How Does Illiquidity Affect Delegated Portfolio Choice? *

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Abstract

In a continuous-time dynamic portfolio choice framework, I study the problem of an investor who exogenously decides to delegate the administration of his or her savings to a risk-averse money manager who trades multiple risky assets in a thin market. I consider a manager who is rewarded for increasing the value of assets under management, which is the product of both the manager's portfolio allocation decisions, taken over the investment period, and the money flows into and out of the fund, as a result of the portfolio performance relative to an exogenous benchmark. The model proposed here shows that, whenever the manager can substitute between more illiquid and less illiquid risky assets, he or she is likely to choose to hold an initial portfolio that is skewed toward more illiquid assets, and to gradually shift toward less illiquid assets over the investment period. The model further shows that several misalignments of objectives between the investor and the manager can lead to large utility costs on the part of the investor, and that these costs decrease with asset illiquidity. Solving for the shadow costs of illiquidity, the model indicates that delegated rather than direct investing is likely to lead to larger price discounts.

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1 Introduction

Continuous and unlimited trading is a key assumption of existing models of delegated portfolio management. Increasingly, however, both academics and practitioners recognize that, in addition to risk and return, liquidity (or the lack of it) is a critical component of the investment equation. In fact, investing in illiquid assets presents a particular set of challenges and risks because their true value is often unknown, their historical performance can often be misleading, they typically cost more to transact, and their illiquidity can eventually result in a failure to transact.

In this paper I relax the assumption of continuous and unlimited trading and, using a dynamic portfolio choice framework, study the problem of an individual investor who exogenously delegates the administration of his savings to a money manager who trades multiple risky assets in a market where prices are abnormally volatile and assets are illiquid. I consider that the exogenous portfolio delegation decision is grounded on the assumption that, in comparison with the individual investor, the money manager is subject to lower transaction costs, lower opportunity costs for engaging in active portfolio management, better information or ability, and/or better investing education. In addition, I consider a risk-averse manager whose compensation scheme is set exogenously and which is proportional to the value of his or her fund's assets under management at some terminal date. This fund's terminal value is the product of both the manager's portfolio allocation decisions, taken over the investment period, and the money flows into and out of the fund which are allowed to happen only once on the investment horizon and which depend on the manager's portfolio performance relative to a specified benchmark.

I model illiquidity according to Longstaff (2001) in which market participants are constrained to trading strategies that are of bounded variation, and investigate the extent to which the restricted ability to initiate or unwind portfolio positions affects the money manager's optimal asset allocation and risk-shifting incentives created by

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benchmarking. Intuitively, it is natural to conjecture that less illiquid assets can give money managers substantially more operating discretion and can offer a greater cushion when things go wrong. However, my analysis shows that in most cases, whenever money managers can choose among multiple risky assets that are subject to asymmetric liquidity constraints, they are likely to choose to hold a lower total initial risk exposure, compared to one that they would have chosen in an unconstrained liquidity setting, but an initial portfolio skewed toward more illiquid assets. The model I propose here also indicates that this initial portfolio composition tilted toward more illiquid assets is likely to subsequently shift toward less illiquid assets over the investment period, while maintaining expected portfolio weights on the more illiquid assets roughly even. According to the model, portfolio weights in less illiquid assets are likely to tilt away from those in the benchmark portfolio significantly more than are portfolio weights of more illiquid assets. These results suggest that a liquidity-constrained money manager, whose portfolio allocations are no longer under his or her complete control, is likely to shift portfolio risk by trading on less illiquid assets, and to use this trading to reduce the uncertainty associated with intertemporal variations of portfolio weights of more illiquid assets.

For ease of exposition, I confine attention to a constant investment opportunity set and a constant absolute risk aversion (CRRA, hereafter) preference structure. I also structure my analysis on two sorts of fund flow-to-performance functions. The first is the unbounded specification used in Browne (1999) and Binsbergen, Brandt, and Koijen (2008), according to which fund flows do not depend on the manager's own performance, but solely on the performance of an exogenous benchmark. The second is the specification used in Basak, Pavlova, and Shapiro (2007), which is capped and calibrated according to the empirical estimations of Chevalier and Ellison (1997). This specification exhibits a local convexity and depends on the manager's own performance relative to an exogenous benchmark.

In addition to their implicit incentives to distort portfolio allocations so as to increase the likelihood of future fund inflows, money managers also have explicit incentives to administer investors' savings according to their own attitudes towards risk, their (usually shorter) investment horizons, and their (eventually more favorable) participation and transaction costs. I investigate how costly it is for an individual investor to delegate portfolio decisions to a money manager when the investor and the manager have conflicting incentives, studying how liquidity constraints affect the potentially costly outcome of this conflict of interests. My analysis indicates that an investor's utility costs due to misalignments of objectives, either explicit or implicit, in a delegated investing relationship, can be quite large and that those costs are inversely related to levels of asset illiquidity. The more liquidity constrained are the decisions of the individual investor and the money manager, the lower are the utility losses resulting from their delegated portfolio relationship.

The results of using the proposed model and solving for the shadow cost of illiquidity indicate that delegated rather than direct investing is likely to lead to larger price discounts, in particular for longer illiquidity periods, and in cases in which money managers are guided by bounded flow-to-performance functions.

Finally, I investigate whether liquidity-constrained money managers adjust the riskiness of their portfolios by taking on unsystematic rather than systematic risk. My analysis, consistent with the results in Basak, Pavlova, and Shapiro (2007), suggests that money managers' risk-taking is exclusively characterized by systematic risk.

Although direct accessibility to financial markets has improved remarkably over time, the tendency of individual investors to delegate the administration of their savings to professional money managers has not decelerated. The Federal Reserve Board (September 2008) documented a steady overall increase of indirect equity holdings for U.S. households and nonprofit organizations between 1945 and 2007. It reported that, following the year 2000, indirect equity holdings surpassed direct equity holdings, and that at year-end 2007, indirect equity holdings made up nearly 70% of the total equity shares at market value held by U.S. households and nonprofit organizations. The report also showed that at year-end 2007, roughly one-third of U.S. households and nonprofit organizations' equities were held indirectly through mutual funds. According to the Investment Company Institute's (ICI, hereafter) official survey, between mid-1987 and the end of 2007, total net assets held in mutual funds increased from just under $818 billion to over $12 trillion, decreasing to about $9.6 trillion by the end of 2008.

One of the principles of mutual fund investing is liquidity. Mutual fund shares may be acquired or liquidated at a moment's notice at a fund's next determined net asset value per share. Unlike managers of closed-end and hedge funds, managers of open-end mutual funds must deal with daily asset flows into and out of their funds. Open-end mutual funds, in particular, provide high liquidity to redeeming investors, even though the stocks they hold possess varying degrees of liquidity. For simplicity, the analysis in this paper focuses on the effects of asset illiquidity on a fund managers' optimal portfolio policies, assuming that flows into and out of open-end mutual funds are not tradable and occur only once on a given manager's investment horizon. Hence, in this work I do not address the possibility of mutual funds being forced to sell into a declining and illiquid market to meet shareholder redemptions, an event that could lead to complete devastation of funds, as exemplified by the 2007-08 subprime-mortage meltdown and liquidity crisis.[1]

This paper closely relates to the strand of research on risk-shifting incentives in delegated portfolio management represented by Chevalier and Ellison (1997), Sirri and Tufano (1998), Arora and Ou-Yang (2001), Basak, Pavlova, and Shapiro (2007), Basak, Pavlova, and Shapiro (2008), and Basak and Makarov (2008). These studies describe a positive convex relationship between past performance relative to a given benchmark or peer group, and subsequent flows into and out of mutual funds, giving money managers, whose compensation is directly linked to the value of the assets they manage, an implicit incentive to distort portfolio allocations so as to increase the likelihood of finishing ahead of a given performance benchmark in order to increase future fund inflows.[2] It is important to remember that benchmarks such as the Russell 2000 and the S&P

500 are simply paper-based calculations of stock prices as they are quoted on the exchange. But as money managers attempt to make a purchase, the prices of stocks can differ significantly, depending on their liquidity, creating a difficult setting for managers who need to grow assets. Sudden liquidity dry-ups result in investors and money managers finding themselves losing control over their portfolio allocations and being forced to sit on their hands for long periods of time, incapable of "trading out of mistakes" they may have made in less liquid assets, potentially putting their short-term performance records at stake. Moreover, as if low liquidity wasn't already difficult to deal with, volatility typically also soars in down markets. For instance, the correlation between the monthly 'traded' liquidity factor of Pastor and Stambaugh (2003), and the CBOE VIX Index (the measure of market expectations of 30-day volatility passed on by S&P 500 Index option prices) as of the beginning of each month, for the period 1990-2006, was close to -0.364. More recently (albeit this was an extreme case), from January 2, 2008 to October 24, 2008, the S&P 500 Index decreased by 36.55% while the CBOE VIX Index skyrocketed from 21.29 to 74.72 points. In the face of such great uncertainty, investors' and money managers' optimal portfolio policies should, unsurprisingly, diverge from the policies they would choose in a less constrained context. Moreover, by hiring a money manager to administer his or her savings, an investor loses control over the composition of a fund's portfolio, and becomes subject to numerous incentive misalignments. For example, according to ICI's official survey results, at year-end 2007, over 90% of U.S. mutual fund-owning households indicated saving-for-retirement as their primary financial goal. However, money managers, who are concerned with their reputations and careers, and who may derive private benefits from being in charge of large funds, may have incentives to take actions that boost measures of short-term relative performance. They may act with the purpose of promoting a fund's new shares sales, which may work at the expense of investors' long-run retirement savings and may also be inconsistent with investors' risk tolerance.

This study is also related to empirical evidence of mutual funds' preference for large liquid stocks (Falkenstein (1996)), evidence of liquidity timing by mutual fund managers (Cao, Simin, and Wang (2007)), evidence that portfolio liquidity is actively managed and chosen as a function of the multiple liquidity needs of a fund (Massa and Phalippou (2005)), and evidence that fund managers tilt their holdings more heavily towards liquid stocks when the market is expected to be more volatile (Huang (2008)). Also relevant is the evidence expressed in Acharya and Pedersen (2005), which suggests that liquid funds should overperform in illiquid periods, and underperform during liquid periods.

Finally, this paper takes as given the standard cross-sectional empirical finding that expected returns decrease in liquidity, as well as the time-series evidence documented by Amihud (2002) and Jones (2002) of a long-term (monthly and yearly) negative relation between liquidity and return, and the evidence in Gervais, Kaniel, and Mingelgrin (2001) of a short-term (up to 20 days) positive relationship between liquidity and returns, leading to the expectation that short-term funds hold relatively more liquid portfolios than do long-term funds.

The paper is organized as follows. In Section 2, I describe the theoretical model setup, which includes the continuous-time economic setting in which investors and money managers decide their optimal dynamic portfolio policies. The setup includes the money managers' unconstrained problem, with and without implicit incentives, and the constrained problem, with constant and stochastic liquidity constraints. In Section 3, I solve the model using numerical methods, and discuss its main results. Conclusions and implications for further research are presented in Section 4.

2 Model setup

2.1 The financial market

Consider two market participants. Let one be a household investor who exogenously decides to directly access the financial market and manage his or her savings by him/herself or, alternatively, to hire a money manager to whom he or she delegates the administration of all his or her savings.[3] Assume the money manager's fund belongs to a peer group consisting of a large number of competing funds, such that there are no incentives for strategic interactions among fund managers.[4] Moreover, let these market participants have constant investment opportunities, and finite investment time horizons, $T_i \in [0, \infty)$, for $i \in \{I, M\}$. Let these participants take asset prices as given.

Consider a nominal economy in which continuous and unlimited trading and short-selling are possible whenever liquidity restrictions are left out. In this economy, participants can invest in $m + 1$ assets, with prices denoted by $S_j(t)$, for $j \in \{0, 1, 2, ..., m\}$. The first asset is a non-redundant nominal riskless money market account, which price dynamics follow the process:

$$\frac{dS_0(t)}{S_0(t)} = rdt \tag{1}$$

where $r \geq 0$ is a constant, continuously compounded interest rate. The remaining m assets are non-redundant risky assets with nominal prices evolving according to the following equation:

$$\frac{dS(t)}{S(t)} = \left(r\boldsymbol{\iota} + \sigma_s^\top \Lambda\right) dt + \sigma_s^\top dZ(t) \tag{2}$$

where $\boldsymbol{\iota}$ is an $m \times 1$ vector of ones, Λ denotes a $d \times 1$ vector of constant prices of risk, and σ_s is a $d \times m$ matrix of constant loadings on the source of uncertainty generated by a d-dimensional standard geometric Brownian motion $Z(t)$.

