The Capital Structure of Financial Institutions and Liquidity Crisis*

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Abstract

In this paper, we present a model that relates the capital structure of modern financial institutions to the recent liquidity crisis. In the model, a shock to market liquidity of a large lender's asset portfolio potentially leads to the lender's own risk management constraint binding, and hence induces the withdrawal of funds from a borrower financial institution. Other lenders realize this possibility and the fear causes them to run. Global game technique enables us to derive the unique equilibrium. Our main findings include: 1) Market liquidity shock suffices for a large lender to withdraw funds. This withdrawal is not due to asymmetric information with regards to the borrower's quality, or to financial contagion. 2) The potential withdrawal by a large lender has amplifying effects. It causes smaller lenders to run first and makes the borrower institution particularly vulnerable. 3) If market liquidity in different markets is correlated, a bank run hits a leveraged financial institution from both the asset and the liability channels. The two channels form a joint force that dramatically increases the probability of collapse of levered financial institutions. 4) The presence of a large lender is an element of stability in good times, but it increases fragility in bad times. We also discuss broad issues raised by this research and the related policy implications of the model.

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"In spite of the television images of long lines of depositors outside its branch offices, the run on Northern Rock was unlike the textbook retail depositor run caused by coordination failure. Also, contrary to received wisdom, its reliance on securitization was not an immediate factor in its failure. Rather, its problems stemmed from its high leverage coupled with reliance on institutional investors for short term funding."

Shin (2008)

"Regulators used to worry about the danger hedge funds might pose to their prime brokers...the risk turned out to be the other way round... The [hedge fund] industry's aggregate leverage has undoubtedly caused it trouble...But there does not appear to have been a systematic withdrawal of bank credit from hedge funds. ... A fuller explanation must include the increasingly jittery nature of hedge funds clients."

The *Economist*, Oct 25th-31st 2008, page 86.

1 Introduction

During 2007 and 2008 the global financial system experienced a severe crisis. Many banks in the USA and in Europe, some of them major players in the industry, either went bankrupt, were taken over, or were rescued by governments. By October 2008, the losses of the banking sector were estimated to have reached $1,000 billion in the USA, and the UK government took significant steps towards nationalization of the banks. In the meantime, the hedge fund industry suffered an incredible shrinkage. Its assets under management fell dramatically. The number of hedge funds, which climbed to over 7,000 at the peak, is estimated to have fallen by half. The financial crisis had its origins in the subprime lending of the housing market, but rapidly spread to every other segment of the credit markets.

This paper presents a theoretical model that explains the crisis in light of the capital structure of modern financial institutions. Our model stresses two recent developments in the balance sheet of financial institutions: the increasingly reliance on leverage, and the changes that occurred in the structure of lenders, with large financial institutional investors occupying a greater role as debtholders. In the last two decades, a greater number of commercial banks have sourced ever greater amounts of funds in the wholesale money market, a market dominated by large institutional investors. Similarly, hedge funds have a close relation with prime brokers at investment banks, which are their most important source of credit, as well as executioners of trades. Hedge funds experienced a very fast growth of assets under management since the previous financial bust of 2001 and 2002. The symbiotic relationship of large institutional investors is indeed a feature of the modern financial markets. Such tight interdependence of large players has both advantages and disadvantages. The disadvantage that we explore in this paper is the following: when a large creditor of a financial institution suffers a shock, it is forced to delever and this results in a sudden withdrawal of credit from a borrower financial institution. In many instances, the mere suspicion that a large creditor is in trouble can trigger a panic by other lenders, who then desert the financial institution and force its rapid collapse. As Shin (2008) noted: "The problems [of Northern Rock] stemmed from its high leverage couple with a reliance on institutional investors for short term funding".

In our model, the failure of a bank happens not because of the deterioration of the bank's portfolio of assets (although this can accelerate its demise), but because of problems affecting one of its large creditors. It is not the lack of confidence in the banks' assets that is at the origin of the problem, but the structure of the banks' creditors with some large and influential creditors establishing their influence. These large and influential creditors, in response to unfolding adverse shocks, may decide to withdraw funds from the bank and thus trigger a run by smaller lenders. Put slightly differently, it is the actions taken by the large creditors that lead to the run of other investors. This is different from a traditional bank run, an accident that results from miscoordination between numerous small lenders.

We believe that the above mechanism can explain many runs in the recent crisis. That runs occurred during 2008 is clearly evidenced by the fact that numerous depositors have withdrew funds from more vulnerable banks and distributed them across different accounts and different banks. Although there were concerns about the quality of the bank's assets, this was not the main reason. Instead, investors worried about the liquidity status of banks, and that liquidity shortages could then force banks into precipitous asset sales at deep discounts. The runs are also evidenced by the hasty increase in deposit insurance amount by many governments. In contrast, the hedge fund industry, without the protection of government guarantees, has suffered severe runs even by long-horizon investors and been forced to conduct fire sales of assets. The large scale of sales by hedge funds largely accounts for the stock market crash in October 2008. Lori Calvasina, Citigroup's strategist in New York, commenting on the stock market fall of October, said :"You had the fundamental reasons of commodity prices falling and lingering concerns about a recession, but the swiftness and the severity came from hedge funds unloading massive stakes." [1]

In this paper, a large lender's withdrawal of credit from a financial institution (a bank or a hedge fund) is due to the lender's own risk management requirement. Sophisticated investors carefully and methodically adjust the risk profile of their positions in adapting to evolving market conditions. When the market experiences an adverse shock, investors become more risk averse and as a result market liquidity becomes low. The probability of mark-to-market losses in asset values, particularly in long-term assets, increases. That is, the risk of mark-to-market leverage increases, which has profound consequences on the ratings of financial operators. In response to unfolding events, the prudent and precautious lender starts to delever and liquidate positions, such as credit extended to other institutions.

This explanation of a large lender's withdrawal is different from other explanations previously offered. One explanation relies on asymmetrical information: if lenders are not certain about the borrower's asset quality and anticipate a deterioration, they rationally deny extending credit and even rush to get out. While this mechanism can explain many situations of inter-banking borrowing during a crisis, it cannot explain well many other instances, namely when concern over the quality of a banks' assets is not the main issue.[2]

The second explanation relies on financial contagion, in the spirit of Allen and Gale (2000)

[1] From Oct. 31 (Bloomberg): Small-Cap Stocks Trail S&P 500 by Most in Six Years: http://www.bloomberg.com/apps/news?sid=aqAeiu7RNS0Y&pid=20601213.

[2] Shin (2008) concludes Northern Rock's status as "with an apparently solid asset book, with virtually no subprime lending". Careful observation would conclude that in the US the crisis had its origins in the housing market, and then spread to the financial sector, while the direction was the opposite in the UK, where it was the liquidity shortage in the financial sector that triggered the problem of the housing market. There is no evidence that shows that there was a systematic quality problem with Northern Rock's mortgage loans. In sum, the run on Northern Rock stems from the liability side of the balance sheet rather than from the asset side.

and Dasgupta (2004). The argument is intuitive. Suppose that lenders incur in some loss in other investments or other places. Then, the micro-prudential action for the lender is to reduce its overall lending, including its lending to some creditworthy clients. Certainly, this is true in many situations in reality, particularly at the later stage of a liquidity crisis. But we would argue this is not the main reason for the run at earlier stages of crisis, as the case of Northern Rock shows. By mid 2007, when Northern Rock was experiencing drastic liquidity problems, few banks all around the world were, at that moment, recognizing significantly losses. It was not until later that a large scale of write-downs begun.

The first part of our model - modelling why a large lender suddenly withdraws credit - is close to Shin (2008). We identify that the change in market liquidity is potentially a fundamental source driving a lender's decisions to adjust its balance sheet. In the second part of the model, using global game techniques, we formalize and quantify the notion that the small investors' fear and jitter, rather than the large lender's actual withdrawal, is sufficient to cause a bank run. Such jitters are reflected in the quote in The Economist at the beginning of this paper.

Our research draws a number of important conclusions. The first is that relying on large lenders to fund financial institutions is a double-edge sword. Due to the size effect of 'self-coordination', large lenders can act as agents of stability if, for some reason, small lenders consider a run on a financial institution. The simple fact that large lenders decide not to run is a sufficient deterrent for small lenders not to run. The threshold for coordination not to run is therefore lower in the presence of large lenders. Our paper, however, shows that large lenders have a downside from the possibility that they themselves are subject to liquidity shocks that can force the rapid deleveraging of financial institutions. Since this is more likely to happen during a period of market upheaval, we can conclude that financial institutions that rely more heavily on wholesale funding and include in their structure of borrowing large lenders, are more secure in good times but also more fragile in bad times. The case of Northern Rock, a bank which sourced its main funds from the wholesale market, contrasts with that of another UK bank, HSBC, which relies on numerous branches to secure deposits, is an interesting example in this respect which is worth further study. This asymmetry of risks in good and bad times dictated by the presence of large lenders in the balance sheet has direct implications to the capital structure of levered financial institutions and their capital requirements.

The second conclusion is that the architecture of the current financial system favours vicious cycles during a crisis. We argue that the great danger of the presence of large financial institutional lenders on banks' and hedge funds' balance sheets is that it scares away spare-liquidity providers and long-term investors. The run on hedge funds is particularly harmful because it can cause a downward spiral to the whole system. The social role of hedge funds is to provide market liquidity, and by their arbitraging activities a reliable price signal necessary to allocate financial resources exists (Gromb and Vayanos (2002, 2008)). A run on hedge funds has two adverse impacts. One, is that in the absence of arbitrage trading, reliable prices disappear, and the fundamental value of banks, especially those that were affected by the subprime mortgage lending, becomes quite fuzzy. Therefore, investors such as sovereign wealth funds, with resources to allocate but lacking valuation expertise, dare not to step in and inject new capital, even if they would like to do so. The dry up of capital in the banking sector, in turn, causes the role of liquidity provision difficult to resume, and thus the recovery in market liquidity is hard. The other, is that the deterioration of market liquidity causes even healthy banks' balance sheet to tighten, and this causes further withdrawals from hedge funds. Thus, a new round of vicious cycle forms.

The chronology of the credit crisis of 2007 well demonstrates the above process. During the early stages of the crisis, many financial institutions resorted to wealthy investors to shore up their capital. Among these were large sovereign wealth funds. What happen subsequently showed that many of these investors, although they had the resources, they did not have the skills to correctly assess the situation and ended up by suffering significant losses. This has tempered their interest in committing further investments. The reason for this is simple: investors, such as the China Investment Corporation (CIC), base their decisions on the soundness of the investments. However, in the absence of stable prices that can reliably reflect fundamental values, it is hard for such investors to commit capital.

The third conclusion is that the collapses of so many financial intuitions was not just an accident. On the contrary, we argue that the vulnerability of financial institutions is a necessary result of the existing financial architecture. Why? Morris and Shin (2008) argue that two factors determine the probability of a bank run: First, the threshold for coordination not to run, and, second, the cost of miscoordination. We argue that the correlated deterioration of market liquidity during crises, plus the increasing importance of large institutional investors as lenders, lead to runs on financial institutions through both channels. For the second channel, if the market liquidity in the asset of the borrower institution is low, a fire sale can depress the price quite a lot. So, the lender who does not run is left with little liquidation value with a high probability. Therefore, the cost of miscoordination is high. For the first channel, if the market liquidity of the assets held by wholesale large lenders deteriorates, the lenders face with a high probability a balance sheet constraint and consequently decide to delever. The withdrawal of funds by large lenders can be anticipated by small investors, and this increases the threshold for small investors to coordinate not to run.

