Riding Bubbles *

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Abstract

We empirically analyze a rational investor's optimal response to asset price bubbles. Our empirical approach provides new insights since theoretical predictions vary between going short, sidelining, and riding bubbles. We propose a new bubble detection method that does not require hindsight information and accounts for uncertainty. It identifies bubbles as a sudden acceleration of price growth beyond the growth in fundamental value, as predicted by conventional asset pricing models. As reference assets, we investigate US industry portfolios. Consistent with the theory of Abreu and Brunnermeier (2003), riding bubbles is optimal. An investor who rides bubbles can earn average abnormal returns of 4% to 8% annually. These high returns come at the expense of a higher risk. Still, upon bubble detection the optimal weight of a power utility investor increases significantly. The portfolio adjustment corresponds to an additional risk-free return of about 3% per year.

Key words: bubbles, crashes, arbitrage, structural breaks
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1 Introduction

Since the early days of financial markets, investors have witnessed periods of bubbles and subsequent crashes. Famous examples include South Sea Bubble, the Roaring Twenties, and the internet bubble. There were also less well known bubbles, often limited to a specific groups of stocks such as the "tronics boom", the conglomerate boom or the concept stock bubble described by Malkiel (1996).

Despite the fact that bubbles have occurred for centuries, we do not know the optimal strategy of a rational investor when she faces an asset bubble. The theoretical literature makes contradictory propositions. The efficient market hypothesis predicts that rational investors go short and "cause these "bubbles" to burst"(see, Fama, 1965, p. 38). The limits to arbitrage literature, for example De Long et al. (1990a), Dow and Gorton (1994), Shleifer and Vishny (1997), posits that rational investors should not actively trade against the mispricing (hereafter: "sideline"). De Long et al. (1990b) and Abreu and Brunnermeier (2003) propose that investors should actively increase their holdings upon the detection of a bubble (i.e., "ride the bubble"). The current empirical literature (e.g. (e.g., Brunnermeier and Nagel, 2004; Griffin et al., 2006; Temin and Voth, 2004; Greenwood and Nagel, 2007; Dass et al., 2008) focuses on describing the trading strategies of different investors during historical bubbles.

We take a normative perspective and analyze empirically which strategy is optimal for a rational investor. Our empirical analysis allows us to refrain from specific assumptions underlying theoretical models. Central to our approach is a new bubble identification method that is applicable to a real-world setting. It uses only a basic information set, does not require hindsight information and allows for uncertainty. Since historical periods of bubbles often started in specific industries (e.g., the railway boom, the electricity boom and the internet bubble), our analysis is based on the sample of 48 US industries from Fama and French (1997).

The bubble definition relies upon two main characteristics of asset price bubbles, described for example by Abreu and Brunnermeier (2003): (1) the growth rate of the price is higher than the growth rate of fundamental value and (2) the growth rate of the price experiences a sudden acceleration. The investor concludes that she discovered a bubble if both conditions are fulfilled. A bubble has ended, when she observes a crash during the previous 6 months. She estimates the growth rate of fundamental value based on one of three different asset pricing models, the Capital Asset Pricing Model (CAPM), the Fama-French (1993) three-factor model (3F-model), and the Carhart (1997) four-factor model (4F-model). To detect a sudden acceleration, she conducts a structural change test as in Andrews (1993). The bubbles we identify are a different phenomenon than industry momentum described by Moskowitz and Grinblatt (1999). They occur at different time horizons (for details see section 3) and our results are robust to the inclusion of a momentum factor. We do not use bubble identification methods such as proposed by Campbell and Shiller (1987) which require a long time-series of prices and dividends. Instead, we only use a basic information set that was available at the respective point in time. Just like a "real" investor, we can therefore not faultlessly identify a bubble.

We find a bubble for 35 out of 846 months on average per industry which constitutes less than 5% of our observations. The distribution of returns after the detection of a bubble differs substantially from the return distribution if no bubble was identified. First, the abnormal returns following bubbles are on average significantly higher than if no bubble was detected. The difference in monthly returns ranges from 0.34% (4F-model), 0.39% (3F-model) to 0.77% (CAPM). Second, we find significant increases for various risk measures. Volatility shows a relative increase of about 15% (from around 4% to 4.60%). Downside risk measures like Value-at-Risk and Expected Shortfall go up as well, indicating the presence of a fat left tail.

To find out whether an investor should ride, sideline or short the asset bubble, we investigate the asset allocation implications of our findings. To isolate the effect of bubbles we assume that all factor exposures are hedged. The investor can allocate his portfolio among two assets, a risky one, which can experience bubbles, and a risk-free asset. An investor with a power-utility function substantially increases the weight allocated to the risky asset if she learn about a bubble. This increase is statistically and economically significant with portfolio weights (as a fraction of wealth) going up by 0.64 (4F-model), 0.91 (3F-model) and 1.61 (CAPM). To determine the economic value of riding bubbles, we determine the strategy's certainty equivalent return. We compute the risk-free return an investor would require as compensation for not increasing the weight upon bubble detection.

On a yearly basis, these compensations add up to 1.12% (4F-model), 2.11% (3F-model) and a stunning 7.04% (CAPM). The relatively extreme estimate for the CAPM may be due to its low explanatory power for industry returns.

Our findings point out that riding bubbles is an attractive strategy for investors, even if they have only information on past returns. This conclusion confirms the theoretical predictions made by Abreu and Brunnermeier (2003) and De Long et al. (1990b). Our findings are neither limited to a few famous cases, nor to very sophisticated investors. As such, we generalize the finding of Temin and Voth (2004), who show that Hoare's Bank was riding the South Sea Bubble, and the evidence by Brunnermeier and Nagel (2004) that a number of hedge funds were riding the technology bubble in the late 90s.

We proceed our analysis as follows. First, we give a short introduction to the literature and describe our data set. In the fourth section, we explain how we identify bubbles. We analyze the relation between the detection of a bubble and future abnormal returns in section 5. In section 6, we analyze how a power-utility investor would allocate her portfolio based on our findings. Section 7 investigates the robustness of our results for different settings in our analysis. Section 8 concludes.

2 Literature Review

Theoretical studies have yet to reach consensus on a rational investor's optimal response to asset price bubbles. From an efficient market perspective, arbitrage by sophisticated traders precludes the existence of bubbles. Fama (1965, p.38) states "If there are many sophisticated traders in the market, however, they may cause these bubbles to burst before they have a chance to really get under way".

Despite the fact that bubbles can theoretically be ruled out by arbitrage, we believe to observe them. To overcome this discrepancy between theory and reality, a second line of research explains why arbitrageurs do not trade against mispricing. The central idea, pointed out by De Long et al. (1990a), is that noise traders can cause prices to divert even further from fundamental value. The risk that the price of the asset does not return to fundamental value, or that the gap widens, outweighs the potential gain from arbitrage trading. De Long et al. (1990a) show that even in the absence of any fundamental risk, it is optimal for risk-averse arbitrageurs with finite horizons to refrain from trading against the mispricing. For assets with fundamental risk, such as equity, not even arbitrageurs with long horizons might be able to profit from trading against the mispricing. Several authors have extended De Long et al. (1990a)'s analysis. Dow and Gorton (1994) introduce transaction costs and show that it is only profitable for an informed trader to act upon his information if arbitrage chains are unbroken. Shleifer and Vishny (1997) show that arbitrageurs, who trade on behalf of uninformed clients, face capital constraints and limited horizons. Therefore, they do not trade against mispricing in the presence of noise-trader risk. Goldman and Slezak (2003) arrive at similar conclusions for mutual fund managers, who inherit portfolios and manage them for a limited time period.

A third line of theoretical research, introduced by De Long et al. (1990b) predicts that rational arbitrageurs actively invest in the asset price bubble. They do not only refrain from trading against the bubble, but instead drive prices further away from fundamental value. Abreu and Brunnermeier (2003) develop a model showing that arbitrageurs have an incentive to ride bubbles in the presence of synchronization risk. If each arbitrageur on her own is not able to burst the bubble and concludes that other arbitrageurs are unlikely to attack it, her optimal choice is to profit from the noise traders by riding the bubble.

To date, only few empirical papers provide, directly or indirectly, evidence in favor or against the different theoretical propositions. The two most closely related papers are Temin and Voth (2004) and Brunnermeier and Nagel (2004). Temin and Voth (2004) document that a very sophisticated investor, Hoare's Bank was well-aware of the South Sea Bubble. The bank actively invested in and profited from the bubble. Brunnermeier and Nagel (2004) show that hedge funds were actively investing in tech stocks during the recent internet bubble. Furthermore, hedge fund managers were able to time the crash and therefore profited substantially from riding the bubble. These two papers support the proposition of Abreu and Brunnermeier (2003) that investors ride bubbles and that it is profitable to do so. Griffin et al. (2006) analyzes the trading behavior of different types of investors during the tech bubble by comparing the direction of the trade to contemporaneous and lagged returns. They classify investors depending on the the brokerage house via which the trade is executed. Their findings suggest that institutional investors, hedge funds and day traders were fueling the bubble and causing its burst. These groups were trading in the same direction as contemporaneous market movements. However, hedge funds and day traders traded contrarian relative to lagged market returns indicating that they were not consistently riding the bubble. The trading behavior of other individual investors as well as derivative traders was overall more contrarian. Based on the results from Griffin et al. (2006), it seems that some investors were riding the bubble while others were actively trading against it.

A couple of studies analyze the behavior of mutual fund managers during the internet bubble. Although these studies do not directly aim at providing evidence in favor or against the different hypotheses outlined above, they still provide interesting insights. Greenwood and Nagel (2007) document that young mutual fund managers were overweighing technology stocks compared to their benchmark as long as the tech market was rising. Old mutual fund managers did not exhibit this trend-chasing behavior. However, unlike hedge fund managers, young mutual fund managers were not successful at predicting the downturn. Dass et al. (2008) analyze the impact of incentive contracts on mutual fund manager's investment decisions during the bubble. Advisory contracts with high performance-based incentives induce managers to divert from the herd and not ride the bubble. The prospect of receiving a high pay-off by being the best outweighs the potential reputation loss of being worse than the average. In line with this result, they also find that managers with low incentive contracts outperformed managers with high incentive contracts during the bubble. However, again it seems that the mutual fund managers were not able to time the burst of the bubble. In the period 2001-2003, the low incentive managers substantially underperformed high incentive managers. Overall, mutual fund managers' investment strategies were more heterogenous than the strategies of hedge fund managers: certain groups were riding the bubble, while others refrained from it. The fund managers that were riding the bubble profited from it as long as the bubble lasted. They incurred large losses when the bubble burst.

The empirical studies provide mixed evidence on the behavior of different types of investors. Based on Griffin et al. (2006), it seems that institutional investors were riding the bubble, but individual investors, except day traders, refrained. However, Greenwood and Nagel (2007) and Dass et al. (2008) document that some mutual funds were riding the bubble while others did not. Brunnermeier and Nagel (2004) document that hedge funds consistently increased their holdings until the peak of the bubble, while Griffin et al. (2006) provide evidence that they traded contrarian relative to the previous day's return.

In contrast to existing empirical studies, we systematically identify bubbles and analyze the optimal strategy for any rational investor. Therefore, our findings are not limited to a specific type of investor nor to a specific bubble period. This systematic approach allows to provide empirical evidence on the contradictory theoretical propositions and shed light on the mixed empirical evidence.

3 Data Description

Throughout our analysis, we use the 48 industry indexes constructed by Fama and French (1997). The data set consists of monthly value-weighted returns from July 1926 to December 2006. Value-weighted returns resemble the portfolio returns earned by real-world investors and ensure that small-caps are not dominating our findings. Table 1 provides descriptive statistics on the industry returns. The average return on the industries is 12.9% per annum and their standard deviation is 26%. The minimum and maximum values indicate that several industries experienced extreme returns. We investigate this issue further and find that most of these extreme values occur during the Great Depression at the beginning of our sample period.

[Table 1 about here.]

In the CAPM estimation, we measure the risk-free rate and the market index by the one-month Treasury bill rate from Ibbotson Associates and the CRSP all share index, respectively. For the Fama and French (1993) model we additionally include the factor portfolios "High-Minus-Low" (HML) and "Small-Minus-Big" (SMB). In the Carhart (1997) model we add "Momentum" (Mom) as a fourth factor. We obtain the data from French's website[1].

[1]The data can be downloaded from `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`. We have used the set of industry returns constructed with the new specifications.

4 Defining Bubbles

As pointed out by Cochrane (2001), "The word "bubble" is widely used to mean very different things...". Although the exact specification of asset bubbles differs among theoretical models, two prominent characteristics of bubbles are common to many models as well as anecdotal descriptions. First, ranging back as far as the origins of the rational bubble literature (see Blanchard and Watson, 1982) asset bubbles are characterized by a price that grows faster than the fundamental value of the asset. Second, a bubble represents a sudden acceleration in the growth rate of the price. Abreu and Brunnermeier (2003) use exactly these two characteristics to define bubbles. Kindleberger (2000, p. 14) characterizes a bubble by a displacement in a Minsky model and Shiller (2000) relates bubbles to "new economy thinking".

In theoretical models an investor either receives a signal that the asset experiences a bubble or simply knows that all along (see e.g., Abreu and Brunnermeier, 2003; De Long et al., 1990b). As our purpose is to investigate the different theoretical predictions in a real-world setting, we require that the investor infers bubbles from publicly available information. Our conclusions should be generalizable to many investors and not limited to few, very sophisticated arbitrageurs, who can afford to spend a substantial amount of time and money on acquiring a superior information set. We therefore restrict the information set to past prices. The investor cannot identify bubbles with certainty: Sometimes she might miss a bubble and at other times she mistakenly believes that there is a bubble. Our real-time bubble identification method introduces noise into our analysis and thereby weakens our findings. We believe that it is realistic as investors face similar uncertainties. The still continuing debate on whether the internet bubble was actually a bubble or whether the high prices were justified by fundamentals (see for example Pástor and Veronesi, 2004) illustrates that one cannot even with hindsight identify bubbles with certainty.

Our bubble definition is designed to capture the two basic characteristics of bubbles outlined in the literature while only using past price information. To adjust returns for the growth rate of fundamental value, we consider three different asset pricing models: the CAPM, the Fama and French (1993) model or the Carhart (1997) model. To identify a sudden acceleration in price growth, we test for a positive structural break in returns which is not explained by the asset pricing models. In addition, to capture the idea that the asset price grows faster than fundamental value, we require significantly positive anomalous returns following the break. Unlike the cointegration approach put forward by Campbell and Shiller (1987) or the regime-switching model proposed by Brooks and Katsaris (2005), this bubble identification method allows us to use a limited history of past price information, which we believe is also available to investors. We also do not need to make any assumptions on investors' expectations of future earnings.