Let $N_0(t)$ denote the number of units of the riskless money market account, and likewise let $N(t)$ denote the $m \times 1$ vector of units of the risky securities held by a market participant at time t, for $0 \leq t \leq T$. This market participant's wealth is therefore given by $W(t) = N_0(t)S_0(t) + N(t)^\top S(t)$, which evolves according to the following equation:

$$dW(t) = N_0(t)rS_0(t)dt + N(t)^\top \text{diag}[S(t)] \left[\left(r\boldsymbol{\iota} + \sigma_s^\top \Lambda \right) dt + \sigma_s^\top dZ(t) \right] \tag{3}$$

where $S(t)$ is an $m \times 1$ vector, and diag$[S(t)]$ puts $S(t)$ on the main diagonal of an $m \times m$ matrix.

Now, take this market participant's holdings in the $m + 1$ assets at each and every time t, to be self-financing and to be constrained to lie in the closed solvency region of:

$$\mathscr{S} = \left\{ (S_0(t), S(t)) \in \mathbb{R}^{m+1} : N_0(t)S_0(t) + N(t)^\top S(t) > 0 \right\} \tag{4}$$

for all t, and $0 \leq t \leq T$.

Take the money manager's compensation to be proportional to the value of the assets under his or her administration, and to be due according to his or her particular investment horizon. In accordance with an empirically well-documented phenomenon (Chevalier and Ellison (1997), Sirri and Tufano (1998)), let money flows follow a positive convex relation with the fund's performance relative to a given benchmark. The combination of a convex flow-performance relation and a compensation scheme directly and proportionally linked to the value of assets under management, creates an implicit incentive for the money manager to distort portfolio allocations in order to increase the likelihood of future flows into the fund. Consider a money manager whose performance is measured relative to the value of a self-designated benchmark, $Y(t)$. Assume this benchmark is a reference portfolio of risky and riskless assets which can be replicated by the money manager. Let the benchmark evolve according to the process:

$$dY(t) = Y(t) \left[\left(r + \beta(t)^\top \sigma_s^\top \Lambda \right) dt + \beta(t)^\top \sigma_s^\top dZ(t) \right] \tag{5}$$

where the $m \times 1$ vector $\beta(t)$ given by $\beta(t) = Y(t)^{-1}\text{diag}[S(t)]M(t)$, denotes the vector of weights of the risky assets on the benchmark portfolio such that the weight on the riskless money market account is given by $1 - \boldsymbol{\iota}^{\top}\beta(t)$. The $m \times 1$ vector $M(t)$ denotes the number of units of risky assets that make up the benchmark at time t, for $0 \leq t \leq T$. Consider a continuously rebalanced (active) benchmark, where $\beta(t)$ is set to be constant, and is determined at time $t = 0$.[5] Note that "self-designated benchmark" does not mean that vector $\beta(t)$ should be included as a control in the manager's problem from which optimal performance benchmarks are derived (Binsbergen, Brandt, and Koijen (2008)). Note that this work is also not concerned about how the incentive to improve fund inflows could drive the money manager to strategically mismatch her fund benchmark, as empirically illustrated in Sensoy (2008). Instead, here my solo focus is on how a money manager allocates and manages resources to achieve investment objectives, given the manager's chosen performance benchmark. The benchmark is assumed to satisfy the manager's participation constraint.

Note that this paper focuses on the description of the continuous-time optimal control problem of the money manager, and only occasionally refers to the investor's problem, which is taken as a special case of the manager's problem. For simplicity of notation, subscripts I and M are used to indicate variables or parameters pertaining to the investor and the money manager, respectively, only when necessary for clarity of exposition.

2.2 The money manager's liquidity-unconstrained problem

As a point of reference, consider first the problem of a money manager who is not subject to liquidity restrictions, and who derives utility from the nominal value of the lump-sum cumulative amount of assets under management at the end of his or her investment horizon, $t = T$. Since admissible allocations require $W(t) > 0$, portfolio weights on risky assets are well defined and are given by the $m \times 1$ vector $\omega(t) = W(t)^{-1}\text{diag}[S(t)]N(t)$, such that the remainder, $1 - \boldsymbol{\iota}^{\top}\omega(t)$, denotes the portfolio weight on the money market account at time t, for $0 \leq t \leq T$. Plugging $\omega(t)$ into Equation (3) we get to the following functional form representing the dynamics of the value of assets under management:

$$dW(t) = W(t)\left[\left(r + \omega(t)^\top \sigma_s^\top \Lambda\right) dt + \omega(t)^\top \sigma_s^\top dZ(t)\right] \tag{6}$$

Subject to Equation (6), a money manager guided by CRRA preferences, dynamically allocates the fund's assets valued initially at $W(0)$ among a money market account and m risky assets, by choosing a vector of controls $\omega(t)$, so as to solve:

$$J(W,t) = \sup_{\omega(t)} E_t\left\{\frac{[W(T)\phi(T)]^{1-\gamma}}{1-\gamma}\right\} \tag{7}$$

where $\gamma > 0$, and $\gamma \neq 1$, denotes the manager's coefficient of relative risk aversion, and $\phi(T)$ denotes the rate at which funds flow into ($\phi(T) > 1$) or out ($\phi(T) < 1$) of the fund at the terminal date, depending on the fund's performance relative to a given benchmark.[6] I assume that the money manager's investment horizon coincides with the date of fund flows T, and that fund flows are nontradeable at that date.[7]

2.2.1 Optimal portfolio policies without implicit incentives

Absent implicit incentive considerations, in which case $\phi(T) = 1$, and under regularity conditions on the value function, the Hamilton-Jacobi-Bellman Partial Differential Equation (HJB PDE, hereafter) representing the problem described above, suppressing time indicators, is given by:

$$J_t + rWJ_W + \sup_{\omega}\left\{WJ_W\left(\omega^\top \sigma_s^\top \Lambda\right) + \frac{1}{2}W^2 J_{WW}\left[\omega^\top\left(\sigma_s^\top \sigma_s\right)\omega\right]\right\} = 0 \tag{8}$$

with terminal condition $J(W,T) = [1/(1-\gamma)]W(T)^{1-\gamma}$. The corresponding optimal portfolio allocations on risky assets are given by the $m \times 1$ vector:

$$\omega^*(t) = \frac{1}{\gamma}\left(\sigma_s^\top \sigma_s\right)^{-1} \sigma_s^\top \Lambda \tag{9}$$

with the remainder, $1 - \boldsymbol{\iota}^{\top}\omega^{*}(t)$, invested in the money market account. In this unconstrained liquidity setting, if $\Lambda < 0_{m\times 1}$, then $\omega^{*}(t) < 0$, the money manager chooses to hold a short position in risky assets. Likewise, if $\Lambda = 0_{m\times 1}$, then $\omega^{*}(t) = 0$, and if $\Lambda > \gamma \left(\sigma_s^{\top}\right)^{-1} \left(\sigma_s^{\top}\sigma_s\right) \boldsymbol{\iota}$, then $\omega^{*}(t) > 1$, and the money manager chooses to hold a leveraged position in risky assets. Moreover, this is so in a complete market setting, where σ_s is invertible, and where $\left(\sigma_s^{\top}\sigma_s\right)^{-1} = \sigma_s^{-1}\left(\sigma_s^{\top}\right)^{-1} = \sigma_s^{-1}\left(\sigma_s^{-1}\right)^{\top}$ is possible. Given that σ_s is assumed to be constant, these optimal investment strategies are independent of the investment horizon, as shown in Merton (1969). Such myopic allocations require continuous trading, and clearly $N^{*}(t) = \gamma^{-1}W(t)\mathrm{diag}[S(t)]^{-1}\left(\sigma_s^{\top}\sigma_s\right)^{-1}\sigma_s^{\top}\Lambda$ is of unbounded variation. Hence, after plugging (9) into (6), to solve the resultant stochastic differential equation, and plugging the solution then into (7), and after linearizing the term involving the Wiener process before placing the expectation operator in front of it, the utility derived by the money manager, as a result of implementing these unconstrained optimal policies, is given by:

$$J(W,t) = \frac{W(t)^{1-\gamma}}{1-\gamma}\exp\left\{\left[r + \frac{1}{2\gamma}A_1\right](1-\gamma)\tau\right\} \tag{10}$$

where $A_1 = \Lambda^{\top}\sigma_s\left(\sigma_s^{\top}\sigma_s\right)^{-1}\sigma_s^{\top}\Lambda$, and $\tau = T - t$. This complete solution to the manager's liquidity-constrained problem coincides with that of the investor whenever any explicit incentives are left out. The money manager has explicit incentives to administer the investor's savings according to her own attitude towards risk, her (usually shorter) investment horizon, and her (eventually more favorable) participation and transaction costs. The implications of these explicit incentives are considered in Section 3.

2.2.2 Optimal portfolio policies with implicit incentives

In the presence of implicit incentives, a money manager experiences money flows into and out of her fund at a rate $\phi(T)$, depending on the manager's relative performance over her investment horizon in relation to a benchmark chosen (exogenously) at time

$t = 0$. I consider two specifications of the flow-performance relationship. First, the specification in line with Browne (1999), and Binsbergen, Brandt, and Koijen (2008), according to which fund flows do not depend on the manager's own performance, but solely on the absolute performance of an exogenous benchmark. In this case, the flow-performance function is given by $\phi(T) = 1/Y(T)$, for a $Y(t)$ that evolves according to Equation (5). The convenience of this configuration is that it allows the derivation of a closed form solution for the optimal portfolio policies whenever trading restrictions are left out. If we let $X(T) = W(T)\phi(T)$, and we apply Ito's rule, we arrive at the dynamics of $X(t)$, for $\phi(t) = 1/Y(t)$, as follows:

$$dX(t) = X(t)[\omega(t) - \beta(t)]^\top \left[\left(\sigma_s^\top \Lambda - \left(\sigma_s^\top \sigma_s\right) \beta(t)\right) dt + \sigma_s^\top dZ(t)\right] \qquad (11)$$

and, under regularity conditions on the value function, suppressing time indicators, and given a continuously rebalanced benchmark, the HJB PDE for this problem is given by:

$$\begin{aligned} \sup_{\omega} \left\{ X J_X \left(\omega^\top A_2\right) + \frac{1}{2} X^2 J_{XX} \left[\omega^\top \left(\sigma_s^\top \sigma_s\right) (\omega - 2\beta)\right] \right\} - \\ - X J_X \left(\beta^\top A_2\right) + \frac{1}{2} X^2 J_{XX} \left[\beta^\top \left(\sigma_s^\top \sigma_s\right) \beta\right] + J_t = 0 \end{aligned} \qquad (12)$$

where $A_2 = \sigma_s^\top \Lambda - \left(\sigma_s^\top \sigma_s\right) \beta$, and with terminal condition $J(X,T) = [1/(1-\gamma)] X(T)^{1-\gamma}$. Hence, when the performance of the money manager is measured relative to an exogenous benchmark, the money manager's optimal portfolio is given by:

$$\omega^{\#}(t) = \frac{1}{\gamma} \left(\sigma_s^\top \sigma_s\right)^{-1} \sigma_s^\top \Lambda + \left(1 - \frac{1}{\gamma}\right) \beta(t) \qquad (13)$$

which is independent of the investment horizon.[8] This optimal portfolio policy contains an actively managed component and a herd component, the latter mimicking the benchmark against which the manager's performance is measured. Note that, in fact, the manager will tend to track the benchmark more and more closely as her risk aversion increases. Clearly, when $\gamma > 1$, if $\beta(t) > \omega^*(t)$, then the manager has an incentive to increase risk exposure. On the contrary, if $\beta(t) < \omega^*(t)$, the manager has an incentive to herd and decrease risk exposure. In either case, the dynamics of the optimal number of units held by the money manager on risky assets, $N^*(t)$, is that of a process of unbounded variation, where both unlimited leveraged and short positions are admissible.