Finally, our model has several implications for financial regulation. In modern capital markets, it is important to consider not just the risk of the assets of a financial institution, but also the risk that stems both from the level of indebtness and from the structure of creditors. Banks that depend in a significant way on funds from the wholesale markets rather than deposits, must be closely monitored, and they may need to satisfy quite different risk control requirements. Financial regulators need to decide whether limits on leverage combined with limits on the relative share of lending by large lenders, when these are financial investors, are a more appropriate course of action than what exists currently. Particularly, financial regulators need to have a systematic view and monitoring may include both the borrowers' behavior and the lenders' actions, since what happens to these lenders may drastically affect not just the borrower institution, but also the actions of the other lenders.

Brunnermeier (2008) deciphers the recent liquidity crisis and touches some of the issues raised by this paper. Particularly, two out of the four amplifying mechanisms outlined by Brunnermeier - lending channel and runs on financial institutions - are addressed in this paper. We present a coherent model to study these mechanisms, and in particular we investigate how recent developments in the credit and capital structures of financial institutions form and strengthen these mechanisms.

In terms of modelling technique, Shin and Morris (1998, 2004a, 2004b), Corsetti, Dasgupta, Morris and Shin (2004) and Goldstein and Pauzner (2000) use the global game technique to study the currency attack, the price of debt, runs on financial markets and bank runs. The strength of the global game technique is that it offers a unique equilibrium, and makes it possible to perform comparative static analysis. Our paper is close to that of Corsetti, Dasgupta, Morris and Shin (2004), in that both papers study the effect of existence of a large player. Corsetti, Dasgupta, Morris and Shin (2004) show that the presence of the large investor makes all other traders more aggressive in the currency attack. One main difference with their paper is that in our model the large lender's decision is based solely on its own situation and is not affected by the strategies of small lenders. We show that the presence of a large lender is a double-edge sword. Depending on market conditions, small investors sometimes become more aggressive, and other times more conservative.

The paper is organized as follows. Section 2 presents the model setup and the equilibrium. Section 3 conducts comparative static analysis and discusses the model implications. Section 4 discusses some broad issues raised by the research. Section 5 concludes.

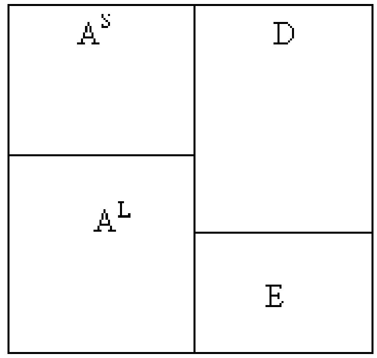
2 The model

Consider a simple model with three agents: financial institution A (hereafter FI-A), financial institution B (FI-B), and "small" investors. FI-A uses its own capital and borrows from FI-B as well as from small investors to fund its portfolio. FI-B is the single "large" lender in FI-A and its loans represent a proportion λ of FI-A's total debt. The remaining proportion, $1-\lambda$ of FI-A's debt is allocated among a continuum of "small" lenders taken together. All debt has the same seniority. FI-A can be interpreted as a hedge fund and FI-B as its prime broker. Alternatively, FI-A is a commercial bank or an investment bank in modern capital markets which heavily rely on the wholesale market for their borrowing needs. In the case of the commercial bank, the group of small lenders correspond to the individual depositors. It is possible to incorporate in the model more than one large lender, However, we abstract from complicating the model, since this is not relevant to address the main ideas of this research. The model has three-dates: T_0, T_1 and T_2. Later, we split T_1 into T_1 and T_{1+}. All the agents in the economy are risk-neutral.

2.1 The agents

2.1.1 The large lender: FI-B

At T_0, FI-B has two types of assets on its balance sheet: short-term assets (A^S) and long-term assets (A^L). These assets are financed with equity, with a value of E, and debt, with a value of D.



FI-B is a prime-broker of a hedge fund or an investor in the wholesale market which lends to banks. We assume that its lending to FI-A is short term. By short term it is meant that FI-B has the right to call back the loan at T_1 or extend it until T_2. As for the long-term assets, for simplicity, we assume that FI-B holds one unit of the long-term asset, denoted by CXO. The fundamental value of CXO at T_0 is v, which equals its expected liquidation value at maturity T_2. The fundamental value v is a constant. At T_0, many investors, including FI-B, hold CXO. However, due to reasons we do not need to specify some investors may decide to sell some units of CXO before the maturity T_2. We denote these traders as noise or liquidity traders. Imperfect market depth translates into a cost to early liquidation. The cost is reflected in a downward-sloping demand curve for CXO, such as:

$$p = v - d_B \cdot s \qquad (1)$$

where d_B is a measure of market depth, s is the aggregate amount of assets CXO investors sell, and p is the price.

The above demand function is a well-established result in the literature (see Grossman and Miller (1988), Campbell, Grossman and Wang (1993), Morris and Shin (2004), and Brunnermeier and Pedersen (2007), Plantin, Sapra, and Shin (2008)).

Liquidity/noise traders trade at any time before the maturity, at T_2. For simplify and without loss of generality, we assume that liquidity/noise traders trade at T_{1+}, where T_{1+} is an instant after T_1. Also, ex-ante, it is assumed that the aggregate demand for liquidity from these investors is normally distributed with $\widetilde{s} \sim N(0, \sigma^2)$.

Given the above setup, the mark-to-market value of FI-B's long-term asset at T_{1+} is distributed as $A^L_{1+} \sim N(v, \sigma^2 d_B^2)$.

It is worth noting that marking-to-market does not affect the value of the short-term asset and the debt value. It only influences the value of the long-term assets and thereby the value of the own capital. That is, the balance sheet equation $A^S + A^L_t = D + E_t$ is always valid at any time point t, where A^L_t and E_t are the mark-to-market values of the long-term asset and equity, respectively.

The central idea we want to model in this subsection is that the decrease in market liquidity can trigger the lender to reduce its exposure in short-term lending (i.e. the increase in d_B leads to the reduce in A^S). As market liquidity becomes low, the probability of mark-to-market losses in long-term asset values increases, that is, the probability of mark-to-market leverage increases. In order to keep the potential leverage lower than a certain level, the prudent and precautious risk management requires the lender to start to delever and liquidate positions, such as credit extended to other institutions.

We suppose that FI-B uses risk management techniques, such as VaR, to take prudent and precautionary actions to unfolding events. Specifically, we assume that FI-B always requires that the probability of its leverage at marking-to-market, beyond an upper limit $\overline{L}$, be less than a small percentage α, say x%. The reason for this precautiounary rule is that financial institutions can face severe consequences if their leverage exceeds certain levels. Debt downgrading would be an example. Another would be costly recapitalization. In an attempt to minimize the probability of such costly events, FI-B takes prudent action beforehand. Specifically, FI-B's risk management imposes the constraint at each time point τ:

$$\Pr\left(\frac{D+E_t}{E_t} > \overline{L} | \mathcal{F}_\tau \ , \tau < t\right) \leq \alpha$$

where $\mathcal{F}_\tau$ is the information FI-B receives by time τ, for a possible adverse event that may happen with a non-zero probability at t.

In our model, the adverse event - the risk of excessive leverage caused by fluctuations in the market price of CXO - occurs at T_{1+}. Therefore, FI-B needs to make sure that the following condition is satisfied before T_{1+}:

$$\Pr(\frac{D+E_{1+}}{E_{1+}} > \overline{L}) \leq \alpha \qquad (2)$$

By applying $A^S + A^L_{1+} = D + E_{1+}$, we can rewrite (2) as

$$\Pr(A^L_{1+} < D\frac{\overline{L}}{\overline{L}-1} - A^S) \leq \alpha \qquad (3)$$

Note that the long-term asset value at T_{1+} is distributed as $A^L_{1+} \sim N(v, \sigma^2 d_B^2)$. We denote α-percentile of A^L_{1+} as C, that is, $\alpha \equiv \Phi(\frac{C-v}{\sigma^2 d_B^2})$. Obviously, $C(d_B)$ is a decreasing function with respect to d_B.

Using d_B, (3) is equivalent to

$$D\frac{\overline{L}}{\overline{L}-1} - A^S < C(d_B) \qquad (4)$$

Suppose that at T_1 FI-B receives *perfect* information about the market depth, d_B. If at T_1, d_B happens to be low, the market is deep and liquid enough, and the right hand side of expression (4) is large, that is, (4) is not binding. In this case, FI-B does not need to do anything. If, on the other hand, d_B happens to be high, the market is illiquid, and the right hand side of (4) is low. In this case, the inequality (4) may be violated. Then, to comply with risk management requirements and satisfy (4), FI-B must deleverage. It does so by calling back the short-term loans extended to FI-A and using the proceeds to repay the debt.[3] To see this, suppose that FI-B calls back the short term loans in the amount of $\triangle A^S > 0$, and repays its outstanding debt. The left hand side of (4) becomes

$$(D - \triangle A^S)\frac{\overline{L}}{\overline{L}-1} - (A^S - \triangle A^S)$$
$$= (D\frac{\overline{L}}{\overline{L}-1} - A^S) - \triangle A^S \cdot \frac{1}{\overline{L}-1}$$

$$< D\frac{\overline{L}}{\overline{L}-1} - A^S.$$

[3] It is not optimal for FI-B to deleverage by selling the long-term assset CXO. The best way to deleverage is to call back the short-term loans. As Brunnermeier (2008) argues, selling some of the long-term assets in a financial crisis would establish a low price and force the holder to mark down remaining holdings. Hence, investors are reluctant to do this—and instead prefer to sell assets with higher market liquidity first.

That is, by deleveraging, the left hand side of (4) goes down, and FI-B is able to satisfy the risk management constraint (4).

From the previous analysis, we can see FI-B's decision to delever at T_1 crucially depends on the comparison between market depth d_B (the right hand side of (4)) and the strength of FI-B balance sheet, which is characterized by D and A^S. If the balance sheet at T_0 is strong enough, even a big negative shock to the market depth d_B does not cause (4) to be violated.

A common way to characterize the strength of the balance sheet is by comparing the value of the assets to the value of the liabilities. In fact, there is a one-to-one map between the pair (D, A^S) and the pair (E_0, L_0): $\begin{cases} D = E_0(L_0 - 1) \\ A^S = E_0 L_0 - v \end{cases}$, where E_0 and L_0 are the equity value and the asset-to-equity ratio at T_0, respectively. We can use leverage L_0 to express the condition (4) and obtain the threshold that characterizes whether FI-B calls or does not call the loans.

Substituting $\begin{cases} D = E_0(L_0 - 1) \\ A^S = E_0 L_0 - v \end{cases}$ into (4), we can re-write (4) as,

$$L_0 < \overline{L} - \frac{v - C(d_B)}{E_0} \cdot (\overline{L} - 1). \qquad (5)$$

Condition (5) expresses the same idea as (4). It states that only when a big adverse shock to d_B is realized and the right hand side of (5) becomes lower than the leverage L_0, does FI-B call back the loans.

