used as the proxy, and which month the rebalancing decision is made. For instance, in the two-asset allocation, the change in the market dividend-price ratio is positively related to allocations to stocks in the second and third month, but not in the first month. The change in the market dividend-price ratio is always negatively associated with the change in allocations to bonds. Moreover, at a given month, the sign and magnitude of the slope coefficient on the state variable are different across different portfolios. This stresses the importance for an investor to take into account the lockup period in his conditional strategy. We test the null hypothesis that the slope coefficients of dividend-price ratio are equal across three months. We can't reject the null hypothesis for coefficients related to stocks or bonds in any case.

Average allocations to stocks and bonds in the three-asset portfolios with a three-month hedge fund lockup period increase monotonically. The most important effect of a three-month lockup period on the asset allocation decision is quite similar to the findings under the unconditional strategy: when there is a lockup period, on average, the first-period tangency investor is going to purchase additional amount of risky assets in the second and third period, financed by a risk-free loan. Moreover, within the mix of risky assets, average allocations to bonds are stable, while increases in the proportion of stocks offset decreases in the proportion of hedge funds in the second and third month. If there is no lockup period, after the first month, the investor will shift some funds to the risk-free asset. Average allocations to the risk-free asset are 21% and 4% in the second and third month, respectively, when the HFRI composite is added to the portfolio. Within the mix of risky assets, there are large decreases in the relative importance of hedge funds, by almost a half. Bonds and stocks gain some importance in the mix of risky assets.

Table 7 shows the decomposition of the total demand for stocks and bonds as the combination of the Markowitz demand and the hedge demand. In the portfolio of stocks, bonds and the HFRI composite, the Markowitz demands for stocks are 25%, 28%, and 24% in the first, second and third month, respectively. Adding the HFRI composite to the portfolio of stocks and bonds induces an increasing average hedge demands for stocks in the first, second and third month of –51%, –42% and –28%, resulting in total demands for stocks of –25%, –14%, and –4%, accordingly. The presence of large, negative hedge demands for stocks is not surprising. Since the investment in hedge funds is locked up for three months, an offsetting position in stocks provides the hedge against possible declining value of hedge funds over the investment horizon. Because hedge funds and stocks are alike, the magnitude of the hedge demands for stocks is large. In this particular case, the inverted U-shape of Markowitz demands for stocks is overwhelmed by the increasing hedge demands, such that the total demands for stocks increase over time. Nevertheless, there is no statistical evidence that total demands differ across the three months at any conventional significance level. Inclusion of the HFRI composite with a three-month lockup period generates positive average hedge demands for bonds. Average hedge demands do not differ too much from month to month. Since average Markowitz demands for bonds are positive and monotonically increasing over time, total demands for bonds are also positive and monotonically increasing.

When the HFRIFOF composite is chosen as the hedge fund proxy, the size of hedge demands for stocks becomes smaller. The hedge demand for stocks is –13% in the first month, and is close to zero in the second and third month. Average hedge demands for bonds are negative. In contrast, we have shown that the average demands for bonds are positive when the HFRI composite is the hedge fund proxy. Since the investments in hedge funds are large and positive in both cases, the opposite signs of hedge demands for bonds can only be the result of difference in the covariances between bond returns and returns of the two hedge fund composite indices.

B. Conditional Strategy vs. Unconditional Strategy

A comparison of the conditional strategy and unconditional strategy reveals some interesting results[7]. One similarity is the patterns of investments in bonds: allocations to bonds increase monotonically over time in all portfolios under both the unconditional strategy and conditional strategy. The conditional strategy seems to reduce allocations to bonds in the portfolios of stocks, bonds and the HFRIFOF composite, compared to the unconditional strategy. The conditional strategy on average allocates more to stocks in every period in the two-asset portfolio and the three-asset portfolios with the HFRIFOF composite (with the HFRI composite, the allocations to stocks under the conditional strategy are larger in the second and third month compared to the unconditional strategy, but not in the first month). This reflects the possibility of portfolio rebalancing in response to changing market conditions. It appears that ability to adjust portfolio weights according to changes in the state variable induces an investor to allocate more aggressively to stocks. Finally, when the conditional strategy includes hedge

[7] All comparisons of results in Subsection V.B. are made between the conditional strategy and the unconditional strategy.

funds with a three-month lockup period, the average allocations to hedge funds become larger compared to the allocations to hedge funds under the unconditional strategy. Together with our results related to stocks above, we can conclude that investors are in general more aggressive under the conditional strategy.

C. Portfolio Efficiency

Table 8 shows portfolio performance of the conditional strategy using different hedge fund indices as a proxy for investments in hedge funds. It also reports certainty equivalents for three-asset portfolios. We perform the Jacque-Bera normality test on all three-period portfolio excess returns, and cannot reject the null hypothesis. Hence, the mean and standard deviation of portfolio excess returns provide useful information on portfolio performance. Three questions arise. First of all, is the two-asset portfolio under the conditional strategy mean-variance efficient or does adding hedge funds to the portfolio improve the portfolio efficiency? Second, are portfolios under the unconditional strategy mean-variance efficient? Third, what difference does a three-month hedge fund lockup period make in terms of the portfolio performance?

We can use the Sharpe ratio test to determine the portfolio efficiency of the two-asset portfolio under the conditional strategy, and various portfolios under the unconditional strategy against the portfolios under the conditional strategy. For each case, we have p-values from four Sharpe ratio tests. For instance, using the HFRIFOF composite index as the hedge fund proxy, the Sharpe ratios of the three-asset portfolio under the conditional strategy with a lockup and without a lockup are 1.57 and 1.77, respectively. The p-value (0.000) to the right of the Sharpe ratio of the three-asset portfolio with a lockup period is based on the Sharpe ratio test in which the two-asset allocation as the benchmark portfolio. The p-value (0.128) next to the Sharpe ratio of the three-asset portfolio without a lockup period is based on the Sharpe ratio test of the difference in Sharpe ratios of two three-asset portfolios, i.e. the portfolio with a lockup period vs. the portfolio without a lockup period. The p-values (0.079) and (0.463) below the Sharpe ratios of the three-asset portfolio with a lockup and without a lockup are based on the Sharpe ratio test of the unconditional strategy vs. conditional strategy.