The solution to the derived utility of the terminal value of assets under management when the money manager chooses a continuously rebalanced exogenous benchmark against which to measure performance, is given by:

$$J(X,t) = \frac{X(t)^{1-\gamma}}{1-\gamma} \exp\left\{ \frac{1}{2\gamma} \left[A_1 - A_3 - \beta^\top A_2 \right] (1-\gamma)\tau \right\} \tag{14}$$

where β is constant, $A_3 = \Lambda^\top \sigma_s \beta$, and $\tau = T - t$. Alternative specifications of the flow-performance relationship are investigated in Basak, Pavlova, and Shapiro (2007). I focus on their collar-type specification, drawn from the estimations of Chevalier and Ellison (1997). For completeness, the functional form of this collar flow-performance specification is given by:

$$\phi(T) = \begin{cases} \phi_L & \text{if } R^W(T) - R^Y(T) < \ell_L \\ \phi_L + \upsilon \left(R^W(T) - R^Y(T) - \ell_L \right) & \text{if } \ell_L \leq R^W(T) - R^Y(T) < \ell_H \\ \phi_L + \upsilon \left(\ell_H - \ell_L \right) & \text{if } R^W(T) - R^Y(T) \geq \ell_H \end{cases} \tag{15}$$

with $\phi_L > 0$, $\upsilon > 0$, $\ell_L \leq \ell_H \in \mathbb{R}$, where $R^W(T) = \ln(W(T)/W(0))$ and $R^Y(T) = \ln(Y(T)/Y(0))$ denote the continuously compounded returns of the money manager's portfolio, and of the exogenous benchmark, respectively, over the investment horizon, where we can normalize $W(0) = Y(0)$ without loss of generality. Since the utility function is homothetic in wealth, we can normalize $W(0) = 1$. Hence, when relative performance reaches ℓ_L, the flow-performance function displays a convex kink followed by an upward-sloping linear segment until reaching ℓ_H, after which it returns to a flat position.

Unlike the specification used in Binsbergen, Brandt, and Koijen (2008), the flow-performance function (15) exhibits a local convexity and assumes that the size of money flows into a fund depends on the fund's own performance. However, compared to using the specification in Binsbergen, Brandt, and Koijen (2008), when using specification (15) a closed form solution becomes much less immediate (see Basak, Pavlova, and Shapiro (2007)). In Section 3, I use numerical methods to solve for the optimal portfolio allocations and related implications of this collar flow-performance specification. Again, absent liquidity constraints, both leveraged and short positions are admissible, as well as unlimited and continuous trading. These assumptions, which are common in delegated portfolio management literature, are relaxed in Section 2.3, as follows.

2.3 The money manager's liquidity-constrained problem

In practice, money managers face multiple constraints. Sudden liquidity dry-ups, like the one that accompanied the 2007-08 meltdown in sub-prime lending, result in money managers finding themselves forced to sit on their hands for long periods of time, unable to deal in any size in shares of even the more liquid large-cap companies. As a result, managers loose control over their portfolio allocations, which potentially puts their short-term performance records (not to mention their jobs) in jeopardy.

Thus, let the money manager choose, at time $t = 0$, an $m \times 1$ portfolio vector $\omega(0)$, which he or she will want to revise later on if she chooses to actively invest. However, for $t > 0$, take the size of the money manager's trades each period, for a given cost, to be out of her complete control, and restricted to lie in a bounded interval. Specifically, and in parallel with Longstaff (2001), assume that the dynamics of the number of units of risky assets that the money manager can hold each time, is given by:

$$dN(t) = \eta(t)dt \tag{16}$$

where $\eta(t)$ is an $m \times 1$ vector, $-\infty < -\psi(t) \leq \eta(t) \leq \psi(t) < \infty$ and $\psi(t) \geq 0$. This specification captures the aspect of depth in markets, which I allow to be either investor-specific or asset-specific, or both. I also allow it to be either constant or time-varying. In this context, the dynamics of the value of a fund's assets under management is given by the expression:

$$dW(t) = rW(t)dt + N(t)^\top \text{diag}[S(t)][\sigma_s^\top \Lambda dt + \sigma_s^\top dZ(t)] \tag{17}$$

which will coincide with the budget constraint of an individual investor, in case we leave out any explicit and implicit incentives provided to the money manager by his or her compensation scheme, risk appetite, or investment horizon. Thus, when liquidity is constrained, the money manager can find him/herself in a situation where portfolio allocations are no longer under his or her control, and it can take a long time for him or her to "trade out of mistakes" in less liquid assets, which may very well lead to bankruptcy. Therefore, in order to guarantee the solvency of the fund, the manager has to abstain from taking leveraged positions or short extensions in the available risky assets. Accordingly, as in Longstaff (2001), in order for portfolio policies to be admissible, optimal controls $\omega(0)$ and $\eta(t)$ need to be such that $0 \leq \omega(t) \leq 1$, for all $0 \leq t \leq T$.

2.3.1 Constant liquidity constraints

Consider first the case in which the liquidity constraint, as measured by the value of the parameter $\psi(t)$, is set to be constant, $\psi(t) = \alpha$, for all $0 < t \leq T$. As a result, and because Equation (17) is now a function of $W(t)$, $N(t)$, and $S(t)$, the problem of our CRRA money manager, who decides on an initial allocation of capital among m risky assets, $\omega(0)$, and a money market account, $1 - \boldsymbol{\iota}^\top \omega(0)$, as well as on the subsequent revisions of that initial portfolio, as denoted by the $m \times 1$ vector of continuous controls

$\eta(t)$, is stated as:

$$J(W,N,S,t) = \sup_{\omega(0),\eta(t)} E_t \left\{ \frac{[W(T)\phi(T)]^{1-\gamma}}{1-\gamma} \right\} \tag{18}$$

subject to the budget constraint (17). Under regularity conditions on the value function, absent implicit incentives, $\phi(T) = 1$, and suppressing time indicators, the HJB PDE for this problem is given by:

$$\begin{gathered} J_t + J_W r W + J_W \left[N^\top \mathrm{diag}[S] \left(\sigma_s^\top \Lambda \right) \right] + \frac{1}{2} J_{WW} \left[N^\top A_4 N \right] + \\ + J_S \left[\mathrm{diag}[S] \left(r\boldsymbol{\iota} + \sigma_s^\top \Lambda \right) \right] + \frac{1}{2} \mathrm{tr}\left[J_{SS} A_4 \right] + J_{WS} \left[A_4 N \right] + \sup_{\eta} \left\{ \eta^\top J_N \right\} = 0 \end{gathered} \tag{19}$$

with terminal condition $J(W,N,S,T) = [1/(1-\gamma)][W(T)\phi(T)]^{1-\gamma}$, where J_S and J_{WS} are $1 \times m$ vectors, J_N is an $m \times 1$ vector, J_{SS} is an $m \times m$ matrix, and $A_4 = \mathrm{diag}[S] \left(\sigma_s^\top \sigma_s \right) \mathrm{diag}[S]$. In this case, because $\eta(t)$ is constrained, the HJB PDE is optimized by choosing $\eta(t)$ so as to maximize the term $\eta^\top J_N$ for a given initial portfolio vector $\omega(0)$. Therefore, the constrained money manager follows a bang-bang control policy, according to which she chooses either $\eta = \alpha$ if $J_N > 0$, or $\eta = -\alpha$ if $J_N < 0$, or $\eta = 0$ if $J_N = 0$, as long as it is guaranteed that $W(t) > 0$, and the trading strategies are admissible, meaning $N_0(t) \geq 0$, $N(t) \geq 0$, and $N_0(t) + \boldsymbol{\iota}^\top N(t) > 0$, for all $0 \leq t \leq T$. Absent implicit incentives, the formal solution of the money manager's derived utility is given by:

$$J(W,N,S,t;\omega(0)) = \frac{W(t)^{1-\gamma}}{1-\gamma} E_t \left[\exp\left\{ \int_t^T A_5(u)du + \int_t^T A_6(u)dZ(u) \right\} \right] \tag{20}$$

where $A_5(t) = r(1-\gamma) + A_6(t)\left(\Lambda - (1/2)\sigma_s \omega(t)\right)$, and $A_6(t) = (1-\gamma)\omega(t)^\top \sigma_s^\top$. Note that the portfolio weight vector $\omega(t)$ enters the derived utility function both linearly and quadratically, which means that liquidity restrictions introduce a second layer of mean-variance analysis into the manager's problem.

This problem coincides with the problem of a liquidity-constrained investor, if we also leave out the money manager's explicit incentives. In Section 3, I use numerical techniques to solve this optimization problem, as well as the problem that accounts for the manager's implicit incentives, as discussed in Section 2.2.2. In particular, I use the methodology suggested in Longstaff (2001). It consists of an application of the Least-Squares Monte Carlo algorithm, proposed by Longstaff and Schwartz (2001). Succinctly, it involves replacing the conditional expectation function in (20) by its orthogonal projection on the space generated by a finite set of basis functions of the values of the state variables that are part of the manager's problem. Next, from that explicit functional approximation, we can then solve for the optimal control variable $\eta(t)$, as defined above, for any given $\omega(0)$. Portfolio weights held in risky assets are then easily retrieved, for each time t, for $0 \leq t \leq T$, from the relation:

$$\omega_j(t) = \omega_j(0) + \int_0^t \frac{S_j(u)}{W(u)} \eta_j(u) du \tag{21}$$

where $\eta_j(0) = 0$, and $j \in \{1, 2, ..., m\}$.

2.3.2 Time-varying liquidity constraints

Existing literature provides evidence of liquidity timing by mutual fund managers (Cao, Simin, and Wang (2007)), evidence that portfolio liquidity is actively managed and is chosen as a function of the multiple liquidity needs of a fund (Massa and Phalippou (2005)), and also evidence that fund managers tilt their holdings more heavily towards liquid stocks when the market is expected to be more volatile (Huang (2008)). Moreover, according to Acharya and Pedersen (2005), liquid funds are likely to overperform in illiquid periods, and to underperform during liquid periods. What's more, stock market downturns have been showing that liquidity has a way of suddenly drying up when it is needed the most, has commonality across assets (Chordia, Roll, and Subrahmanyam (2000)), is related to volatility, and co-moves with the market.

In order to capture some of these aspects of market liquidity, including the "flight-to-quality" incentive induced by the liquidity uncertainty in the risky assets' market, consider the following mean-reversion process:

$$d\alpha(t) = K(\pi - \alpha(t))dt + \sigma_{\alpha}^{\top} dZ(t) \tag{22}$$

where $\alpha(t)$ and π, are $m \times 1$ vectors, K is an $m \times m$ diagonal matrix of speed of reversion parameters, and σ_{α} is a $d \times m$ matrix of loadings on the sources of uncertainty, generated by the same d-dimensional geometric Brownian motion used in previous sections, $Z(t)$. I then allow the money manager to be able to hedge against liquidity risk. Note, however, that we need the stochastic liquidity parameter $\psi(t)$, as defined above, to assume only positive values, and to have a long-run equilibrium level denoted by θ. In this case, an adequate mean-reverting model for $\psi(t)$ may be represented by:

$$\psi(t) = \exp\left\{\alpha(t) - \frac{1}{2}\left(\boldsymbol{\iota} - \frac{1}{2}\left[\exp\left\{-2Kt\right\} K^{-1}\right] D_{\alpha}\right)\right\} \tag{23}$$

where $\psi(t)$ is an $m \times 1$ vector, K^{-1} is the inverse matrix of K, and D_{α} is an $m \times 1$ vector that contains the diagonal of the matrix $\left(\sigma_{\alpha}^{\top} \sigma_{\alpha}\right)$. The long-run equilibrium level for $\psi(t)$ is related to the equilibrium level for $\alpha(t)$, and is given by the relation $\theta = \exp\{\pi\}$. Here, like in the previous section, the money manager's optimal portfolio strategy is to trade as much as possible, whenever possible.