We define $L_B^*(d_B) \equiv \overline{L} - \frac{v - C(d_B)}{E_0} \cdot (\overline{L} - 1)$, which is a decreasing function of d_B. Thus, (5) becomes $L_0 < L_B^*(d_B)$.

The above inequity characterizes FI-B's decision rule, which is a central result of this subsection. We summarize it in Theorem 1.

Theorem 1 *After FI-B receives information about market depth, d_B, its decision rule at T_1 is* $(L_0, d_B) \longmapsto \begin{cases} Call & L_0 \geq L_B^*(d_B) \\ Hold & L_0 < L_B^*(d_B) \end{cases}$.

2.1.2 The borrower financial institution: FI-A

Financial institution FI-A funds its investments with own capital (equity), and borrowing from FI-B and from a continuum of small lenders. The investments in question are normalized to one unit of the long-term asset. FI-A's long-term asset has a fundamental value of f at T_0 (i.e. the asset's expected liquidation value at maturity T_2). The sale price of the asset at the intermediate date T_1 is downward sloping. Specifically, $p = f - d_A \cdot s$, where d_A is the depth of the market for the asset held by FI-A. In order to highlight the effects of a quick sale in a market without perfect liquidity, we assume that the trading actions of FI-A affect by themselves the price of the asset. What this implies is that FI-A's sales alone can bring the market price of the asset down. This assumption can be rationalized by the fact that in many segments of the capital markets, the trading in securities is dominated by a handful of vary large players. To emphasize the effect of the actions of FI-A in the market for the asset, we assume that FI-A is a monopolist seller of the asset, or that the sale s that influences the price in the demand curve comes from the actions of FI-A's attempting to unload the asset. With this setup, the revenue function of FI-A at T_1 (from selling) can be written as $\pi(s) = (f - \frac{1}{2}d_A \cdot s) \cdot s$.

On the liability side of the balance sheet, FI-A has debt with total face value K. The debt is short-term and the debtholders have the right to call back the loans at T_1. If they call in the loans, FI-A is liable to repay the loan at face value. If lenders do not recall the loans at T_1, then the loans are automatically extended until T_2, when FI-A is required to repay the debtholders with interest. The total amount of repayment at T_2 is KR, where R is the gross interest rate ($R > 1$). It is more appropriate to interpret R as the opportunity cost to the lender when it decides to call back the loans at T_1, instead of letting them run until T_2. The opportunity cost can be particularly high if account is taken also for the loss of reputation that results from calling the loans early.

As mentioned before, the debt of FI-A is held in a proportion λ by FI-B, and this debt is identical to FI-B's short-term assets A^S. The remaining proportion, $1 - \lambda$, is held by a continuum of "small" lenders.

We assume further that if the debt holders withdraw their loans at time T_1, the only way that FI-A can fulfill the debt repayment obligation is by liquidating assets. It cannot find new creditors that are ready to replace the old creditors. This assumption is present in Schleifer and Vishny (1997) and is both documented by the empirical evidence, as well as by the corporate finance literature on relationship banking. The idea is that it takes time for a creditor to build firm-specific lending relationships with a borrower, either because of adverse selection related to the quality of the borrower' s assets, or because of moral hazard related to monitoring the borrower.

From a social welfare perspective, it is never optimal for FI-A to sell any unit of asset at T_1. However, FI-A might be forced to do so because its creditors may decide not to extend their short term loans.

We also make the following assumptions.

$f > KR$:

It implies that the liquidation value of the asset held by FI-A is, at T_2, sufficient to cover the debt obligations.

$f - \frac{1}{2}d_A < K$:

We assume that $\pi(1) = f - \frac{1}{2}d_A < K$. That is, even if FI-A liquidates the whole asset at T_1, the revenue is not sufficient to cover the debt at face value.

$f > d$:

Finally, we assume that even if the whole unit of the asset were to be sold, the price would not drop to a negative value, that is, $p = f - 1 \cdot s > 0$.

Similar with previous work, such as Diamond and Dybvig (1983), Morris and Shin (2004), and Goldstein and Pauzner (2000), we write the payoffs of a creditor when it decides either to call the loans at time T_1 and forego the interest, or extend the loans until time T_2. In the event of a run on the financial institution, the payoff is a function of the aggregate number of creditors calling. This is because the higher the number of creditors calling early, the less is left for those who decide to hold on, and because the volume of loans withdrawn affects the amount of the asset the borrower financial institution must sell at a discount, due to price pressure.

We use u to denote the proportion of creditors who decide to call the loan at T_1, where $0 \leq u \leq 1$. We have the following result.

Lemma 1 *The payoff function of calling the loan at* T_1 *is* $w^C(u) = \begin{cases} K & \textit{if } 0 \leq u \leq \frac{f-\frac{1}{2}d}{K} \\ \frac{f-\frac{1}{2}d}{u} & \textit{if } \frac{f-\frac{1}{2}d}{K} < u \leq 1 \end{cases}$,

while the payoff function of extending the loan until T_2 *is*

$$w^H(u) = \begin{cases} \min(KR, \frac{[(d_A-f)+\sqrt{f^2-2u\,\mathrm{K}d_A}]\cdot f}{(1-u)d_A}) & \text{if } 0 \leq u \leq \frac{f-\frac{1}{2}d}{K} \\ 0 & \text{if } \frac{f-\frac{1}{2}d}{K} < u \leq 1 \end{cases}.$$

Proof: We divide u into two regions: low u and high u. In the first region where u is low, FI-B does not need to liquidate the whole asset to repay the early-withdrawing creditors. The staying creditors still obtain a positive payoff at T_2. In this case, the payoff of calling is K, while the payoff of holding is $\min(KR, \frac{(1-s)\cdot f}{(1-u)})$, where s is the proportion of the asset that is sold to honor the creditors who decide to withdraw early. Note that s solves $u \cdot K = \pi(s)$. Therefore, $s = \frac{f-\sqrt{f^2-2u\,\mathrm{K}d_A}}{d_A}$. Using $s = \frac{f-\sqrt{f^2-2u\,\mathrm{K}d_A}}{d_A}$, we have the payoff function for extending the loan. That is, $w^H(u) = \min(KR, \frac{[(d_A-f)+\sqrt{f^2-2u\,\mathrm{K}d_A}]\cdot f}{(1-u)d_A})$. In the second region, where u is high, the creditors who stay obtain 0, and the calling creditors divide all the liquidation value. Each creditor who calls the loan at T_1 obtains $\frac{f-\frac{1}{2}d}{u}$. The threshold between the first and the second region is the u that solves $\frac{f-\sqrt{f^2-2u\,\mathrm{K}d_A}}{d_A} = 1$. The threshold value of u is, after simple manipulation, equal to $\frac{f-\frac{1}{2}d}{K}$. Q.E.D.

We also denote $\triangle w(u)$ to measure the difference in payoffs between holding and calling the loan at T_1, as a function of u. That is,

$$\triangle w(u) = \begin{cases} \min(KR, \frac{[(d_A-f)+\sqrt{f^2-2u\,\mathrm{K}d_A}]\cdot f}{(1-u)d_A}) - K & \text{if } 0 \leq u \leq \frac{f-\frac{1}{2}d}{K} \\ -\frac{f-\frac{1}{2}d}{u} & \text{if } \frac{f-\frac{1}{2}d}{K} < u \leq 1 \end{cases}$$

Figure 1(1) and Figure 1(2) depict the payoff functions of calling and holding, as a function of u.

Figure 1(1): The payoff function of holding and calling

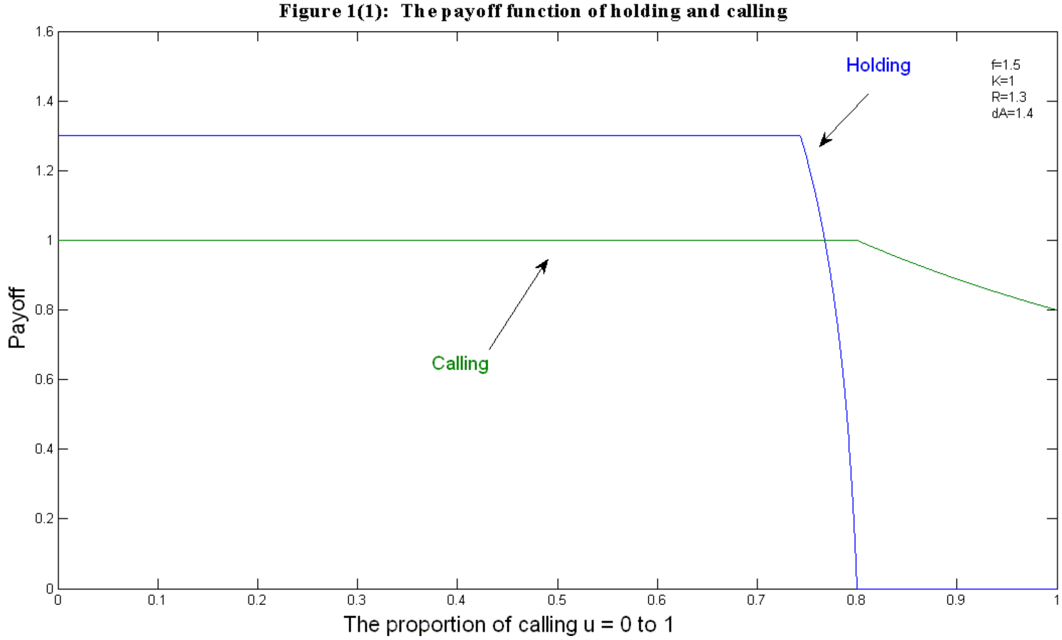
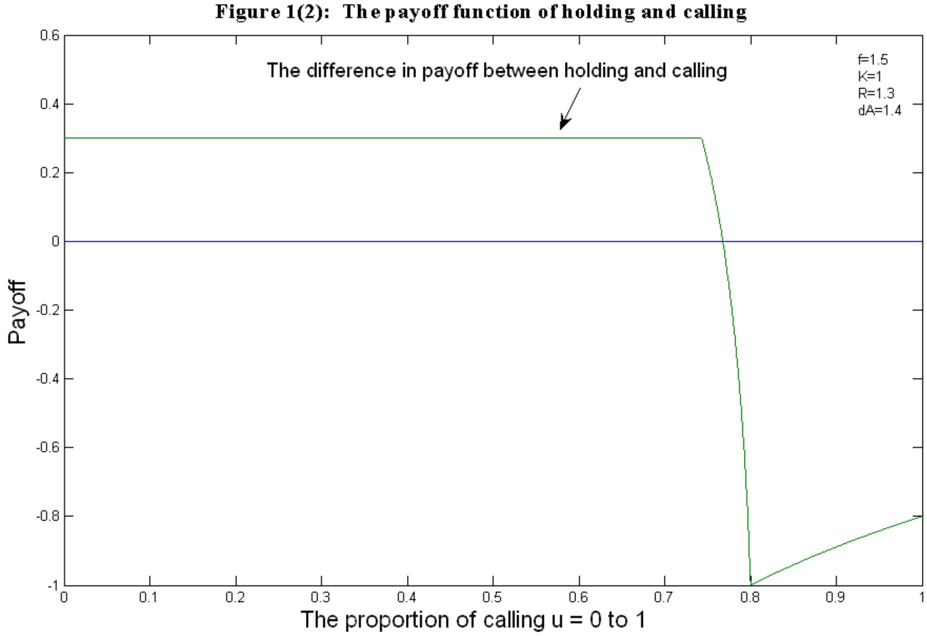


Figure 1(2): The payoff function of holding and calling



2.1.3 Small lenders

Small lenders, in our model, should be interpreted in a broad sense. They are not just the typical debt holder. If FI-A is a hedge fund, then small lenders can also be interpreted as the hedge fund's limited partners, such as high-net-worth individuals and funds-of-hedge-funds, as long as these are allowed to early redeem their investments (Shleifer and Vishny (1997)). In fact, many hedge funds have, before the 2007-08 crisis, reduced considerably and even lifted the lock-up period to as short as three months. Although nominally characterized as equity partners, investors in funds that are able to redeem at short notice have a payoff structure that appears more like that of a debtholder. Brunnermeier (2008) provides detailed arguments why a first-mover advantage can make financial institutions in general, not only banks, subject to runs.

