# Strategic IPOs and Product Market Competition*

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**Abstract**

We examine firms' incentives to go public in the presence of product market competition. As a result of their greater ability to diversify risk in capital markets, public firms' owners tolerate higher profit variability than owners of private firms. Consequently, public firms adopt riskier and more aggressive output market strategies than private firms, which improves the competitive position of the former vis-à-vis the latter. This strategic benefit of being public, and thus, the proportion of public firms in an industry are shown to be positively related to the degree of competitive interaction among firms in output markets. Additional empirical predictions concern the effect of a firm's initial public offering on its market share and on its rivals' valuations. We also examine the interaction between the decision to go public and alternative ways of reducing risk, such as hedging.

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# 1 Introduction

In this paper we analyze firms' decision to go public in the presence of product market competition. In particular, we are interested in the relation between the degree of competitive interaction in an industry and the equilibrium proportion of public firms in it, and in the effects of a firm's IPO on its industry rivals' product market strategies and valuations.

Issuing public equity has numerous benefits. In addition to facilitating the financing of new investments, an IPO subjects a firm to outside monitoring (e.g., Holmström and Tirole (1993)); improves its liquidity (e.g., Amihud and Mendelson (1986)); reduces valuation uncertainty (e.g., Benveniste and Spindt (1989) and Dow and Gorton (1997)), which in turn lowers the costs of subsequent seasoned equity offerings (e.g., Derrien and Kecskeś (2007)), allows consumers to infer the firm's product quality from its stock price (e.g., Stoughton, Wong and Zechner (2001)), and improves the firm's mergers and acquisitions policy (e.g., Lyandres, Zhdanov and Hsieh (2009)); increases the firm's likelihood to become an acquisition target (e.g., Zingales (1995)); and increases the dispersion of its ownership (e.g., Chemmanur and Fulghieri (1999)). Importantly, an underdiversified risk-averse entrepreneur may benefit from an IPO because diversified investors assign higher valuations to a risky asset (firm equity) than the entrepreneur herself (e.g., Bodnaruk, Kandel, Massa and Simonov (2008)). Furthermore, transferring firm ownership from a risk-averse entrepreneur to diversified investors may improve profitability because risk considerations generally prevent profit maximization (e.g., Rothschild and Stiglitz (1971)).

All of the aforementioned studies examining various reasons for going public abstract from the competitive interaction among firms in output markets. Our model shows that product market competition is an important factor in the decision to go public. The intuition is as follows. Owners of public firms tend to hold more diversified portfolios than owners of private firms. For instance, Moskowitz and Vissing-Jørgensen (2002) find that about three-fourths of all private equity is owned by individuals for whom such investment constitutes at least half of their total net worth; Bodnaruk et al. (2008) show that an IPO substantially increases the degree of diversification of private firm's controlling shareholders. As a result, public firms tend to be less concerned with (idiosyncratic) profit variability, and hence, tend to pursue more aggressive operating strategies than otherwise similar private firms (e.g., Shah and Thakor (1988)).

When firms compete in quantities (à la Cournot), a public firm's commitment to a more aggressive product market strategy has an important *strategic* benefit: it reduces the equilibrium aggressiveness of the firm's rivals. Specifically, a public firm's commitment to a larger output results in a lower equilibrium output of its competitors, and thus, in a higher residual demand for the firm's own product.

Because the strategic benefit of going public increases with the degree of competitive interaction among product market rivals, there is a positive relationship between the equilibrium number and proportion of public firms in an industry and the degree of competitive interaction in this industry. Other implications of the greater product market aggressiveness of public firms are the adverse effects that an IPO has on the market shares and valuations of the firm's product market rivals.