For all cases, the difference in Sharpe ratios of the three-asset portfolio with a three-month lockup period and the two-asset portfolio is significant at the 1% significance level. Hence, we conclude that the portfolio of stocks and bonds is not mean-variance efficient under the conditional strategy. An investor should add hedge funds to the portfolio even though there is a lockup period of three months.

The two p-values below the Sharpe ratios of three-asset portfolios come from the Sharpe ratio test of the unconditional strategy vs. conditional strategy. For the three-asset portfolios with a lockup period and using any hedge fund index, the Sharpe ratios of the portfolios under the conditional strategy do not differ from those of the portfolios under the unconditional strategy at the 5% significance level. Therefore, even if the market dividend yield predicts returns of stocks, bonds and hedge funds, and generates allocations different from those under the unconditional strategy, the investor does not benefit from using the conditional strategy. However, in terms of the certainty equivalent, in some cases, the difference between the unconditional strategy and the conditional strategy is quite large. For instance, the certainty equivalent is 3.4% for the three-asset portfolio of stocks, bonds and the HFRIFOF composite with a lockup under the unconditional strategy, and 5.8% under the conditional strategy.

To answer the last question, we perform the Sharpe ratio test of the three-asset portfolio with a three-month lockup period vs. the three-asset portfolio without a lockup period, in order to assess the effect of a three-month hedge fund lockup on the portfolio performance. When the HFRIFOF composite and four HFRIFOF strategy indices are considered as the proxy for hedge funds, the difference in the Sharpe ratios of the three-asset portfolios with a lockup period and the three-asset portfolios without a lockup period is not significant for 4 cases. The difference is significant at the 10% level only if the HFRIFOF Conservative index is used as the hedge fund proxy in the three-asset portfolio. When the HFRI composite index or the HFRI strategy indices are used as the proxy for hedge funds, the difference is much larger and significant at the 10% significance level for all cases. Therefore, having a three-month lockup period implies a significant lower Sharpe ratio of the three-asset portfolio of stocks, bonds, and the HFRI composite (and HFRI strategy indices). We can also compare the certainty equivalents for portfolios with or without a lockup. The difference is in the range of 4% to 5% when the HFRI composite is the hedge fund proxy (the difference is in the range of 2% to 4% when the HFRIFOF composite is the hedge fund proxy). Hence, the utility cost of having a lockup is large for an investor under the conditional strategy.

As the results of the Sharpe ratio tests indicate, a three-month lockup period has smaller negative effect on the performance of the three-asset portfolios of stocks, bonds and funds of funds. Funds of funds seem to be better able to suppress the effect of lockup periods on the portfolio performance of the conditional strategy than individual hedge funds do. Possible explanations which require further research include: a fund of funds typically has more frequent subscriptions and can use new money to pay off redemption requests. Moreover, a fund of funds manager can actively manage the lockup periods of the underlying individual hedge funds, such that each fund has a different lockup expiration date. In this way, a fund of funds can still invest in many individual hedge funds with long lockup periods, while imposing a shorter lockup period for fund of funds investors. From the perspective of an investor following the conditional strategy, when he decides to add funds of funds to the portfolio of stocks and bonds, a three-month hedge fund lockup period should not cause great concerns. In contrast, the investor should not overlook the effect of a three-month lockup period on the portfolio performance when individual hedge funds are considered. He would get a wrong impression of the incremental benefits of investing in individual hedge funds if he ignores the existence of a lockup period.

VI. Bootstrap Samples with a One-Year Lockup Period

Three-month hedge fund lockup periods are plausible for many funds of funds, but some funds of funds and individual hedge funds have longer lockup periods. The estimation is problematic with longer lockup periods since the history of hedge fund indices is relatively short. For instance, for the one-year horizon, we have only 18 non-overlapping samples to estimate parameters of interest whose number can be more than 70. Using quarterly returns or fewer state variables will reduce the number of parameters, without decreasing the sample size. We use the bootstrap method to obtain a larger sample size in order to examine the effect of a long lockup period.

We follow the stationary bootstrap method by Politis and Romano (1993) and Sullivan, Timmermann and White (1999) to obtain 5000 bootstrap samples of quarterly data. The smoothing parameter is chosen to be 0.2, so the mean block length is 5 quarters. The choice of the smoothing parameter affects the portfolio weights and performance, but the results of the Sharpe ratio tests are not too sensitive to the smoothing parameter.

Table 9 gives the results of the portfolio performance under the unconditional strategy, using various hedge fund indices as a proxy for hedge funds. The significantly higher Sharpe ratios and large certainty equivalents for the three-asset portfolios justify the inclusion of hedge funds into an investors' portfolio. Nevertheless, a one-year lockup period seems to make little impact on the performance of the three-asset portfolios of stocks, bonds and the HFRIFOF (or HFRIFOF strategy indices), as the difference in Sharpe ratios of the portfolios with or without a lockup period is not significant. Nevertheless, a certainty equivalent of 1.4% per annum is a non-trivial cost. Adding the HFRI composite or HFRI strategy indices to the portfolio also increases the Sharpe ratio significantly. However, having a one-year lockup period causes the significant difference in the Sharpe ratios of the three-asset portfolios when the HFRI Event-Driven or the HFRI Relative Value is used as the hedge fund proxy. In these two cases, the costs of having a lockup in terms of certainty equivalents are large.

Table 10 reports the analysis of the portfolio performance under the conditional strategy. The Sharpe ratios are significantly higher under the conditional strategy than those under the unconditional strategy in all cases. Therefore, an investor can benefit from using conditional information in the portfolio decision with one-year investment horizon. Relative to the two-asset portfolio, adding hedge funds to the portfolios improves the portfolio payoff in terms of Sharpe ratios under the conditional strategy. However, a portfolio investor would overestimate the portfolio performance when he ignores the presence of a one-year hedge fund lockup period. A one-year lockup period has a significant impact on the portfolio performance whichever hedge fund index is chosen as the hedge fund proxy. It seems that if the lockup period is long, an investor should be concerned with the effect of a lockup period on the performance of his portfolio under the conditional strategy, for investments in funds of funds as well as individual hedge funds.