Unlike the large lender FI-B, small lenders' decisions of calling back their loans or not depend on their beliefs about other lenders' actions. Other than that, we assume that small lenders are not themselves constrained by additional considerations.

2.2 Information, decisions and timeline

At T_1, both the large lender FI-B and the small lenders receive *perfect* information about market depth d_A and d_B.[4] However, the balance sheet strength of FI-B is not in the public domain. Only FI-B knows the strength of its balance sheet, L_0. Other investors receive signals about L_0.

We believe that the assumption that information about the balance sheet, valued at market prices, or book value adjusted for impairments in the case of commercial banks, being non-public is realistic. First, mark-to-market value changes frequently, and in the case of adjusted book value for impairments requires information that is not easily gathered by outsiders. It is virtually impossible for outside investors to perfectly infer the market-value-based leverage of a financial institution either by looking at ratings from rating agencies, or by analyzing financial statements that are made public. Second, financial institutions differ significantly from non-financial firms, in that their capital structure and financial positions can quickly and significantly be altered by trading and risk management as well as by financial commitments. Hence, it is impossible for outsiders to get a true picture of the financial situation of a financial institution in real time. Third, events after the collapse of Lehman Brothers in September 2008 have shown that the main difficulty in financial markets was that nobody had a clue about the strength of the balance sheet of big market

[4] It is worth noting that we assume that small lenders also observe d_B. A weaker assumption that small lenders receive imperfect information (signals) about d_B, does not change the model results. In reality, small investors can learn about market depth by observing spreads and other material trading information. Investors can obtain a lot of information about market liquidity and depth even if they are not experts on that market.

players, such as Morgan Stanley or UBS, as well as many large commercial banks. The wholesale market where most of these banks find funding for their activities was paralyzed by fear, fed by rumor about the leverage of many financial institutions.

Specifically, we assume that small lender i's signal about the leverage of FI-A is given by $L_0^i = L_0 + \epsilon^i$, where ϵ^i is uniformly distributed with support $[-\epsilon, \epsilon]$, and ϵ^i is independent from ϵ^j for $i \neq j$. A small lender only knows his own signal and does not know the signal of other small investors. However, he can infer the signals of other investors based on his knowledge of the distribution of signals, and on his own signal. Furthermore, we assume that L_0's prior distribution is $L_0 \sim N(\overline{L_0},\, \delta^2)$, where $\overline{L_0} > 0$ and $0 < \delta \leq +\infty$. At this stage, we assume $\delta = +\infty$, which is equivalent to saying that L_0 has an improper prior over the real line. Later, we return to this assumption.

Given the above information structure, the large lender FI-B's strategy is a map

$(L_0,\, d_A, d_B) \longmapsto$ (Call, Hold).

The large lender's equilibrium strategy is already given in Theorem 1. FI-B's decision is only based on its own situation: whether its risk management constraint is binding or not. In particular, the size of FI-B among the other creditors of FI-A allows FI-B to behave independent of what other small creditors' strategies. This is one of the main features of our model. We use this setup for two reasons.

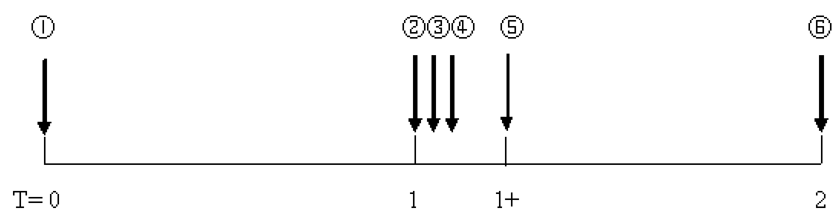
First, we want to highlight that, in modern financial markets, large creditors' own financial constraints can trigger runs by other creditors that cripple and even force the collapse of borrower financial institutions. A prime broker curtailing credit to a hedge fund provokes other lenders and investors to withdraw their money. A large fund that refuses to extend loans to a bank in the money market can cause other lenders and depositors to run for cover. In this work, we wish to distinguish our model from the traditional bank run mechanism, which is purely due to a coordination failure. While it is possible, in our model, to capture the effect that lender FI-B's decision is also influenced by the actions of small lenders, this effect is not the focus of this paper. Second, FI-B's decision of calling or not, independent of small lenders' decisions, can be an endogenous result in our model rather than an assumption. In fact, as we will show later, if large lender FI-B has sufficient large loans to FI-A (i.e. λ is sufficient large), whether small lenders call their loans or not does not really matter: FI-B's optimal decision is holding, conditional on that its own risk management constraint is not binding. This is because with a downward-sloping demand curve for the asset, calling and forcing the borrower to incur in a fire sale would put additional pressure on the price of the asset, and, therefore, it would hurt the large lender FI-B itself.

For each small lender, its decision depends on its beliefs about the large lender's action, as well as on the actions of other small lenders. Specifically, small lender i's strategy is a map

$$(L_0^i,\, d_A, d_B) \longmapsto \text{(Call, Hold)}.$$

Figure 2 describes the timeline.

Figure 2: The timeline



① FI-A borrows from FI-B and small lenders to invest on its long-term asset.
② FI-B and small lenders observe market depth d_A and d_B. Small lenders receive signals about FI-B's balance sheet strength.
③ FI-B and small lenders decide simultaneously whether to call loans.
④ If lenders call loans, FI-A liquidates its asset based on the repayment requirement.
⑤ Some noise traders may come out and sell asset CXO.
⑥ If FI-A has not liquidated its entire asset at T_1, the residual asset realizes its final liquidation value and FI-A repays its remaining lenders based on limited liability rule.

2.3 The equilibrium

We use global game technique to solve the unique equilibrium of the game. The main mechanism underlying uniqueness is the lack of common knowledge about FI-B's balance sheet strength, L_0.

Specifically, we find the unique switch equilibrium of the game, where every small lender sets a threshold $L_S^*(d_A, d_B)$ and uses the switching strategy $(L_0^i, d_A, d_B) \longmapsto \begin{cases} Call\ L_0^i \geq L_S^*(d_A, d_B) \\ Hold\ L_0^i < L_S^*(d_A, d_B) \end{cases}$, such that, given that all other small lenders set the threshold as $L_S^*(d_A, d_B)$, it is optimal for a small lender itself to do the same.

We prove that, if the size of the loans provided by the large lender is sufficiently large, such an equilibrium exists and is unique.

Theorem 2 *When $\lambda \geq \underline{\lambda}(K, R, f, d_A)$, the model has a unique equilibrium in which a small lender calls the loan if and only if it observes a signal above the threshold $L_S^*(d_A, d_B)$.*

We proceed by presenting the main idea and intuition to prove Theorem 2. Discussing the idea of proof here is necessary because it is key to understand the mechanisms driving comparative static analysis in next section.

In the first step to solve the equilibrium, we work out the aggregate proportion of lenders who call loans for a given L_0, conditional on that every small lender uses switch strategy with threshold as L_S^*. We obtain the aggregate calling function:

$$u(L_0) = \begin{cases} (1-\lambda)\frac{L_0+\epsilon-L_S^*}{2\epsilon} & L_0 < L_B^* \\ \lambda + (1-\lambda)\frac{L_0+\epsilon-L_S^*}{2\epsilon} & L_0 \geq L_B^* \end{cases}, \qquad (6)$$

where $u(L_0)$ is the aggregate calling for a given L_0. It is important to note that the aggregate calling is not a continuous function on L_0. There exists a discrete jump at $L_0 = L_B^*$. This feature plays a crucial role in deriving the uniqueness of equilibrium.

In the second step, we wonder: for the small lender i who receives the signal L_0^i, what is the distribution of L_0 in his eyes?

Because of the improper prior distribution of L_0, small lender i's posterior density over L_0 is uniform over the interval $[L_0^i - \epsilon,\, L_0^i + \epsilon]$.

In the third step, considering the lender at the margin who just receives the signal as L_S^*, he should be indifferent between holding and calling. That is, the expectation of his net payoff from holding (or calling) should be 0. That is,

$$\tfrac{1}{2\epsilon}\int_{L_S^*-\epsilon}^{L_S^*+\epsilon} \triangle\, w(u(L_0))\, dL_0 = 0 \qquad (7)$$

From (7), we can work out the unique L_S^*.

The above procedures are typical ones to solve the global game equilibrium. However, for a uniform distribution of signal, the more intuitive way to solve the equilibrium is the approach developed in Morris and Shin (2004). It gives better insight.