Formally, we investigate whether an asset experiences a bubble at time t by estimating the following model:

$$r_\tau = \alpha_\tau + \boldsymbol{\beta}'\boldsymbol{f}_\tau + \varepsilon_\tau,\ \mathrm{E}[\varepsilon_\tau] = 0,\ \mathrm{E}[\varepsilon_\tau^2] = \sigma^2, \quad \tau = t - T + 1, \ldots, t \tag{1}$$

where r_τ is the asset's excess return and T is the estimation window, which typically equals 120 months. The vector $\boldsymbol{f}_\tau$ represents the set of risk factors. In our first specification, we estimate the CAPM developed by Sharpe (1964) and Lintner (1965). Second, we augment the model by the Fama and French (1993) factors SMB and HML, resulting in a three-factor model (3F-model). Third, we estimate the Carhart (1997) four-factor model (4F-model), which additionally includes a momentum factor.

Our test procedure concentrates on α_τ. To capture the two basic characteristics of a bubble we interpret a bubble (i) as a structural break in α_τ, (ii) after which α_τ is significantly positive. Our setup closely follows the structural break literature (see Andrews, 1993; Hansen, 2001) and the null hypothesis of no bubbles implies that α_τ does not change significantly during the test period:

$$H_0 : \alpha_\tau = \alpha_0 \quad \text{for all } \tau. \tag{2}$$

The alternative is that we observe a structural break in α_τ. As we have no a priori expectations of when a bubble starts, we test for different breakpoints. Since we are interested in recent bubbles, we require that the bubble lasts until time t. In addition, we require a bubble to be a prolonged acceleration in price growth and set its minimum length to twelve months. Its maximum length is five years. Formally, the alternative hypothesis reads:

$$H_{1T}(\zeta) : \alpha_\tau = \begin{cases} \alpha_1(\zeta) & \text{for } \tau = t - T + 1, \ldots, t - \zeta \\ \alpha_2(\zeta) & \text{for } \tau = t - \zeta + 1, \ldots, t, \end{cases} \tag{3}$$

with $\alpha_2(\zeta) > \alpha_1(\zeta)$, where ζ ranges from 12 to 60, $\alpha_1(\zeta)$ refers to the first part of our test period and $\alpha_2(\zeta)$ to the second part. For each value of ζ we calculate the Wald test-statistic for the hypothesis $\alpha_1(\zeta) = \alpha_2(\zeta)$. We select the breakpoint ζ with the largest test statistic and determine the critical value for it based on the tables in Andrews (1993).[2] If we reject H_0 in favor of $\alpha_2(\zeta)$ being significantly larger than $\alpha_1(\zeta)$, we subsequently test whether $\alpha_2(\zeta)$ is significantly larger than zero. If both criteria are fulfilled, we conclude that the asset experiences a bubble. We consider low (95%), medium (97.5%) and high (99%) confidence levels to determine the critical values. We present our results for the medium level in the following sections, and consider the other levels in Section 7.2. Every time we detect a bubble, we count the number of months since inception and accordingly define the length of the bubble. In addition, we define the strength of the bubble as the t-statistic of $\alpha_2(\zeta)$.

Finally, the investor has to determine whether the bubble has not ended recently. In many models, ranging from Blanchard and Watson (1982) to Abreu and Brunnermeier (2003), bubbles end with a crash. Therefore, we take a crash in the innovations ε_τ during the last κ months as a signal that the bubble has ended. We define a crash as a value of ε_τ below a threshold, a multiple k of the standard deviation σ. Typically we take κ equal to 6 months, and the threshold multiple equal to -2. As both choices are arbitrary, we conduct robustness checks but find that our results are only marginally affected (results are available from the authors on request). If bubbles do not end with a crash, but deflate, the estimate for α_2 will decrease over time. If the deflation is strong enough, α_2 will not pass the second test. In that case the investor will simply not detect the bubble anymore.

[2] Andrews (1993) actually discusses two-sided tests for the detection of a structural break, whereas our alternative hypothesis is one-sided. Estrella and Rodriguez (2005) derive the corresponding asymptotic distribution of the Wald-statistic. They show in their Figure 1 that the common approach of halving the p-value for a given critical value when moving from a two-sided to a one-sided alternative gives a good approximation. Therefore, we use the tables provided by Andrews (1993).

Figure 1(a) shows the number of bubbles per industry. It is computed as the unbroken sequence of months a bubble is detected. On average, each industry experiences about two bubbles. We find slightly less bubbles for the 3F-model and 4F-model than for the CAPM. Figure 1(b) shows the distribution of bubble detections across industries. A detection is counted every time we identify a bubble. Thus, "one" bubble can be detected many times. Both, the number of bubbles as well as the number of bubble detections are well-spread across industries and we can conclude that our findings will not be driven by a specific bubble or industry.

[Figure 1 about here.]

To investigate whether our detection method discovers economically meaningful bubbles, we compute standardized abnormal returns as well as "raw" returns during bubbles. For the abnormal returns, we use the same factor models as in the bubble identification. We estimate the respective model over the previous 120 months to compute the abnormal return for the following month according to:

$$\eta_{t+1} = r_{t+1} - \boldsymbol{\beta}' \boldsymbol{f}_{t+1}, \tag{4}$$

where the r_{t+1} is the excess return at $t + 1$, and $\boldsymbol{\beta}'$ is a vector of estimates based on the regression in equation (1) under the null hypothesis. To accommodate time-varying volatilities and different volatilities across industries, we standardize the abnormal returns by dividing them by the residual standard deviation: $\tilde{\eta}_{i,t} \equiv \eta_{i,t}/\sigma_{i,t}$. Tables 11 to 13 in the Appendix A provide summary statistics of the standardized abnormal returns for each of the three different factor models per industry. Overall, we find that the standardized abnormal returns are on average close to zero. The volatility differs statistically significantly from one. However, the deviations are economically small ranging from 0.04 for the CAPM to 0.08 for the 4F-model.

Although the average abnormal returns for the complete sample are close to zero, the raw and abnormal returns during bubbles are economically large as shown in Table 2. The standardized abnormal returns during bubbles for the different models are 0.47, 0.49 and 0.51 per month for the CAPM, the 3F-model and the 4F-model, respectively. Assuming an average idiosyncratic return volatility of 4%[3], these returns translate into annual abnormal returns of about 23%. We also find that the residual volatility of the standardized abnormal returns is substantially larger than one. These findings indicate that we indeed observe economically meaningful deviations from the null hypothesis. Although our bubble identification method is inherently based on abnormal returns, we have for illustrative purposes computed the raw returns during bubbles as well. The raw return of the average industry during a bubble is 30.6% per year when we apply the CAPM or the 4F-model, and 32.4% when using the 3F-model.

[Table 2 about here.]

Just like the returns, we find that the average strength varies only slightly across models ranging from 3.22 for the CAPM to 3.08 for the 3F-model and 3.07 for the 4F-model. The average bubble length per detection is about 2.5 to 3 years depending on the asset pricing model. In Appendix A, Tables 8 to 10, we provide detailed information on the bubble statistics per industry.

One can clearly distinguish between the bubbles we find and several well-documented anomalies such as price and earnings momentum and drift following news. First, the momentum factor we use should account at least for the typical stock momentum reported by Jegadeesh and Titman (1993). Second, the reported anomalies generally occur at different horizons than the bubbles we find. The average length of the bubbles we detect is about 30 months. At this time interval, buy-side industry momentum, as reported by Moskowitz and Grinblatt (1999), actually reverses and profits become negative. This finding is in line with Fama and French (1988), who report a negative autocorrelation for industry returns at similar horizons. Similarly, although earnings momentum does not reverse, returns to the high and low momentum portfolio are effectively indistinguishable already after 2 years (Chan et al. (1996)). The stock return reaction to positive news is limited to a couple of months (Chan (2003)).

[3]The average idiosyncratic volatility ranges from 4.14% for the CAPM to 3.83% for the 3F-model and 3.79% for the 4F-model.

5 Risk-Return Implications of Bubbles

To examine the profitability of riding bubbles, we compare the abnormal returns in month $t + 1$, given that the investor has detected a bubble up to month t, to the abnormal returns if no bubble has been detected. Table 3, Panel (a) shows the characteristics of return distribution if the bubble detection (equation 1) and abnormal return construction (equation 4) is based on the CAPM. Panel (b) and panel (c) show the results for the 3F-model and 4F-model, respectively. Throughout all specifications, we observe significantly positive abnormal returns after the detection of a bubble as indicated by the confidence intervals.[4] The returns are also economically large: Assuming an idiosyncratic return volatility of about 4%, the mean abnormal returns range from 0.77% ($0.19 \times 4.14\%$) per month if we base our estimate on the CAPM to 0.34% for the 4F-model. The estimates of median abnormal returns are somewhat smaller ranging from 0.58% per month for the CAPM to 0.23% per month for the 3F-model. If no bubble was detected, which is obviously the case for most of our sample period, the abnormal returns are close to zero, or even negative. Again we observe that the median is smaller the the average. The p-values in the final column of each panel indicate that the differences in abnormal returns depending on the presence or absence of a bubble are significantly different below the 1% (CAPM and 3F-model) and 5% (4F-model) significance level. On an annual basis, the return differentials ranges from 3.7% for the 4F-model to 8.8% for the CAPM. To ensure that our findings are not driven by a specific bubble or a specific industry, Table 14 in Appendix A shows the abnormal returns for each of the factor models per industry.

[Table 3 about here.]

The evidence so far supports the idea that riding bubbles is a highly profitable strategy. However, the different risk measures presented in Table 3 indicate that it is also a risky strategy. For all three asset pricing models, we find that the abnormal return volatility following a bubble is significantly larger than if no bubble was detected. Furthermore, especially for the abnormal returns based on the 3F-model and the 4F-model, we observe significant differences in downside risk. For example, assuming again an idiosyncratic

[4]We use 10,000 bootstraps to construct confidence intervals and conduct tests.

volatility of about 4%, the Value-at-Risk (Expected Shortfall) at the 95% level based on the 4F-model is 7.31% (9.94%) if we detected a bubble versus 6.28% (8.65%) if there was no bubble. When we compute abnormal returns based on the CAPM or the 3F-model, we observe – in contrast to Chen et al. (2001) – that skewness is larger after the detection of a bubble. Once we account for return momentum, the relation between skewness and bubbles diminishes.

To get a better impression of the complete distribution of abnormal returns, Figures 2(a), 2(c) and 2(e) plot the distribution of abnormal returns based on the three different asset pricing models with and without bubble detection. An investor riding bubbles does not only face the upside potential of positive abnormal returns but also the downside risk of extremely negative returns. In line with the positive abnormal returns we document above, the distribution after the detection of a bubble is shifted to the right. However, especially for two standard deviations below zero and lower, we observe that the return distribution after bubbles has a prominent left tail (see also Figures 2(b), 2(d) and 2(f)). These relatively rare but extremely negative returns suggest that bubbles are followed by crashes. We therefore investigate the risk-return trade-off in more detail in the next section.

[Figure 2 about here.]

The evidence of large positive abnormal returns is consistent with the theoretical propositions of Abreu and Brunnermeier (2003) that investors have a strong incentive to ride bubbles. Given our definition of bubbles, we can conclude that this finding does not only apply to very sophisticated arbitrageurs, who have a special information set, but is generalizable to many investors.

6 The optimal asset allocation

This section analyzes how a power-utility investor would adjust her asset allocation if she learns about a bubble. It provides an answer to whether the investor would short, sideline or ride the bubble. We assume that the investor can allocate her wealth W at t between a single risky asset, which is represented by the typical industry, and a risk-free asset. If the weight allocated to the risky asset (w) increases significantly upon the detection of a bubble, we conclude that riding bubbles is the optimal response. If learning about a bubble does not lead to a significant change in the optimal weight, our results support the predictions of the limits-to-arbitrage literature and sidelining is the optimal strategy. A significant decrease in optimal weight upon bubble detection indicates that short-selling is optimal. Short-selling can already be ruled out as the abnormal returns upon bubble detection are significantly higher than if no bubble was detected.

In line with our bubble definition, we derive the optimal asset allocation in terms of abnormal returns. This approach allows us to isolate the impact of the bubble detection on the optimal weight. The risky asset earns a return r_i and the return on the risk-free asset is denoted as r_{f}. We define the portfolio return r_{p} as:

$$r_{\mathrm{p}} = wr_i - w\boldsymbol{\beta}_i'\boldsymbol{f} + (1-w)r_{\mathrm{f}} = r_{\mathrm{f}} + w\eta_i \tag{5}$$

where w is the fraction of wealth invested in the risky asset and η_i is the abnormal return of the risky asset.

The investor's preferences over wealth are described by a power utility function:

$$U(W) = \frac{W^{1-\gamma}-1}{1-\gamma} \quad \gamma \neq 1, W \leq 0 \tag{6}$$

where γ is the coefficient of risk aversion. For $W \leq 0$, the investor is bankrupt and we set utility to $-\infty$.

Based on equation (5), next period's wealth is given by $W_{t+1} = W_t(1 + r_{\mathrm{f}} + w\eta_i)$ and the investor's problem is to solve:

$$\max_w \mathrm{E}_t \left[\frac{\left(W_t \left(1 + r_{\mathrm{f}} + w\eta_i\right)\right)^{(1-\gamma)}}{1-\gamma} \right] \tag{7}$$

which results in the first order condition

$$\mathrm{E}\left[(1 + r_{\mathrm{f}} + w^*\eta_i)^{-\gamma}\eta_i\right] = 0. \tag{8}$$

We refrain from making assumptions on the distribution of abnormal returns η and use its empirical distribution. The solution of Equation (8) is therefore based on numerical approximations.

We also compare the expected utility in the presence and absence of a bubble, i.e. $V_{\mathrm{B}} \equiv \max \mathrm{E}[U(1+r_F+w\eta_i)|I_{\mathrm{B}}]$ and $V_{\mathrm{NB}} \equiv \max \mathrm{E}[U(1+r_f+w\eta_i)|I_{\mathrm{NB}}]$. We use I to denote the information set, with I_{B} (I_{NB}) indicating that a bubble has (not) been detected. If riding bubbles is the optimal strategy, expected utility should be significantly higher upon bubble detection than if no bubble was detected. If sideling is optimal, the expected utility of both scenarios should be about equal.

To determine the economic significance of trading on bubble information, we compute the certainty equivalent. Specifically, we calculate the risk-free return that the investor would require as a compensation for not changing her portfolio upon bubble detection. The optimal weight with(out) prior bubble detection is denoted as w_{B} (w_{NB}). The certainty equivalent return λ satisfies the condition

$$\mathrm{E}[U(1 + r_{\mathrm{f}} + w_{\mathrm{B}}\eta_i)|I_{\mathrm{B}}] = \mathrm{E}[U(1 + r_{\mathrm{f}} + w_{\mathrm{NB}}\eta_i + \lambda)|I_{\mathrm{B}}] \tag{9}$$

and we approximate λ numerically.