While our model is not specific to any particular industry, it may be useful to put the theory in the context of existing anecdotal evidence. An interesting illustration of the possible link between a firm's decision to go public and the competitive structure of its industry is provided by the evolution of the U.S. investment banking industry in the mid-eighties. During this period the fraction of firms switching underwriters between their IPOs and SEOs increased considerably (e.g., Burch, Nanda and Warther (2005)). Following transaction cost economics literature (e.g., Williamson (1979)), the decreased "brand-loyalty" suggests an increase in the competitive intensity in the investment banking industry. According to Eccles and Crane (1988), the U.S. investment banking industry was also exposed to increased competition from U.S. commercial banks and foreign banks during the same period. Interestingly, this period of intensifying competition coincided with a wave of IPOs: 27 investment banks went public between 1983 and 1987, compared to just one bank that was taken public in the preceding ten years (e.g., Muscarella and Vetsuypens (1989)). It is difficult to fully explain such a dramatic increase in the number of IPOs in the investment banking industry by the generally hot IPO market in the mid-eighties. While the coincidence of increased competition and an IPO wave in the same industry does not establish a causal link, it motivates us to explore possible rationale for the existence of such a link.

The strategic benefit of IPO is similar to the "strategic benefit of debt" (e.g., Titman (1984), Brander and Lewis (1986), Maksimovic (1988), Poitevin (1989), and Glazer (1994)). Because of limited liability, increasing leverage raises a firm's incentive to pursue a more aggressive output market strategy. Similar to issuing debt, performing an IPO allows the firm to commit to a product market strategy that reduces the equilibrium aggressiveness of its industry rivals. Thus, our key result – the positive relationship between the degree of competitive interaction and firms' equilibrium propensity to go public – is parallel to the positive relationship between the degree of competitive interaction and optimal financial leverage (e.g., Lyandres (2006)).

Interestingly, the benefit of going public is decreasing in the number of firms that have already done so. As a result, some firms go public while others remain private in equilibrium, despite being ex-ante identical. The reason is that the benefit of going public drops below the cost of doing so once a sufficient number of firms have gone public. Thus, our model is similar to other models of corporate

finance decisions in the presence of product market competition in that ex-ante identical firms are likely to choose different strategies in equilibrium (e.g., Maksimovic and Zechner (1991) for the case of capital structure and Adam, Dasgupta and Titman (2007) for the case of hedging strategies).

Whereas going public facilitates risk sharing at the shareholder level, risk can be also reduced at the corporate level by means of hedging, insurance, or corporate diversification. In order to examine the interaction between a firm's decision to go public, its product market strategy, and its activities aimed at mitigating risk, we extend our model by allowing firms to hedge.[1] The effect of hedging on the equilibrium proportion of public firms depends on the relative magnitudes of the following two effects. On one hand, because going public and hedging are substitute ways of mitigating risk, the benefit of going public is decreasing in the effectiveness of hedging. On the other hand, allowing firms to hedge makes them more aggressive in output markets, which may reduce firms' equilibrium profits and induce exit from the industry. The latter effect can potentially lead to a positive relation between the proportion of public firms in an industry and the availability and effectiveness of hedging opportunities. We show that the relative magnitudes of the two effects and the resulting incentives to go public depend on industry characteristics, such as the barriers to entry into the industry. Our paper is one of the first to link two extensive strands of corporate finance literature: the IPO literature and the hedging literature (e.g., Smith and Stulz (1985), Froot, Scharfstein and Stein (1993), Nance, Smith and Smithson (1993)).

While our paper focuses on Cournot-type competition, a similar strategic benefit of going public exists under Bertrand-type competition as well. When firms are price-setters, profit variability decreases with product market aggressiveness.[2] As a result, a price-setting public firm pursues a less aggressive product market strategy than a similar private firm. However, because under Bertrand-type competition firms' competitive actions are strategic complements, the less aggressive strategy of a public firm reduces the aggressiveness of its competitors, which increases the residual demand for the firm's product. In other words, going public has a strategic benefit under both Cournot and Bertrand-type competition, although the driving forces behind this benefit are different in the two situations.

To summarize, this paper's contribution to the IPO literature is as follows. First and most importantly, it is, to the best of our knowledge, the first paper to demonstrate the strategic benefit of going public. Second, we show that this benefit, and thus, firms' propensity to go public increase with the degree of competitive interaction in the output market. In addition, we show that, under

[1] We consider hedging as an example of any action that reduces profit variability at the firm level.

[2] Under price-setting, profit variance is increasing in price and decreasing in output.