VII. Conclusion

A lockup period is a realistic feature of investments in hedge funds, private equities and venture capital. This paper considers the impact of a hedge fund lockup period on the asset allocation decisions of an investor who re-adjusts the portfolio weights periodically. Due to the presence of a hedge fund lockup period, the investor can only adjust the allocation of stocks and bonds. The framework in this paper serves to illustrate the effect of hedge fund lockup periods on multi-period asset allocation, with the potential to extend to other asset classes with similar lockup or illiquid constraints. The empirical analysis indicates that the investor is better off by investing in portfolios of stocks, bonds and hedge funds, relative to a portfolio of stocks and bonds only. In addition, the three-asset portfolios under the unconditional strategy seem to be mean-variance efficient with a three-month horizon and monthly frequency. The conditional strategy can achieve better outcomes in terms of Sharpe ratios than the unconditional strategy only with a longer horizon and quarterly frequency. Most importantly, the presence of a lockup period is not trivial, especially when investing in individual hedge funds. An investor may overstate the benefit from adding individual hedge funds to the portfolio when he overlooks the existence of a hedge fund lockup period. Nevertheless, funds of funds seem to be able to partially suppress the effect of a short lockup period on the portfolio performance under the conditional strategy and the effect of a long lockup period on the portfolio performance of the unconditional strategy.

References

Agarwal, Vikas. and Narayan Y. Naik, 2004, Risks and portfolio decisions involving hedge funds, *Review of Financial Studies* 17, 63-98.

Ang, Andrew and Nicolas P.B. Bollen, 2008, Locked up by a lockup: valuing liquidity as a real option, available at http://ssrn.com/abstract=1291842.

Aragon, George O., 2007, Share restrictions and asset pricing: evidence from the hedge fund industry, *Journal of financial economics 83*, 33-58.

Brandt, Michael W., and Pedro Santa-Clara, 2006, Dynamic portfolio selection by augmenting the asset space, *Journal of Finance* 61, 2187-2217.

Brandt, Michael W., Pedro Santa-Clara, and Rossen Valkanov, 2009, Parametric portfolio policies: Exploiting characteristics in the cross-section of equity returns, *Review of Financial Studies*, forthcoming.

Britten-Jones, Mark, 1999, The sampling error in estimates of mean-variance efficient portfolio weights, *Journal of Finance* 54, 655-671.

Campbell, John Y., 1987, Stock returns and the term structure, *Journal of Financial Economics* 18, 373-399.

Campbell, John Y., and Samuel B. Thompson, 2007, Predicting excess stock returns out of sample: can anything beat the historical average? *Review of Financial Studies*, forthcoming.

Campbell, John Y., and Robert J. Shiller, 1988a, The dividend-price ratio and expectations of future dividends and discount factors, *Review of Financial Studies* 1, 195-228.

Campbell, John Y., and Robert J. Shiller, 1988b, Stock prices, earnings, and expected dividends, *Journal of Finance* 43, 661-676.

Campbell, John Y., and Luis M. Viceira, 1999, Consumption and portfolio decisions when expected returns are time varying, *Quarterly Journal of Economics* 114, 433-492.

Campbell, John Y., and Luis M. Viceira, 2002, Strategic asset allocation, *Oxford University Press*.

Cochrane, John H., 2007, The dog that did not bark: a defense of return predictability. *Review of Financial Studies*, forthcoming.

De Roon, Frans A., and Theo E. Nijman, 2001, Testing for mean-variance spanning: a survey, Journal of Empirical Finance 8, 111-155.

Derman, Emanuel, 2007, A simple model for the expected premium for hedge fund lockups, *Journal of Investment Management 5.*

Fama, Eugene F., and Kenneth R. French, 1988, Dividend yields and expected stock returns, *Journal of Financial Economics* 22, 3-27.

Fama, Eugene F., and Kenneth R. French, 1989, Business conditions and expected returns on stocks and bonds, *Journal of Financial Economics* 25, 23-49.

Fung, William, and David A. Hsieh, 2000, Performance characteristics of hedge funds and commodity funds: natural vs. spurious biases, *Journal of Financial and Quantitative Analysis* 35, 291-307.

Fung, William, David A. Hsieh, Narayan Y. Naik, and Tarun Ramadorai, 2008, Hedge funds: performance, risk and capital formation, *Journal of Finance 63*, 1777-1803.

Goyal, Amit, and Ivo Welch, 2007, A comprehensive look at the empirical performance of equity premium prediction, *Review of Financial Studies*, forthcoming.

Lars Peter Hansen and Kenneth J. Singleton, 1982, Generalized instrumental variables estimation of nonlinear rational expectations models, *Econometrica 50(5),* 1269-86.

Hodrick, Robert J., 1992, Dividend yields and expected stock returns: Alternative procedures for inference and measurement, *Review of Financial Studies* 5, 357-386.

Jegadeesh, Narasimhan, and Sheridan Titman, 1993, Returns to buying winners and selling losers: implications for stocks market efficiency, *Journal of Finance* 48, 65-91.

Jobson, J. Dave, and Bob Korkie, 1982, Potential performance and tests of portfolio efficiency, Journal of Financial Economics 10, 433-466.

Keim, Donald, and Robert Stambaugh, 1986, Predicting returns in stock and bond markets, *Journal of Financial Economics* 17, 357-390.

Kosowski, Robert, Narayan Y. Naik, and Melvyn Teo, 2005, Do hedge funds deliver alpha? A bayesian and bootstrap analysis, *Journal of Financial Economics* 84, 229-264.

Lettau, Martin, and Sydney C. Ludvigson, 2005, Consumption, aggregate wealth, and expected stock returns, *Journal of Finance* 56, 815-849.

Liang, Bing and Hyuna Park, 2008, Share restrictions, liquidity premium and offshore hedge funds, available at http://ssrn.com/abstract=967788.

Malkiel, Burton G., and Atanu Saha, 2005, Hedge funds: risk and return, *Financial Analyst Journal 61*, 80-88.

Mayers, David, 1976, Nonmarketable assets, market segmentation, and the level of asset prices, Journal of Financial and Quantitative Analysis 11, 1-12.