Morris and Shin (2004) look at the conditional density of selling directly rather than the conditional density of fundamental. In our context, for the small lender i who receives the signal L_0^i, we have worked out the density of L_0 in his eyes. Of course, we can work out the density of a function of L_0 in his eyes. In particular, $u(L_0)$ is a function of L_0. Hence, the density of $u(L_0)$ in the margin lender's eyes can be found out. It is

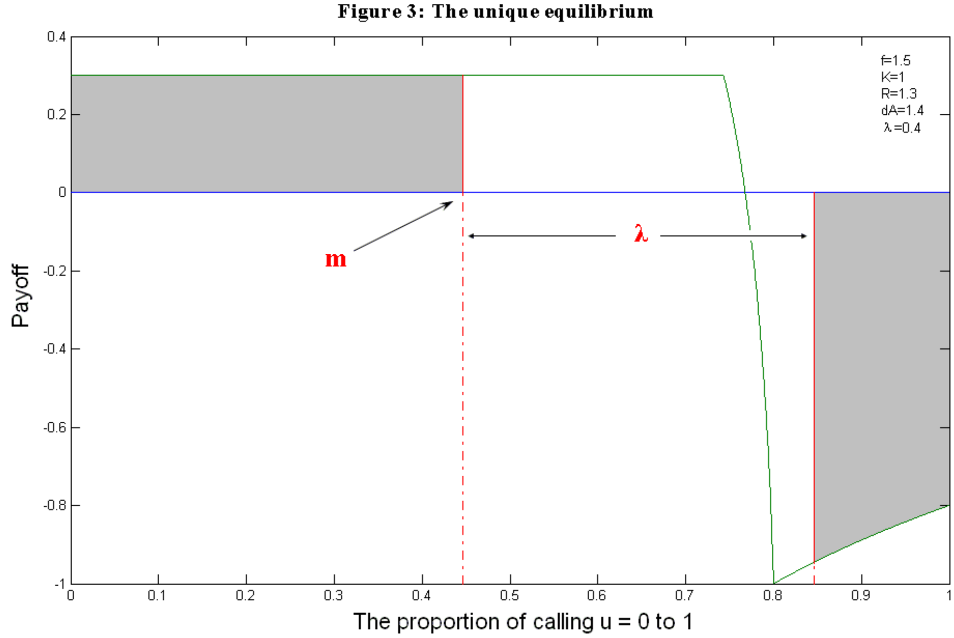
$$g(u) = \begin{cases} \frac{1}{1-\lambda} & u \in [0,\ (1-\lambda)\frac{L_B^*+\epsilon-L_S^*}{2\epsilon}\ ] \cup [\lambda + (1-\lambda)\frac{L_B^*+\epsilon-L_S^*}{2\epsilon},\ 1] \\ 0 & u \in [(1-\lambda)\frac{L_B^*+\epsilon-L_S^*}{2\epsilon},\ \lambda + (1-\lambda)\frac{L_B^*+\epsilon-L_S^*}{2\epsilon}] \end{cases}. \quad (8)$$

The discontinuity in the support of $g(u)$ is due to the jump in function of (6).

After we work out (8), the equation of (7) is transferred to

$$\int_0^1 \triangle\, w(u) \cdot g(u)\ du = 0. \quad (9)$$

Figure 3: The unique equilibrium



Importantly, the equation (9) gives a geometrical presentation of finding an equilibrium. In $\triangle w - u$ space of Figure 3, geometrically, the equilibrium means that we need to locate a (starting) point in u-axis such that if we cut a λ-width block horizontally starting from that point, after cutting, the area of the remaining part above u-axis (i.e. the above shaded area) should be equal to that below u-axis (i.e. the below shaded area). We denote the u-coordinate of this starting point as m, which is a deterministic function of K, R, f, d_A and λ.

Further, from (8), we know $m = (1-\lambda)\frac{L_B^* + \epsilon - L_S^*}{2\epsilon}$. Hence we solve out

$$L_S^* = L_B^*(d_B) + \epsilon - \frac{2\epsilon}{1-\lambda} \cdot m(K,\, R,\, f,\, d_A,\, \lambda). \tag{10}$$

The condition to guarantee the existence of equilibrium is direct based on the above analysis. That is, λ should be sufficiently big to guarantee finding out a starting point m. Figure 4 illustrates how to calculate the minimum λ to guarantee the existence of equilibrium. In Figure 4(1), it is for the case that the area above the u-axis is bigger than the below. So the minimum $\underline{\lambda}$ is the width of cutting block such that the sum of remaining areas after cutting is zero (i.e. two shaded areas are cancelled). In Figure 4(2), we show the case that the below area is bigger than the above. The idea is same as the previous case of the proof for Figure 4(1).

Figure 4(1): The existence of equilibrium

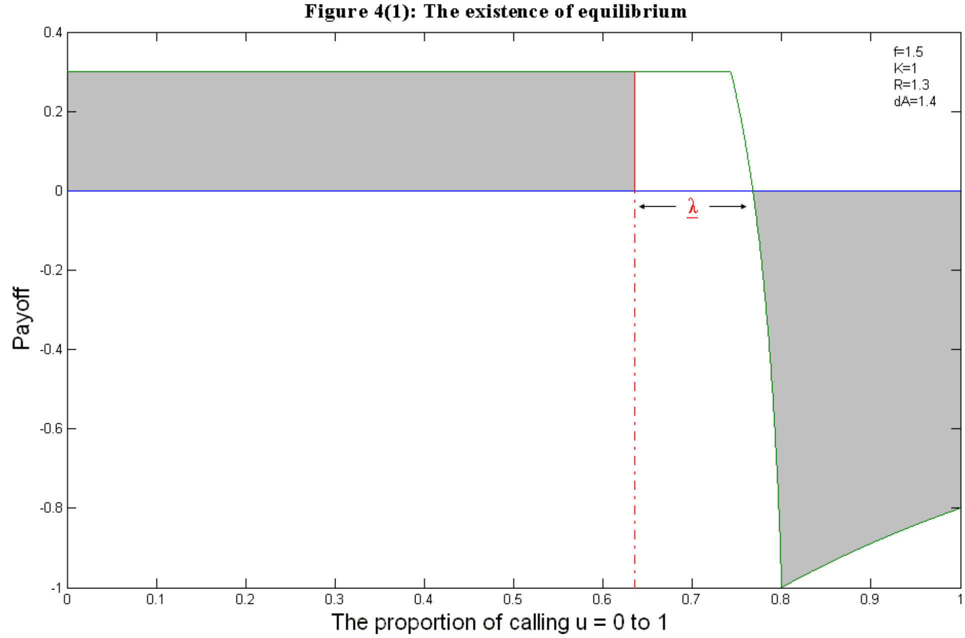
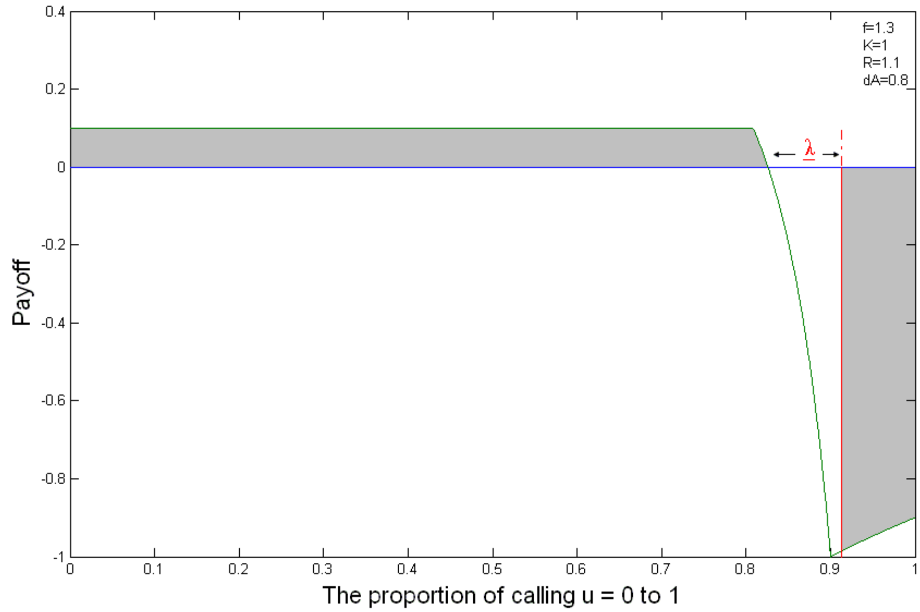


Figure 4(2): The existence of equilibrium



As for the uniqueness of equilibrium, the proof is similar with Goldstein and Pauzner (2000). In our model, $\triangle w(u)$ is not strictly monotonic decreasing in u. But it satisfies so-called 'single crossing property' in Goldstein and Pauzner (2000). So there exists a unique equilibrium. In fact, for our model, in Figure 3, suppose there is another start point $m^{'}$, where $m^{'} < m$, and we start the cutting block from $m^{'}$ rather than m. In this case, the shaded area left to the cutting block decreases while the right side increases. So the sum of two shaded areas is less than 0, which means the start point $m^{'}$ doesn't form an equilibrium. Similar argument applies for the opposite.

3 Comparative static analysis and model implications

In this section, we perform comparative static analysis and discuss the main implications of the model. We separate the findings into four main results.

3.1 The cause of credit withdrawal by a large lender

The financial crisis of 2007 and 2008 has witnessed some of the most unusual events in the history of financial markets. Many prestigious names, be they commercial banks, investment banks, not to mention many hedge funds, have collapsed or were rescued either in a hasty arranged merger or by the government. One common feature in many of these failures has been that creditors in the failed institutions suddenly decided to withdraw their lines of credit, and in doing so they have exposed the borrower institutions to unbearable liquidity pressures. In the UK, commercial bank Northern Rock is a typical example of such pressures.

Received wisdom usually associates the problem of Northern Rock to is its heavy reliance on securitization of mortgages. The trigger for its failure is that creditors suddenly declined to roll over the securitized notes because they suspected of the quality of Northern Rock's securitized portfolio, especially after the large scale of defaults of similar products in the USA. However, Shin (2008) has written an interesting case study providing details as well as convincing evidence that this is not the main reason for Northern Rock's failure. Northern Rock's securitized notes were mostly medium and long-term, with an average maturity of over one year, and faced almost no rolling-over problem. Rather, Shin (2008) argues:

"Contrary to received wisdom, [Northern Rock's] reliance on securitization was not an immediate factor in its failure. Rather, its problems stemmed from its high leverage coupled with reliance on institutional investors for short term funding. When the de-leveraging in the credit markets began in August 2007, it was uniquely vulnerable to the shrinking of lender balance sheets arising from the tick-up in measured risks."

From Shin (2008)'s argument, it is Northern Rock's creditors rather than Northern Rock itself that are at the root of the failure of the bank.

Many hedge funds have also suffered a similar fate to that of Northern Rock. In a recent article, The Economist wrote:"Regulators used to worry about the danger hedge funds might pose to their prime brokers ... the risk turned out to be the other way round."

After getting the point that a large institutional creditor can be the origin of a run on a financial institution, we need to understand a crucial question: why do creditors decide to withdraw their investments in the first place? The answer to this question is truly the micro-foundation of the recent financial crisis.

As we argue in the introduction, the two available explanations - the first based on information asymmetry, and the second based on financial contagion - do not seem to be able to explain many situations in the current crisis. In contrast to the above hypotheses, we argue that FI-B's withdrawal of funds from FI-A is entirely caused by FI-B's own risk management strategy. Such action is necessary from FI-B's own interests and is induced by regulation. The intuition in Shin (2008) has touched the point. He writes :"Many of the creditors to Northern Rock would have been sophisticated investors that tailor their risk-taking strategy to unfolding events. When measured risks are low, risk constraints on capital do not bind, and such investors will be able to expand balance sheets, meaning that they are willing to lend and are looking for borrowers. However, when a crisis strikes, risk constraints bind and lenders cut back their exposures in response. This will be the case especially if the investor is leveraged. For a leveraged institution, prudent risk management dictates the cutting back of exposures when market turmoil strikes."

The first part of our model is close to Shin's intuition. We identify that the change in market liquidity can potentially be the driving force for a lender to decide to adjust the balance sheet.

3.2 The amplifying effect of fearing a *potential* withdrawal

The run on Bear Stearns in March 2008 highlights another critical channel through which some financial institutions might suddenly find themselves stifled by a liquidity shortage. The open letter by Christopher Cox, the Chairman of the Securities and Exchange Commission (SEC), clearly summarizes this process:

"[T]he fate of Bear Stearns was the result of a lack of confidence, not a lack of capital. When the tumult began last week, and at all times until its agreement to be acquired by JP Morgan Chase during the weekend, the firm had a capital cushion well above what is required to meet supervisory standards calculated using the Basel II standard."