Cournot-type competition, an IPO has an adverse effect on market shares and valuations of the firm's product market rivals. Finally, we examine the effect of risk reduction at the corporate level on firms' incentives to go public.

The remainder of the paper is organized as follows. In the next section, we outline the model, characterize its equilibrium, and analyze the effects of a firm's IPO on the equilibrium strategies and valuations of its rivals. In Section 3, we discuss how the equilibrium industry structure, particularly, the equilibrium proportion of firms deciding to go public, depends on the nature of product market competition. In Section 4, we extend the model by considering financial hedging as an alternative to IPO in mitigating risk. We examine how the availability of hedging affects the decision to go public, and how the equilibrium proportions of firms that hedge and those that go public depend on the characteristics of the industry. Section 5 concludes. All proofs are relegated to Appendix.

## 2 Model

Our model of going public in the presence of oligopolistic product market competition has three decision stages, as illustrated in Figure 1. Initially, the demand for firms' products is uncertain. In the first stage, firms owned by risk-averse entrepreneurs (private firms henceforth) enter the industry. In the second stage, each of the firms decides whether to sell its shares to investors through an IPO. In the third stage, while the demand for firms' products is still uncertain, each firm chooses its output level. Finally, demand uncertainty is resolved, the product market clears and the firms realize profits. [3]Firms' decisions at each stage are made non-cooperatively. Throughout the paper, we abstract from various agency conflicts between firms' managers and shareholders, assuming that their interests are aligned. We assume that all firms have symmetric information. In what follows, we formulate each of the decision stages and characterize its equilibrium, starting with the third stage and going backwards.

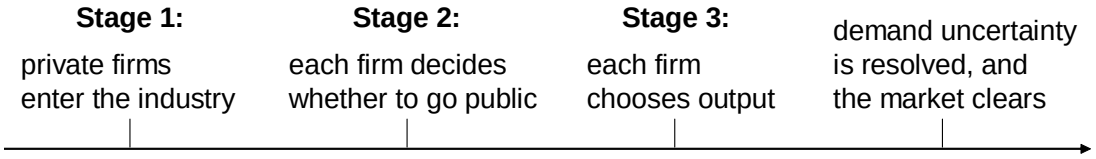


Figure 1: The sequence of events.

[3]Note that the game is divided into three stages for expositional purposes only. Identical results are obtained within a model in which all of the decisions are made simultaneously.

### 2.1 Stage 3: Product market competition

Consider $N$ firms competing in quantities in a heterogeneous product market. Assuming linear demand curves,[4] the market-clearing price of firm $i$'s product is given by

$$p_i(\mathbf{q}, \alpha_i) = \alpha_i - \beta q_i - \gamma \sum_{j=1,\ j\neq i}^{N} q_j,\ \ i = 1, ..., N, \tag{1}$$

where $\mathbf{q} = (q_1, ..., q_N)$ is the output vector of the $N$ firms, and $\alpha_i$, $\beta$, and $\gamma$ are the demand curve parameters.

To reflect demand uncertainty, we assume that demand curves' intercepts, $\alpha_1, ..., \alpha_N$, are stochastic, and refer to them as "demand shocks." For analytical convenience, we assume that the vector of demand shocks, $\boldsymbol{\alpha} = (\alpha_1, ..., \alpha_N)$, is normally distributed with means and variances symmetrical across firms, i.e. $\mathbb{E}(\alpha_i) \equiv \mu$ and $\mathbb{V}\mathtt{ar}(\alpha_i) \equiv \sigma^2$ for all $i$.[5]

The coefficients $\beta > 0$ and $\gamma \geq 0$ measure the sensitivity of the market clearing price of a firm's product to the firm's own output and to its rivals' outputs, respectively. For simplicity, these parameters are also assumed to be symmetrical across firms. Because in a heterogeneous product market, the own-price effect has to be larger than the cross-price effect, we have $\beta > \gamma$.[6] Parameter $\gamma$ is of particular interest because it measures product substitutability, and thus, the degree of "competitive interaction" among firms. When $\gamma = 0$, there are no cross-price effects, i.e. each firm is a monopolist in its own market. As $\gamma \to \beta$, the competitive interaction intensifies.