Newey, Whitney K. and Kenneth D. West, 1987, A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix, *Econometrica* 55, 703–708.

Nicholas, Joseph G., 2004, Hedge fund of funds investing, Bloomberg Press.

Politis, Dimitris N., and Joseph P. Romano, 1994, The stationary bootstrap, *Journal of the American Statistical Association* 89, 1303-1313.

Rouwenhorst, K. Geert, 1998, International Momentum Strategies, *Journal of Finance* 53, 267-284.

Sullivan, Ryan, Allan Timmermann, and Halbert White, 1999, Data-snooping, technical trading rule performance, and the bootstrap, *Journal of Finance* 54, 1647-1691.

Table 1
Descriptive Statistics of Returns of Stocks, Bonds and Hedge Funds

This table gives summary statistics of risky assets from January 1990 to December 2007. The value-weighted NYSE index is proxy for stocks, and Fama Bond Portfolio (Treasuries) with maturities greater than 10 years is proxy for bonds. For hedge funds, various indices are considered: HFRI Fund of Funds composite index (HFRIFOF), HFRIFOF sub-strategy indices, HFRI Fund Weighted Composite Index (HFRI) and HFRI sub-strategy indices. Means, standard deviations, maximums and minimums are expressed in percentages. We annualize means, standard deviations and Sharpe ratios, while the remaining statistics are on a monthly basis.

	Mean	Std	Sharpe	Max	Min	Skew	Kurtosis
Stocks	11.4%	12.6%	0.581	10.7%	-14.7%	-0.531	1.347
Bonds	8.5%	7.9%	0.563	7.2%	-8.3%	-0.430	0.845
HFRIFOF	9.7%	5.5%	1.033	6.9%	-7.5%	-0.284	4.049
-Conservative	8.3%	3.2%	1.332	4.0%	-3.9%	-0.506	3.144
-Diversified	9.1%	5.8%	0.870	7.7%	-7.8%	-0.134	4.182
-Market Defensive	9.4%	5.8%	0.939	7.4%	-5.4%	0.148	1.234
-Strategic	12.7%	8.6%	1.010	9.5%	-12.1%	-0.389	3.827
HFRI	13.2%	6.6%	1.383	7.7%	-8.7%	-0.590	2.940
-Equity Hedge	15.7%	8.5%	1.376	10.9%	-7.7%	0.193	1.551
-Event-Driven	13.5%	6.4%	1.475	5.1%	-8.9%	-1.251	4.630
-Macro	14.3%	8.0%	1.295	7.9%	-6.4%	0.394	0.784
-Relative Value	11.2%	3.5%	2.079	5.7%	-5.8%	-0.804	10.433

Table 2
Correlation Matrix of the State Variable and Asset Returns

This table displays the correlation matrix of the lagged state variable and risky asset returns from January 1990 to December 2007. The data frequency is monthly. State variables include the market dividend-price ratio.

	Dividend price ratio	Stocks	Bonds	HFRIFOF composite	FOF Conservative	FOF Diversified	FOF Market defensive	FOF Strategic	HFRI composite	Equity Hedge	Event-Driven	Macro	Relative Value
Dividend price ratio	1												
Stocks	0.08	1											
Bonds	0.05	0.05	1										
HFRIFOF composite	0.12	0.43	0.02	1									
FOF Conservative	0.13	0.44	0.05	0.89	1								
FOF Diversified	0.10	0.43	0.00	0.97	0.84	1							
FOF Market Defensive	0.07	0.04	0.11	0.69	0.62	0.63	1						
FOF Strategic	0.19	0.48	0.01	0.93	0.82	0.87	0.55	1					
HFRI composite	0.14	0.69	-0.01	0.83	0.74	0.81	0.35	0.85	1				
Equity Hedge	0.13	0.64	0.00	0.77	0.69	0.75	0.35	0.80	0.93	1			
Event-Driven	0.08	0.67	-0.03	0.67	0.62	0.65	0.26	0.70	0.88	0.78	1		
Macro	0.21	0.40	0.28	0.72	0.64	0.71	0.52	0.68	0.69	0.61	0.56	1	
Relative Value	0.20	0.39	-0.03	0.53	0.54	0.51	0.27	0.53	0.63	0.55	0.65	0.41	1

Table 3
Asset Allocation under the Unconditional Strategy
[Lockup: Three-month]

This table reports results of asset allocations under the unconditional strategy (rescaled such that each portfolio is the tangency portfolio in the first month). The data frequency is monthly. Column 4 to 7 show optimal unconditional weights for various portfolios at each month using the HFRI composite index as the proxy for hedge funds. Column 8 to 11 show optimal unconditional weights for various portfolios at each month using the HFRIFOF composite index as the proxy for hedge funds. Absolute values of t-statistics for the portfolio weights are in square brackets. For each portfolio, we test the hypothesis that portfolio weights are equal across three months, and report the p-values of Wald test.

			HFRI as Hedge Fund Proxy				HFRIFOF as Hedge Fund Proxy			
Period	**Two-Asset**		**Three-Asset with a Lockup**		**Three-Asset No Lockup**		**Three-Asset with a Lockup**		**Three-Asset No Lockup**	
Column	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Stocks										
Month 1	0.524	[1.504]	-0.229	[0.904]	-0.289	[1.330]	0.087	[0.410]	0.005	[0.401]
Month 2	0.503	[1.404]	-0.180	[1.061]	-0.028	[1.134]	0.137	[0.607]	0.156	[1.280]
Month 3	0.371	[1.013]	-0.124	[0.672]	0.070	[1.446]	0.122	[0.675]	0.130	[1.279]
Wald Test		(0.682)		(0.546)		(0.067)		(0.635)		(0.162)
Bonds										
Month 1	0.476	[0.835]	0.312	[0.931]	0.224	[0.904]	0.293	[0.878]	0.239	[0.905]
Month 2	0.647	[1.186]	0.350	[1.107]	0.264	[1.122]	0.367	[1.166]	0.277	[1.147]
Month 3	0.854	[1.535]	0.500	[1.541]	0.375	[1.535]	0.482	[1.498]	0.348	[1.384]
Wald Test		(0.834)		(0.813)		(0.813)		(0.740)		(0.541)
Hedge Funds										
Month 1			0.917	[3.912]	1.065	[2.396]	0.619	[2.686]	0.756	[1.786]
Month 2					0.577	[1.316]			0.359	[1.616]
Month 3					0.316	[1.563]			0.222	[1.611]
Wald Test						(0.022)				(0.011)