3 Numerical results and discussion

Let the investor and the money manager trade in two risky assets ($m = 2$), which may have distinct degrees of liquidity. We could think of one of those risky assets to be a liquid well-known publicly traded large-cap stock, while the other would be, for instance, real estate or a small-cap stock from an emerging economy. I addition, let uncertainty be generated by a two-dimensional Brownian motion ($d = 2$), such that the price dynamics for these two risky assets is to include the following 2×2 matrix of loadings on the sources of risk:

$$\sigma_s = \begin{pmatrix} \sigma_{11} & \sigma_{21} \\ \sigma_{12} & \sigma_{22} \end{pmatrix} \tag{24}$$

where the term σ_{jk} denotes the loading that asset j puts on the source of risk k. From (24) we get that the variance of asset j is given by $\sigma_j^2 = \sigma_{j1}^2 + \sigma_{j2}^2$ and the correlation between the two assets is given by $\rho = (\sigma_{11}\sigma_{21} + \sigma_{12}\sigma_{22}) / (\sigma_1\sigma_2)$. In addition, let the 2×1 vector of expected returns for this pair of risky assets be given by $\mu = r\boldsymbol{\iota} + \sigma_s^\top \Lambda$. The numerical results in this section are based on 100,000 runs and 20 time steps per year. Investment horizons T_i, for $i \in \{I, M\}$, are expressed in years. Initial prices of assets (risky and riskless) are set to unity, such that $S_j(0) = 1$, where $j \in \{0, 1, 2\}$. Furthermore, let the riskless money market account earn a constant riskless interest rate $r = 0.02$.

3.1 Analysis of illiquidity and explicit incentives

The money manager has an explicit incentive to administer the investor's savings according to his or her own attitude towards risk, his or her (usually shorter) investment horizon, and his or her (eventually more favorable) participation and transaction costs. In this section, I assess the economic significance of these explicit incentives, in particular when they diverge from their investor's counterparts, and to what extent the presence of liquidity constraints affects their likely outcome.

3.1.1 The case of symmetric asset liquidity constraints

First, consider the case where the two risky assets are identical in every respect, including their liquidity characteristics. Table 1 reports the optimal initial portfolio weights for the unconstrained, and the constrained liquidity cases, $\omega_j^u(0)$, and $\omega_j^c(0)$, respectively, for different values of T_i, ρ, σ_j, and γ_i, when in the absence of implicit incentives ($\phi(T) = 1$). Take the two risky assets to be identical in every respect, so we can isolate the effects of the misalignment of incentives between the investor and the money manager. Therefore, I set both risky assets to have the same expected return, $\mu_j = 0.10$, and to be non-tradeable ($\alpha_j = 0$), for $t > 0$, throughout the period T_i. The result is that, in general, under liquidity constraints, either the investor, or the money manager, choose to hold initial portfolio weights that are lower than those that would be chosen under no liquidity restrictions, even when the unconstrained weights would be admissible in the constrained problem. These differences can be seen as hedging demands for the expected changes in portfolio weights through time, due to the limited tradability of the risky assets, as suggested in Longstaff (2001). Table 1 reports values for the simulated cross-sectional variation of the constrained portfolio weights, CSV_j, at $t = T_i$. For this set of parameters, note that CSV_j increases with σ_j for independent and negatively correlated risky assets, while it decreases for positively correlated risky assets. Not unexpectedly, constrained portfolio weights' variation increases with the investment horizon. These simulated variations range from .0105 (for $\rho = 0.5$, $\gamma_i = 10$, $\sigma_j = 0.5$, and $T_i = 1$), to .2258 (for $\rho = 0.5$, $\gamma_i = 1$, $\sigma_j = 0.5$, and $T_i = 1$), while they are all null for the unconstrained portfolio weights, by construction. To trade-off the expected value of $\omega_j^c(t)$ with its variation is also part of the problem of a liquidity constrained market participant. Note also that, when the unconstrained investor holds a leveraged position, the constrained one restricts her portfolio weights to lie in the interval $0 \leq \omega_1^c(0) + \omega_2^c(0) \leq 1$, to prevent against insolvency. Finally, Table 1 also reports the number of basis points we would have to discount the prices of the identical risky assets so as to make the investor or the money manager indifferent between holding the constrained portfolio, and the constrained one. For this set of parameters, illiquidity discounts (ID) increase with T_i, and decrease with γ_i, ρ, and σ_j. For this set of parameters, illiquidity discounts range from 1.23 to 1,380.80 basis points. In particular, for $T_i = 2$, $\sigma_j = 0.3$, $\rho = 0$, and $\gamma_i = 1$, the price of the identical risky assets would have to be discounted by 2.87% so as to compensate the investor for holding a totally illiquid portfolio, instead of one he or she could rebalance without restrictions. Not surprisingly, the largest discounts occur when endogenous constraints on leverage are binding.

3.1.2 The case of asymmetric asset liquidity constraints

Consider now the case where we let the maximum number of shares of asset 2, that can be traded per year, be $\alpha_2 = 0.10$, while keeping $\alpha_1 = 0$. Table 2 reports the optimal initial portfolio weights for the unconstrained, and the constrained liquidity cases, $\omega_j^u(0)$, and $\omega_j^c(0)$, respectively, for $T_i = 1$, $\sigma_j = 0.5$, and different values of ρ, and γ_i, where implicit incentives are still left out ($\phi(T) = 1$). Because the risky assets are not identical in terms of tradability anymore, I cannot directly assign to each of them the responsibility for the total cost of the illiquidity effect. Therefore, I report instead the total illiquidity cost (IC), which denotes the percent (measured in basis points) of the investor's (or, likewise, the manager's) initial wealth, that one would have to give him or her in order to compensate him or her for holding a liquidity constrained portfolio, instead of a portfolio that she could revise with no restrictions. Table 2 shows that illiquidity costs are significantly larger for the case where $\alpha_2 = 0.10$ than for the case where $\alpha_j = 0$. Given that the risky assets are no longer identical, Table 2 reports individualized information for the two assets. Note that the optimal initial unconstrained portfolio holdings, $\omega_j^u(0)$, are exactly the same as in Table 1, but now the initial constrained portfolio weights, $\omega_j^c(0)$, differ for the two assets, and they do so in an interesting way. Hence, when we relax the liquidity constraint on asset 2, the investor (and the manager alike) chooses, on the whole, a lower risk exposure, and (somewhat surprisingly) skewed towards the more illiquid asset, which is to say $\omega_1^c(0) > \omega_2^c(0)$. Note that the ratio of the more liquid asset initial portfolio weight, $\omega_2^c(0)$, to the one for the less liquid asset, $\omega_1^c(0)$, decreases monotonically with γ_i, meaning that, a more risk averse investor or money manager optimally chooses to hold relatively more of the less liquid asset. The ratio $\omega_2^c(0)/\omega_1^c(0)$ can be thought of as a measure of the level of portfolio liquidity, and Table 2 shows that the optimal initial portfolio liquidity decreases, in general, with the correlation of assets' returns. It also shows that these initial optimal portfolios shift liquidity levels over the investment period. The variable $E[\omega_j^c(T)]$ denotes the expected value of the constrained portfolio weights at time $t = T_i$. Note that $E[\omega_2^c(T)]$ is generally larger than $\omega_2^c(0)$ by, approximately, $\alpha_2 = 0.10$, the maximum number of shares of asset 2 that can be traded per year. Note also that the cross-sectional variation of $\omega_2^c(T)$, given by CSV_2, is generally larger than the one we discussed above for Table 1, where asset 2 could not be traded, while CSV_1 remains of the same kind as in Table 1. Thus, the constrained money manager, in trading off the expected value of $\omega_2^c(T)$ against its variation, does so by taking much smaller initial positions and by trading asset 2, so as to keep the non-tradeable asset 1's portfolio weights' expected value, and variation, under control. However, the optimal utility levels that an investor/manager attains when actively trading asset 2, turn out to be lower than those she would obtain under a passive buy-and-hold strategy as the one shown in Table 1. Table 2 shows the variable CSV_W, which denotes the cross-sectional variation of the value of assets under management at $t = T_i$. Generally, these variations of $W(T)$ are larger when trading for asset 2 is allowed than when it is not. For brevity, I do not tabulate the simulation results for the case where, leaving everything else constant, I let $T_i = 2$, or alternatively, I let $\sigma_j = 0.3$. Succinctly, for longer investment horizons, initial portfolio weights decrease substantially, but illiquidity costs, and portfolio value variations, increase drastically. For instance, when $\rho = 0$, and $\gamma_i = 10$, then IC$= 2.022\%$, $\mathrm{CSV}_2 = 0.1254$. If, alternatively, I let $\sigma_j = 0.3$, then illiquidity costs increase dramatically for $\gamma_i = 1$, while they decrease for $\gamma_i > 1$, which is the result of having endogenous borrowing constraints to bind. Differently, when I keep all the same parameters used in Table 2 except that I also allow asset 1 to trade during the investment period, with $\alpha_1 = 0.10$, then the assets return to identical, as in Table 1, but optimal initial portfolio risk exposure decreases, while illiquidity discounts rise.

3.1.3 The case of divergence in appetites for risk

We observe the SEC regularly advising investors to carefully read fund prospectus and shareholder reports, to learn about its investment strategy and the risks it takes to achieve its returns, so that these risks can be factored in and be tested for consistency with the investor's financial goals and risk tolerance. In fact, significant misalignments of incentives can be derived from the extensive differences in appetites for risk, between investor and money manager. Figure 1 reveals the shape of the shadow costs associate with this particular explicit incentive, as a function of both the investor's and the money manager's risk appetites. Table 3 reports these costs for investment horizons $T_i = 1$, and where parameter values are as in Table 1. These costs are expressed as the percent of wealth the suboptimal investor would be willing to give away in return for being allowed to follow the optimal strategy. Panel A reports the shadow costs for the unconstrained liquidity case. Panel B reveals the results for the case where liquidity is totally constrained ($\alpha_j = 0$). Not unexpectedly, when investor and manager have the same attitude towards risk, given implicit incentives are left out, the money manager is acting in the best interest of the investor and, as a result, no losses are derived from this delegated portfolio relationship. However, in case of divergence in risk attitudes, utility costs can become very significant. These costs range from 0.08% to 250.33% of the investor's initial wealth, for the unconstrained liquidity case, while they range from 0% to 32.66% in the constrained liquidity case. Largest costs occur for higher values of γ_I, and lower values of γ_M, which is what we should find, professional money managers to be much less risk averse than fund investors. What is interesting to see in these results is that utility costs become significantly lower when in the presence of liquidity constraints. This has to do with the endogenous leveraging constraints, and hedging demands, which restrain portfolio weights to fall into a closed limited set of values, in order to prevent for bankruptcy. These costs significantly increase with the investment horizon T_i. When the investment horizon increases to 2 years, utility losses derived from the difference in appetites for risk between the investor and the money manager, for the same parameter values as in Table 3, range from 0.17% (for $\gamma_I = 5$, $\gamma_M = 10$, $\rho = 0.5$, and $\sigma_j = 0.5$) to 709.27% (for $\gamma_I = 10$, $\gamma_M = 1$, $\rho = -0.5$, and $\sigma_j = 0.3$), in the unconstrained liquidity case, while they range from 0% (for $\gamma_I = 1$, $\gamma_M = 2$, $\rho = -0.5$, and $\sigma_j = 0.3$) to 59.59% (for $\gamma_I = 10$, $\gamma_M = 1$, $\rho = -0.5$, and $\sigma_j = 0.5$), in the constrained liquidity case.

For brevity, I do not tabulate either the results for $T_i = 2$, or the results for $\alpha_1 = 0$ and $\alpha_2 = 0.10$. When I let the maximum number of shares of asset 2, that can be traded per year, be $\alpha_2 = 0.10$, then utility losses for a constrained investor whose risk appetite may not be consistent with that of the money manager to whom she delegates the management of all her savings, are generally reduced. They range from 0.04% to 32.35%, when $\sigma_j = 0.5$. For instance, when $\gamma_M = 10$, $\gamma_I = 2$, $\rho = 0$, and $\sigma_j = 0.5$, and $T_i = 1$, the utility loss is equal to 0.63%, instead of 0.74%. These utility costs increase for longer investment horizons and for lower asset return volatilities, everything else constant.