Table 4 presents the results. We put the idiosyncratic volatility of the industry equal to the pooled averages of the respective models, and we use the long run average of the risk-free rate of 30.5 basis points per year (3.67% per year). The first columns show that the optimal weight assigned to the risky asset increases substantially upon bubble detection. For an investor with a moderate risk aversion level of 2, the optimal weight increases from 0.10 to 1.71 for the abnormal returns based on the CAPM. For 3F-model and 4F-model the optimal weight rises from 0.00 to 0.91, and from 0.11 to 0.74, respectively. The standard errors (shown in parentheses) indicate that the weight allocated to the risky asset in the absence of a bubble is indistinguishable from zero. Upon the detection of a bubble, it becomes statistically significantly different from zero. The p-values in column 4 reveal that the difference in weight is statistically significant at the 5% level for all specifications. These results show that riding bubbles is the optimal strategy.

[Table 4 about here.]

We also investigate how the change in optimal weight is affected by different risk aversion levels and find an inverse relation. This finding is intuitively appealing as the risk measures in section 5 consistently indicated a higher risk after bubble detection. It is comforting to observe that even for rather high levels of risk aversion, the difference in portfolio weights is economically significant. Even for a risk aversion level of 10, our most conservative estimate (which is in this case based on the 4F-model) indicates an increase in the optimal weight of 13%. Using the approximation $\mathrm{E}[(1+r_f+w^*\eta_i)^{-\gamma}\eta_i] \approx \mathrm{E}[\exp(-\gamma(r_f+w^*\eta_i))\eta_i]$ for the first order condition in Equation (8) illustrates that a multiplication of γ with a constant should lead to division of w^* with about the same factor to maintain optimality. It also explains why the p-values do not depend on the level of risk aversion. To test $w_{\mathrm{B}} = w_{\mathrm{NB}}$, we construct a series of portfolio differences, based on a bootstrap. Since both weights are inversely related to risk aversion, the same holds for the difference. Risk aversion therefore only has an impact on the scale of the distribution of weight differences, and the statistical significance is unaffected.

In line with our results for the optimal weight, we find that riding bubbles is associated with significant increases in expected utility. For an investor with a risk-aversion of $\gamma = 2$, expected utility rises from typical values around $3.06 \cdot 10^{-3}$ to $4.29 \cdot 10^{-3}$ for the 4F-model and $9.55 \cdot 10^{-3}$ for the CAPM.

Investors require a sizable compensation for not updating their portfolio upon bubble detection. In case of CAPM, the investor would require a risk-free return of 7% annually. The certainty equivalent returns for the 3F-model and 4F-model are 2.11% and 1.12% for an investor with a risk-aversion level of 2. As risk aversion decreases, the certainty equivalent increases. The confidence intervals indicate that the certainty equivalent is statically significantly different from zero for the CAPM and the 3F-model.

Our results show strong statistical and economic support for the riding bubbles hypothesis. Increases in portfolio weights are large, and the certainty equivalent returns for a bubble riding strategy are non-trivial. The increases in portfolio weights are quite extreme, with investors choosing positions exceeding their initial wealth. However, that is actually what we saw during the IT bubble, with leveraged investors like hedge funds speculating on further price increases.

7 Robustness checks

Our analysis contains some arbitrary choices. In this section, we show that our main conclusions are not affected by changing the settings in our analysis.

7.1 Misspecification

The bubble detection we propose allows for a structural break in α and assumes that the exposures to the risk factors is constant over the estimation period. It is possible, of course, that in reality α is constant, while there is a structural break in one of the factor exposures β. In this subsection, we analyze how this situation affects our bubble detection method.

Suppose that the true data generating process is given by

$$r_\tau = \alpha + \beta_\tau f_{1\tau} + \gamma' \tilde{f}_\tau + \varepsilon_\tau,\ \mathrm{E}[\varepsilon_\tau],\ \mathrm{E}[\varepsilon_\tau^2] = \sigma^2 \quad \tau = t - T + 1, \ldots, t \tag{10}$$

$$\beta_\tau = \begin{cases} \beta_1 & \text{for } \tau = t - T + 1, \ldots, t - \zeta \\ \beta_2 & \text{for } \tau = t - \zeta + 1, \ldots, t \end{cases}. \tag{11}$$

So the true process exhibits a structural break in one factor exposure which we assume without loss of generality to be the first factor. The intercept and the exposures to the other factors $\tilde{f}$ are assumed to be constant.

Our bubble detection method would estimate the misspecified model in Equation (1), and would subsequently test for a break in α_τ. In Appendix B we show that the misspecification leads to a bias in the difference of $\alpha_1(\zeta)$ and $\alpha_2(\zeta)$, and its variance:

$$\mathrm{E}[\hat{\alpha}_2(\zeta) - \hat{\alpha}_1(\zeta)] = (\beta_2 - \beta_1)\,\mathrm{E}[f_{1\tau}] \tag{12}$$

$$\mathrm{Var}[\hat{\alpha}_2(\zeta) - \hat{\alpha}_1(\zeta)] = \frac{1}{T}\left((\beta_2 - \beta_1)^2\,\mathrm{Var}[f_{1\tau}] + \frac{1}{\xi(1-\xi)}\sigma^2\right), \tag{13}$$

with $\xi = 1 - \zeta/T$ the fraction of observations before the structural break. Assuming $\beta_2 > \beta_1$, and $\mathrm{E}[f_{1\tau}] > 0$, the expected difference $\mathrm{E}[\hat{\alpha}_2 - \hat{\alpha}_1]$ will be positively biased. However, the misspecification also increases the variance with $(\beta_2 - \beta_1)^2\,\mathrm{Var}[f_{1\tau}]/T$. The Wald statistic χ that we use to test for a structural break is given by

$$\chi = \frac{\mathrm{E}[\hat{\alpha}_2 - \hat{\alpha}_1]}{\sqrt{\mathrm{Var}[\hat{\alpha}_2 - \hat{\alpha}_1]}}. \tag{14}$$

It is easy to see that both the numerator and the denominator of the statistic are affected by the misspecification.

To gauge the effect of misspecification in our setting we conduct the following exercise. For each of the different factors in our model, we determine its long-run average, and variance, and investigate how the size of a potential structural break $\beta_2 - \beta_1$ would influence χ. In the appendix, we show that this effect is maximized when the structural break is located at the middle of the estimation period, and we set $\xi = 1/2$. We set the residual variance equal to the overall average of 4%.

Figure 3(a) shows the bias in the Wald statistic χ as a function of the true structural break size in β. The solid line corresponds with the case in which the structural break is in the exposure to the market factor. We see that the bias in χ increases if the true structural break becomes larger. For a break in the CAPM-β of 0.5, the bias equals 0.42. If the break becomes larger, say 1, the bias rises to 0.74. We can compare this to the critical values as tabulated by Andrews (1993). For a confidence level of 97.5%, the critical value for bubble detection is 2.82. Even for implausibly large structural breaks exceeding 2, the bias in χ is well below this value.

[Figure 3 about here.]

Figure 3(a) also shows the bias for the other factors we consider. The bias arising from breaks in the exposure to the size factor SMB or value factor HML are well below those for the market return. A break in the exposure to the momentum factor can have a larger the effect, but also this bias is well below the critical value of 2.82.

It can of course be, that the long-run average understates the possible bias effect of misspecification. Therefore, we repeat the exercise above, using the largest 10-year factor average that shows up in our sample. We show the relation between bias and the true structural break size in Figure 3(b). The largest bias is produced by a break in the exposure to the market return. Over the period July 1949 – June 1959, the average market return equalled 1.47% per month, with a volatility of only 3.20%. If the structural break in CAPM-β is 0.5, the bias would be 0.98. For a break of 1, the bias would be 1.87. Only for implausibly large breaks exceeding 1.75 would the bias exceed the critical value of 2.82.

Based on this analysis we conclude that it is very unlikely that our bubble detection method would identify a bubble, when an industry actually exhibits a structural change in its exposure to a risk factor. Even when we choose the parameters values such that the effect of misspecification is maximum, we need a very large structural break in the factor exposure to surpass the critical values. Therefore, structural breaks in factor exposures are unlikely to influence our results.

7.2 Confidence levels

The confidence levels we apply in the tests of the bubble detection procedure are 97.5%. Increasing the confidence level makes the detection of bubbles stricter. As a consequence, we should find less bubbles, but the bubbles that are detected should be more extreme. So, we expected a further increase in average abnormal returns, and an increase in risk. It is not a priori clear what this should mean for the allocation, as these effects may cancel out. If we lower the confidence level, the reverse should apply. As a robustness check, we consider confidence levels of 95% and 99% here.

We find that the confidence levels have quite some effect on the number of bubble detections. Using the CAPM leads to 65 detections per industry, when we choose 95% for the confidence level, 45 when we choose 97.5%, and 27 when we choose 99%. For the 3F-model, the respective detections are 50, 31 and 15; for the 4F-model they are 49, 30 and 14. Table 5(a) shows that lowering the confidence level leads to less extreme bubbles. The average raw and standardized returns become lower, and also the strength of a bubble decreases. Being less strict naturally leads to longer periods marked as a bubble. The opposite is also true, as is shown by Table 5(b). Bubbles are more extreme, as they correspond with higher returns, higher strength and lower length.

[Table 5 about here.]

In Table 6 we see a somewhat positive relation between the average abnormal returns after bubble detection and the confidence level used for the tests. On the 99% level, average returns are higher for the CAPM (up from 0.19), and the 4F-model (up from 0.09). For the 3F model, the average remains 0.10. Comparing the 95% level with the 97.5% level in Table 3 shows a decrease for the CAPM to 0.17 and increases for the other two models (0.13 and 0.11 respectively). Judged by the standard errors on the estimates, these changes are hardly significant. In all six settings, we maintain our conclusion that average returns are higher after bubble detection, with all *p*-values well below the conventional 5%.

[Table 6 about here.]

If we compare the different risk measures under the different confidence levels, we see a monotonic increase if we apply stricter tests to detect bubbles. Volatility, Value-at-Risk and Expected Shortfall estimates all go up, moving from 95% to 97.5% and 99%. Again, these increases may be not be significant. We find for all confidence levels that volatility is significantly higher after bubble detection. For Value-at-Risk and Expected Shortfall, we find that they are higher though not always significant. Since Value-at-Risk and Expected Shortfall are based on fewer data points this is not surprising.

In Table 7 we report optimal portfolios for a power utility-investor with moderate risk aversion ($\gamma = 2$) for different settings. Overall, the changes are minor, in particular if we take the the standard errors into account. The changes we have observed earlier in the average abnormal returns and their risks seem to cancel to a large extent. In case of the CAPM, the settings with 95% and 99% lead to somewhat lower weights allocated to the bubbly industry, for the 3F-model, a 95% confidence level results in a larger allocation, and 99% in a smaller allocation, and for the 4F-model weights increase for both settings. For all settings, our conclusion that an investor should increase her allocation when she detects a bubble is supported. All p-values are below 2.5% and seven out of nine below 1%. The same applies to the resulting expected utility.

[Table 7 about here.]

Finally, we consider the risk-free compensation with which a strategy of riding bubbles corresponds. The changes in compensation largely follow the changes we have seen for the weights. For the CAPM differences between the different confidence levels are small. For the 3F model we observe an increase in compensation for a lower level, and a reduction for a higher level. For the 4F model, we find increases for the lower and the higher level. Perhaps, the results in Table 4 are a bit understated for the 4F-model and overstated for the CAPM. All estimates show that the strategy of riding bubble has a non-trivial economic value.

8 Conclusion

In this paper we make an empirical contribution to the debate on bubbles. One of the central questions in this field deals with the optimal response of a rational investor when confronted with an asset price bubble. Theoretical studies do not provide a unanimous answer, but make diverging predictions. From an efficient market perspective, it is argued that the investor should sell the overvalued asset (see e.g., Fama, 1965). The limits-to-arbitrage literature predicts the investor to sideline (see e.g., Shleifer and Vishny, 1997; Dow and Gorton, 1994) and not trade on this information. Other authors, such as Abreu and Brunnermeier (2003), propose that an investor should ride the bubble and try to profit from it.

We take an empirical approach to this question. Instead of making assumptions on the behavior of bubbles, we base our evidence on bubble behavior observed in US industries. Moreover, we require that the investor infers the presence of a bubble from publicly available information. Although this approach results in noisy bubble identification, it describes a real-world setting and is applicable by many investors.

To find out how a bubble affects an investor's optimal portfolio, we propose a novel bubble detection method. This procedure leads to the positive detection of a bubble, if a structural break in the α of an asset pricing model is present in the recent past, after which the α becomes significantly positive. The bubble has ended when a crash takes place. This approach enables us to split the set of returns after bubble detection in those for which we have, and those for which we have not found bubbles. We consider the CAPM, the three-factor model of Fama and French (1993) and Carhart (1997)'s four-factor model as asset pricing models.

We find that the abnormal returns after the detection of a bubble offer a favorable risk-return trade-off. A rational investor equipped with a power utility function exhibiting moderate risk aversion, will increase her portfolio significantly, when she learns that an asset currently encounters a bubble. We conclude that the additional average abnormal return that are present after bubble detection more than compensate for the rise in risk that we also found. The rise in expected utility that a strategy of riding bubbles implies corresponds with a risk-free return of about 3% per year, highlighting the economic significance of our results.

Our findings lead to the conclusion that riding bubbles is a rational investor's optimal behavior. Our results are in line with the theoretical model of Abreu and Brunnermeier (2003). We show that a strategy of riding bubbles is attractive to a relatively unsophisticated investor, who only has past prices to determine the presence of a bubble. This means that riding bubbles is not limited to sophisticated investors as the hedge funds in Brunnermeier and Nagel (2004), or well-informed market participants such as Hoare's bank in Temin and Voth (2004). Instead, our results indicate that the widespread trading in IT stocks by both private and institutional investors during the IT bubble had a rational foundation, and was not just madness of the crowds.

A Industry specific information

A.1 Bubbles

[Table 8 about here.]

[Table 9 about here.]

[Table 10 about here.]

A.2 Abnormal returns

[Table 11 about here.]

[Table 12 about here.]

[Table 13 about here.]

[Table 14 about here.]

B Structural break in factor exposure

In this appendix, we derive the effect of an ignored structural break in a factor exposure β on our bubble detection method. We use the setup of Andrews (1993). Suppose, that the true model reads

$$y_t = \alpha + \beta_t x_t + \gamma w_t + \upsilon_t,\ \mathrm{E}[\upsilon_t] = 0,\ \mathrm{E}[\upsilon_t^2] = \sigma_\upsilon^2, \mathrm{E}[x_t \upsilon_t] = 0, \mathrm{E}[w_t \upsilon_t] = 0 \tag{15}$$

$$\beta_t = \begin{cases} \beta_1 & \text{for } t \leq \xi T \\ \beta_2 & \text{for } t > \xi T, \end{cases} \tag{16}$$

where $\xi \in (0, 1)$. So, the true model exhibits a structural break in the exposure to x_t instead of the intercept. Note that this setup applies to models with more factors as well. We can write any multi-factor model

$$\begin{aligned} y_t &= \alpha + \beta_t x_t + \gamma w_t + \boldsymbol{\delta}' \boldsymbol{f}_t + \upsilon_t = \alpha + \beta_t x_t + \gamma(w_t + (\boldsymbol{\delta}'/\gamma)\boldsymbol{f}_t) + \upsilon_t \\ &= \alpha + \beta_t x_t + \gamma \tilde{w}_t + \upsilon_t, \end{aligned}$$

with $\tilde{w}_t \equiv w_t + (\boldsymbol{\delta}'/\gamma)\boldsymbol{f}_t$.