Table 4
Hedge Demands for Stocks and Bonds under the Unconditional Strategy
[Lockup: Three-month]

This table displays, for the three-asset portfolio (rescaled such that each portfolio is the tangency portfolio in the first month) under the unconditional strategy with a three-month hedge fund lockup period, the decomposition of portfolio weights of stocks and bonds at each month into two parts: a Markowitz demand or speculative demand and a hedge demand. Absolute values of t-statistics for those portfolio weights are in square brackets. For each asset, we test the hypothesis that portfolio weights are equal across three months, and report the p-values of Wald test.

	Markowitz Demand [M]		Hedge Demand [H]		Optimal Demand = M + H	
HFRI as Hedge Fund proxy:						
Stocks						
Month 1	0.240	[1.504]	-0.469	[5.355]	-0.229	[0.904]
Month 2	0.231	[1.404]	-0.410	[4.702]	-0.180	[1.061]
Month 3	0.170	[1.013]	-0.295	[3.319]	-0.124	[0.672]
Wald Test		(0.682)		(0.172)		(0.546)
Bonds						
Month 1	0.219	[0.835]	0.094	[0.643]	0.312	[0.931]
Month 2	0.297	[1.186]	0.053	[0.681]	0.350	[1.107]
Month 3	0.392	[1.535]	0.108	[0.785]	0.500	[1.541]
Wald Test		(0.834)		(0.755)		(0.813)
HFRIFOF as Hedge Fund proxy:						
Stocks						
Month 1	0.270	[1.504]	-0.183	[2.912]	0.087	[0.410]
Month 2	0.260	[1.404]	-0.123	[1.943]	0.137	[0.607]
Month 3	0.191	[1.013]	-0.069	[1.063]	0.122	[0.675]
Wald Test		(0.682)		(0.258)		(0.635)
Bonds						
Month 1	0.246	[0.835]	0.048	[0.636]	0.293	[0.878]
Month 2	0.334	[1.186]	0.033	[0.875]	0.367	[1.166]
Month 3	0.441	[1.535]	0.041	[0.432]	0.482	[1.498]
Wald Test		(0.834)		(0.411)		(0.740)

Table 5
Performance of the Unconditional Strategy
[Lockup: Three-month]

This table reports performance of various portfolios under the unconditional strategy (rescaled such that each portfolio is the tangency portfolio in the first month). There are ten hedge fund indices that are considered one at a time as the proxy for hedge funds. Mean returns, standard deviations and Sharpe ratios are annualized. We report p-values (in parenthesis) of Sharpe ratio tests. The benchmark portfolio for the three-asset with a lockup is the two-asset portfolio, and the three-asset with a lockup is the benchmark portfolio for the three-asset with no lockup. Certainty equivalent or equalization fee for the three-asset portfolio with a lockup is calculated as the difference in the utilities of the three-asset portfolio with a lockup and the two-asset portfolio, for the investor with a mean-variance utility function and the risk aversion, $\hat{\gamma} = 10$. For the same investor, the certainty equivalent for the three-asset portfolio without a lockup is the difference in utilities of the three-asset portfolio without a lockup and the three-asset portfolio with a lockup (this is the utility cost of a lockup). The data frequency is monthly.

	Two-Asset	Three-Asset with a Lockup		Three-Asset No Lockup		Three-Asset with a Lockup		Three-Asset No Lockup	
Hedge Fund Proxy:		**HFRI Composite**				**HFRIFOF Composite**			
Mean excess returns	7.1%	9.5%		8.5%		6.6%		7.2%	
Std. excess returns	7.8%	6.1%		4.7%		5.4%		4.7%	
Sharpe ratio	0.907	1.549	(0.000)	1.782	(0.012)	1.225	(0.001)	1.533	(0.004)
Certainty equivalent		7.9%		3.9%		3.4%		4.2%	
Hedge Fund Proxy:		**HFRI Equity Hedge**				**HFRIFOF Conservative**			
Mean excess returns	7.1%	11.4%		11.8%		5.2%		5.4%	
Std. excess returns	7.8%	7.1%		6.5%		3.7%		3.3%	
Sharpe ratio	0.907	1.591	(0.000)	1.814	(0.015)	1.389	(0.000)	1.622	(0.014)
Certainty equivalent		8.6%		3.8%		5.5%		3.5%	
Hedge Fund Proxy:		**HFRI Event-Driven**				**HFRIFOF Diversified**			
Mean excess returns	7.1%	9.8%		8.7%		6.4%		6.5%	
Std. excess returns	7.8%	5.8%		4.6%		5.5%		4.7%	
Sharpe ratio	0.907	1.694	(0.000)	1.893	(0.023)	1.153	(0.006)	1.384	(0.019)
Certainty equivalent		10.3%		3.7%		2.5%		2.9%	
Hedge Fund Proxy:		**HFRI Macro**				**HFRIFOF Market Defensive**			
Mean excess returns	7.1%	10.9%		11.2%		6.8%		7.5%	
Std. excess returns	7.8%	8.2%		7.5%		5.1%		4.9%	
Sharpe ratio	0.907	1.314	(0.000)	1.481	(0.054)	1.317	(0.000)	1.540	(0.018)
Certainty equivalent		4.6%		2.3%		4.6%		3.2%	
Hedge Fund Proxy:		**HFRI Relative Value**				**HFRIFOF Strategic**			
Mean excess returns	7.1%	7.8%		6.9%		8.2%		9.2%	
Std. excess returns	7.8%	3.9%		3.3%		6.8%		6.4%	
Sharpe ratio	0.907	1.994	(0.000)	2.083	(0.195)	1.201	(0.002)	1.434	(0.017)
Certainty equivalent		15.8%		1.8%		3.1%		3.1%	

Table 6
Asset Allocation under the Conditional Strategy
[Lockup: Three-month; State Variable: Market Dividend-Price Ratio]

This table reports results of asset allocations under the conditional strategy (rescaled such that the portfolio is the tangency portfolio in the first month). The data frequency is monthly. Column 5 to 8 show intercepts and coefficients of state variables by which optimal conditional weights for various portfolios are determined using the HFRI composite index as the proxy for hedge funds. Column 9 to 12 show intercepts and coefficients of state variables by which optimal conditional weights for various portfolios are determined using the HFRIFOF composite index as the proxy for hedge funds. Absolute values of t-statistics for the intercepts and coefficients are in square brackets. For each portfolio, we test the hypothesis that portfolio weights are equal across three months, and report the p-values of Wald test.