Cox's remarks imply that even at the time of its sale, Bear Stearns' capital, as wells as its broker-dealers' capital, exceeded regulatory requirements. The fate of the bank was decided by counterparty withdrawals and credit denials, resulting in a loss of liquidity, not inadequate capital. A keyword in Cox's comments was confidence. The confidence in this and other similar cases is not about capital adequacy, solvency, or, equivalently, asset quality; It is about the liquidity situation of Bear Sterns. While, in general, confidence refers to an qualification about the health of the assets, in this case, it referred to confidence in the liability side of the balance sheet, rather than to the asset side.

Modern financial institutions typically have a capital structure with multiple creditors, some of them quite prominent. Such capital structure has meaningful and important implications in the credit-renewing cycle. In the presence of multiple creditors, a creditor not only needs to look at the quality of the borrower's assets (the left hand side of balance sheet), but also pay attention to the liquidity status of the borrower (the right hand side of balance sheet). Even if the quality of the asset is good (and thus the adverse selection problem is small), the expectation that one or several large creditors may withdraw their funding due to their own problems, can cause a smaller creditor himself to withdraw funds. This is true in particular during a crisis. In a crisis, the market liquidity typically becomes very low. Therefore, risk constraints bind many financial institutions and lenders often need to cut back their exposures. All creditors realize such a possibility. The fear is sufficient to lead to a creditor run. This is the fate of modern financial markets: the jittery nature of investors of financial institutions due to influential lenders being on balance sheets.

The global game modeling technique enables us to formalize and quantify this idea. Let us go back to the model and do comparative analysis.

First, let us look at the impact of market liquidity d_B on small lenders' decisions. From (10), we have

$$\frac{\partial L_S^*}{\partial d_B} < 0. \tag{11}$$

Inequality (11) says that an increase in d_B (i.e. the market becomes less liquid) leads to small lenders lowering their threshold to run (withdraw their funds from the borrowing bank). That is, the fear that market liquidity drying-up causes other creditors to withdraw, results in a creditor withdrawing himself. It is important to emphasize that this kind of run is purely due to the small lenders' fear, rather than the large lender's actual withdrawal. It is quite possible that FI-B will not withdraw at all.

Theorem 3 *An investor observing a high d_B and thus fearing that other investors may withdraw sufficiently leads the investor himself to run.*

We believe that the mechanism shown in Theorem 3 largely accounts for failures of many financial institutions during the crisis. In the case of the hedge fund industry, The Economist writes: “The industry’s aggregate leverage has undoubtedly caused it trouble. [...] But there does not appear to have been a systematic withdrawal of bank credit from hedge funds. [...] A fuller explanation must include the increasingly jittery nature of hedge funds clients.”

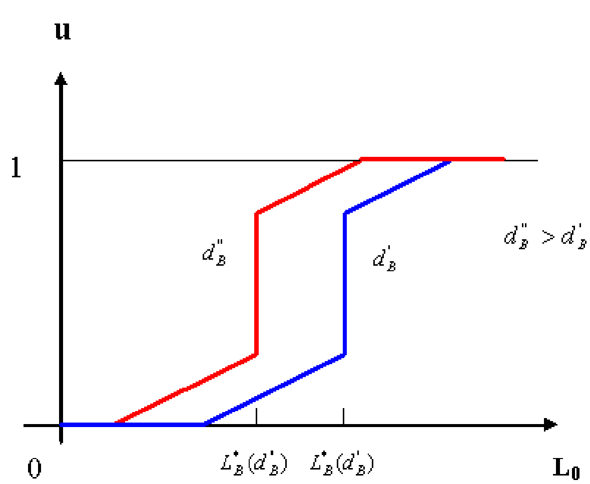
Secondly, let us look at the total effect of d_B ex-post. That is, for a given realized market liquidity d_B, and the true balance sheet strength L_0, we want to know the exact proportion of lenders running ex-post. We rewrite (6) as

$$u(L_0) = \begin{cases} (1-\lambda)\frac{L_0+\epsilon-L_S^*(d_A,d_B)}{2\epsilon} & L_0 < L_B^*(d_B) \\ \lambda + (1-\lambda)\frac{L_0+\epsilon-L_S^*(d_A,d_B)}{2\epsilon} & L_0 \geq L_B^*(d_B) \end{cases} . \tag{12}$$

From (12), we can see that an adverse shock on d_B has two effects on the likelihood of a run. The first one is the amplifying effect discussed above: a higher d_B decreases the threshold for small lenders to run, and hence increases the proportion of small lenders running. The second one is that it increases the probability of run by the large lender, FI-B, itself.

A good way to express the idea of (12) is to graph it, which is done in Figure 5.

Figure 5: The proportion of lenders running



In Figure (5), the red curve corresponds to a higher d_B. The red curve is above the blue curve. In the region of low L_0, the vertical distance between two curves is small and the distance is purely caused by the small lenders' effect (of fearing). But in the regions where L_0 becomes high, the large lender FI-B, which until then would not run, may also withdraw. As a result, the vertical distance increases even further.

3.3 Why are financial institutions so vulnerable?

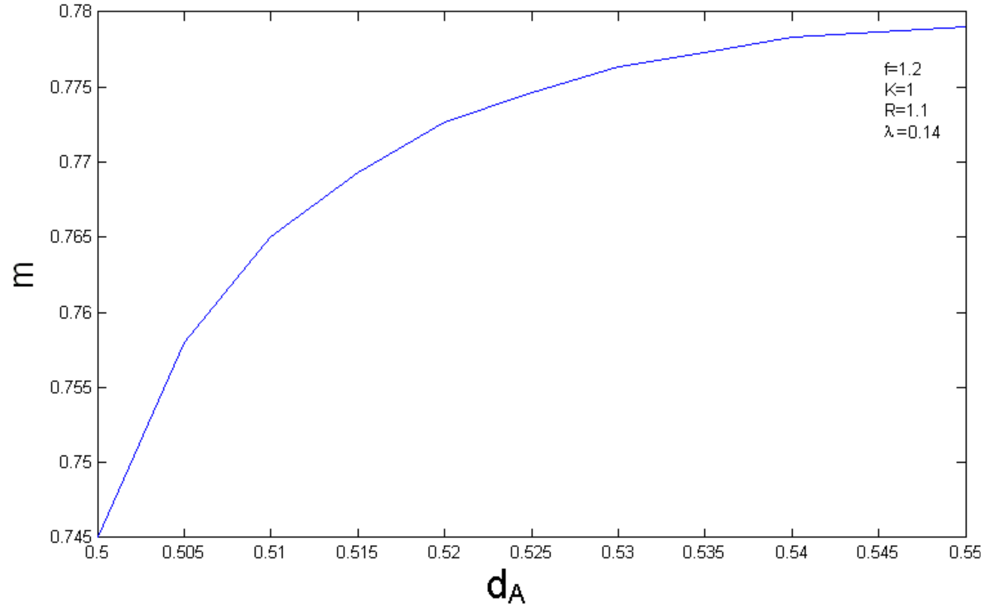
To some extent, it is still a big surprise why so many financial institutions collapsed within a short time period. It is then natural to ask why financial institutions are so vulnerable.

Morris and Shin (2008) argue that there are two components that determine the probability of a bank run. The first component is the threshold for coordination not to run, and the second component is the cost of miscoordination. In our model we show that these two components correspond to the two sides of the balance sheet of a financial institution.

Let's look at the first component, which in our model is characterized by d_B. If d_B is high, for a given L_0, small lenders anticipate that the large lender FI-B will run with a high probability. So the threshold for small lenders to run is low. That is (11): $\frac{\partial L_S^*}{\partial d_B} < 0$.

Turning to the second component, which in our model is characterized by d_A, if d_A is high, asset sales by FI-A depress the price. So a lender who does not run is left with zero liquidation value with a high probability. That is, the cost of miscoordination is high. We would like to show that $\frac{\partial L_S^*}{\partial d_A} < 0$. Unfortunately, we cannot obtain a closed-form solution because the functions involved are implicit. Nevertheless, simulation results confirm the above analysis for relevant parameter values. Figure 6 presents the simulations result.

Figure 6: m increases with d_A

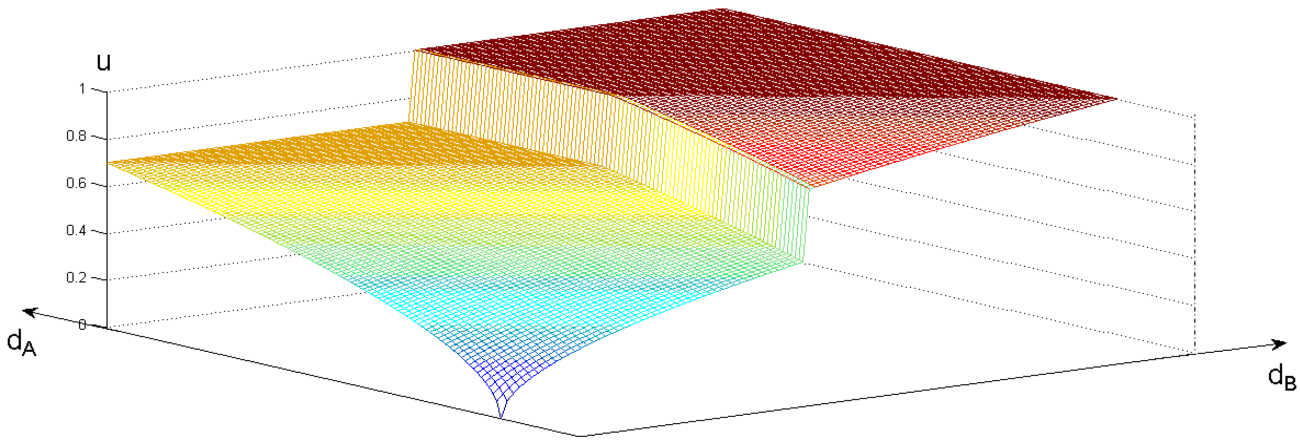


Theorem 4 *For some parameters, the higher is the value of d_A, the lower L_S^* is. That is, market illiquidity of FI-A's assets leads to a low threshold for small lenders to run (a run is more likely).*

Note that $\frac{\partial L_S^*}{\partial d_A} < 0$ is not always true (for our setup where there is a large lender), and this result is valid only for some parameters. We discuss this in the Appendix.

The central idea in this subsection is that a financial institution, such as FI-A, can be hit by a bank run through two sides: the liability side and the asset side, characterized by d_B and d_A, respectively. If d_B increases, the bank run hits the financial institution through the liability side, because d_B increases the probability that a large lender's constraint is binding. Thus, d_B increases the threshold for coordination not to run for small lenders. Therefore, small lenders run more often. In contrast, if d_A is high, the financial institution's asset becomes less liquid and can only be sold at a low value, and the larger the sale, the lower the realized value. The direct consequence of this is that the cost of miscoordination is high. Therefore, lenders are apt to run, as shown in Theorem 4. We argue that if adverse shocks occur simultaneously (i.e. d_A and d_B increase jointly), then a financial institution like FI-A can suffer a run that is triggered both from the asset and the liability sides. When this happens, a collapse is almost certain. Based on (12), Figure 7 depicts the effect of the two forces hitting a financial institution.