Before the resolution of demand uncertainty, firm $i$, $i = 1, ..., N$, produces $q_i$ units of output at a constant marginal cost $c$. After that, demand shocks are realized and the product market clears. Thus, firm $i$ realizes profit

$$\pi_i(\mathbf{q}, \alpha_i) = p_i(\mathbf{q}, \alpha_i)\, q_i - c q_i, \tag{2}$$

where the market-clearing price $p_i(\mathbf{q}, \alpha_i)$ is given in (1).[7,8]

[4] Linear demand follows from the second-order approximation of consumer utility function that is additively separable, linear in money, and concave in consumption of all other goods.

[5] In our one-period model, the correlation structure of demand shocks has no impact on the results.

[6] Formally, this inequality follows from the strict concavity of the consumer utility function.

[7] Note that under extreme demand shock realizations, the linear demand model can lead to negative prices. As is common in the industrial organization literature (e.g., Vives (1984), page 77), we assume the variance of demand intercepts to be sufficiently small to make the probability of negative prices negligible.

[8] Also note that we do not allow firms to withhold any output from the market after observing the demand shock realization. It can be shown that even if firms were allowed to withhold output from the market, they would never do so in equilibrium as long as $\alpha_i \geq \mu - c$, $i = 1, ..., N$. The probability of this condition being satisfied can be made arbitrarily close to one by restricting demand shock variance.

When choosing optimal output, each firm's objective depends on its status as a private firm or a public one. Because owners of public firms hold more diversified portfolios than owners of private firms (e.g., Hansen and Lott (1996), Moskowitz and Vissing-Jørgensen (2002), Bodnaruk et al. (2008)), public firms' owners are less concerned with firm-specific risk than private firms' owners. To reflect this empirical observation in a most parsimonious fashion, we assume that all risk is idiosyncratic and that public firms' shareholders are fully diversified whereas private firms' owners (entrepreneurs) have their wealth closely tied in the stock of their firms. As a result, public firms are assumed to be expected profit maximizers, whereas private firms maximize the expected utility that their owners derive from the firms' profits.[9] The assumption that all risk is idiosyncratic, and thus can be ignored by diversified investors, is common in the corporate finance literature.[10] Importantly, even if risk has a systematic component, the product market strategy of a public firm is affected by overall uncertainty less than that of a private firm and, therefore, all of the qualitative results of the model go through as long as public firms' owners are more diversified than the private firms' owners.

For analytical convenience, we assume that all entrepreneurs have the same exponential utility given by

$$u(\pi_i) = r^{-1} - r^{-1}\exp(-r\pi_i), \tag{3}$$

where $r = -u''/u'$ is the Arrow-Pratt coefficient of absolute risk aversion, which measures entrepreneurs' attitude towards risk.[11] Because demand shocks are normally distributed, so are profits, and a private firm's owner's expected utility simplifies to the mean-variance criterion, i.e.,

$$\mathbb{E}u\left(\pi_i\right) = \mathbb{E}\pi_i - \frac{r}{2}\mathbb{V}\mathtt{ar}\left(\pi_i\right). \tag{4}$$

Thus, if we let $V_i$ to be the cash equivalent, or "value" of firm $i$ to its owner(s), we have

$$V_i = \begin{cases} \mathbb{E}\pi_i - \frac{r}{2}\mathbb{V}\mathtt{ar}\left(\pi_i\right) & \text{if firm } i \text{ is private, and} \\ \mathbb{E}\pi_i & \text{if firm } i \text{ is public.} \end{cases} \tag{5}$$

Each firm chooses its output to maximize its value while taking into account the output decisions of its rivals. If we let $n \in \{0, ..., N\}$ denote the number of firms that are public, the equilibrium output

[9] Another difference between the objective functions of public and private firms is the potentially myopic behavior of the former, resulting from public firms' concerns about short-term stock price movements. However, such dynamic considerations are outside the scope of our static model.