The first step of our bubble detection method only allows for a structural break in the intercept, so it estimates the model

$$y_t = a_t + bx_t + c_t w_t + e_t,\ \mathrm{E}[e_t] = 0 \tag{17}$$

$$a_t = \begin{cases} a_1 & \text{for } t \leq \xi T \\ a_2 & \text{for } t > \xi T \end{cases} \tag{18}$$

with OLS. To estimate this model, a sample of size T is available, with T_1 observations before the structural breakpoint ξT, and T_2 observations thereafter. We use Y, X and W to denote the vector of observations and U for the vector of error terms. A subscript 1 (2) denotes the subvectors before (after) the structural break. First we derive the OLS estimates. We define the auxiliary matrix

$$\boldsymbol{Z}_T = \begin{pmatrix} \boldsymbol{\imath}_{T_1} & 0 & X_1 & W_1 \\ 0 & \boldsymbol{\imath}_{T_2} & X_2 & W_2 \end{pmatrix},$$

, where $\boldsymbol{\iota}_m$ denotes a vector of length m filled with ones. Standard regression theory gives the estimates for the coefficients

$$\begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{b} \\ \hat{c} \end{pmatrix} = \left(\boldsymbol{Z}_T' \boldsymbol{Z}_T\right)^{-1} \boldsymbol{Z}_T' \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}. \tag{19}$$

Next, we use asymptotic theory to derive the properties of these estimates. We use m_x^n to denote the n^{th} raw moment of the variable x_t, and similar for the other variables; and m_{xw} for the raw comoment of x and w. We assume that the moments of the explanatory variables are constant over time, and do not change with the structural break. We calculate

$$\boldsymbol{Z}_T' \boldsymbol{Z}_T = \begin{pmatrix} T_1 & 0 & \boldsymbol{\imath}_{T_1}' X_1 & \boldsymbol{\imath}_{T_1}' W_1 \\ 0 & T_2 & \boldsymbol{\imath}_{T_2}' X_2 & \boldsymbol{\imath}_{T_2}' W_2 \\ \boldsymbol{\imath}_{T_1}' X_1 & \boldsymbol{\imath}_{T_2}' X_2 & X'X & X'W \\ \boldsymbol{\imath}_{T_1}' W_1 & \boldsymbol{\imath}_{T_2}' W_2 & X'W & W'W \end{pmatrix},$$

and use this to define

$$\boldsymbol{\Sigma}_{zz} \equiv \lim_{T\to\infty} \frac{1}{T}\boldsymbol{Z}_T'\boldsymbol{Z}_T = \begin{pmatrix} \xi & 0 & \xi m_x^1 & \xi m_w^1 \\ 0 & (1-\xi) & (1-\xi)m_x^1 & (1-\xi)m_w^1 \\ \xi m_x^1 & (1-\xi)m_x^1 & m_x^2 & m_{xw} \\ \xi m_w^1 & (1-\xi)m_w^1 & m_{xw} & m_w^2 \end{pmatrix}. \tag{20}$$

In a similar fashion we calculate

$$\boldsymbol{Z}_T'\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = \begin{pmatrix} \boldsymbol{\imath}_{T_1}'Y_1 \\ \boldsymbol{\imath}_{T_2}'Y_2 \\ X_1'Y_1 + X_2'Y_2 \\ W_1'Y_1 + W_2'Y_2 \end{pmatrix} = \begin{pmatrix} T_1\alpha + \beta_1\boldsymbol{\imath}_{T_1}'X_1 + \boldsymbol{\imath}_{T_1}'W_1 + \boldsymbol{\imath}_{T_1}'U_1 \\ T_2\alpha + \beta_2\boldsymbol{\imath}_{T_2}'X_2 + \boldsymbol{\imath}_{T_2}'W_2 + \boldsymbol{\imath}_{T_2}'U_2 \\ \boldsymbol{\imath}_T'X\alpha + \beta_1X_1'X_1 + \beta_2X_2'X_2 + \gamma X'W + X'U \\ \boldsymbol{\imath}_T'W\alpha + \beta_1X_1'W_1 + \beta_2X_2'W_2 + \gamma W'W + W'U \end{pmatrix},$$

where we have substituted the true model for y_t. We use this result to define

$$\boldsymbol{\Sigma}_{zy} \equiv \lim_{T\to\infty} \frac{1}{T}\boldsymbol{Z}_T'\boldsymbol{Y}_T = \begin{pmatrix} \xi\big(\alpha + \beta_1 m_x^1 + \gamma m_w^1 + m_u^1\big) \\ (1-\xi)\big(\alpha + \beta_2 m_x^1 + \gamma m_w^1 + m_u^1\big) \\ \alpha m_x^1 + \big(\xi\beta_1 + (1-\xi)\beta_2\big)m_x^2 + \gamma m_{xw} + m_{xu} \\ \alpha m_w^2 + \big(\xi\beta_1 + (1-\xi)\beta_2\big)m_{xw} + \gamma m_w^2 + m_{wu} \end{pmatrix}. \tag{21}$$

Consequently, we find

$$\operatorname{plim}_{T\to\infty} \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{b} \\ \hat{c} \end{pmatrix} = \begin{pmatrix} \alpha + (1-\xi)(\beta_1-\beta_2)m_x^1 \\ \alpha - \xi(\beta_1-\beta_2)m_x^1 \\ \xi\beta_1 + (1-\xi)\beta_2 \\ \gamma \end{pmatrix}. \tag{22}$$

As expected, $\hat{b}$ converges to a weighted average of β_1 and β_2, where the weight depends on the proportions of the sample before and after the structural break. The deviations of a_1 and a_2 from the true intercept α reflect the size of the structural break $\beta_1 - \beta_2$, the proportion ξ and the average value of x_t.

The structural break test in our bubble detection method also needs the variance of the estimator. Therefore, we derive the variation of the residuals e_t. We have

$$e_t = \begin{cases} (1-\xi)(\beta_1-\beta_2)(x_t - m_x^1) + \upsilon_t & \text{for } t \le \xi T \\ -\xi(\beta_1-\beta_2)(x_t - m_x^1) + \upsilon_t & \text{for } t > \xi T \end{cases}$$

so σ_e^2 becomes

$$\sigma_e^2 = \xi(1-\xi)(\beta_1-\beta_2)^2\sigma_x^2 + \sigma_v^2, \tag{23}$$

where σ_x^2 is the (population) variance of x_t. This expression shows that the misspecification leads to an increase in the residual variance. Applying standard regression theory gives the desired result

$$\sqrt{T}\left(\begin{pmatrix}\hat{a}_1\\ \hat{a}_2\\ \hat{b}\\ \hat{c}\end{pmatrix} - \begin{pmatrix}\alpha + (1-\xi)(\beta_1-\beta_2)m_x^1\\ \alpha - \xi(\beta_1-\beta_2)m_x^1\\ \xi\beta_1 + (1-\xi)\beta_2\\ \gamma\end{pmatrix}\right) \to N\left(\mathbf{0}, \boldsymbol{\Sigma}_{zz}^{-1}\sigma_e^2\right). \tag{24}$$

The test statistic for a structural break is based on the difference $a_2 - a_1$, for which we have

$$\begin{aligned}\text{plim}_{T\to\infty}\, a_2 - a_1 &= (\beta_2-\beta_1)m_x^1\\ \sqrt{T}\left(a_2 - a_1 - (\beta_2-\beta_1)m_x^1\right) &\to N\left(0, \frac{1}{\xi(1-\xi)}\sigma_e^2\right).\end{aligned}$$

This means that the expected value of the statistic for the structural break test on the intercept is given by

$$\chi = \frac{\sqrt{T\xi(1-\xi)}(\beta_2-\beta_1)m_x^1}{\sigma_e^2} = \frac{\sqrt{T}(\beta_2-\beta_1)m_x^1}{\sqrt{(\beta_2-\beta_1)^2\sigma_x^2 + \frac{1}{\xi(1-\xi)}\sigma_v^2}}, \tag{25}$$

and we conclude that the statistic depends on the average value of the factor, its variance, the residual variance of the returns, the size of the true structural break, and the location of the structural break.

We finish this appendix by analyzing the sensitivities of the statistic for the different inputs. As the size and location of the structural break show up in both the numerator and the denominator, we rewrite the statistic as

$$\chi = m_x^1\sqrt{T}\left(\sigma_x^2 + \sigma_v^2\left(\xi-\xi^2\right)^{-1}\Delta^{-2}\right)^{-1/2},$$

where we use $\Delta \equiv \beta_2 - \beta_1$ for the size of the structural break. It is straightforward to see that the statistic is increasing in the factor average m_x^1 and in the absolute size of the structural break Δ, and decreasing in the factor and residual variances σ_x^2 and σ_v^2. To find the effect of the location of the structural break, we differentiate χ with respect to ξ:

$$\frac{\mathrm{d}\chi}{\mathrm{d}\xi} = \frac{1}{2} m_x^1 \sqrt{T} \left(\sigma_x^2 + \sigma_v^2 \left(\xi - \xi^2 \right)^{-1} \Delta^{-2} \right)^{-3/2} \sigma_v^2 \left(\xi - \xi^2 \right)^{-2} \Delta^{-2} (1 - 2\xi).$$

We conclude that the statistic is maximized for $\xi = 1/2$, so when the structural break is located in the middle of the sample.

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Table 1: Descriptive statistics of industry returns

industry	start date	# obs	average	volatility	skewness	kurtosis	min.	max.
Agric	1926:07	966	11.4	26.6	1.69	22.0	−36.5	91.3
Food	1926:07	966	12.1	17.0	0.05	5.9	−28.0	32.8
Soda	1936:07	846	14.0	23.8	0.30	3.3	−29.6	38.8
Beer	1926:07	966	15.0	26.1	1.82	21.8	−29.2	89.2
Smoke	1926:07	966	14.0	20.4	0.07	3.5	−24.9	33.3
Toys	1926:07	966	13.0	35.6	2.86	37.1	−43.3	140.4
Fun	1926:07	966	15.4	32.2	0.68	10.0	−44.5	69.8
Books	1926:07	966	13.2	26.7	0.87	7.5	−34.9	56.0
Hshld	1926:07	966	11.1	19.4	0.15	10.4	−33.7	52.2
Clths	1926:07	966	13.2	24.2	0.76	7.9	−31.4	54.2
Hlth	1963:07	522	13.0	36.1	0.47	3.4	−43.6	48.7
MedEq	1926:07	966	13.6	22.1	−0.07	1.8	−26.6	30.5
Drugs	1926:07	966	13.4	20.5	0.26	7.5	−35.6	40.3
Chems	1926:07	966	12.7	21.8	0.41	7.3	−33.3	47.0
Rubbr	1944:07	750	13.2	19.4	−0.26	1.8	−30.6	19.3
Txtls	1926:07	966	11.4	26.5	0.87	8.6	−32.4	57.1
BldMt	1926:07	966	12.0	23.4	0.35	6.0	−31.8	41.8
Cnstr	1926:07	966	12.8	33.4	0.95	7.1	−38.0	67.8
Steel	1926:07	966	12.0	29.0	1.58	15.1	−31.1	80.7
FabPr	1963:07	522	7.8	23.9	−0.19	1.4	−27.2	26.0
Mach	1926:07	966	13.0	25.0	0.55	7.8	−33.4	51.9
ElcEq	1926:07	966	14.6	26.7	0.64	9.0	−34.5	59.6
Autos	1926:07	966	13.5	27.2	1.24	15.2	−34.9	81.9
Aero	1926:07	966	17.5	33.2	0.94	7.7	−40.4	72.0
Ships	1926:07	966	11.8	27.6	0.82	7.9	−34.4	63.4
Guns	1963:07	522	14.2	23.5	−0.02	1.8	−30.1	32.6
Gold	1963:07	522	14.4	36.1	0.88	6.2	−31.3	79.6
Mines	1926:07	966	12.2	23.4	0.26	4.4	−33.2	45.6
Coal	1926:07	966	14.7	30.5	1.11	7.8	−30.1	77.5
Oil	1926:07	966	13.2	21.1	0.34	4.3	−30.0	39.2
Util	1926:07	966	10.9	19.7	0.15	7.3	−29.8	43.2
Telcm	1926:07	966	10.4	16.0	0.04	3.2	−21.6	28.2
PerSv	1927:07	954	12.1	33.3	1.54	13.2	−39.3	84.8
BusSv	1926:07	966	13.0	26.5	0.48	7.8	−40.4	56.7
Comps	1926:07	966	15.0	25.6	0.09	4.6	−34.6	53.4
Chips	1926:07	966	14.2	30.8	0.48	6.2	−42.1	64.7
LabEq	1926:07	966	13.2	24.3	−0.24	1.8	−33.2	25.4
Paper	1936:07	846	13.9	23.0	0.28	5.4	−35.8	47.9
Boxes	1926:07	966	13.0	21.5	0.24	6.0	−29.3	43.4
Trans	1926:07	966	11.2	25.1	1.15	13.8	−34.5	65.4
Whlsl	1926:07	966	9.9	26.3	0.61	11.4	−44.5	59.2
Rtail	1926:07	966	12.3	21.0	0.02	5.1	−30.3	37.8
Meals	1926:07	966	13.4	24.2	−0.24	2.3	−31.4	31.5
Banks	1926:07	966	15.7	24.5	0.16	5.7	−34.4	42.3
Insur	1926:07	966	13.3	26.1	1.14	17.2	−45.4	73.7
RlEst	1926:07	966	9.8	33.3	0.73	6.5	−52.6	58.6
Fin	1926:07	966	13.7	26.4	0.63	10.8	−39.2	67.2
Other	1926:07	966	8.8	25.6	0.07	3.6	−32.9	41.1
Pooled	-	44,124	12.9	26.0	0.86	12.2	−52.6	140.4

This table reports summary statistics on the 48 US industries as defined in Fama and French (1997). For each industry we report its start date, the number of available return observations, their average (in % per year), volatility (in % per year), skewness, kurtosis, minimum (in %) and maximum (in %).