3.1.4 The case of divergence in investment horizons

According to ICI's survey, in year-end 2007, over 90% of U.S. mutual fund-owning households indicate saving-for-retirement as their primary financial goal. However, money managers concerned with their reputations and careers, and whose remuneration schemes usually induce, may have incentives to take actions that boost measures of short-term relative performance, which may work at the expense of the investor's saving-for-retirement value. Therefore, investment horizons of the investor and the money manager may very likely be mismatched, which may produce utility losses for an investor who decides to delegate the administration of all her savings. For instance, when the risky assets are totally illiquid ($\alpha_j = 0$), and therefore cannot be traded for $t > 0$, when $\sigma_j = 0.5$, both investor and money manager have the same risk aversion, $\gamma_i = 2$, and $\rho = -0.5$, for the case where $T_I = 5$, and $T_M = 1$, the investor's utility cost, as percent of her initial wealth, is 0.987%, which increases slightly to 1.035% if we let $\gamma_i = 1$. These costs slightly decrease when we let one of the assets, asset 2, to trade throughout the investment period. If we let the number of shares of asset 2, that can be traded per year, be $\alpha_2 = 0.10$, then, for the parameter values given above, the investor's utility cost would be, instead, 0.590% for $\gamma_i = 2$, and 0.891% for $\gamma_i = 1$. Larger costs would be found for wider differences between T_I and T_M, and smaller costs would be given by larger values of ρ, not necessarily in a monotonic fashion.

3.1.5 The case of divergence in participation constraints

It may also be the case that the money manager has access to the financial markets in more favorable terms than the investor. The manager may have lower transaction, market participation, or informational costs, lower opportunity costs of time to engage in active portfolio management, better information or ability, or better investing education. Some of these could simply translate into flexible manager-specific liquidity constraints. Therefore, consider the case here the investor is unable to trade in any of the assets ($\alpha_j = 0$), for $t > 0$, while, at the same time, the manager is allowed to trade in asset 2, throughout the year, $\alpha_2 = 0.1$. The shadow costs resulting from this difference in participation costs appear to be not very significant. For $T_i = 1$, and $\rho = -0.5$, they range from 0%, when $\gamma_i = 1$, to 0.28%, when $\gamma_i = 10$.

3.1.6 Implications of time-varying liquidity constraints

Furthermore, I examine the implications of allowing marketability bounds to follow a stochastic process like the one described by expression (23), in Section 2.3.2. Table 4 reports optimal initial investment policies that the investor, and the money manager alike, would choose, absent implicit incentives ($\beta_j = 0$), for the case where asset 2 is allowed to trade throughout the year, and the following parameter values: volatility of the marketability bound for asset 2, $\sigma_{\alpha_2} = 0.2$, speed of reversion $\kappa_2 = 0.1$ (where κ_h denotes the h^{th} element of the diagonal matrix K), initial value $\psi_2(0) = 0.1$ (annualized), long-run equilibrium levels $\theta_2 = 0.1$ (annualized), in Panel A, $\theta_2 = 0.2$, in Panel B, $\sigma_j = 0.5$, and $T_i = 1$. Succinctly, these results suggest that illiquidity costs slightly increase when we allow marketability to be stochastic, and increase more for larger long-run equilibrium levels. In addition, cross-sectional variations for $\omega_2^c(T)$ rise, while they remain roughly level for $\omega_1^c(T)$. Furthermore, Table 4 also reports estimates for the parameter λ_{j2}, which denotes the simulated average time-series correlation coefficients between $\omega_j^c(t)$ and the stochastic $\psi_2(t)$, for $j \in \{1, 2\}$. The values for these coefficients suggest that swings in asset 2's marketability are directly accompanied by $\omega_j^c(t)$, progressively more closely the more risk tolerant is the investor/manager, and the larger the equilibrium level θ. Finally, on the whole, shadow costs of explicit incentives, under stochastic liquidity, generally decrease.

3.2 Analysis of illiquidity and implicit incentives

Empirical evidence of a positive convex relationship between a fund's flows and its performance relative to a benchmark or peer group (Chevalier and Ellison (1997), Sirri and Tufano (1998)), together with the fact that a money manager's compensation is directly linked to the value of assets under management, tend to create an implicit incentive towards the distortion of portfolio allocations so as to increase the likelihood of future fund inflows. In this section, I assess the economic significance of a money manager's undesirable behavior due to her implicit incentives, when in the presence of liquidity restrictions. Therefore, in order to isolate the implications of implicit incentives, I expunge from this analysis any circumstance that could lead to any sort of adverse explicit incentives, as discussed in Section 3.1. Therefore, assume that both investor and money manager guide their portfolio allocations by similar risk appetites, investment horizons, and participation constraints.

3.2.1 Implications of benchmarking

Table 5 reports optimal initial portfolio weights a money manager would choose in case his or her fund's performance was to be measured against the performance of a benchmark portfolio, $Y(t)$, for $\beta_j = 0.5$, where risky assets are set to be identical, independent ($\rho = 0$), and totally illiquid ($\alpha_j = 0$). Panel A shows the results for the case where $T_M = 1$, while Panel B shows the results for $T_M = 2$. Each panel also splits the results between the case where the fund flow-performance is given by $\phi(T) = 1/Y(T)$ (e.g. Browne (1999), Binsbergen, Brandt, and Koijen (2008)), and where it is of the collar-type, for parameter values $\phi_L = 0.8$, $\upsilon = 4.375$, $\ell_L = -0.08$, and $\ell_H = 0.08$ (see Basak, Pavlova, and Shapiro (2007)). Figure 2 illustrates how the shape of the money manager's derived utility function looks like, for a particular set of parameters, and for $\phi(T) = 1/Y(T)$. Optimal initial portfolio weights are, therefore, those that attain the maximum of this function. In a similar way, Figure 3 reveals the shape of the derived utility function for the case where $\phi(T)$ is of the collar-type. When compared to the results in Table 1, Table 5 shows that the money manager's optimal initial portfolio policies have now an actively managed component, and an herd component. It just confirms the result of Equation (13), in Section 2.2.2, for the case where $\phi(T) = 1/Y(T)$, while it appears much less obvious for the case of a collar-type flow-performance specification. If we take the difference between the optimal initial portfolio weights of Table 5, and those in Table 1, and then divide these differences by the latter values, we get a potential measure of herding demands. From these computations, it becomes clear that, generally, herding incentives increase for more conservative money managers, and that they are considerably lower for the constrained liquidity case, when we consider the collar-type flow-performance specification. However, for $\phi(T) = 1/Y(T)$, herding incentives become slightly larger for the constrained liquidity case. One potential explanation for the weaker herding incentives in the collar-type flow-performance, may be the presence of limited liability, which may reinforce the motivation to gamble to finish ahead of the benchmark. Somewhat surprisingly, these results show that under a collar-type flow-performance relation, a more and more risk averse money manager tends to choose portfolio allocations that deviate more and more from the benchmark, while they converge to the benchmark portfolio weights when $\phi(T) = 1/Y(T)$. For longer investment horizons, these results get amplified for $\phi(T) = 1/Y(T)$, but become smoother for $\phi(T)$ of the collar-type.

Table 5 also shows that cross-sectional variations of the portfolio weights, CSV_j, increase in the presence of implicit incentives. It also shows the simulated cross-sectional variations for the value of assets under management, and the benchmark portfolio ($\beta_j = 0.5$), at the horizon T_M, which are denoted by CSV_W and CSV_Y, respectively. These results show that, generally, $\text{CSV}_W < \text{CSV}_Y$, given that the benchmark is continuously rebalancing in order to preserve $\beta_j = 0.5$, while the risky assets in the manager's portfolio are totally illiquid, which makes it a buy-and-hold portfolio. Nevertheless, the tracking error of Y, TE_Y, is relatively small. As a tracking error measure, I use the square root of the non-central second moment of the deviations between the money manager's portfolio and benchmark returns, which is the measure that is frequently used in practice. The tracking error for β_j appear to be significantly larger, ranging from 10.07% to 19.71% when $T_M = 1$, and from 13.40% to 27.13% when $T_M = 2$. In addition, TE_β appears to decrease with γ_M for $\phi(T) = 1/Y(T)$, while they increase with γ_M for $\phi(T)$ of the collar-type. As a result, illiquidity discounts (ID) for our identical illiquid risky assets, come out dramatically larger for the collar-type flow-performance case, which, as in previous sections, get amplified for longer investment horizons. Finally, Table 5 also reports the simulated probabilities that the money manager's optimal portfolio values end up below that of the benchmark, by the terminal date, T_M, for a given optimal control $\omega_j^c(0)$. It is interesting to see that, for the case of a collar-type flow-performance relation, the more conservative the money manager, the lower the probability of under-performing the benchmark at T_M. Bear in mind that we normalized $W(0) = Y(0) = 1$. Thus, the apparent gambling conduct eventually pays off. For brevity, I do not tabulate the results I obtain for $\beta_j = 0.2$, in which case the benchmark has a money market exposure of $\beta_0 = 0.6$. Obviously, in this case, the cross-sectional variations of $Y(T)$ become significantly lower, where $\text{CSV}_Y = 0.151$ for $T_M = 1$, and $\text{CSV}_Y = 0.225$ for $T_M = 2$. Moreover, the probabilities of under-performing the benchmark rise, despite the reductions in TE_Y, and TE_β.

3.2.2 Shadow costs of implicit incentives created by benchmarking

Table 6 reports the results for the shadow costs of implicit incentives, measured in percent points of the investor's initial wealth, for the case of independent, identical ($\sigma_j = 0.5$, $\mu_j = 0.1$), and non-tradeable risky assets ($\rho = 0$, $\alpha_j = 0$). Figure 4 reveals the shape of this shadow cost function, for a particular set of parameters. Shadow costs range from 0% to 188.76% in the unconstrained liquidity case, while ranging from 0% to 136.21% for the constrained liquidity one. Clearly, implicit incentives appear to be significantly more costly for the investor in the case where $\phi(T) = 1/Y(T)$ than in the case where $\phi(T)$ is of the collar-type. For $\gamma_i = 10$, a liquidity constrained investor, with an investment horizon of 2 years, requires 14.66% extra initial wealth in order to be indifferent between delegating the management of her savings to the professional money manager and directly accessing the financial markets to do it him/herself, when the manager is guided by the incentives of a collar flow-performance function, and $\beta_j = 0.5$. I also investigate whether a constrained professional money manager adjusts her portfolio riskiness through taking on unsystematic rather than systematic risk. Thus, I let $\mu_1 = r = 0.02$, so that asset 1 does not command any risk premium, and set $\sigma_{12} = 0$, so that asset 1 is solely driven by the source of risk $Z_1(t)$. Moreover, let $\beta_1 = 0$ and $\beta_2 = 1$, such that the benchmark portfolio is given by asset 2 alone, and is solely driven by the source of risk $Z_2(t)$, so that $\sigma_{21} = 0$, which also implies that $\rho = 0$. The results obtained under this setup show that, the money manager optimally chooses to hold asset 2 only, in her portfolio, which means that risk-taking happens only through systematic risk. This is the case either when both risky assets are non-tradeable ($\alpha_j = 0$), or when we let asset 2 trade during the year ($\alpha_1 = 0$ and $\alpha_2 = 0.20$).

Consider now the case where we let the maximum number of shares of asset 2, that can be traded per year, be non-zero. In particular, let $\alpha_2 = 0.20$. Table 7 shows that, not unexpectedly, in this case the optimal initial portfolio risk exposure (the fraction of assets under management invested in risky assets) declines, when compared to the results in Table 5, for both flow-performance specifications. Note that now it becomes more likely that the money manager will under-perform the benchmark. This result seems to relate with the empirical evidence that actively managed funds have, on average, an inferior performance than that of index funds (e.g. Gruber (1996)). In addition, the cross-sectional variations of $\omega_2^c(T)$ slightly increase, while CSV_W slightly decrease, for both flow-performance specifications. Furthermore, illiquidity costs significantly increase when we allow asset 2 to trade, even for limited amounts, during the year. I also include in this Table 7 estimates of the parameters π and ν, which denote the simulated time-series correlation coefficients of the portfolio liquidity (ratio of $\omega_2^c(t)$ to $\omega_1^c(t)$), and the portfolio risk exposure ($\omega_2^c(t) + \omega_1^c(t)$), respectively, with respect to the ratio of the assets under management to the benchmark portfolio (W(t)/Y(t)). These results suggest that, the presence of limited liability reduces the sensitivity of the portfolio allocations to the liquidity differentials between the two risky assets. Note that π switches from a negative figure to a positive one, as the money manager becomes more conservative. A negative π means that, when the performance of the manager's portfolio deteriorates relative to the benchmark, then on average, contemporaneously, the manager optimally chooses to distort her portfolio composition towards the more liquid risky asset, which occurs for the more risk loving managers when $\phi(T) = 1/Y(T)$, and indiscriminately for the collar-type flow-performance relation. These results would be consistent with risk-shifting behavior to take place in the more liquid asset class. Note, however, that these correlations don't appear specially strong in either case. The negative signs of ν all over Table 7 suggest that increasing fund distress implies escalating risk exposure. Shadow costs of implicit incentives to a liquidity constrained investor, when asset 2 is tradeable, $\alpha_j = 0.20$, range from 0% to 25.87% (of his or her initial wealth) for $\phi(T) = 1/Y(T)$, and from 0.05% to 6.17% for a collar flow-performance function.