[10] For example, Gomes (2000) and Bitler, Moskowitz and Vissing-Jørgensen (2005) examine the sale of firm equity by a risk-averse entrepreneur to diversified investors, explicitely assuming that all risk is idiosyncratic; Kirilenko (2001) considers a venture between a risk-averse nondiversified entrepreneur and a fully diversified, and therefore profit-maximizing, investor.

[11] Note that this is the only utility function in which the coefficient of absolute risk aversion, $r = -u''/u'$, does not depend on $\pi_i$, and where $u(\pi_i) \to \pi_i$ as $r \to 0$.

vector $\mathbf{q}^*(N,n)$ is given by the following system of $N$ equations:

$$q_i^*(N,n) = \arg\max_{q_i} V_i\left(q_i, \mathbf{q}_{-i}(N,n)\right), \ i = 1, ..., N, \tag{6}$$

where $\mathbf{q}_{-i}$ is the output vector of the $N-1$ firms other than $i$. To simplify the notation, we will use $\pi_i^*(N,n) = \pi_i(\mathbf{q}^*(N,n))$ and $V_i^*(N,n) = V_i(\mathbf{q}^*(N,n))$ to denote the profit and value of firm $i$ evaluated at the equilibrium output vector, respectively.

We begin by characterizing the equilibrium output vector and the resulting firm values conditional on pre-determined industry structure. The next result establishes that the equilibrium is symmetric, in the sense that all public firms produce identical output quantities in equilibrium, as do all private firms. Thus, in characterizing the equilibrium, we only need to distinguish between private and public firms, whose indexes we replace with *pri* and *pub*, respectively.

**Lemma 1** *For any given industry structure* $(N,n)$, *there exists a unique equilibrium output vector* $\mathbf{q}^*(N,n)$, *which is given by*

$$q_{pub}^*(N,n) = \frac{(\mu - c)(2\beta - \gamma + r\sigma^2)}{(2\beta - \gamma)(2\beta - \gamma + N\gamma + r\sigma^2) + n\gamma r\sigma^2}, \tag{7}$$

$$q_{pri}^*(N,n) = \frac{(\mu - c)(2\beta - \gamma)}{(2\beta - \gamma)(2\beta - \gamma + N\gamma + r\sigma^2) + n\gamma r\sigma^2}, \tag{8}$$

*and results in the following equilibrium firm values:*

$$V_{pub}^*(N,n) = \beta\left(q_{pub}^*(N,n)\right)^2, \tag{9}$$

$$V_{pri}^*(N,n) = \left(\beta + \frac{1}{2}r\sigma^2\right)\left(q_{pri}^*(N,n)\right)^2. \tag{10}$$

The following result characterizes the effects of demand uncertainty, risk-aversion and the degree of competitive interaction on the equilibrium output levels of public and private firms.

**Proposition 1** *(i) For a given industry structure* $(N,n)$, *the equilibrium output of a public firm is always greater than that of a private firm. Furthermore, the equilibrium output of a private firm is decreasing while that of a public firm is increasing in risk aversion* $r$ *and in demand uncertainty* $\sigma^2$.

*(ii) For a given industry structure* $(N,n)$, *the equilibrium output of either type of firm is decreasing in the degree of competitive interaction* $\gamma$. *Furthermore, as the degree of competitive interaction increases, the market share of public firms increases, while that of private firms decreases.*

*(iii) For a given number of public firms* $n$, *the equilibrium output of either type of firm is decreasing in the total number of firms* $N$.

*(iv) For a given total number of firms $N$, when a firm goes public, its equilibrium output increases while the equilibrium output of each of its rivals decreases. The relative increase in the IPO firm's market share is larger the larger the degree of competitive interaction $\gamma$.*

The intuition behind Proposition 1 is as follows. Taking rivals' output decisions as given, private firms choose lower output levels than public firms because their owners are risk-averse and because profit variance increases with output. In a competitive equilibrium, the relative gap between public firm's output and private firm's output is further widened as public firms take advantage of their private rivals' lower outputs by increasing their own output levels. The higher the risk aversion and the higher the demand uncertainty, the lower the optimal output levels of private (risk-averse) firms, which in turn encourages higher outputs of public (risk-neutral) firms.