Table 2: Bubble statistics

	CAPM			3F			4F		
	mean	med.	std. dev.	mean	med.	std. dev.	mean	med.	std. dev.
raw return	30.6	28.3	23.5	32.4	31.1	23.0	30.6	29.5	22.7
st. abn. return	0.468	0.395	1.16	0.494	0.424	1.18	0.509	0.439	1.23
strength	3.22	3.16	0.752	3.08	2.89	0.763	3.07	2.94	0.700
length	34.5	33.0	15.3	31.2	28.0	14.7	31.3	28.0	14.9

This table reports the mean, median and standard deviation of four bubble statistics: raw returns during bubbles (in % per year), standardized abnormal returns during bubbles, strength at detection (t-statistic), and length at detection. An investor detects a bubble, if a 10-year series of industry returns shows evidence of an upward structural break in the constant, when regressing the industry returns on a constant and the market return (column CAPM), Fama and French (1993)'s three factors (column 3F), or Carhart (1997)'s four factors. Moreover, after the structural break, the constant should be significantly larger than zero, and the associated t-statistic gives the strength of the bubble. A bubble has ended if a crash has occurred in the last 6 months, where a crash is defined as a residual below -2 times its standard deviation. Critical values for the structural break test correspond with a 97.5% confidence level, and have been taken from Andrews (1993).

Table 3: Standardized abnormal returns with and without prior bubble detection.

(a) abnormal returns based on the CAPM

	bubble detected		no bubble detected		p-value
# obs	2071		36293		
mean	0.19	(0.03)	0.01	(0.01)	< 0.0001
median	0.14	(0.02)	−0.01	(0.00)	< 0.0001
volatility	1.17	(0.03)	1.03	(0.01)	< 0.0001
skewness	0.52	(0.21)	0.18	(0.04)	0.032
kurtosis	5.80	(1.46)	5.46	(0.20)	0.42
VaR(0.95)	1.63	(0.07)	1.59	(0.01)	0.31
ES(0.95)	2.18	(0.07)	2.23	(0.02)	0.27
VaR(0.975)	1.96	(0.09)	2.01	(0.02)	0.35
ES(0.975)	2.57	(0.10)	2.67	(0.03)	0.18

(b) abnormal returns based on Fama and French (1993)'s three factor model

	bubble detected		no bubble detected		p-value
# obs	1406		36958		
mean	0.10	(0.03)	0.00	(0.01)	0.0010
median	0.06	(0.03)	−0.02	(0.01)	0.0035
volatility	1.21	(0.03)	1.06	(0.01)	< 0.0001
skewness	0.18	(0.10)	0.20	(0.05)	0.44
kurtosis	4.00	(0.20)	5.69	(0.30)	< 0.0001
VaR(0.95)	1.84	(0.08)	1.65	(0.01)	0.021
ES(0.95)	2.44	(0.10)	2.28	(0.02)	0.055
VaR(0.975)	2.24	(0.11)	2.05	(0.02)	0.057
ES(0.975)	2.88	(0.14)	2.72	(0.03)	0.13

(c) abnormal returns based on Carhart (1997)'s three factor model

	bubble detected		no bubble detected		p-value
# obs	1379		36745		
mean	0.09	(0.03)	0.01	(0.01)	0.0080
median	0.06	(0.03)	−0.02	(0.01)	0.0082
volatility	1.26	(0.03)	1.07	(0.01)	< 0.0001
skewness	0.04	(0.10)	0.22	(0.05)	0.065
kurtosis	3.94	(0.22)	5.59	(0.30)	< 0.0001
VaR(0.95)	1.93	(0.08)	1.65	(0.01)	0.00050
ES(0.95)	2.62	(0.12)	2.28	(0.02)	0.0018
VaR(0.975)	2.38	(0.07)	2.06	(0.02)	0.0012
ES(0.975)	3.10	(0.18)	2.72	(0.03)	0.013

This table reports summary statistics and downside risk measures for the pooled set of standardized abnormal returns. The abnormal returns are based on rolling regressions of the CAPM (panel a), Fama and French (1993)'s three factor model (panel b) and Carhart (1997)'s four factor model (panel c) in Eq. (1) with a 120-month estimation window. For each regression, we construct an abnormal return for the period after the estimation window as in Eq. (4). To correct for time-varying volatility, we standardize the abnormal return by a division by the residual volatility of the regression model. We split the abnormal returns according to the detection of a bubble. For each statistic, we construct a 95% confidence interval based on 10,000 bootstraps. The column p-values reports the results of tests for equality of the statistics for the cases "bubble detected" and "no bubble detected".

Table 4: Optimal portfolio choice and expected utility

(a) abnormal returns based on the CAPM

γ	w_B		w_{NB}		p-value	V_B ($\times 10^{-3}$)		V_{NB} ($\times 10^{-3}$)		p-value	λ (in % p.a.)	
0.5	6.02	(0.52)	0.39	(0.25)	< 0.0001	28.16	(6.36)	3.12	(0.09)	< 0.0001	26.88	[13.61,43.01]
1	3.36	(0.45)	0.19	(0.12)	< 0.0001	16.07	(3.54)	3.08	(0.05)	< 0.0001	14.03	[6.96,23.35]
2	1.71	(0.24)	0.10	(0.06)	< 0.0001	9.55	(1.79)	3.06	(0.02)	< 0.0001	7.04	[3.48,11.75]
3	1.14	(0.16)	0.06	(0.04)	< 0.0001	7.36	(1.19)	3.05	(0.02)	< 0.0001	4.69	[2.32,7.82]
5	0.69	(0.10)	0.04	(0.02)	< 0.0001	5.60	(0.71)	3.04	(0.01)	< 0.0001	2.81	[1.39,4.68]
10	0.34	(0.05)	0.02	(0.01)	< 0.0001	4.27	(0.35)	3.01	(0.00)	< 0.0001	1.42	[0.70,2.34]

(b) abnormal returns based on Fama and French (1993)'s three factor model

γ	w_B		w_{NB}		p-value	V_B ($\times 10^{-3}$)		V_{NB} ($\times 10^{-3}$)		p-value	λ (in % p.a.)	
0.5	3.53	(1.03)	0.00	(0.25)	0.0010	9.98	(4.36)	3.05	(0.04)	0.00050	8.34	[1.30,22.38]
1	1.81	(0.57)	0.00	(0.13)	0.0010	6.54	(2.25)	3.05	(0.02)	0.00050	4.21	[0.65,11.51]
2	0.91	(0.29)	0.00	(0.06)	0.0010	4.79	(1.12)	3.05	(0.01)	0.00050	2.11	[0.33,5.78]
3	0.61	(0.19)	0.00	(0.04)	0.0010	4.20	(0.75)	3.04	(0.01)	0.00050	1.40	[0.22,3.85]
5	0.36	(0.12)	0.00	(0.03)	0.0010	3.72	(0.45)	3.03	(0.00)	0.00050	0.85	[0.13,2.31]
10	0.18	(0.06)	0.00	(0.01)	0.0010	3.35	(0.22)	3.01	(0.00)	0.00050	0.42	[0.07,1.17]

(c) abnormal returns based on Carhart (1997)'s three factor model

γ	w_B		w_{NB}		p-value	V_B ($\times 10^{-3}$)		V_{NB} ($\times 10^{-3}$)		p-value	λ (in % p.a.)	
0.5	2.88	(1.02)	0.41	(0.26)	0.012	8.01	(3.88)	3.12	(0.10)	0.0087	4.41	[0.12,15.48]
1	1.47	(0.56)	0.21	(0.13)	0.012	5.55	(2.00)	3.09	(0.05)	0.0087	2.23	[0.00,8.00]
2	0.74	(0.29)	0.10	(0.07)	0.012	4.29	(1.00)	3.06	(0.02)	0.0087	1.12	[0.00,4.02]
3	0.49	(0.19)	0.07	(0.04)	0.012	3.87	(0.67)	3.05	(0.02)	0.0087	0.75	[0.00,2.68]
5	0.30	(0.11)	0.04	(0.03)	0.012	3.53	(0.40)	3.04	(0.01)	0.0087	0.45	[0.00,1.61]
10	0.15	(0.06)	0.02	(0.01)	0.012	3.25	(0.20)	3.01	(0.00)	0.0087	0.22	[0.00,0.81]

This table reports the optimal response of an investor when learning that a bubble is (subscript B) or is not (subscript NB) present. We assume that the investor has a power utility function. We report optimal portfolios for varying levels of relative risk aversion γ for both cases. The investment opportunity is the typical industry with idiosyncratic volatilities equal to their pooled averages (CAPM: 4.14%, 3F: 3.83%, 4F 3.79%). We set the risk-free rate equal to its long term average of 0.305% per month. The optimal portfolios are reported as fractions of wealth. Based on the optimal portfolios we calculate the respective expected utilities, denoted by V_B and V_{NB}. We also calculate the certainty equivalent return λ that an investor requires for not changing his portfolio from w_{NB} to w_{NB}. We use 10,000 bootstraps to calculate standard errors (reported between parentheses) and to construct a 95% confidence interval around λ (reported between brackets). We also use these bootstraps to test $w_{NB} = w_{NB}$ and to test $V_B = V_{NB}$, for which we report p-values.

Table 5: Bubble statistics for different confidence levels

(a) 95%

	CAPM			3F			4F		
	mean	med.	std. dev.	mean	med.	std. dev.	mean	med.	std. dev.
raw return	29.2	27.1	23.1	30.2	28.8	23.8	29.0	27.0	23.1
st. abn. return	0.419	0.351	1.12	0.422	0.356	1.14	0.435	0.367	1.18
strength	2.97	2.89	0.774	2.81	2.69	0.756	2.80	2.68	0.706
length	35.2	34.0	15.6	32.5	29.0	15.2	32.5	29.0	15.2

(b) 99%

	CAPM			3F			4F		
	mean	med.	std. dev.	mean	med.	std. dev.	mean	med.	std. dev.
raw return	33.0	32.1	23.5	36.6	34.0	23.4	35.2	32.3	22.8
st. abn. return	0.572	0.509	1.19	0.593	0.487	1.23	0.620	0.535	1.27
strength	3.54	3.44	0.717	3.51	3.46	0.772	3.48	3.43	0.677
length	33.2	31.0	14.8	28.1	24.0	13.7	27.9	23.0	14.0

This table reports the mean, median and standard deviation of four bubble statistics: raw returns during bubbles (in % per year), standardized abnormal returns during bubbles, strength at detection (t-statistic), and length at detection. An investor detects a bubble, if a 10-year series of industry returns shows evidence of an upward structural break in the constant, when regressing the industry returns on a constant and the market return (column CAPM), Fama and French (1993)'s three factors (column 3F), or Carhart (1997)'s four factors. Moreover, after the structural break, the constant should be significantly larger than zero, and the associated t-statistic gives the strength of the bubble. A bubble has ended if a crash has occurred in the last 6 months, where a crash is defined as a residual below -2 its standard deviation. Critical values for the structural break test correspond with a 95% (panel a) or a 99% (panel b) confidence level, and have been taken from Andrews (1993).

Table 6: Standardized abnormal returns with and without prior bubble detection, using different confidence levels.

(a) CAPM, 95%

detection	yes		no		p-value
# obs	2987		35377		
mean	0.17	(0.02)	0.01	(0.01)	< 0.001
median	0.13	(0.02)	−0.01	(0.00)	< 0.001
volatility	1.13	(0.02)	1.03	(0.01)	< 0.001
VaR(0.95)	1.63	(0.06)	1.59	(0.01)	0.221
ES(0.95)	2.14	(0.06)	2.23	(0.02)	0.925

(b) CAPM, 99%

detection	yes		no		p-value
# obs	1238		37126		
mean	0.20	(0.03)	0.01	(0.01)	< 0.001
median	0.15	(0.03)	−0.01	(0.01)	< 0.001
volatility	1.19	(0.03)	1.04	(0.01)	< 0.001
VaR(0.95)	1.70	(0.10)	1.59	(0.01)	0.188
ES(0.95)	2.29	(0.10)	2.22	(0.02)	0.254

(c) 3F-model, 95%

detection	yes		no		p-value
# obs	2284		36080		
mean	0.13	(0.03)	−0.01	(0.01)	< 0.001
median	0.09	(0.03)	−0.03	(0.01)	< 0.001
volatility	1.19	(0.02)	1.06	(0.01)	< 0.001
VaR(0.95)	1.69	(0.07)	1.65	(0.01)	0.271
ES(0.95)	2.31	(0.08)	2.28	(0.02)	0.366

(d) 3F-model, 99%

detection	yes		no		p-value
# obs	683		37681		
mean	0.10	(0.05)	0.00	(0.01)	0.025
median	0.07	(0.05)	−0.02	(0.01)	0.015
volatility	1.30	(0.04)	1.06	(0.01)	< 0.001
VaR(0.95)	1.94	(0.10)	1.65	(0.01)	0.001
ES(0.95)	2.54	(0.14)	2.28	(0.02)	0.032

(e) 4F-model, 95%

detection	yes		no		p-value
# obs	2246		35878		
mean	0.11	(0.03)	0.01	(0.01)	< 0.001
median	0.09	(0.03)	−0.02	(0.01)	< 0.001
volatility	1.24	(0.02)	1.07	(0.01)	< 0.001
VaR(0.95)	1.82	(0.06)	1.65	(0.01)	< 0.001
ES(0.95)	2.52	(0.09)	2.28	(0.02)	0.004

(f) 4F-model, 99%

detection	yes		no		p-value
# obs	674		37450		
mean	0.13	(0.05)	0.01	(0.01)	0.010
median	0.10	(0.05)	−0.02	(0.01)	0.012
volatility	1.36	(0.04)	1.07	(0.01)	< 0.001
VaR(0.95)	1.94	(0.12)	1.66	(0.01)	0.001
ES(0.95)	2.67	(0.17)	2.29	(0.02)	0.008

This table reports summary statistics and downside risk measures for the pooled set of standardized abnormal returns. The bubble detection procedure uses a confidence level of 95% (left panels) or 99% (right panels). The abnormal returns are based on rolling regressions of the CAPM (panels a and b), Fama and French (1993)'s three factor model (panels c and d) and Carhart (1997)'s four factor model (panels e and f) in Eq. (1) with a 120-month estimation window. For each regression, we construct an abnormal return for the period after the estimation window as in Eq. (4). To correct for time-varying volatility, we standardize the abnormal return by a division by the residual volatility of the regression model. We split the abnormal returns according to the detection of a bubble. For each statistic, we construct a 95% confidence interval based on 10,000 bootstraps. The column p-values reports the results of tests for equality of the statistics for the cases "bubble detected" and "no bubble detected".