			HFRI as Hedge Fund Proxy					HFRIFOF as the Hedge Fund Proxy			
Period	**State Variables**	**Two-Asset**		**Three-Asset with a Lockup**		**Three-Asset No Lockup**		**Three-Asset with a Lockup**		**Three-Asset No Lockup**	
Column	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Stocks											
Month 1	Constant	0.526	[1.374]	-0.254	[0.882]	-0.276	[1.078]	0.124	[0.508]	0.021	[0.757]
	DP ratio	-0.064	[1.000]	-0.227	[0.868]	-0.018	[0.529]	0.036	[0.765]	0.047	[0.611]
Month 2	Constant	0.588	[1.494]	-0.142	[0.652]	0.084	[1.258]	0.244	[0.991]	0.249	[1.339]
	DP ratio	0.157	[1.306]	-0.075	[0.681]	-0.036	[0.566]	0.220	[0.996]	0.090	[0.502]
Month 3	Constant	0.501	[1.202]	-0.043	[0.620]	0.069	[1.180]	0.254	[1.034]	0.207	[1.274]
	DP ratio	0.201	[1.160]	0.053	[0.623]	-0.008	[0.119]	0.246	[1.133]	0.123	[0.718]
Wald Test of Constant			(0.298)		(0.483)		(0.115)		(0.476)		(0.149)
Wald Test of DP ratio			(0.145)		(0.315)		(0.800)		(0.304)		(0.690)
Bonds											
Month 1	Constant	0.474	[0.767]	0.321	[0.843]	0.194	[0.643]	0.107	[0.295]	0.080	[0.279]
	DP ratio	-0.255	[1.285]	-0.169	[0.400]	-0.169	[0.736]	-0.360	[0.788]	-0.184	[0.538]
Month 2	Constant	0.529	[0.860]	0.330	[0.882]	0.263	[0.916]	0.191	[0.556]	0.174	[0.665]
	DP ratio	-0.239	[1.401]	-0.215	[0.591]	-0.232	[0.810]	-0.251	[0.505]	-0.218	[0.595]
Month 3	Constant	0.654	[1.088]	0.434	[1.171]	0.381	[1.326]	0.220	[0.616]	0.193	[0.715]
	DP ratio	-0.294	[1.110]	-0.214	[0.692]	-0.137	[0.552]	-0.359	[0.768]	-0.248	[0.703]
Wald Test of Constant			(0.592)		(0.922)		(0.793)		(0.893)		(0.817)
Wald Test of DP ratio			(0.194)		(0.744)		(0.650)		(0.974)		(0.922)
Hedge Funds											
Month 1	Constant			0.933	[3.672]	1.083	[2.116]	0.769	[2.986]	0.899	[1.940]
	DP ratio			0.191	[0.805]	-0.106	[0.290]	0.436	[1.677]	0.244	[0.479]
Month 2	Constant					0.442	[1.128]			0.297	[1.151]
	DP ratio					0.030	[0.342]			0.228	[0.419]
Month 3	Constant					0.508	[1.172]			0.431	[1.389]
	DP ratio					0.213	[0.556]			0.410	[0.767]
Wald Test of Constant							(0.220)				(0.159)
Wald Test of DP ratio							(0.897)				(0.721)

Table 7
Average Hedge Demands for Stocks and Bonds under the Conditional Strategy
[Lockup: Three-month; State Variable: Market Dividend-Price Ratio]

This table displays, for the three-asset portfolio (rescaled such that the portfolio is the tangency portfolio in the first month) under the conditional strategy with a three-month hedge fund lockup period, the decomposition of average portfolio weights of stocks and bonds at each month into two parts: a Markowitz demand or speculative demand and a hedge demand. Absolute values of t-statistics for those portfolio weights are in square brackets. For each asset, we test the hypothesis that portfolio weights are equal across three months, and report the p-values of Wald test.

	Markowitz Demand [M]		**Hedge Demand [H]**		**Optimal Demand = M + H**	
HFRI as Hedge Fund proxy:						
Stocks						
Month 1	0.252	[1.374]	-0.506	[4.513]	-0.254	[0.882]
Month 2	0.282	[1.494]	-0.424	[3.913]	-0.142	[0.652]
Month 3	0.240	[1.202]	-0.282	[2.696]	-0.043	[0.620]
Wald Test		(0.298)		(0.252)		(0.483)
Bonds						
Month 1	0.227	[0.767]	0.094	[0.830]	0.321	[0.843]
Month 2	0.253	[0.860]	0.077	[0.980]	0.330	[0.882]
Month 3	0.313	[1.088]	0.121	[0.748]	0.434	[1.171]
Wald Test		(0.592)		(0.598)		(0.922)
HFRIFOF as Hedge Fund proxy:						
Stocks						
Month 1	0.254	[1.374]	-0.130	[2.258]	0.124	[0.508]
Month 2	0.284	[1.494]	-0.040	[1.414]	0.244	[0.991]
Month 3	0.241	[1.202]	0.013	[0.635]	0.254	[1.034]
Wald Test		(0.298)		(0.229)		(0.476)
Bonds						
Month 1	0.229	[0.767]	-0.122	[0.548]	0.107	[0.295]
Month 2	0.255	[0.860]	-0.065	[0.715]	0.191	[0.556]
Month 3	0.316	[1.088]	-0.096	[0.394]	0.220	[0.616]
Wald Test		(0.592)		(0.537)		(0.893)