Higher degree of competitive interaction results in lower prices, and hence, lower marginal revenues, which in turn lead to lower equilibrium outputs. In other words, as products become closer substitutes, overall market size decreases, resulting in lower equilibrium output levels. Recall that public firms take advantage of the lower output levels of their private rivals by increasing their own output levels. Importantly, this relative advantage of public firms increases with the degree of competitive interaction.

As the number of firms in the industry increases, the demand for each firm's product, and thus, each firm's equilibrium output decline. Finally, a firm that goes public becomes more aggressive and increases its output. As the residual demand decreases, the equilibrium output of all other firms decreases. This effect is stronger the stronger the degree of competitive interaction among the firms. The last part of Proposition 1 leads to our first empirical prediction.

**Empirical Prediction 1** *(i) Under competition in strategic substitutes (e.g., quantities), a firm's IPO is expected to increase the firm's market share.*
*(ii) The relative increase in the newly public firm's market share is expected to be larger the larger the degree of competitive interaction in the industry.*

The empirical evidence in a recent paper by Hsu, Rocholl and Reed (2009) is generally consistent with the first part of the prediction. Industry rivals of firms that go public experience significantly negative sales growth around IPOs. This, combined with the increased post-IPO sales of newly public firms implies that going public tends to increase firms' market shares. There also exists some anecdotal evidence supporting the first part of our prediction. Killian, Smith and Smith (2001) cite a number a examples in which firms that went public ahead of its competitors gained a considerable market share (e.g., Handspring, Affymetrix, Petsmart). Of course, there are reasons why the going public decision is likely to have a positive impact on the firm's market share other than the one captured by our

model. Most importantly, a firm may be able to use the capital raised through its IPO to expand its production capacity. In addition, Chemmanur and He (2008) argue that going public may increase the firm's market share because of the enhanced credibility of a public firm with customers, its greater ability to acquire related firms, or its ability to hire quality employees and motivate them using stock options. Thus, to validate the *strategic* effect of the firm's mode of incorporation predicted by our model in an empirical setting, it would be useful to test the second part of our prediction, i.e., the relation between the effect of an IPO on the firm's market share and the competitive intensity in the industry.[12,13]

The next proposition presents the comparative statics of firm values.

**Proposition 2** *(i) For a given industry structure $(N, n)$, the equilibrium value of a public firm is always greater than that of a private firm. Furthermore, the equilibrium value of a public firm is increasing while that of a private firm may be increasing or decreasing in risk aversion $r$ and in demand uncertainty $\sigma^2$.*

*(ii) For a given industry structure $(N, n)$, the equilibrium value of either type of firm is decreasing in the degree of competitive interaction $\gamma$. Furthermore, as the degree of competitive interaction increases, the relative benefit of being public, $V^*_{pub}(N, n)/V^*_{pri}(N, n)$, increases.*

*(iii) For a given number of public firms $n$, the equilibrium value of either type of firm is decreasing in the total number of firms $N$.*

*(iv) For a given total number of firms $N$, when a firm goes public, the equilibrium value of each of its rivals decreases. The relative decrease in the rival firms' values, $(V_i^*(N, n) - V_i^*(N, n+1))/V_i^*(N, n)$, is larger the larger the degree of competitive interaction $\gamma$.*

A private firm's value is lower than that of a public firm for three reasons. First, a risk-averse owner of the former, unlike public firm's shareholders, cannot diversify risk in the capital market, and thus, her valuation of a risky firm is lower. Second, because public firms' product market strategies are not distorted by risk considerations, public firms achieve higher expected profits than otherwise

[12] Hsu, Rocholl and Reed (2009) examine the relation between sales growth of rivals of IPO firms and their industry's Herfindahl index. However, the theoretical relation between Herfindahl index and the degree of competitive interaction ($\gamma$) is indeterminate. We are not aware of any study that examines the relation between the degree of competitive interaction in an industry and the effects of firms' IPOs on their own and their rivals' market shares.

[13] Furthermore, to control for the effect of the capital raised through an IPO, one could compare the effect of IPOs and that of seasonal equity offerings in which firms raise capital without changing the mode of incorporation.

similar private firms. Finally, because firms' products are substitutes, the greater product market aggressiveness of public firms reduces the equilibrium aggressiveness of their private rivals.