Table 7: Optimal portfolio choice and expected utility, with different confidence levels for bubble detection.

setting	w_B		w_{NB}		p-value	V_B ($\times 10^{-3}$)		V_{NB} ($\times 10^{-3}$)		p-value	λ (in % p.a.)	
CAPM, 95%	1.65	(0.19)	0.06	(0.06)	< 0.001	8.71	(1.33)	3.05	(0.02)	< 0.001	6.38	[3.46, 10.03]
CAPM, 97.5%	1.71	(0.24)	0.10	(0.06)	< 0.001	9.55	(1.79)	3.06	(0.02)	< 0.001	7.04	[3.48, 11.75]
CAPM, 99%	1.71	(0.31)	0.14	(0.06)	< 0.001	9.95	(2.46)	3.08	(0.03)	< 0.001	7.11	[2.79, 13.64]
3F, 95%	1.24	(0.24)	−0.07	(0.07)	< 0.001	6.16	(1.20)	3.05	(0.02)	< 0.001	4.22	[1.67, 7.90]
3F, 97.5%	0.91	(0.29)	0.00	(0.06)	0.001	4.79	(1.12)	3.05	(0.01)	< 0.001	2.11	[0.33, 5.78]
3F, 99%	0.78	(0.39)	0.00	(0.06)	0.021	4.53	(1.62)	3.05	(0.01)	0.010	1.79	[0.00, 7.14]
4F, 95%	0.96	(0.23)	0.07	(0.06)	< 0.001	5.06	(0.96)	3.05	(0.02)	< 0.001	2.12	[0.47, 4.87]
4F, 97.5%	0.74	(0.29)	0.10	(0.07)	0.012	4.29	(1.00)	3.06	(0.02)	0.009	1.12	[0.00, 4.02]
4F, 99%	0.94	(0.37)	0.11	(0.06)	0.008	5.35	(1.91)	3.07	(0.02)	0.006	2.17	[0.00, 7.68]

This table reports the optimal response of an investor when learning that a bubble is (subscript B) or is not (subscript NB) present. We assume that the investor has a power utility function with coefficient of relative risk aversion $\gamma = 2$. The optimal portfolios are reported as fractions of wealth. We consider different confidence levels for the bubble detection procedure. Based on the optimal portfolios we calculate the respective expected utilities, denoted by V_B and V_{NB}. We also calculate the certainty equivalent return λ that an investor requires for not changing his portfolio from w_{NB} to w_{NB}. We use 10,000 bootstraps to calculate standard errors (reported between parentheses) and to construct a 95% confidence interval around λ (reported between brackets). We also use these bootstraps to test $w_{NB} = w_{NB}$ and to test $V_B = V_{NB}$, for which we report p-values.

Table 8: Bubble statistics based on the CAPM

industry	number of bubbles	number of detections	average return	average abn. return	average strength	average length
Agric	1	4	35.0	0.44	2.61	29.8
Food	3	76	23.4	0.57	4.46	30.2
Soda†	2	21	37.2	0.43	2.92	22.8
Beer	2	15	41.0	0.40	2.87	26.1
Smoke	4	50	39.5	0.74	3.91	29.1
Toys	3	58	41.8	0.59	3.37	25.4
Fun	3	36	20.0	0.43	3.42	32.7
Books	4	72	35.1	0.39	3.08	38.1
Hshld	2	35	21.7	0.50	3.60	37.0
Clths	3	68	22.4	0.33	2.83	37.5
Hlth†	3	50	37.1	0.32	2.96	38.5
MedEq	5	47	34.9	0.45	2.98	36.6
Drugs	4	63	30.0	0.57	3.66	38.4
Chems	3	33	19.2	0.36	2.60	38.2
Rubber†	3	10	25.4	0.43	2.78	39.2
Txtls	4	59	26.5	0.40	2.55	33.8
BldMt	3	69	22.5	0.33	2.71	41.1
Cnstr	4	85	40.5	0.43	3.33	34.4
Steel	3	20	32.8	0.60	3.23	21.8
FabPr†	0	0	–	–	–	–
Mach	4	36	18.7	0.39	2.62	30.0
ElcEq	2	54	19.6	0.42	2.91	35.8
Autos	2	13	30.8	0.34	2.81	24.9
Aero	4	78	38.3	0.47	2.82	37.5
Guns†	2	19	33.6	0.76	3.44	21.8
Gold†	1	3	96.1	0.78	3.48	14.7
Ships	2	13	36.1	0.88	3.75	20.5
Mines	3	62	25.0	0.41	3.18	35.0
Coal	5	49	51.3	0.66	3.48	21.7
Oil	2	6	39.8	0.81	2.99	18.0
Util	0	0	–	–	–	–
Telcm	3	33	35.8	0.97	4.39	27.1
PerSv†	3	97	32.0	0.46	3.25	39.7
BusSv	2	54	28.8	0.45	2.84	37.9
Comps	2	45	40.9	0.49	2.99	37.5
Chips	4	53	53.2	0.52	3.36	28.4
LabEq	3	22	29.4	0.54	3.37	33.2
Paper†	2	12	20.1	0.69	3.45	17.3
Boxes	3	69	19.6	0.27	2.70	38.9
Trans	3	46	12.7	0.48	2.70	31.9
Whlsl	3	41	25.0	0.44	3.59	45.7
Rtail	4	78	22.9	0.47	3.35	35.9
Meals	3	57	38.2	0.41	3.72	35.8
Banks	4	58	28.2	0.48	3.29	36.4
Insur	2	6	40.1	0.86	3.42	18.0
RlEst	3	105	28.4	0.30	3.04	39.0
Fin	3	86	19.6	0.41	3.45	34.8
Other	2	5	43.5	0.81	3.59	14.8
Pooled	2	45	30.6	0.47	3.22	34.5

This table reports for each of the 48 industries in our sample the number of bubbles found, their average length, the average raw return and average standardized abnormal return during the bubble (in % per year), the number of times the investor detects the bubbles, and the average strength (t-statistic) at detection. An investor detects a bubble, if a 10-year series of industry returns shows evidence of an upward structural break in the constant, when regressing the industry returns on a constant and the market return. Moreover, after the structural break, the constant should be significantly larger than zero, and the associated t-statistic gives the strength of the bubble. A bubble has ended if a crash has occurred in the last 6 months, where a crash is defined as a residual below -2 times its standard deviation. Critical values for the structural break test correspond with a 97.5% confidence level, and have been taken from Andrews (1993).

Table 9: Bubble statistics based on Fama and French (1993)'s three factor model

industry	number of bubbles	number of detections	average return	average abn. return	average strength	average length
Agric	0	0	–	–	–	–
Food	5	58	27.1	0.59	4.20	27.1
Soda†	2	18	37.3	0.43	2.94	20.9
Beer	2	19	40.8	0.46	2.78	41.2
Smoke	3	47	35.1	0.61	3.57	31.1
Toys	2	31	50.7	0.85	4.06	19.1
Fun	3	7	39.9	0.48	2.72	22.4
Books	4	42	42.0	0.46	3.02	28.3
Hshld	2	43	21.9	0.51	3.30	38.0
Clths	2	18	30.0	0.40	2.40	26.4
Hlth†	2	16	39.0	0.37	2.75	16.8
MedEq	4	50	34.1	0.55	2.96	36.6
Drugs	4	54	24.9	0.61	3.64	34.5
Chems	2	19	18.6	0.36	2.43	41.3
Rubber†	3	5	20.5	0.65	2.69	19.6
Txtls	4	54	33.1	0.40	2.98	39.6
BldMt	3	21	23.6	0.52	2.47	25.9
Cnstr	4	21	52.1	0.56	2.66	19.1
Steel	4	30	27.4	0.55	2.73	20.1
FabPr†	0	0	–	–	–	–
Mach	3	39	23.2	0.45	2.80	26.9
ElcEq	3	20	23.6	0.53	2.95	22.3
Autos	2	6	42.4	0.61	2.57	14.2
Aero	5	42	31.9	0.39	2.75	37.1
Guns†	0	0	–	–	–	–
Gold†	1	2	113.	0.92	3.51	14.5
Ships	4	7	36.6	0.61	3.00	18.6
Mines	7	57	32.7	0.48	2.78	24.7
Coal	5	39	54.8	0.70	3.50	20.8
Oil	2	6	48.1	0.82	3.94	20.2
Util	0	0	–	–	–	–
Telcm	3	28	33.8	0.81	4.05	25.6
PerSv†	2	60	30.7	0.40	2.89	28.9
BusSv	1	7	32.8	0.39	2.56	25.1
Comps	4	98	37.5	0.44	2.88	43.9
Chips	3	56	41.2	0.56	3.69	35.4
LabEq	3	15	26.1	0.87	3.62	19.8
Paper†	2	18	21.7	0.55	2.78	18.9
Boxes	3	43	20.0	0.38	2.73	33.6
Trans	4	12	29.8	0.52	2.38	17.8
Whlsl	2	16	31.5	0.23	2.32	40.9
Rtail	5	86	23.1	0.44	3.01	38.1
Meals	3	33	42.5	0.42	3.07	36.4
Banks	3	44	23.2	0.36	2.96	27.9
Insur	1	3	45.9	0.77	2.88	13.3
RlEst	1	41	27.0	0.31	2.76	35.0
Fin	2	70	20.2	0.27	2.84	32.9
Other	2	5	25.6	0.41	2.64	29.6
Pooled	2	31	32.4	0.49	3.08	31.2

This table reports for each of the 48 industries in our sample the number of bubbles found, their average length, the average raw return and average standardized abnormal return during the bubble (in % per year), the number of times the investor detects the bubbles, and the average strength (t-statistic) at detection. An investor detects a bubble, if a 10-year series of industry returns shows evidence of an upward structural break in the constant, when regressing the industry returns on a constant and Fama and French (1993)'s three factors. Moreover, after the structural break, the constant should be significantly larger than zero, and the associated t-statistic gives the strength of the bubble. A bubble has ended if a crash has occurred in the last 6 months, where a crash is defined as a residual below -2 times its standard deviation. Critical values for the structural break test correspond with a 95% confidence level, and have been taken from Andrews (1993).

Table 10: Bubble statistics based on Carhart (1997)'s four factor model

industry	number of bubbles	number of detections	average return	average abn. return	average strength	average length
Agric	0	0	–	–	–	–
Food	4	48	26.4	0.66	4.18	25.6
Soda†	2	17	37.3	0.50	3.10	20.2
Beer	2	14	38.6	0.41	2.48	38.3
Smoke	4	47	33.2	0.65	3.50	29.8
Toys	2	32	46.9	0.74	3.75	20.9
Fun	3	7	35.7	0.50	2.73	21.1
Books	4	49	40.7	0.48	3.09	29.4
Hshld	2	42	22.5	0.49	2.94	37.1
Clths	2	14	29.1	0.52	2.79	24.0
Hlth†	2	15	39.0	0.43	2.86	17.1
MedEq	4	51	30.4	0.52	2.88	35.7
Drugs	4	50	24.9	0.59	3.44	33.2
Chems	3	25	22.7	0.45	2.77	37.4
Rubber†	2	6	30.3	0.71	2.79	16.7
Txtls	4	59	30.7	0.43	2.95	37.3
BldMt	3	20	25.6	0.55	2.52	24.6
Cnstr	4	13	53.8	0.63	2.64	18.9
Steel	5	30	19.8	0.54	2.75	19.9
FabPr†	1	4	19.1	0.25	2.02	54.3
Mach	4	47	20.9	0.48	2.70	29.7
ElcEq	3	24	24.1	0.52	2.67	24.8
Autos	2	13	43.0	0.62	2.85	15.5
Aero	4	51	28.5	0.34	2.70	40.6
Guns†	0	0	–	–	–	–
Gold†	0	0	–	–	–	–
Ships	4	7	29.3	0.57	2.99	19.1
Mines	6	57	27.0	0.41	2.68	27.0
Coal	5	32	57.4	0.74	3.64	19.8
Oil	2	7	45.1	0.72	3.28	20.9
Util	1	1	25.1	0.83	3.87	22.0
Telcm	3	34	32.4	0.78	3.84	24.3
PerSv†	2	62	31.1	0.45	3.01	29.5
BusSv	2	7	30.4	0.52	2.85	21.4
Comps	3	77	32.4	0.42	3.01	46.9
Chips	3	50	41.1	0.62	3.84	35.0
LabEq	2	12	20.9	0.86	3.40	21.1
Paper†	3	19	22.9	0.66	2.82	19.0
Boxes	3	42	21.7	0.42	2.86	34.5
Trans	3	13	29.6	0.50	2.33	19.2
Whlsl	3	17	28.8	0.32	2.37	34.4
Rtail	5	72	23.1	0.51	3.26	39.8
Meals	2	19	45.2	0.48	3.14	31.7
Banks	3	48	23.5	0.39	3.06	27.3
Insur	2	3	39.9	0.76	3.00	13.3
RlEst	1	41	27.0	0.34	2.81	35.0
Fin	2	76	21.9	0.32	2.90	36.5
Other	2	5	25.6	0.48	2.94	29.6
Pooled	2	30	30.6	0.51	3.07	31.3

This table reports for each of the 48 industries in our sample the number of bubbles found, their average length, the average raw return and average standardized abnormal return during the bubble (in % per year), the number of times the investor detects the bubbles, and the average strength (t-statistic) at detection. An investor detects a bubble, if a 10-year series of industry returns shows evidence of an upward structural break in the constant, when regressing the industry returns on a constant and Carhart (1997)'s four factors. Moreover, after the structural break, the constant should be significantly larger than zero, and the associated t-statistic gives the strength of the bubble. A bubble has ended if a crash has occurred in the last 6 months, where a crash is defined as a residual below -2 its standard deviation. Critical values for the structural break test correspond with a 95% confidence level, and have been taken from Andrews (1993).