Figure 7: The proportion of lender running as a function of d_A and d_B for a given L_0



3.4 Is the presence of large wholesale lenders in the balance sheet of a bank good?

Finally, we want to discuss the role of large lenders in the stability of the financial system. It has become accepted wisdom that traditional bank runs are an accident and the result of miscoordination. Intuitively, then, the presence of a large lender in the balance sheet of a bank should reduce the threshold for coordination. In the extreme case of only one lender, there is, of course, no coordination problem for the lender with itself. This is the advantage of having a large lender.

In this work, however, we also show the downside for the existence of a large lender to a financial institution. The large lender can become constrained and be forced to withdraw the credit extended to the borrower, for reasons that have nothing to do with the financial health of the borrower. Because of the significant size of the large lender, the possible withdrawal by the large lender will be sufficient to lower the threshold level for a run by other smaller lenders.

In contrast, small lenders face little constraints themselves, and recent evidence has shown that individual depositors and lenders have became the most stable funding source for many financial institutions. Banks that relied more on a large and geographically diversified pool of small depositors fared much better than banks that relied more on the wholesale markets, usually dominated by large institutions. HSBC and Santander are examples of the first case. Deutsche Bank and many investment banks are examples of the second case. Perhaps this explains why some investment banks are now considering opening branches to attract deposits. Ironically, before the crisis, during 2005-07, the management of HSBC was put under severe pressure by activist investors (mainly hedge funds) for being a geographically diversified local bank, a business model that, according to the activists required too much capital invested and depressed the return on equity capital.

In the following, we conduct the above analysis in a rigorous way. Keeping other things equal, we choose two different levels of d_B, d_B^H and d_B^L (where $d_B^H > d_B^L$). d_B^L represents that the market is in good times, while d_B^H means that the market is in bad times. Then, we conduct comparative static analysis $\frac{\partial L_S^*}{\partial \lambda}$. We show that $\frac{\partial L_S^*}{\partial \lambda}|_{d_B=d_B^L} > 0$ and $\frac{\partial L_S^*}{\partial \lambda}|_{d_B=d_B^H} < 0$, meaning that in, good times, the larger the lender is the less likely there is a bank run, while in bad times, the larger the lender is the more likely there is a bank run. That is, a large lender is a source of stability in good times. The flip side of this, is that the large lender can itself suffer from a shock that forces it to delever and in doing so cause the collapse of the bank it lends money to.

One technical matter required to prove the above result is that we need to strengthen the assumption about the prior distribution of L_0. The standard deviation of the prior distribution cannot be $+\infty$, and must be a finite positive number. With this added restriction, we obtain the desired result for some parameter values. We discuss the details in the Appendix.

Figure 8(1) and Figure 8(2) illustrates the simulation result.

Figure 8(1): The threshold for small lenders to run in bad times

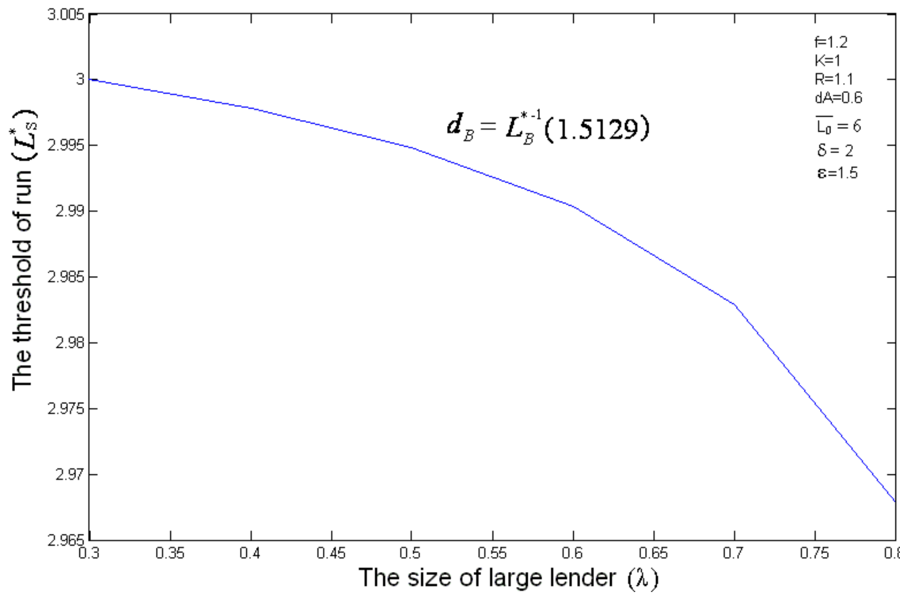
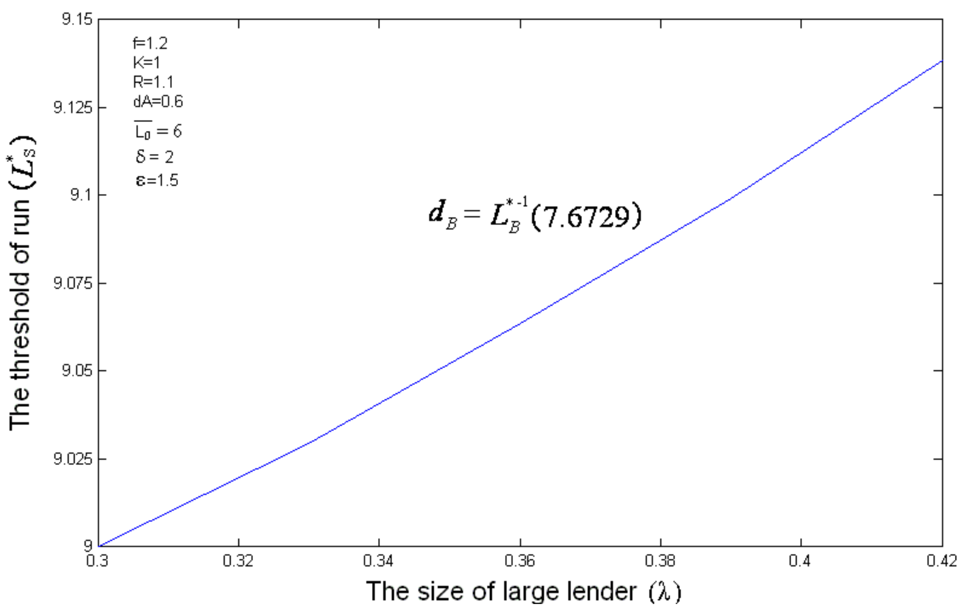


Figure 8(2): The threshold for small lenders to run in good times



4 Discussion: the vicious cycle caused by the structure of the financial system

It is commonly accepted that the recent liquidity crisis had its origins in the subprime mortgage lending. However, the triggering effect, mortgage defaults, cannot explain by themselves the wide spread devastation of the financial sector. As Brunnermeier (2008) argues, subprime mortgages losses represent a relatively modest destruction of wealth when compared to the banking industry as a whole. But the original loss of several hundred billion dollars in the mortgage market has triggered an extraordinary series of worldwide financial and economic consequences. In our view, the fundamental reasons of a systemic crisis of such wide proportions are rooted in the business model of many financial institutions, as well as in the structure of the modern financial system. Some recent developments in financial industry not only amplify the mortgage crisis, but also cause a vicious circle, which accounts for large dislocations and turmoil in the financial markets.

In this paper we show that it is the combination of high leverage supporting risky activities of financial institutions (business model) and the new creditor structure of financial institutions that explain the crash of the financial system.[5] Leverage obviously plays a great role in amplifying the mortgage defaults. High leverage means that any small loss to the value of the assets can become a high percentage loss to the equity. In order to keep leverage below a certain level, a financial institution needs to either inject new equity or deleverage by selling its assets with a multiplier of leverage. This can lead to a loss spiral - a decline in asset values erodes a leveraged financial institution's equity much faster, forcing a fire sale of assets that, in turn, depresses the price even further. This loss spiral is already examined by Adrian and Shin (2007) and Brunnermeier and Pedersen (2008).

In this paper, we emphasize that high leverage alone is not enough to cause systemic turmoil in the recent crisis. We believe that there is a second important reason: the structure of the creditors of financial institutions. The presence of large creditors can, at times, make the system degenerate into a vicious cycle during a crisis. Our argument is as follows: suppose the hedge funds, which in recent times have become the main providers of market liquidity by being the most active traders in the capital market, had enough funds and did not suffer too much during the crisis. In that case, hedge funds with their expertise in spotting good investment opportunities would take advantage of any mispricings and helped form an efficient capital market. Therefore, even if banks' equity shrunk significantly due to the losses in mortgage loans, after clearing their balance sheets of the

[5] The explosion of trading in complex securities, many without a liquid market, the compensation models of key players (traders and executives), the fault of risk assessors (rating agencies and loan guarantors), the lack of regulation and poor governance are other reasons offered for the crisis.

bad assets, banks would regain a value, and it would be possible for these banks to find some long-term well funded investors to inject new capital. Note that, as long as a reliable price indicator for the banks' assets exists, new investors can have a good idea of the value of banks, and are willing to participate in the recapitalization. Such investors would not need to be experts in assessing the value of the assets, as long as there was enough skilled investors (hedge funds) that would actively participate in the formation of efficient prices.

The question is why did this not happen in the recent crisis? Because hedge funds were themselves in trouble. They faced severe funding problems, which prevented them from performing their usual role of market-liquidity providers. The root of their funding problems was the banks themselves, on which many hedge funds critically relied for loans. This vicious cycle is the argument we wish to put forward as an important reason for the crisis: a shock to the market liquidity of a prime-broker's asset leads that broker to withdraw funds, and that creates a funding problem to the hedge funds. Hedge funds are thus unable to perform their usual arbitrage activities and stop providing market liquidity. Typically market liquidity is highly correlated across different asset markets. This will worsen market liquidity of brokers' assets even further, reinforcing the downward spiral. Furthermore, the disappearance of market liquidity makes it impossible for banks to obtain new equity to shore up their capital reserves. The outcome of such cycle is a market breakdown.

As mentioned before, a fundamental reason for the breakdown is that during the recent crisis there were no true 'long-term' investors. The deep pocket and long-horizon investors, such as sovereign wealth funds and wealthy individuals, had the money but lacked the necessary expertise. Hedge funds, on the other hand, had the expertise but faced serious funding problems. Why? Because, in the last decade or so, gradually but inexorably assets under management grew exponenationally fuelled by leverage, and with this leverage, the structure of the hedge funds' creditors changed dramatically. Moreover, hedge funds also widened their investor base beyond the original core of very wealthy and long term investors to include short-term investors. The result was that hedge funds became too dependent on large lenders such as prime-brokers, on one hand, and at the same time relaxed the horizon of the lock up period to attract short-term investors, on the other hand. Their capital structure became short term and their ever greater reliance on debt was pursued by borrowing more and more from the wholesale markets. In short, it is the change of capital and creditor structure of hedge funds that cripples their role as arbitrageurs during the recent crisis.