Public firms' values are increasing in risk aversion and in demand uncertainty as public firms benefit from the declining output of their private rivals. Somewhat surprisingly, private firm values are not monotonic in demand uncertainty and in risk aversion. For a given output vector $\mathbf{q}$, a private firm's value is decreasing in both demand uncertainty and risk aversion, as one would expect. However, as demand uncertainty and/or risk aversion increase, each private firm reduces its output, which brings the total combined output closer to the monopoly level, benefitting all firms. In other words, risk aversion mitigates the cost of excess production (relative to the monopoly level) resulting from competition. This effect dominates, i.e., more uncertainty or stronger risk aversion increase each private firm's value when there is a large number of private firms and/or when the degree of competitive interaction is high.

Stronger competitive interaction results in lower prices, and therefore, in lower firm values. However, public firms suffer less as their strategic commitment to larger outputs becomes more valuable. Thus, the relative benefit of being public increases in the degree of competitive interaction. As one would expect, when market entry occurs (i.e., when $N$ increases), the equilibrium profits and values of all incumbents decline. Finally, the last result in Proposition 2 stems from the adverse effect that the greater aggressiveness of a public firm has on the competitors. It leads to an interesting empirical prediction.

> **Empirical Prediction 2** *(i) Under competition in strategic substitutes (e.g., quantities), a firm's IPO is expected to adversely affect the values of other firms operating in the industry, including the market values of its publicly traded rivals.*
> *(ii) The effect of an IPO on rivals is expected to be stronger the stronger the degree of competitive interaction in the industry.*

The first part of this prediction is consistent with Slovin, Sushka and Ferraro (1995) and Hsu, Reed and Rocholl (2009) who report that a firm's IPO results in abnormally negative returns to the firm's competitors, and with Slovin, Sushka, and Benedeck (1991) who find positive valuation effects for industry rivals of a firm going private. However, none of these papers tests whether product market competition is one of the driving forces behind this relation as suggested by the second part of the prediction.

Having characterized the equilibrium of the product-market-competition stage of the game, in the next subsection we consider the preceding stage of the game, in which firms decide whether to go public or stay private.

## 2.2 Stage 2: The decision to go public

In the second stage, each of the $N$ firms is owned by a risk-averse entrepreneur who decides whether to sell her firm to the public through an IPO. The entrepreneurs decide whether to go public or stay private in an arbitrary sequence. Going public has benefits as well as costs. We discuss its benefits first. By selling her firm to investors who can diversify risk in the capital market, an entrepreneur receives cash equalling the expected firm value, which becomes public and is valued accordingly. Thus, for a given industry structure $(N, n)$, the benefit of IPO, $B_{IPO}(N, n)$, is given by

$$B_{IPO}(N, n) = V^*_{pub}(N, n+1) - V^*_{pri}(N, n) > 0. \tag{11}$$

The IPO benefit can be further decomposed into three positive components as follows:

$$B_{IPO}(N, n) = B_{\text{diversification}} + B_{\text{profit}} + B_{\text{strategic}}, \text{ where} \tag{12}$$

$$B_{\text{diversification}} = \frac{r}{2}\mathbb{V}\mathtt{ar}\pi^*_{pri}(N, n), \tag{13}$$

$$B_{\text{profit}} = \max_{q_i} \mathbb{E}\pi_i\left(q_i, \mathbf{q}^*_{-pri}(N, n)\right) - \mathbb{E}\pi^*_{pri}(N, n), \tag{14}$$

$$B_{\text{strategic}} = \mathbb{E}\pi^*_{pub}(N, n+1) - \max_{q_i} \mathbb{E}\pi_i\left(q_i, \mathbf{q}^*_{-pri}(N, n)\right), \tag{15}$$

and where $\mathbf{q}^*_{-pri}(N, n)$ is the equilibrium output vector of all but one private firm for a given $(N, n)$.