4 Conclusions

This study was limited in several ways. For ease of exposition, I confined attention to CRRA preferences, continuously rebalanced benchmarks, and totally passive investors. Nevertheless, it suggests that asset illiquidity can significantly affect money managers' risk-shifting incentives as well as the investors' utility costs of misaligned objectives.

It would be useful in future research to include a more complicated preference structure in the analysis, extending it to include the use of buy-and-hold benchmarks, and allowing the investors to endogenously decide how much of their savings to invest directly and how much to indirectly hold in risky assets through financial intermediaries. Other interesting extensions could focus on deriving optimal performance benchmarks that would account for asset illiquidity and could then be used to better align incentives between investors and money managers in more illiquid markets. Future work could also focus on studying the implications of time-varying investment opportunities and their interaction with asset illiquidity, as well as on the sensitivity of the results presented here to alternative measures of asset liquidity. Examples of those alternative measures of liquidity include the bid-ask spread (Amihud and Mendelson (1986)), the price impact of trade (Brennan and Subrahmanyam (1996)), turnover (Datar, Naik, and Radcliffe (1998)), trading volume (Brennan, Chordia, and Subrahmanyam (1998)), and transaction costs (Liu and Loewenstein (2002), Liu (2004), Dai and Yi (2006), Jang, Koo, Liu, and Loewenstein (2007), and Dai, Jin, and Liu (2008)). Lastly, given that only partial equilibrium results are presented here, they should be taken only as suggestive. Future research could focus on assessing the asset pricing implications of liquidity restrictions in a delegated portfolio general equilibrium setting. Examples of related recent literature on this topic are Longstaff (2005) and Leippold and Rohner (2008). The former studies the asset-pricing implications of illiquidity in a two-asset exchange economy with heterogeneous agents, where one asset is always liquid and the other can be traded initially, but then not again until after a restricted period. Longstaff finds that, in such a framework, agents abandon diversification as a strategy and choose highly polarized portfolios instead. In the latter, using a model with endogenous delegation, Leippold and Rohner show that delegation under benchmarking leads to more informative prices and to lower equity risk premia.

Notes

[1] See Brunnermeier (2008) and Allen and Carletti (2008) for excellent accounts of the sequence of events that have mapped out the 2007-08 financial crisis, focusing on a wide range of factors, among which the typical fragility of market liquidity. See also Brunnermeier and Pedersen (2008) for a notable discussion on the mutually reinforcing effects of market liquidity and funding liquidity.

[2] Huang, Sialm, and Zhang (2008) show that funds that shift risk end up performing worse, which is consistent with risk-shifting being driven by money managers' opportunistic behavior, rather than their active portfolio management ability.

[3] In a more general model, the investor could be allowed to dynamically choose how much of her portfolio of risky assets to hold directly, and how much to hold indirectly through mutual funds, pension funds, and the like. I am grateful to Joshua Shemesh for this insight.

[4] See Basak and Makarov (2008) for an analysis of the dynamic portfolio choice implications of strategic interaction among money managers.

[5] In a more general setting, we would consider also the implications of using a (passive) buy-and-hold benchmark portfolio, where $M(t)$ would be constant, and where $\beta(t)$ would become rather random.

[6] In Section 3, for comparison purposes, I also present results for the simpler case of logarithmic preferences (CRRA utility with $\gamma = 1$).

[7] In a more general model, the investment horizon would not coincide with the fund flows date (e.g. Basak and Makarov (2008)), in which case fund flows would be tradeable after the flow date, and $\phi(t < T)$ would then enter the problem through the budget constraint, and not directly through the utility function.

[8] This is the case because our money manager chooses to be compared to a continuously rebalanced benchmark, and $\beta(t)$ is set to be constant. As mentioned before, in a more general model, we would allow the manager to choose to have her performance be measured against a (passive) buy-and-hold benchmark, in which case her optimal portfolio weights, as give by Equation (13), would be rather time-varying, because the vector of weights on the benchmark portfolio becomes a moving target, for a money manager who chooses active investing.

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Table 1: Optimal buy-and-hold ($\alpha_j = 0$) investment policies and illiquidity discounts, with no influence of either explicit or implicit incentives. Illiquidity discount (ID) is defined as the number of basis points the price of the identical risky assets would have to be reduced in order to make the investor/manager indifferent between holding the liquidity-constrained ($\omega_j^c(0)$) and the liquidity-unconstrained ($\omega_j^u(0)$) portfolios. CSV_j denotes the cross-sectional volatility of the simulated constrained portfolio weights, at the terminal date, $\omega_j^c(T)$, for $j \in \{1, 2\}$. Parameters σ_j, ρ, and $\text{RA}(\gamma_i)$, denote the return volatility of the risky assets, the correlation coefficient between the risky assets' returns, and the coefficient of relative risk aversion, respectively.

Panel A: Constrained liquidity, $\alpha_j = 0$, T_i=1									
		$\sigma_j = 0.3$				$\sigma_j = 0.5$			
ρ	$\text{RA}(\gamma_i)$	$\omega_j^u(0)$	$\omega_j^c(0)$	CSV_j	ID(bp)	$\omega_j^u(0)$	$\omega_j^c(0)$	CSV_j	ID(bp)
-0.5	1	1.778	0.500	0.1197	704.03	0.640	0.500	0.1838	83.99
	2	0.889	0.500	0.1197	149.17	0.320	0.268	0.1111	44.48
	5	0.356	0.328	0.0833	20.29	0.128	0.103	0.0499	19.34
	10	0.178	0.165	0.0464	10.71	0.064	0.050	0.0261	9.99
0	1	0.889	0.500	0.0996	136.55	0.320	0.305	0.1075	12.82
	2	0.444	0.440	0.0891	5.73	0.160	0.153	0.0649	8.93
	5	0.178	0.175	0.0444	3.54	0.064	0.060	0.0298	4.39
	10	0.089	0.085	0.0243	2.15	0.032	0.030	0.0158	2.40
0.5	1	0.593	0.500	0.0718	11.70	0.213	0.210	0.0713	5.76
	2	0.296	0.293	0.0527	1.61	0.107	0.100	0.0430	4.92
	5	0.119	0.115	0.0291	1.92	0.043	0.040	0.0199	2.61
	10	0.059	0.058	0.0164	1.23	0.021	0.020	0.0105	1.41

Panel B: Constrained liquidity, $\alpha_j = 0$, T_i=2									
		$\sigma_j = 0.3$				$\sigma_j = 0.5$			
ρ	$\text{RA}(\gamma_i)$	$\omega_j^u(0)$	$\omega_j^c(0)$	CSV_j	ID(bp)	$\omega_j^u(0)$	$\omega_j^c(0)$	CSV_j	ID(bp)
-0.5	1	1.778	0.500	0.1620	1,380.80	0.640	0.473	0.2258	285.84
	2	0.889	0.500	0.1620	336.82	0.320	0.230	0.1351	158.06
	5	0.356	0.313	0.1124	76.34	0.128	0.088	0.0645	68.84
	10	0.178	0.155	0.0645	40.49	0.064	0.043	0.0347	35.52
0	1	0.889	0.500	0.1364	286.80	0.320	0.293	0.1419	63.84
	2	0.444	0.430	0.1209	29.70	0.160	0.140	0.0873	41.65
	5	0.178	0.170	0.0634	16.78	0.064	0.053	0.0405	19.67
	10	0.089	0.083	0.0356	9.80	0.032	0.025	0.0210	10.43
0.5	1	0.593	0.500	0.0999	29.83	0.213	0.205	0.0984	28.23
	2	0.296	0.298	0.0754	9.41	0.107	0.095	0.0605	21.92
	5	0.119	0.115	0.0431	8.38	0.043	0.038	0.0292	10.96
	10	0.059	0.058	0.0250	5.24	0.021	0.018	0.0149	5.86

Table 2: Optimal investment policies and costs of constant illiquidity, with no influence of implicit incentives ($Y(t) = 1$), when asset 1 is non-tradeable ($\alpha_1 = 0$), and asset 2 has limited trading per year ($\alpha_2 = 0.1$), $\sigma_j = 0.5$, and $T_i = 1$. Illiquidity cost (IC) is defined as the amount of initial wealth (in basis points) that we would have to give the investor/manager in order to make her indifferent between holding the liquidity constrained and the liquidity unconstrained portfolios. CSV_W denotes the cross-sectional volatility of the simulated value of assets under management, under liquidity constraints, at the terminal date, W(T). The variable $\mathrm{E}[\omega_j^c(T)]$ denotes the expected value of the constrained portfolio weight for asset j, at the terminal date.

ρ	RA(γ_i)	IC(bp)	CSV_W	Asset j	$\omega_j^u(0)$	$\omega_j^c(0)$	$\mathrm{E}[\omega_j^c(T)]$	CSV_j
-0.5	1	84.70	0.3195	1	0.640	0.500	0.500	0.1838
				2	0.640	0.500	0.500	0.1838
	2	52.27	0.1660	1	0.320	0.276	0.279	0.1100
				2	0.320	0.184	0.287	0.1230
	5	28.61	0.0671	1	0.128	0.105	0.110	0.0505
				2	0.128	0.045	0.147	0.0693
	10	23.63	0.0389	1	0.064	0.054	0.057	0.0278
				2	0.064	0.006	0.106	0.0493
0	1	16.18	0.2583	1	0.320	0.285	0.284	0.1039
				2	0.320	0.285	0.376	0.1271
	2	12.45	0.1266	1	0.160	0.150	0.154	0.0641
				2	0.160	0.100	0.200	0.0837
	5	11.58	0.0508	1	0.064	0.059	0.062	0.0291
				2	0.064	0.007	0.107	0.0501
	10	17.70	0.0368	1	0.032	0.030	0.032	0.0157
				2	0.032	0	0.097	0.0435
0.5	1	7.82	0.2132	1	0.213	0.222	0.222	0.0745
				2	0.213	0.148	0.244	0.0842
	2	8.30	0.1027	1	0.107	0.105	0.108	0.0450
				2	0.107	0.045	0.145	0.0606
	5	9.49	0.0479	1	0.043	0.040	0.042	0.0199
				2	0.043	0	0.097	0.0440
	10	19.88	0.0352	1	0.021	0.010	0.011	0.0053
				2	0.021	0	0.098	0.0430

Table 3: Shadow costs of explicit incentives derived from differences in risk appetites between the investor and the money manager, where $T_i = 1$. Shadow cost is defined as the additional percentage of the investor's initial wealth that we would have to give her in order to make her indifferent between delegating the administration of her savings to a money manager, and administering those savings herself. Using standard working practice, these utility losses are computed via taking the ratio of the annualized certainty equivalent rates of return, achieved under the investor's portfolio delegated and centralized problems, after which I subtract one and multiply by 100 to express the losses in percent points per year. Parameter values are as in Table 1.