Table 11: Summary Statistics of Abnormal Returns based on the CAPM

industry	$\bar{\beta}$	mean	vol.	skew.	kurt.	min.	max.
Agric	0.93	0.01	1.00	0.41*	5.14	−3.39	5.57
Food	0.75	0.08*	1.10*	0.39*	5.72	−4.85	5.76
Soda†	0.83	0.04	1.12*	0.33*	5.93	−4.99	6.00
Beer	0.92	0.06	1.03	0.41*	7.04	−4.22	7.13
Smoke	0.67	0.09*	1.10*	−0.04	3.93	−4.98	3.89
Toys	1.21	−0.01	1.00	0.18*	6.04	−5.51	5.02
Fun	1.27	0.05	0.99	0.00	4.70	−5.14	3.60
Books	1.15	0.02	0.98	−0.20*	5.49	−6.16	3.31
Hshld	0.88	0.04	1.05*	−0.22*	6.65	−6.17	5.04
Clths	1.06	0.01	1.06*	0.18*	5.75	−4.94	4.67
Health†	1.29	−0.01	0.98	−0.58*	5.90	−4.83	2.94
MedEq	0.95	0.05	0.98	0.09	4.35	−3.95	5.24
Drugs	0.90	0.08*	1.06*	0.05	6.26	−5.58	6.31
Chems	1.02	−0.03	1.05*	0.29*	5.27	−4.07	6.24
Rubbr†	1.04	0.02	1.10*	0.03	6.19	−5.53	5.32
Txtls	1.10	−0.01	1.05*	0.04	5.07	−3.88	4.98
BldMt	1.14	−0.03	1.05*	−0.08	4.60	−4.65	4.20
Cnstr	1.37	−0.02	1.01	0.63*	5.23	−3.58	5.63
Steel	1.25	−0.05	1.05*	0.67*	5.99	−3.16	7.07
FabPr†	1.06	−0.07	1.07	0.15	4.41	−3.59	4.98
Mach	1.17	−0.04	1.04	0.31*	3.70	−3.23	3.73
ElcEq	1.21	0.03	1.01	0.10	3.57	−4.01	4.16
Autos	1.10	−0.01	1.04	0.01	6.21	−6.41	5.26
Aero	1.15	0.04	1.01	0.09	5.11	−4.64	3.79
Guns†	0.90	0.06	1.12*	−0.79*	8.83	−7.17	4.46
Gold†	0.68	0.03	1.12*	1.31*	10.61	−2.90	8.48
Ships	1.11	−0.02	1.05*	0.76*	6.81	−3.66	6.62
Mines	0.96	0.01	1.05*	0.23*	3.78	−3.53	4.08
Coal	0.99	0.05	1.12*	0.94*	8.07	−4.06	8.15
Oil	0.88	0.06	1.05*	0.29*	4.09	−3.60	4.27
Util	0.67	0.04	1.04	0.09	3.73	−3.35	3.80
Telcm	0.64	0.08*	1.09*	1.04*	10.12	−3.99	9.37
PerSv†	1.10	−0.02	1.00	0.03	4.51	−4.27	3.67
BusSv	1.08	0.04	0.99	0.44*	4.79	−3.24	4.33
Comps	1.13	0.05	1.06*	0.33*	4.60	−4.13	4.55
Chips	1.34	0.00	1.02	0.08	4.29	−4.66	5.04
LabEq	1.12	0.03	1.04	0.25*	4.82	−3.97	5.30
Paper†	1.12	−0.01	1.03	0.54*	5.44	−3.39	5.97
Boxes	0.98	0.03	1.05*	−0.18*	3.81	−4.07	3.40
Trans	1.13	−0.03	1.02	0.49*	4.94	−3.39	5.45
Whshl	1.10	−0.01	1.01	0.03	6.66	−6.27	5.21
Rtail	0.96	0.04	1.05*	−0.15	4.32	−4.06	4.55
Meals	1.05	0.05	1.02	0.12	4.69	−4.73	3.90
Banks	0.96	0.06	0.98	−0.15	5.28	−4.84	4.33
Insur	0.96	0.03	1.02	0.08	6.26	−5.83	5.56
RlEst	1.20	−0.06	1.03	0.44*	7.09	−4.82	5.63
Fin	1.18	0.04	1.03	−0.02	4.33	−4.09	4.02
Other	1.12	−0.06	1.08*	−0.01	4.50	−4.48	4.42
Pooled	1.04	0.02*	1.04*	0.22*	5.52	−7.17	9.37

This table reports the results of the rolling regressions of the market model in Eq. (1) with a 120-month estimation window. For each regression, we construct an abnormal return for the period after the estimation window as in Eq. (4). To correct for time-varying volatility, we standardize the abnormal return by a division by the residual volatility of the regression model. A dagger after an industry name indicates that less observations are available. An asterisk denotes a significant difference from zero in case of means and skewness coefficients, and a significant difference from one in case of volatility, all at a 5% confidence level. Standard errors of the skewness coefficient are calculated as $\sqrt{6/T}$ (see Tabachnick and Fidell, 2001).

Table 12: Summary Statistics of Abnormal Returns based on Fama and French (1993)

industry	$\bar{\beta}_{RM}$	$\bar{\beta}_{SMB}$	$\bar{\beta}_{HML}$	mean	vol.	skew.	kurt.	min.	max.
Agric	0.88	0.50	−0.06	0.00	1.01	0.62*	5.79	−3.26	6.20
Food	0.79	0.01	−0.03	0.08*	1.12*	0.44*	5.56	−4.81	5.73
Soda†	0.86	0.25	0.13	0.05	1.14*	0.60*	6.67	−4.32	6.40
Beer	0.90	0.29	−0.01	0.05	1.07*	0.29*	6.52	−3.95	7.09
Smoke	0.73	−0.05	0.03	0.08*	1.12*	−0.11	4.08	−5.56	3.92
Toys	1.08	0.84	−0.20	−0.02	1.01	0.17*	5.72	−5.57	5.29
Fun	1.18	0.58	−0.05	0.04	1.01	−0.17*	6.13	−6.77	3.78
Books	1.06	0.52	−0.05	0.02	1.00	−0.26*	5.64	−6.63	2.96
Hshld	0.90	0.06	−0.22	0.07	1.09*	−0.60*	12.60	−9.70	5.30
Clths	0.96	0.59	0.16	−0.04	1.11*	0.17*	7.13	−5.66	5.85
Health†	1.04	0.94	−0.28	−0.04	1.07	−1.09*	9.28	−7.06	3.05
MedEq	0.86	0.20	−0.29	0.09*	1.03	0.02	4.69	−4.88	5.23
Drugs	0.91	−0.13	−0.43	0.14*	1.09*	0.03	4.98	−5.19	5.01
Chems	1.12	−0.12	0.04	−0.06	1.06*	0.27*	5.04	−3.95	5.95
Rubbr†	0.95	0.70	0.14	−0.04	1.12*	0.14	4.98	−4.34	4.87
Txtls	1.05	0.71	0.39	−0.08*	1.08*	0.27*	5.34	−4.19	6.29
BldMt	1.15	0.31	0.01	−0.08*	1.08*	0.07	4.31	−4.56	4.50
Cnstr	1.25	0.70	0.22	−0.06	1.02	0.43*	5.06	−3.76	5.74
Steel	1.21	0.26	0.51	−0.12*	1.09*	0.52*	5.47	−3.87	6.79
FabPr†	1.02	0.60	0.07	−0.10	1.09*	0.10	4.49	−3.81	5.09
Mach	1.14	0.31	0.15	−0.08*	1.05*	0.27*	3.82	−3.58	4.42
ElcEq	1.19	0.08	−0.20	0.05	1.03	0.09	3.27	−3.57	4.26
Autos	1.19	0.11	0.32	−0.07*	1.06*	0.12	5.26	−5.42	5.41
Aero	1.05	0.57	0.18	0.02	1.03	0.13	4.76	−4.98	4.18
Guns†	0.95	0.24	0.36	0.00	1.14*	−0.50*	8.84	−7.27	5.68
Gold†	0.69	0.56	0.26	0.01	1.13*	1.49*	12.08	−3.37	8.84
Ships	1.04	0.35	0.38	−0.04	1.09*	0.74*	6.00	−4.05	6.20
Mines	0.94	0.42	0.27	−0.02	1.05*	0.21*	3.80	−3.63	4.58
Coal	0.86	0.35	0.39	0.04	1.14*	0.90*	7.37	−4.15	7.52
Oil	0.96	−0.38	0.30	0.03	1.07*	0.13	3.96	−3.56	4.07
Util	0.78	−0.11	0.21	−0.01	1.06*	0.03	3.98	−3.64	3.86
Telcm	0.70	−0.12	0.04	0.08*	1.11*	1.09*	10.80	−3.71	9.77
PerSv†	0.96	0.75	−0.08	−0.03	1.02	−0.23*	5.74	−5.75	3.61
BusSv	0.89	0.49	−0.20	0.09*	1.02	0.41*	4.89	−3.67	5.04
Comps	1.06	0.07	−0.49	0.11*	1.06*	0.36*	4.47	−3.98	4.54
Chips	1.22	0.38	−0.30	0.02	1.05*	0.01	3.81	−4.46	4.19
LabEq	1.05	0.31	−0.46	0.07	1.05*	0.21*	4.25	−3.76	4.40
Paper†	1.18	0.14	0.24	−0.06	1.04	0.40*	5.12	−3.55	5.62
Boxes	1.00	0.01	−0.07	0.04	1.08*	−0.22*	3.94	−4.20	3.60
Trans	1.04	0.25	0.53	−0.10*	1.06*	0.13	4.20	−4.37	4.94
Whshl	1.04	0.51	0.00	−0.04	1.03	−0.14	6.57	−5.52	5.11
Rtail	0.98	0.17	−0.15	0.05	1.08*	−0.13	4.54	−4.54	4.83
Meals	0.96	0.59	−0.03	0.04	1.05*	0.05	4.43	−4.96	4.11
Banks	1.07	0.13	0.05	0.04	1.00	0.28*	4.15	−3.81	4.26
Insur	1.05	−0.09	−0.01	0.02	1.02	0.27*	4.31	−3.75	4.49
RlEst	1.07	1.05	0.25	−0.13*	1.04	0.53*	6.95	−4.79	5.67
Fin	1.18	0.17	0.29	−0.01	1.04	0.08	3.99	−4.00	3.71
Other	1.06	0.48	−0.09	−0.07	1.15*	0.20*	5.38	−4.72	5.98
Pooled	1.01	0.31	0.04	0.00	1.07*	0.20*	5.60	−9.70	9.77

This table reports the results of the rolling regressions of Fama and French (1993)'s three factor model in Eq. (1) with a 120-month estimation window. For each regression, we construct an abnormal return for the period after the estimation window as in Eq. (4). To correct for time-varying volatility, we standardize the abnormal return by a division by the residual volatility of the regression model. A dagger after an industry name indicates that less observations are available. An asterisk denotes a significant difference from zero in case of means and skewness coefficients, and a significant difference from one in case of volatility, all at a 5% confidence level. Standard errors of the skewness coefficient are calculated as $\sqrt{6/T}$ (see Tabachnick and Fidell, 2001).

Table 13: Summary Statistics of Abnormal Returns based on Carhart (1997)

industry	$\bar{\beta}_{\text{RM}}$	$\bar{\beta}_{\text{SMB}}$	$\bar{\beta}_{\text{HML}}$	$\bar{\beta}_{\text{UMD}}$	mean	vol.	skew.	kurt.	min.	max.
Agric	0.88	0.51	−0.04	0.05	−0.02	1.02	0.61*	5.82	−3.22	6.27
Food	0.79	0.01	−0.02	0.00	0.10*	1.14*	0.51*	5.03	−3.49	5.61
Soda†	0.86	0.27	0.12	0.00	0.06	1.16*	0.61*	6.59	−4.42	6.49
Beer	0.91	0.29	0.02	0.10	0.04	1.08*	0.31*	6.49	−3.90	7.27
Smoke	0.73	−0.05	0.01	−0.06	0.10*	1.14*	−0.07	3.92	−5.45	3.89
Toys	1.09	0.86	−0.16	0.07	−0.02	1.02	0.21*	5.22	−4.97	5.16
Fun	1.18	0.60	−0.04	0.08	0.02	1.03	−0.25*	6.75	−7.38	3.79
Books	1.06	0.51	−0.08	−0.04	0.04	1.03	−0.16	5.37	−6.61	3.43
Hshld	0.90	0.08	−0.20	0.05	0.06	1.11*	−0.57*	13.31	−10.04	5.42
Clths	0.96	0.56	0.14	−0.15	0.01	1.12*	0.36*	6.39	−5.36	6.10
Health†	1.05	0.95	−0.27	0.08	−0.05	1.09*	−0.85*	7.50	−6.05	3.16
MedEq	0.87	0.21	−0.28	0.06	0.08*	1.04	0.03	4.79	−4.71	5.27
Drugs	0.91	−0.11	−0.43	0.05	0.12*	1.11*	−0.03	4.99	−5.22	5.17
Chems	1.11	−0.12	0.02	−0.06	−0.04	1.09*	0.13	4.89	−4.30	5.28
Rubbr†	0.95	0.71	0.15	0.04	−0.04	1.14*	0.10	4.91	−4.24	4.62
Txtls	1.05	0.69	0.38	−0.10	−0.05	1.09*	0.34*	5.00	−3.66	6.22
BldMt	1.15	0.30	0.01	−0.03	−0.06	1.09*	0.15	4.45	−4.45	4.69
Cnstr	1.26	0.69	0.21	−0.04	−0.05	1.03	0.45*	4.97	−3.56	5.72
Steel	1.20	0.26	0.48	−0.10	−0.10*	1.11*	0.49*	5.16	−3.77	6.30
FabPr†	1.01	0.63	0.04	−0.06	−0.10	1.09*	0.22	3.88	−3.87	4.46
Mach	1.13	0.30	0.14	−0.08	−0.05	1.06*	0.28*	3.74	−3.57	4.21
ElcEq	1.19	0.08	−0.19	−0.01	0.04	1.04	0.06	3.52	−4.44	4.54
Autos	1.18	0.07	0.29	−0.18	−0.01	1.06*	0.02	5.35	−5.78	5.33
Aero	1.05	0.60	0.21	0.08	0.00	1.04	0.12	4.70	−4.58	4.28
Guns†	0.95	0.24	0.35	−0.04	0.01	1.15*	−0.39*	8.52	−7.21	5.84
Gold†	0.71	0.62	0.27	0.13	−0.01	1.14*	1.39*	11.68	−3.66	8.78
Ships	1.04	0.35	0.37	−0.02	−0.04	1.11*	0.70*	6.56	−4.78	6.35
Mines	0.94	0.42	0.25	−0.07	−0.02	1.06*	0.18*	3.72	−3.75	4.56
Coal	0.87	0.33	0.38	−0.02	0.05	1.15*	0.86*	7.12	−4.24	7.49
Oil	0.96	−0.38	0.30	0.04	0.02	1.10*	0.06	3.77	−3.58	4.04
Util	0.77	−0.11	0.20	−0.02	0.00	1.07*	0.06	3.99	−3.90	3.72
Telcm	0.69	−0.12	0.03	−0.06	0.10*	1.13*	1.02*	10.17	−3.69	9.68
PerSv†	0.97	0.72	−0.08	−0.06	−0.01	1.03	−0.21*	5.47	−5.28	3.67
BusSv	0.89	0.51	−0.19	0.03	0.08*	1.03	0.40*	5.40	−4.95	5.28
Comps	1.07	0.08	−0.49	0.00	0.11*	1.07*	0.42*	4.32	−3.59	4.69
Chips	1.22	0.38	−0.31	−0.04	0.04	1.06*	0.02	3.88	−4.48	4.32
LabEq	1.06	0.34	−0.44	0.06	0.05	1.06*	0.29*	4.55	−3.77	4.66
Paper†	1.18	0.13	0.22	−0.06	−0.03	1.06*	0.30*	5.29	−5.20	4.90
Boxes	0.99	0.03	−0.08	−0.03	0.04	1.10*	−0.17*	4.03	−4.49	4.60
Trans	1.03	0.24	0.51	−0.07	−0.07*	1.07*	0.12	4.09	−4.20	4.78
Whshl	1.04	0.52	0.01	0.05	−0.04	1.05	−0.06	6.28	−5.43	5.25
Rtail	0.97	0.16	−0.18	−0.10	0.10*	1.09*	−0.11	4.35	−4.92	4.48
Meals	0.96	0.60	−0.04	−0.05	0.06	1.06*	0.14	4.12	−4.88	4.07
Banks	1.06	0.12	0.03	−0.11	0.08*	1.00	0.25*	3.87	−3.45	3.92
Insur	1.05	−0.10	−0.02	−0.05	0.03	1.03	0.30*	4.22	−3.77	4.46
RlEst	1.07	1.07	0.24	0.03	−0.13*	1.04	0.49*	6.77	−4.78	5.72
Fin	1.17	0.16	0.26	−0.06	0.02	1.06*	0.08	4.06	−4.12	3.64
Other	1.06	0.47	−0.09	0.00	−0.06	1.16*	0.18*	5.23	−4.71	5.93
Pooled	1.01	0.31	0.04	−0.02	0.01*	1.08*	0.21*	5.50	−10.04	9.68