Table 8
Performance of the Conditional Strategy
[Lockup: Three-month; State Variable: Market Dividend-Price Ratio]

This table reports performance of portfolios under the conditional strategy (rescaled such that each portfolio is the tangency portfolio in the first month). There are ten hedge fund indices that are considered one at a time as the proxy for hedge funds. Mean returns, standard deviations and Sharpe ratios are annualized. We report p-values (in parenthesis) of Sharpe ratio tests. For each three-asset portfolio, two p-values are reported using two benchmark portfolios. In the ‘Sharpe ratio’ row, the benchmark portfolio for the three-asset with a lockup is the two-asset portfolio, and the three-asset with a lockup is the benchmark portfolio for the three-asset with no lockup. In the ‘Unconditional vs. Conditional’ row, the benchmark portfolio for each three-asset portfolio is the corresponding unconditional portfolio. Certainty equivalent or equalization fee follows the same definition as in Table 5. The data frequency is monthly.

	Two-Asset	**Three-Asset with a Lockup**		**Three-Asset No Lockup**		**Three-Asset with a Lockup**		**Three-Asset No Lockup**	
Hedge Fund Proxy:		**HFRI Composite**				**HFRIFOF Composite**			
Mean excess returns	9.9%	10.9%		9.4%		9.0%		7.6%	
Std. excess returns	8.6%	6.3%		4.8%		5.7%		4.3%	
Sharpe ratio	1.149	1.740	(0.000)	1.960	(0.078)	1.572	(0.000)	1.765	(0.128)
Unconditional vs. Conditional			(0.421)		(0.667)		(0.079)		(0.463)
Certainty equivalent		8.5%		4.1%		5.8%		3.3%	
Hedge Fund Proxy:		**HFRI Equity Hedge**				**HFRIFOF Conservative**			
Mean excess returns	9.9%	13.2%		14.0%		6.2%		6.5%	
Std. excess returns	8.6%	7.6%		7.0%		3.6%		3.4%	
Sharpe ratio	1.149	1.737	(0.000)	2.011	(0.031)	1.704	(0.000)	1.910	(0.098)
Unconditional vs. Conditional			(0.613)		(0.589)		(0.104)		(0.270)
Certainty equivalent		8.5%		5.1%		8.0%		3.7%	
Hedge Fund Proxy:		**HFRI Event-Driven**				**HFRIFOF Diversified**			
Mean excess returns	9.9%	11.0%		10.5%		8.5%		7.0%	
Std. excess returns	8.6%	5.8%		4.8%		5.8%		4.2%	
Sharpe ratio	1.149	1.891	(0.000)	2.155	(0.035)	1.462	(0.004)	1.643	(0.159)
Unconditional vs. Conditional			(0.391)		(0.343)		(0.142)		(0.384)
Certainty equivalent		11.3%		5.6%		4.2%		2.8%	
Hedge Fund Proxy:		**HFRI Macro**				**HFRIFOF Market Defensive**			
Mean excess returns	9.9%	12.3%		13.5%		8.6%		8.0%	
Std. excess returns	8.6%	8.2%		7.6%		5.3%		4.5%	
Sharpe ratio	1.149	1.502	(0.002)	1.769	(0.040)	1.607	(0.000)	1.766	(0.210)
Unconditional vs. Conditional			(0.467)		(0.283)		(0.153)		(0.479)
Certainty equivalent		4.7%		4.4%		6.3%		2.7%	
Hedge Fund Proxy:		**HFRI Relative Value**				**HFRIFOF Strategic**			
Mean excess returns	9.9%	8.4%		7.2%		11.9%		10.9%	
Std. excess returns	8.6%	3.7%		2.9%		7.7%		6.5%	
Sharpe ratio	1.149	2.255	(0.000)	2.472	(0.085)	1.535	(0.001)	1.685	(0.241)
Unconditional vs. Conditional			(0.186)		(0.082)		(0.097)		(0.408)
Certainty equivalent		18.9%		5.1%		5.2%		2.4%	

Table 9
Performance of the Unconditional Strategy
[Lockup: One-year; Bootstrap Samples]

This table reports performance of various portfolios under the unconditional strategy (rescaled such that each portfolio is the tangency portfolio in the first month). There are ten hedge fund indices that are considered one at a time as the proxy for hedge funds. Mean returns, standard deviations and Sharpe ratios are annualized. We report p-values (in parenthesis) of Sharpe ratio tests. The benchmark portfolio for the three-asset with a lockup is the two-asset portfolio, and the three-asset with a lockup is the benchmark portfolio for the three-asset with no lockup. Certainty equivalent or equalization fee follows the same definition as in Table 5. The data frequency is quarterly. The number of bootstrap samples is 5000.

	Two-Asset	Three-Asset with a Lockup		Three-Asset No Lockup		Three-Asset with a Lockup		Three-Asset No Lockup	
Hedge Fund Proxy:		**HFRI Composite**				**HFRIFOF Composite**			
Mean excess returns	8.4%	9.7%		8.3%		7.0%		6.5%	
Std. excess returns	10.4%	6.6%		5.2%		5.7%		4.9%	
Sharpe ratio	0.825	1.468	(0.000)	1.586	(0.104)	1.225	(0.000)	1.325	(0.133)
Certainty equivalent		9.1%		1.9%		5.9%		1.4%	
Hedge Fund Proxy:		**HFRI Equity Hedge**				**HFRIFOF Conservative**			
Mean excess returns	8.4%	11.5%		10.3%		5.5%		4.7%	
Std. excess returns	10.4%	8.0%		6.7%		4.3%		3.4%	
Sharpe ratio	0.825	1.428	(0.000)	1.532	(0.142)	1.286	(0.000)	1.384	(0.147)
Certainty equivalent		8.6%		1.0%		6.7%		1.4%	
Hedge Fund Proxy:		**HFRI Event-Driven**				**HFRIFOF Diversified**			
Mean excess returns	8.4%	11.6%		8.5%		6.7%		6.6%	
Std. excess returns	10.4%	7.0%		4.7%		5.6%		5.1%	
Sharpe ratio	0.825	1.661	(0.000)	1.827	(0.039)	1.195	(0.000)	1.297	(0.128)
Certainty equivalent		12.3%		3.0%		5.5%		1.4%	
Hedge Fund Proxy:		**HFRI Macro**				**HFRIFOF Market Defensive**			
Mean excess returns	8.4%	12.2%		11.6%		7.1%		6.9%	
Std. excess returns	10.4%	10.0%		8.6%		4.8%		4.4%	
Sharpe ratio	0.825	1.223	(0.000)	1.332	(0.108)	1.475	(0.000)	1.590	(0.107)
Certainty equivalent		5.8%		1.5%		9.4%		1.9%	
Hedge Fund Proxy:		**HFRI Relative Value**				**HFRIFOF Strategic**			
Mean excess returns	8.4%	8.5%		6.9%		8.6%		8.2%	
Std. excess returns	10.4%	4.9%		3.6%		7.4%		6.5%	
Sharpe ratio	0.825	1.728	(0.000)	1.902	(0.034)	1.164	(0.000)	1.257	(0.155)
Certainty equivalent		13.4%		3.3%		5.1%		1.2%	