Panel A: Unconstrained liquidity, T_i=1

		$\gamma_I = 1$		$\gamma_I = 2$		$\gamma_I = 5$		$\gamma_I = 10$	
	ρ	$\sigma_j = 0.3$	0.5	0.3	0.5	0.3	0.5	0.3	0.5
$\gamma_M = 1$	-0.5	0	0	7.78	2.72	64.60	18.40	250.33	55.13
	0	0	0	3.67	1.30	26.20	8.54	82.92	22.96
	0.5	0	0	2.41	0.86	16.47	5.57	46.52	14.54
$\gamma_M = 2$	-0.5	3.51	1.22	0	0	6.79	2.41	27.12	8.71
	0	1.77	0.63	0	0	3.26	1.17	12.12	4.17
	0.5	1.19	0.42	0	0	2.15	0.77	7.83	2.74
$\gamma_M = 5$	-0.5	9.38	3.24	2.55	0.89	0	0	1.46	0.54
	0	4.63	1.63	1.28	0.45	0	0	0.72	0.26
	0.5	3.07	1.09	0.86	0.31	0	0	0.47	0.17
$\gamma_M = 10$	-0.5	12.05	4.14	4.61	1.61	0.70	0.24	0	0
	0	5.90	2.08	2.29	0.81	0.35	0.12	0	0
	0.5	3.91	1.39	1.53	0.54	0.24	0.08	0	0

Panel B: Constrained liquidity, $\alpha_j = 0$, T_i=1

		$\gamma_I = 1$		$\gamma_I = 2$		$\gamma_I = 5$		$\gamma_I = 10$	
	ρ	$\sigma_j = 0.3$	0.5	0.3	0.5	0.3	0.5	0.3	0.5
$\gamma_M = 1$	-0.5	0	0	0	1.62	0.69	11.70	5.50	32.66
	0	0	0	0.07	1.17	4.96	7.06	16.95	16.28
	0.5	0	0	1.13	0.81	10.37	4.62	29.93	10.12
$\gamma_M = 2$	-0.5	0	1.00	0	0	0.69	1.80	5.50	5.82
	0	0.44	0.59	0	0	3.19	1.03	11.46	3.24
	0.5	1.09	0.43	0	0	1.93	0.61	6.59	1.97
$\gamma_M = 5$	-0.5	2.04	2.71	1.35	0.74	0	0	1.21	0.35
	0	3.20	1.54	1.24	0.41	0	0	0.66	0.23
	0.5	2.97	1.06	0.86	0.28	0	0	0.40	0.15
$\gamma_M = 10$	-0.5	4.31	3.46	3.19	1.34	0.04	0.20	0	0
	0	4.48	1.96	2.26	0.74	0.35	0.10	0	0
	0.5	3.79	1.34	1.51	0.51	0.23	0.07	0	0

Table 4: Optimal investment policies and costs of time-varying illiquidity, with no influence of implicit incentives ($Y(t) = 1$), for independent risky assets ($\rho = 0$), where asset 1 is non-tradeable ($\alpha_1(t) = 0$), and asset 2 has limited time-varying trading per year, for a liquidity volatility parameter value of $\sigma_{\alpha_2} = 0.2$, speed of reversion $K_2 = 0.1$, initial level $\psi_2(0) = 0.1$ (annualized), long-run equilibrium level $\theta_2 = 0.1$ (annualized), $\sigma_j = 0.5$, and $T_i = 1$. The parameter λ_{j2} denotes the simulated average time-series correlation between $\omega_j^c(t)$ and the stochastic liquidity boundary $\psi_2(t)$, for $j \in \{1, 2\}$.

Panel A: Stochastic constrained liquidity, $\theta_2 = 0.1$								
RA(γ_i)	IC(bp)	CSV$_W$	Asset j	$\omega_j^u(0)$	$\omega_j^c(0)$	λ_{j2}	E[$\omega_j^c(T)$]	CSV$_j$
1	17.00	0.2580	1	0.320	0.280	-0.3646	0.282	0.1033
			2	0.320	0.280	0.8221	0.377	0.1340
2	13.59	0.1283	1	0.160	0.150	-0.1551	0.154	0.0641
			2	0.160	0.100	0.6649	0.205	0.0930
5	13.46	0.0508	1	0.064	0.054	-0.0556	0.057	0.0271
			2	0.064	0.006	0.2548	0.111	0.0603
10	20.65	0.0386	1	0.032	0.030	-0.0476	0.032	0.0157
			2	0.032	0	0.2276	0.102	0.0525

Panel B: Stochastic constrained liquidity, $\theta_2 = 0.2$								
RA(γ_i)	IC(bp)	CSV$_W$	Asset j	$\omega_j^u(0)$	$\omega_j^c(0)$	λ_{j2}	E[$\omega_j^c(T)$]	CSV$_j$
1	17.18	0.2586	1	0.320	0.280	-0.3452	0.282	0.1034
			2	0.320	0.280	0.9035	0.380	0.1352
2	13.89	0.1289	1	0.160	0.150	-0.1554	0.154	0.0641
			2	0.160	0.100	0.8528	0.208	0.0946
5	13.99	0.0516	1	0.064	0.054	-0.0645	0.057	0.0271
			2	0.064	0.006	0.5012	0.115	0.0622
10	22.06	0.0394	1	0.032	0.030	-0.0572	0.032	0.0157
			2	0.032	0	0.4760	0.105	0.0543

Table 5: Optimal investment policies and illiquidity discounts, under the influence of implicit incentives, for a benchmark with $\beta_j = 0.5$, and independent ($\rho = 0$) non-tradeable ($\alpha_j = 0$) risky assets. Illiquidity discounts are defined as the number of basis points the price of the identical risky assets would have to be reduced in order to make the manager indifferent between holding the liquidity constrained and the liquidity unconstrained portfolios. CSV_j denotes the cross-sectional volatility of the simulated constrained portfolio weights, at the terminal date, $\omega_j^c(T)$, for $j \in \{1, 2\}$.

Panel A: Benchmark with $\beta_j = 0.5$, $\alpha_j = 0$, T_M=1, $\rho = 0$ (CSV_Y=0.404)								
	$\phi(T) = 1/Y(T)$							
RA(γ_M)	$\omega_j^u(0)$	$\omega_j^c(0)$	CSV_j	TE_β	ID(bp)	CSV_W	TE_Y	P[W<Y]
1	0.320	0.305	0.1075	19.71	12.82	0.2543	3.09	0.5173
2	0.410	0.395	0.1289	12.60	39.27	0.3294	2.03	0.5344
5	0.464	0.450	0.1424	10.07	108.95	0.3752	1.56	0.5864
	$\phi(T)$ of the collar-type							
RA(γ_M)	$\omega_j^u(0)$	$\omega_j^c(0)$	CSV_j	TE_β	ID(bp)	CSV_W	TE_Y	P[W<Y]
1	0.455	0.360	0.1206	15.10	429.33	0.3002	2.41	0.5232
2	0.420	0.325	0.1123	17.97	478.06	0.2710	2.83	0.5189
5	0.385	0.315	0.1099	18.83	644.70	0.2627	2.96	0.5185
Panel B: Benchmark with $\beta_j = 0.5$, $\alpha_j = 0$, T_M=2, $\rho = 0$ (CSV_Y=0.655)								
	$\phi(T) = 1/Y(T)$							
RA(γ_M)	$\omega_j^u(0)$	$\omega_j^c(0)$	CSV_j	TE_β	ID(bp)	CSV_W	TE_Y	P[W<Y]
1	0.320	0.290	0.1412	22.01	63.84	0.4049	3.55	0.5264
2	0.410	0.385	0.1685	15.42	156.63	0.5375	2.54	0.5503
5	0.464	0.445	0.1862	13.40	360.01	0.6213	2.12	0.6099
	$\phi(T)$ of the collar-type							
RA(γ_M)	$\omega_j^u(0)$	$\omega_j^c(0)$	CSV_j	TE_β	ID(bp)	CSV_W	TE_Y	P[W<Y]
1	0.455	0.290	0.1412	22.01	617.24	0.4049	3.55	0.5264
2	0.380	0.240	0.1254	26.25	596.62	0.3351	4.17	0.5240
5	0.290	0.230	0.1221	27.13	716.54	0.3211	4.30	0.5240

Table 6: Shadow costs of implicit incentives, with independent ($\rho = 0$) non-tradeable ($\alpha_j = 0$) risky assets, measured in percentage points, for different investment horizons T_i, different benchmark portfolio weights β_j, and different flow-performance specifications $\phi(T)$.

Panel A: Unconstrained liquidity								
	$\phi(T) = 1/Y(T)$				$\phi(T)$ of the collar-type			
	$\beta_j = 0.2$		$\beta_j = 0.5$		$\beta_j = 0.2$		$\beta_j = 0.5$	
RA(γ_i)	$T_i = 1$	2	1	2	$T_i = 1$	2	1	2
1	0	0	0	0	0.20	0.39	0.47	0.94
2	0.51	1.02	3.20	6.50	0.05	0.10	3.46	5.00
5	3.26	6.64	22.25	50.31	1.42	2.87	13.77	13.70
10	8.39	17.78	67.11	188.76	4.50	9.30	25.73	24.54

Panel B: Constrained liquidity, $\alpha_j = 0$								
	$\phi(T) = 1/Y(T)$				$\phi(T)$ of the collar-type			
	$\beta_j = 0.2$		$\beta_j = 0.5$		$\beta_j = 0.2$		$\beta_j = 0.5$	
RA(γ_i)	$T_i = 1$	2	1	2	$T_i = 1$	2	1	2
1	0	0	0	0	0.25	0.40	0.07	0
2	0.44	0.85	3.00	5.82	0.06	0.12	1.50	0.94
5	2.81	4.82	20.32	41.28	1.32	1.64	7.68	6.51
10	6.44	10.71	58.25	136.21	3.90	4.83	19.05	14.66

Table 7: Optimal investment policies and costs of constant illiquidity, under the influence of implicit incentives, for a benchmark with $\beta_j = 0.5$, when asset 1 is non-tradeable ($\alpha_1 = 0$), and asset 2 has limited trading per year ($\alpha_2 = 0.2$), $\sigma_j = 0.5$, and $T_i = 1$. Illiquidity cost (IC) is defined as the additional amount of initial wealth (in basis points) that we would have to give the money manager in order to make her indifferent between holding the liquidity constrained and the liquidity unconstrained portfolios. CSV_Y denotes the cross-sectional volatility of the simulated value of the rebalanced benchmark. These panels consider the case of independent risky assets ($\rho = 0$), which results in $\mathrm{CSV}_Y = 0.4040$. Each panel also includes values for the parameters π and ν, which denote the simulated time-series correlation coefficients of the portfolio liquidity (ratio of $\omega_2^c(t)$ to $\omega_1^c(t)$), and the portfolio risk exposure ($\omega_2^c(t) + \omega_1^c(t)$), respectively, with the ratio of the assets under management to the benchmark portfolio (W(t)/Y(t)).

Panel A: $\phi(T) = 1/Y(T)$, T_i=1

γ_M	IC(bp)	CSV_W	TE_Y	P[W<Y]	j	$\omega_j^u(0)$	$\omega_j^c(0)$	$\mathrm{E}[\omega_j^c(T)]$	CSV_j	TE_β
1	19.32	0.2572	3.22	0.5215	1	0.320	0.306	0.306	0.1073	19.63
					2	0.320	0.204	0.395	0.1409	21.57
2	54.18	0.3318	2.21	0.5367	1	0.410	0.414	0.411	0.1293	11.29
					2	0.410	0.276	0.465	0.1575	16.39
5	138.21	0.4061	1.51	0.6476	1	0.464	0.455	0.457	0.1514	10.28
					2	0.464	0.455	0.539	0.1529	10.28

	$\gamma_M = 1$	2	5	10
π	-0.0660	-0.1923	0.1626	0.1376
ν	-0.3804	-0.2620	-0.2571	-0.2507

Panel B: $\phi(T)$ of the collar-type, T_i=1

γ_M	IC(bp)	CSV_W	TE_Y	P[W<Y]	j	$\omega_j^u(0)$	$\omega_j^c(0)$	$\mathrm{E}[\omega_j^c(T)]$	CSV_j	TE_β
1	471.25	0.2945	2.68	0.5258	1	0.455	0.360	0.358	0.1188	15.07
					2	0.455	0.240	0.430	0.1498	18.82
2	552.34	0.2572	3.22	0.5215	1	0.420	0.306	0.306	0.1073	19.63
					2	0.420	0.204	0.395	0.1409	21.57
5	826.08	0.2475	3.36	0.5207	1	0.385	0.294	0.295	0.1044	20.69
					2	0.385	0.196	0.382	0.1374	22.30

	$\gamma_M = 1$	2	5	10
π	-0.1180	-0.0660	-0.0525	-0.0606
ν	-0.3316	-0.3804	-0.3908	-0.4060