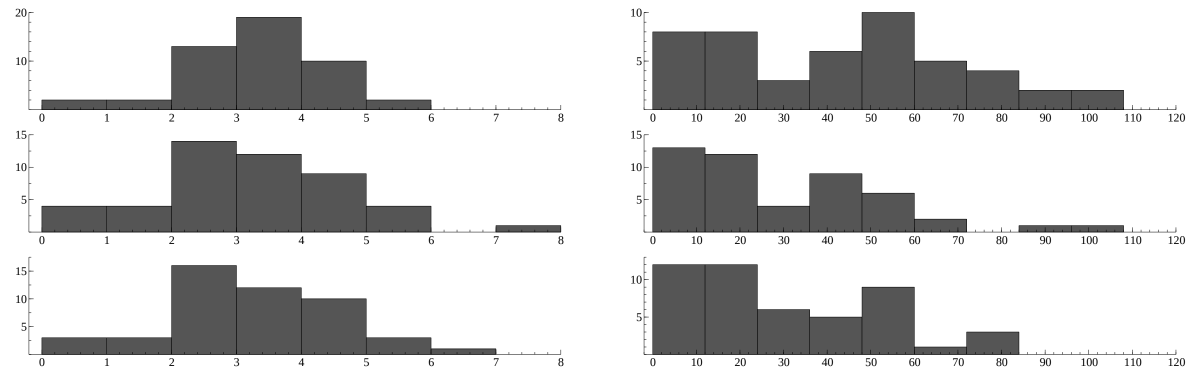
This table reports the results of the rolling regressions of Fama and French (1993)'s three factor model in Eq. (1) with a 120-month estimation window. For each regression, we construct an abnormal return for the period after the estimation window as in Eq. (4). To correct for time-varying volatility, we standardize the abnormal return by a division by the residual volatility of the regression model. A dagger after an industry name indicates that less observations are available. An asterisk denotes a significant difference from zero in case of means and skewness coefficients, and a significant difference from one in case of volatility, all at a 5% confidence level. Standard errors of the skewness coefficient are calculated as $\sqrt{6/T}$ (see Tabachnick and Fidell, 2001).

Table 14: Standardized abnormal returns per industry after the detection of a bubble

	CAPM	3F	4F
Agric	−0.26	–	–
Food	0.52	0.51	0.44
Soda†	0.11	−0.20	0.06
Beer	0.22	0.21	0.03
Smoke	0.48	0.46	0.42
Toys	0.25	0.52	0.44
Fun	0.13	−0.76	−0.69
Books	0.16	0.20	0.17
Hshld	0.18	0.23	0.09
Clths	0.04	−0.23	−0.22
Health†	0.05	−0.05	−0.24
MedEq	0.21	0.21	0.20
Drugs	0.29	0.20	0.22
Chems	−0.05	−0.43	−0.43
Rubbr†	−0.23	−0.76	−1.15
Txtls	0.05	0.14	0.18
BldMt	0.14	−0.26	−0.20
Cnstr	0.15	−0.22	−0.57
Steel	0.46	0.11	0.40
FabPr†	–	–	−0.03
Mach	−0.24	−0.01	−0.27
ElcEq	0.16	−0.01	−0.15
Autos	−0.33	−0.71	−0.30
Aero	0.16	0.01	0.13
Guns†	0.20	–	–
Gold†	−1.30	−0.78	–
Ships	0.00	−1.68	−1.48
Mines	0.20	0.03	−0.14
Coal	0.50	0.12	0.11
Oil	−0.48	0.62	0.31
Util	–	–	0.31
Telcm	0.79	0.01	0.16
PerSv†	0.22	0.15	0.23
BusSv	0.30	−0.29	−0.16
Comps	0.04	0.20	0.16
Chips	0.19	0.40	0.48
LabEq	−0.12	−0.21	−0.19
Paper†	0.22	0.08	−0.34
Boxes	0.15	0.19	0.32
Trans	0.26	−0.50	−0.15
Whshl	0.01	−0.38	−0.67
Rtail	0.33	0.18	0.21
Meals	0.14	0.01	0.06
Banks	0.16	−0.11	−0.01
Insur	−0.08	−0.34	−0.31
RlEst	0.18	0.14	0.16
Fin	0.24	0.21	0.26
Other	−0.73	−2.07	−2.00
Pooled	0.19	0.10	0.09

This table report the standardized average abnormal returns per industry after the detection of a bubble. In the bubble detection procedure and the construction of the abnormal returns, we use the CAPM (column CAPM), Fama and French (1993)'s three factor model (column 3F) or Carhart (1997)'s four factor model (column 4F).

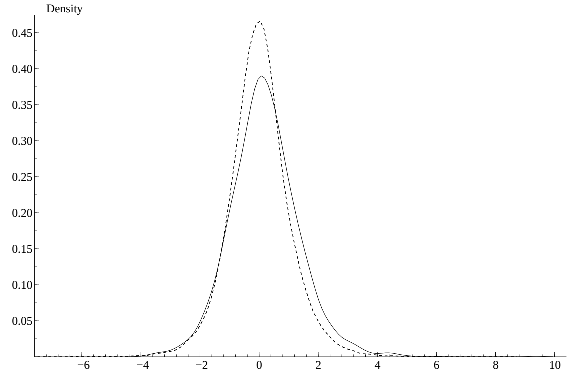
Figure 1: Number of bubbles and bubble detections per industries



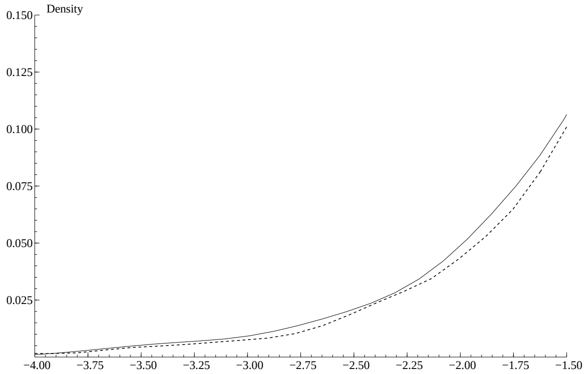
(a) bubbles (b) detections

This figure shows how bubbles and bubble detections are distributed over the 48 industries. An investor detects a bubble, if a 10-year series of industry returns shows evidence of an upward structural break in the constant, when regressing the industry returns on a constant and the market return (top histograms), Fama and French (1993)'s three factors (middle histogram), or Carhart (1997)'s four factors (bottom histograms). Moreover, after the structural break, the constant should be significantly larger than zero. A bubble has ended if a crash has occurred in the last 6 months, where a crash is defined as a residual below -2 times its standard deviation. Critical values for the structural break test correspond with a 97.5% confidence level, and have been taken from Andrews (1993). We count the unbroken sequence of months belonging to the subperiod after the structural break as one bubble.

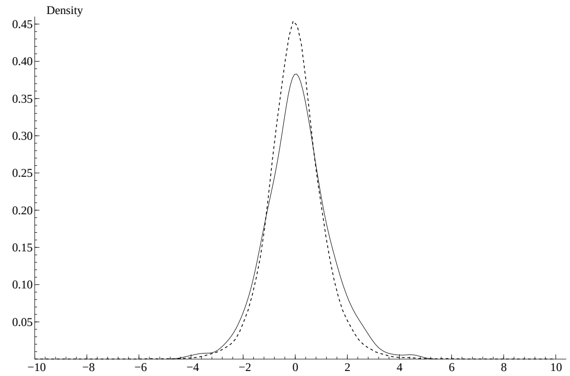
Figure 2: Distribution of standardized abnormal returns with and without prior bubble detection.



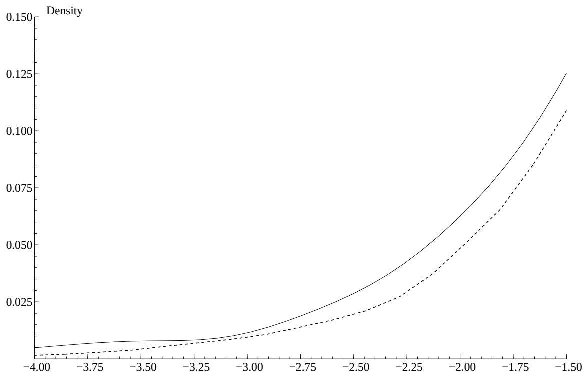
(a) whole distribution, CAPM



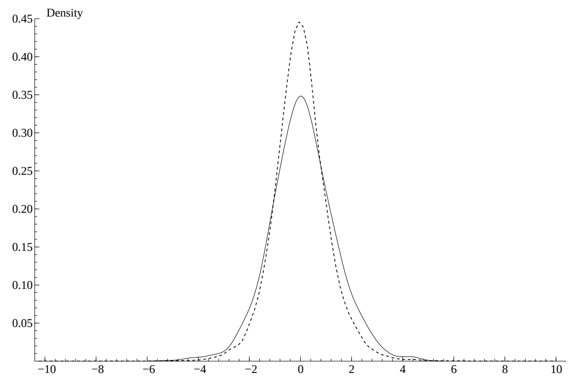
(b) left tail, CAPM



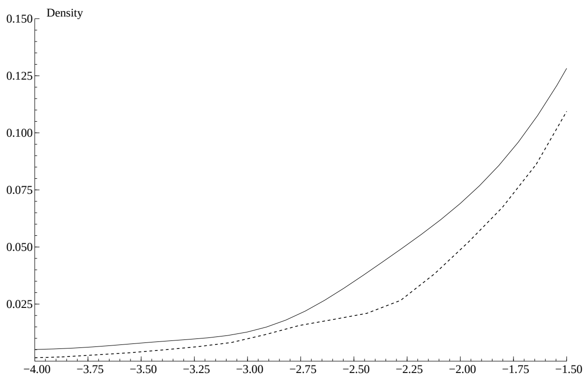
(c) whole distribution, Fama and French (1993)



(d) left tail, Fama and French (1993)



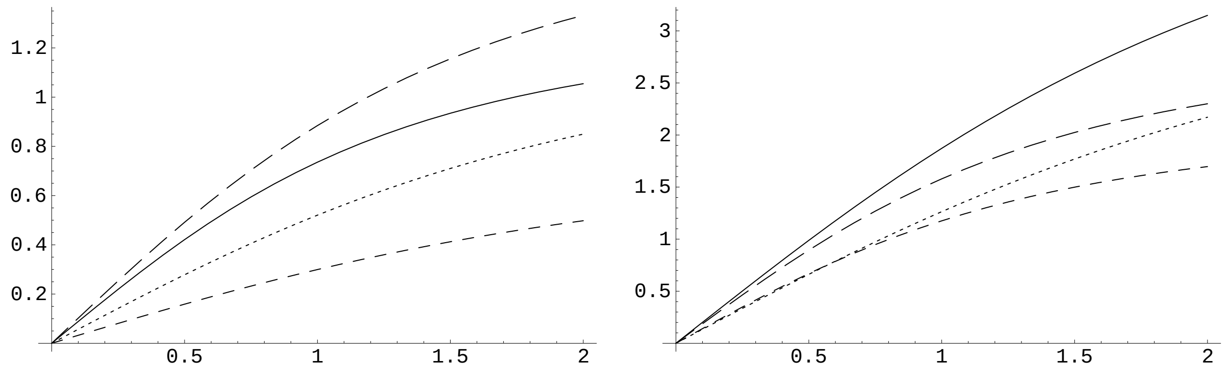
(e) whole distribution, Carhart (1997)



(f) left tail, Carhart (1997)

This figure shows the density of the standardized abnormal returns, based on rolling regressions of the CAPM (panels a and b), Fama and French (1993)'s three factor model (panels c and d) and Carhart (1997)'s four factor model (panels e and f) in Eq. (1) with a 120-month estimation window. We split the abnormal returns according to the detection of a bubble. The solid (dashed) line gives the density for the abnormal returns for which a (no) preceding bubble has been detected. We show the complete density (panels a, c and e) and the density in the left tail (panels b, d and f). The densities are constructed with the Ox function `DrawDensity`, which smoothes the graphs based on kernel density estimation with a Gaussian kernel (see Doornik, 1998, p. 221).

Figure 3: Effect of misspecification



(a) long-run factor moments (b) maximum factor average

This figure shows how the asymptotic bias in the Wald statistic (y-axis) for a structural break in the intercept is affected by the size of the structural break (x-axis) in a factor exposure in the true model. We assume that the true model is given by

$$r_\tau = \alpha + \beta_\tau f_{1\tau} + \gamma' \tilde{f}_\tau + \varepsilon_\tau,\ \mathrm{E}[\varepsilon_\tau],\ \mathrm{E}[\varepsilon_\tau^2] = \sigma^2 \quad \tau = t-T+1,\dots,t$$

$$\beta_\tau = \begin{cases} \beta_1 & \text{for } \tau = t-T+1,\dots,t-\zeta \\ \beta_2 & \text{for } \tau = t-\zeta+1,\dots,t \end{cases},$$

while the estimated model is given by

$$r_\tau = \alpha_\tau + \beta f_{1\tau} + \gamma' \tilde{f}_\tau + e_\tau,\ \mathrm{E}[e_\tau], \quad \tau = t-T+1,\dots,t$$

$$\alpha_\tau = \begin{cases} \alpha_1 & \text{for } \tau = t-T+1,\dots,t-\zeta \\ \alpha_2 & \text{for } \tau = t-\zeta+1,\dots,t \end{cases}.$$

We plot the expected value of the Wald statistic on the difference $\alpha_2 - \alpha_1$

$$\chi = \frac{\sqrt{T}\Delta\,\mathrm{E}[f_{1\tau}]}{\sqrt{\Delta^2\,\mathrm{Var}[f_{1\tau}] + \frac{1}{\xi(1-\xi)}\sigma^2}},$$

as a function of the size of the true structural break $\Delta = \beta_2 - \beta_1$. Panel a shows this relation for the long-run factor moments: CAPM, solid ($\mu = 0.65\%$, $\sigma = 5.43\%$); SMB, dashed ($\mu = 0.24\%$, $\sigma = 3.35\%$), HML, dotted ($\mu = 0.42\%$, $\sigma = 3.58\%$) and MOM, long dashed ($\mu = 0.75$, $\sigma = 4.67\%$). In panel b, we use the largest 10-year average in our sample: CAPM, solid ($\mu = 1.47\%$, $\sigma = 3.20\%$); SMB, dashed ($\mu = 1.03\%$, $\sigma = 5.32\%$), HML, dotted ($\mu = 0.98\%$, $\sigma = 2.93\%$) and MOM, long dashed ($\mu = 1.37$, $\sigma = 5.18\%$). We have $\xi = 1/2$, $T = 120$ and $\sigma^2 = 4\%$.