Table 10
Performance of the Conditional Strategy
[Lockup: One-year; State Variable: Market Dividend-Price Ratio; Bootstrap Samples]

This table reports performance of portfolios under the conditional strategy (rescaled such that each portfolio is the tangency portfolio in the first month). There are ten hedge fund indices that are considered one at a time as the proxy for hedge funds. Mean returns, standard deviations and Sharpe ratios are annualized. We report p-values of Sharpe ratio tests. For each three-asset portfolio, two p-values are reported using two benchmark portfolios. In the 'Sharpe ratio' row, the benchmark portfolio for the three-asset with a lockup is the two-asset portfolio, and the three-asset with a lockup is the benchmark portfolio for the three-asset with no lockup. In the 'Unconditional vs. Conditional' row, the benchmark portfolio for each three-asset portfolio is the corresponding unconditional portfolio. Certainty equivalent or equalization fee follows the same definition as in Table 5. The data frequency is quarterly. The number of bootstrap samples is 5000.

	Two-Asset	Three-Asset with a Lockup		Three-Asset No Lockup		Three-Asset with a Lockup		Three-Asset No Lockup	
Hedge Fund Proxy:		**HFRI Composite**				**HFRIFOF Composite**			
Mean excess returns	11.9%	10.9%		11.8%		9.1%		11.2%	
Std. excess returns	11.6%	4.6%		4.2%		4.6%		4.7%	
Sharpe ratio	1.081	2.382	(0.000)	2.817	(0.005)	1.998	(0.000)	2.413	(0.002)
Unconditional vs. Conditional			(0.000)		(0.000)		(0.000)		(0.000)
Certainty equivalent		29.6%		12.3%		20.9%		9.9%	
Hedge Fund Proxy:		**HFRI Equity Hedge**				**HFRIFOF Conservative**			
Mean excess returns	11.9%	14.7%		15.7%		6.7%		7.9%	
Std. excess returns	11.6%	6.8%		6.2%		3.4%		3.3%	
Sharpe ratio	1.081	2.153	(0.000)	2.573	(0.003)	1.976	(0.000)	2.358	(0.005)
Unconditional vs. Conditional			(0.000)		(0.000)		(0.000)		(0.000)
Certainty equivalent		24.0%		10.7%		20.3%		9.0%	
Hedge Fund Proxy:		**HFRI Event-Driven**				**HFRIFOF Diversified**			
Mean excess returns	11.3%	12.0%		9.9%		8.6%		10.5%	
Std. excess returns	11.1%	4.9%		3.4%		4.4%		4.5%	
Sharpe ratio	2.081	2.435	(0.000)	2.902	(0.003)	1.955	(0.000)	2.370	(0.002)
Unconditional vs. Conditional			(0.000)		(0.000)		(0.000)		(0.000)
Certainty equivalent		30.9%		13.5%		20.0%		9.8%	
Hedge Fund Proxy:		**HFRI Macro**				**HFRIFOF Market Defensive**			
Mean excess returns	11.3%	14.8%		18.3%		8.9%		10.5%	
Std. excess returns	11.1%	7.5%		7.6%		4.4%		4.3%	
Sharpe ratio	2.081	1.961	(0.000)	2.323	(0.007)	2.021	(0.000)	2.419	(0.004)
Unconditional vs. Conditional			(0.000)		(0.000)		(0.000)		(0.000)
Certainty equivalent		20.0%		8.4%		21.2%		9.7%	
Hedge Fund Proxy:		**HFRI Relative Value**				**HFRIFOF Strategic**			
Mean excess returns	11.9%	9.0%		8.5%		12.2%		15.3%	
Std. excess returns	11.6%	3.4%		2.7%		6.2%		6.4%	
Sharpe ratio	1.081	2.621	(0.000)	3.202	(0.001)	1.959	(0.000)	2.374	(0.002)
Unconditional vs. Conditional			(0.000)		(0.000)		(0.000)		(0.000)
Certainty equivalent		35.8%		18.3%		20.0%		9.8%	

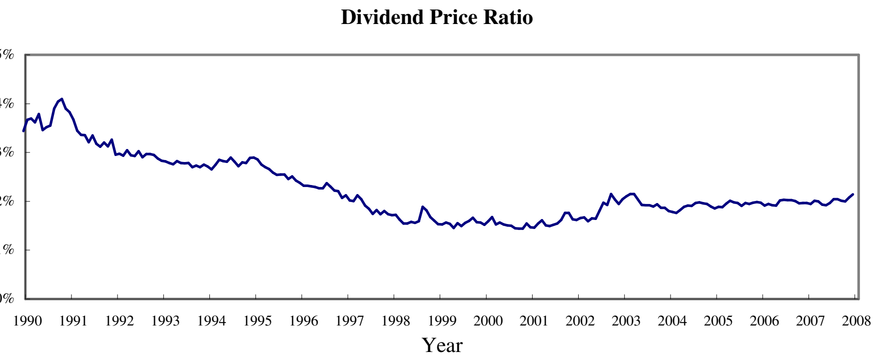

Figure 1. Evolution of the Market Dividend-Price Ratio