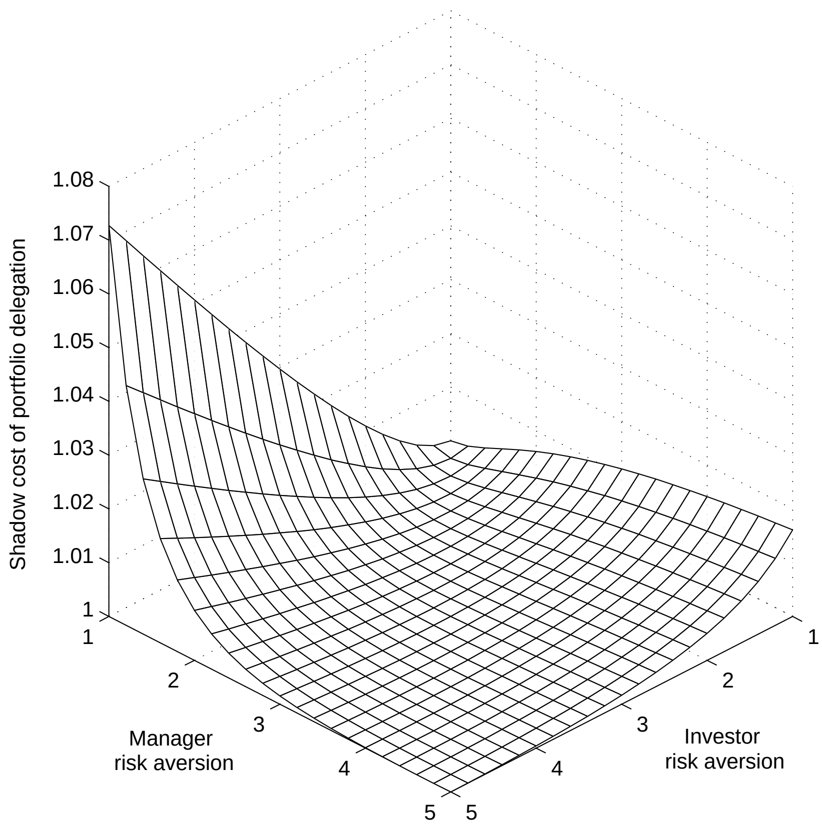

Figure 1: Shadow cost derived from explicit incentives only, due to differences in risk appetites between the investor and the money manager. These costs are measured as factors we would have to multiply the investor's initial wealth with, in order to compensate her for the effect of suboptimal policies derived from portfolio delegation. This is also the case of identical, independent, and non-tradeable ($\alpha_j = 0$) risky assets, where $r = 0.02$, $\mu_j = 0.10$, and $\sigma_j = 0.5$, for $j \in \{1, 2\}$, and $T_i = 1$.

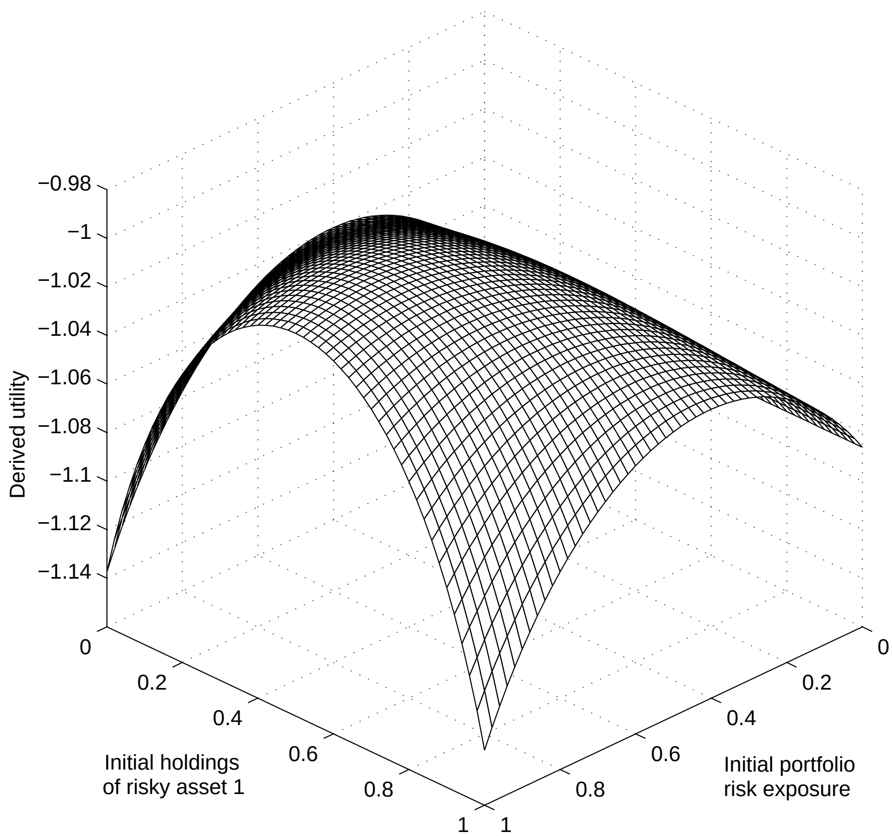

Figure 2: Derived utility of terminal wealth function, for a CRRA manager, with $\gamma_M = 2$, whose performance is measured relative to a benchmark, with $\beta_j = 0.5$, and where the flow-performance specification is given by: $\phi(T) = 1/Y(T)$. This manager chooses, at time $t = 0$, to hold $\omega_1(0)$ and $\omega_2(0)$, on risky assets 1 and 2, respectively, and these initial allocations cannot be revised for $t > 0$ ($\alpha_j = 0$). The values on the axis for initial holdings of risky asset 1 are fractions of the initial total portfolio risky exposure ($\omega_1(0)+\omega_2(0)$). These are identical and independent risky assets, where $r = 0.02$, $\mu_j = 0.10$, and $\sigma_j = 0.5$, for $j \in \{1, 2\}$.

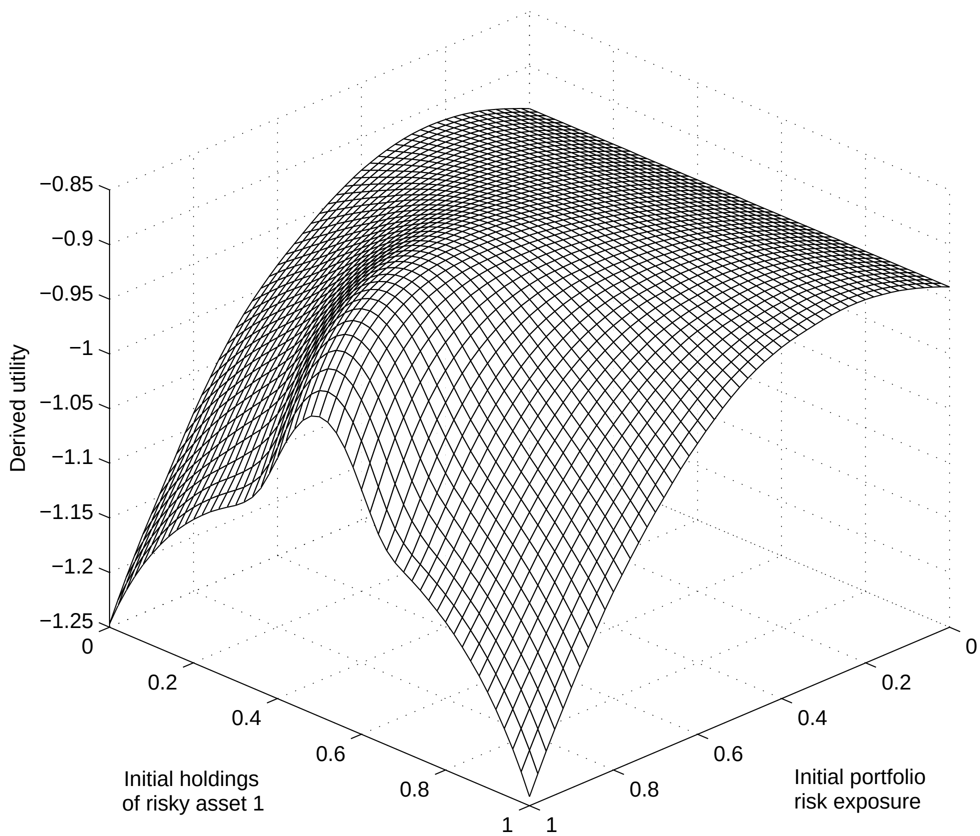

Figure 3: Derived utility of terminal wealth function, for a CRRA manager, with $\gamma_M = 2$, whose performance is measured relative to a benchmark, with $\beta_j = 0.5$, and where the flow-performance specification, $\phi(T)$, is of the collar-type. This manager chooses, at time $t = 0$, to hold $\omega_1(0)$ and $\omega_2(0)$, on risky assets 1 and 2, respectively, and these initial allocations cannot be revised for $t > 0$ ($\alpha_j = 0$). The values on the axis for initial holdings of risky asset 1 are fractions of the initial total portfolio risky exposure ($\omega_1(0)+\omega_2(0)$). These are identical and independent risky assets, where $r = 0.02$, $\mu_j = 0.10$, and $\sigma_j = 0.5$, for $j \in \{1, 2\}$.

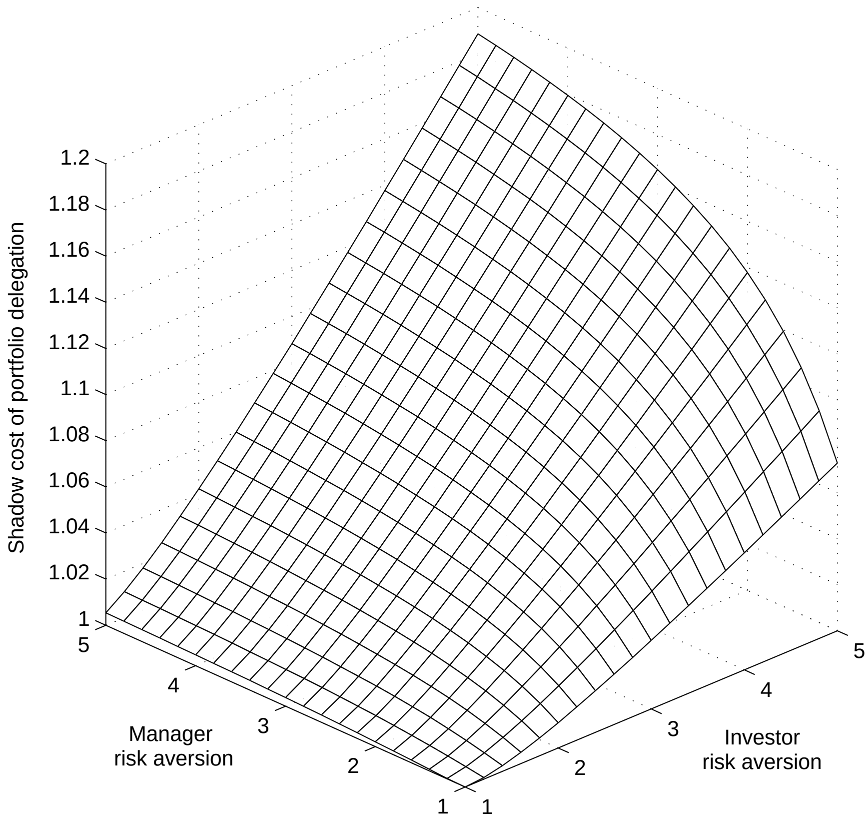

Figure 4: Shadow cost (vertical axis) derived from explicit and implicit incentives, together, due to portfolio delegation, as a function of the risk aversion parameters for the investor and the money manager (horizontal axes). This is the case of a manager whose performance is measured relative to a benchmark $Y(t)$, with $\beta_j = 0.5$, and where the fund flow-performance specification is given by: $\phi(T) = 1/Y(T)$. These costs are measured as factors we would have to multiply the investor's initial wealth with, in order to compensate her for the effect of suboptimal policies derived from portfolio delegation. This is also the case of identical, independent, and non-tradeable ($\alpha_j = 0$) risky assets, where $r = 0.02$, $\mu_j = 0.10$, and $\sigma_j = 0.5$, for $j \in \{1, 2\}$, and $T_i = 1$.