The above vicious cycle is in the same spirit but differs from Brunnermeier and Pedersen (2008) argument of 'margin spiral'. In our work, the adverse shock of market liquidity has an impact on the broker's financial constraint (i.e. the lender channel), while in Brunnermeier and Pedersen (2008) it affects the borrower's margin (i.e. the borrower channel). Note that for the lender channel, it is not necessary that the broker actually withdraws the funds. The fear of that happening is sufficient to scare other investors and thus cripple the arbitrageur's ability to conduct arbitrage trading.

The recent financial crisis has highlighted the fact that financial institutions nowadays are deeply linked and tightly interdependent of one another. One main linkage is that many financial institutions became large creditors to each other. This kind of interdependence makes the whole financial system particularly vulnerable. When an adverse shock hits one bank, the withdrawal of funds by one bank can trigger a chain reaction in the system. When prime brokers withdraw funds from hedge funds, it is particularly damaging because it reduces market liquidity and with it the price discovery necessary for an optimal allocation of financial resources in the economy. The great danger of financial institutions being the large lenders is that during a crisis those institutions scare away the true long-term investors and spare-liquidity providers. Traditionally, banks attract the spare liquidity from individual depositors. Hedge funds attract spare liquidity from wealthy individuals and institutions. Nowadays, due to the presence of a large influential creditor in the balance sheet of many financial institutions, the mere suspicion that a large creditor is in trouble can trigger a panic that easily spreads to the spare-liquidity providers.

5 Conclusion

The crisis of 2007 and 2008 is one of the most severe in financial history. Many investment banks, which were previously considered the ideal model of modern financial institutions, collapsed (Bear Sterns and Lehman Brothers) or lost its independence (Merrill Lynch). Major commercial banks either went bankrupt, were taken over or had to rescued by national governments. To have an idea of the scale of the losses, between the second quarter of 2007 and October 2008, Citigroup, the largest commercial bank in the world, shrank from $255 bn to $82 bn, and Morgan Stanley, one of the very few large investment banks that so far has remained independent, went from $49 bn to just \$20 bn.[6] Hedge funds suffered a similar fate. Assets under management were reduced by over 60% and the number of funds was reduced dramatically.

Interestingly, some banks did rather well under the severe circumstances. For example, JP Morgan Chase, a mixture of a commercial bank and an investment bank, over the same period of time, only lost from \$165 bn to \$147 bn in market capitalization. HSBC, the largest UK bank, went from \$215 bn to \$169 bn, when other large British banks, such as Royal Bank of Scotland and Barclays, lost 80% and two thirds of its value, respectively.

This paper presents an explanation for the crisis that complements reasons already offered. We claim that one of the problems of the modern financial system lies with the capital structure of many financial entities. First, there is just too much leverage in the balance sheet. Leverage increases the return on equity, but it magnifies the risk of the business. Second, the structure of creditors has changed over the years, and most banks (investment and commercial) use the wholesale market, a market where lenders are large institutional investors.[7] These large institutional lenders are themselves subject to shocks, and when these shocks bind their risk management constraints they are forced to delever. When they delever, they withdraw funds from institutions that borrow from them, forcing these, in turn, to sell assets at discounted prices. A chain effect that simultaneously operates through the asset and the liability sides of the balance sheet is then set in motion with dangerous consequences. Although our work is exploratory and more work needs to be done, we can show that the contemporaneous collapse of many financial institutions is not an accident. On the contrary, we argue that the vulnerability of financial institutions is an expected result of the existing financial architecture.

In particular, the potential actions taken by large lenders influence smaller lenders, because the small lenders know that if the large lender delevers, the liquidation value of the assets of the borrower financial institution is likely to not be sufficient to repay them all. This makes small lenders more jittery and run with a higher probability. Financial institutions which rely on wholesale markets are therefore a lot more fragile than regulators would believe. Regulation of financial institutions needs to be improved to take into account the capital structure and the structure of creditors of

[6] Data from Bloomberg, October 20 2008.

[7] The 'national bankruptcy' of Iceland is also attributed to its domestic banks dependence on foreign wholesale funds. It has been reported that a large fraction of the assets were healthy.

financial institutions. In modern capital markets, it is important to consider not just the risk of the assets of a financial institution, but also the risk that stems from both the level of indebtness and the structure of lenders. Financial regulators' concerns may need to include limits on leverage combined with limits on the relative share of lending by large lenders, when these are financial investors. Perhaps, financial regulators need to consider whether their monitoring activities should include both the borrowing financial institutions as well as their large lenders, which often might include non-banking institutions.

6 Appendix

Proof of section 3.3:

Figure 9 (1) : m may decrease with d_A

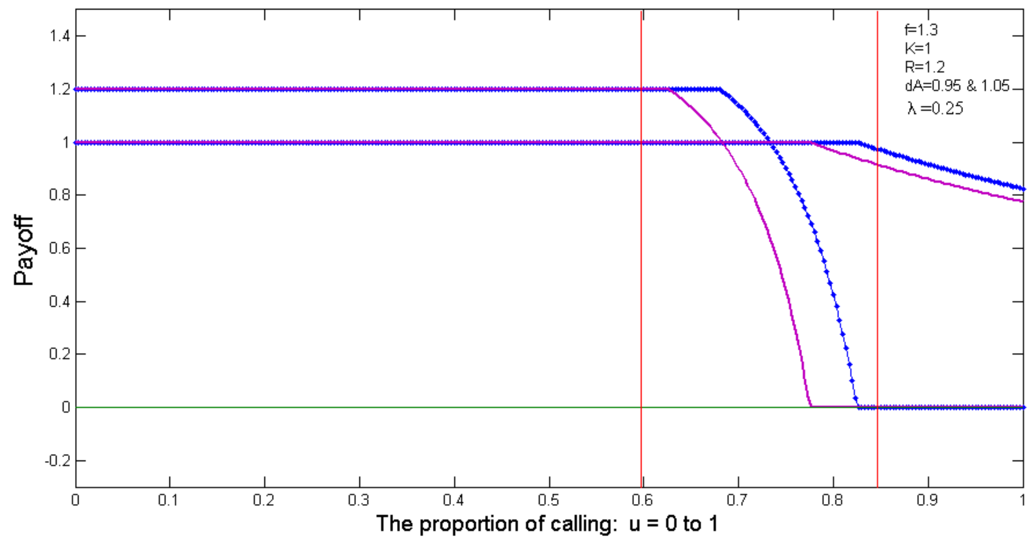
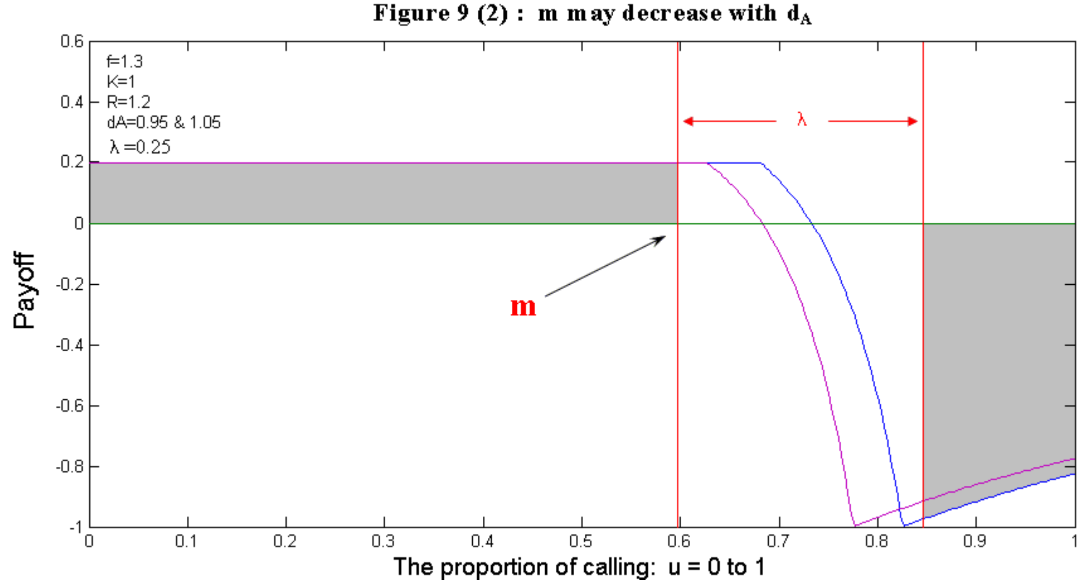


Figure 9 (2) : m may decrease with d_A



The comparative static result $\frac{\partial L_S^*}{\partial d_A} < 0$ is not always true in the presence of a large lender. Figure 9 illustrates the idea. In Figure 9(2), the increase of d_A actually leads to m decreasing, hence L_S^* increasing. Geometrically, if we increase d_A but keep the starting point of the cutting block m unchanged (i.e. the interval of the cutting block is still $[m, m+\lambda]$), then the shaded area below becomes smaller, while the shaded area above remains the same, which means that the original equilibrium is not valid. In the new equilibrium, the two shaded areas need to cancel out. So the shaded area above needs to shrink. That is, m has to move to the left and become smaller.

The economic intuition is as follows: Because there exists a large lender, the proportion of small lenders calling u is either very small or very large. (Recalling (8), the probability measure of the intermediate value of u is 0.) As Figure 9(2) shows, when u is very small, the net payoff of calling compared with holding $\triangle w(u)$ under the high d_A and the low d_A is same. However, when u is very large, the absolute value of net payoff $\triangle w(u)$ declines as market depth d_A increases. Aggregating the two possibilities, the increase of d_A may actually lead to the small lenders having a lower incentive to call early.

Proof of section 3.4:

We strengthen the assumption of L_0's prior distribution. Specifically, we assume that the standard deviation δ is a finite positive number, that is, $0 < \delta < +\infty$.

In this case, for the small lender i who receives the signal L_0^i, his posterior assessment of L_0 is no longer uniform over the interval $[L_0^i - \epsilon,\ L_0^i + \epsilon]$. Rather, the posterior density is $\varphi(L_0) = \frac{\phi(L_0)}{\Phi(L_0^i+\epsilon)-\Phi(L_0^i-\epsilon)}$ for $L_0 \in [L_0^i - \epsilon,\ L_0^i + \epsilon]$, where $\phi(\cdot)$ and $\Phi(\cdot)$ are pdf and cdf of the normal distribution $L_0 \sim N(\overline{L_0},\ \delta^2)$ respectively.

In particular, for the small lender at the margin who just receives the signal L_S^*, his posterior assessment over density of L_0 is

$$\varphi(L_0) = \begin{cases} \frac{\phi(L_0)}{\Phi(L_S^*+\epsilon)-\Phi(L_S^*-\epsilon)} & L_0 \in [L_S^* - \epsilon, L_S^* + \epsilon] \\ 0 & Otherwise \end{cases}.$$

As the margin lender is indifferent between holding and calling and its expected net payoff from holding (or calling) is 0, we have

$$\int_{L_S^*-\epsilon}^{L_S^*+\epsilon} \triangle\, w(u(L_0)) \cdot \varphi(L_0)\ dL_0 = 0.$$

From the above equation, we can work out the unique L_S^*.

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