The first benefit, $B_{\text{diversification}}$, reflects the fact that, ceteris paribus, stochastic profit is worth more to diversified investors than to an underdiversified risk-averse entrepreneur. The second benefit, $B_{\text{profit}}$, reflects the fact that a public firm pursues profit maximization, and thus, achieves a higher profit than a utility-maximizing private firm, taking the output of all other firms as given. Finally, the third, strategic, benefit of going public, $B_{\text{strategic}}$, exists only in the presence of interaction among firms in product markets and stems from the effect that an IPO has on the equilibrium strategies of a firm's product market rivals. Because firms' products are substitutes, private as well as public competitors respond to the greater aggressiveness of the firm that has gone public by reducing their own outputs, i.e., $q^*_{pri}(N, n+1) < q^*_{pri}(N, n)$ and $q^*_{pub}(N, n+1) < q^*_{pub}(N, n)$. As the competitors' combined output declines, the equilibrium value of the IPO firm increases, i.e., $\mathbb{E}\pi^*_{pub}(N, n+1) > \max_{q_i} \mathbb{E}\pi_i\left(q_i, \mathbf{q}^*_{-pri}(N, n)\right)$.

The following result shows how the benefit of IPO depends on several key variables.

**Proposition 3** *(i) For a given industry structure $(N, n)$, the benefit of IPO, $B_{IPO}(N, n)$, is increasing in risk aversion $r$ and in demand uncertainty $\sigma^2$.*

*(ii) For a given number of public firms $n$, the benefit of IPO is decreasing in the total number of firms $N$.*

*(iii) For a given total number of firms $N$, the benefit of IPO is decreasing in the number of public firms $n$.*

*(iv) For a given industry structure $(N, n)$, the relative benefit of IPO, $V^*_{pub}(N, n+1)/V^*_{pri}(N, n)$, is increasing in the degree of competitive interaction $\gamma$.*

The benefit of IPO stems from the ability of public firms' shareholders to diversify risk in capital market. Thus, the higher the underlying uncertainty or the higher the risk aversion, the larger the benefit of public incorporation. As the number of firms in the industry increases, all firm values decline due to lower residual demand for each firm's product, and so does the incremental value stemming from going public. Similarly, as the number of public (and thus more aggressive) firms increases, the values of both types of firms decline, and so does the incremental value of an IPO.

Finally, recall that stronger competitive interaction leads to a larger market share of public firms and a smaller market share of private firms (see Proposition 1). As a result, stronger competitive interaction hurts public firms *relatively* less than private firms, and increases the relative benefit of going public, $V^*_{pub}(N, n+1)/V^*_{pri}(N, n)$. A natural question arises whether stronger competitive interaction also increases the *absolute* benefit of going public, $B_{IPO}(N, n) = V^*_{pub}(N, n+1) - V^*_{pri}(N, n)$. For a given industry structure $(N, n)$, this effect depends on model parameters. However, as we show below, this effect becomes unambiguous once we endogenize the number of firms going public $n$ as well as the total number of firms in the industry $N$.

We now turn to the costs of going public. A substantial part of this cost is the underwriting fee, which is a result of the interaction between firms going public and underwriters (investment banks). We assume that the market for underwriting is competitive and is characterized by a non-decreasing supply curve denoted as $F(n)$ (i.e., $F(n)$ is the underwriting fee if exactly $n$ firms go public.) We assume that all other costs associated with going public (loss of private benefits of control, cost of disclosure, underpricing, etc.) are exogenous to our model and are identical for all firms, in which case, they can be normalized to zero without any loss of generality.[14,15] It is possible to show that all

[14] To account for indirect costs of going public, one can simply shift $F(n)$ up by the amount of these indirect costs.

[15] There is extant empirical evidence that suggests that the direct cost of going public is related to IPO size (e.g., Chen and Ritter (2000). It can be shown that all of the model's results hold under proportional (as opposed to fixed) IPO costs.

of the model's results hold in the case of heterogenous IPO costs.[16] When there are $N$ firms in the industry, $n$ of which are public, any of the remaining private firms has incentive to go public if, and only if, the benefit of IPO exceeds its cost,[17] i.e.,

$$B_{IPO}(N,n) > F(n+1). \tag{16}$$

The next result characterizes the equilibrium number of IPOs, $n^*(N)$, for a given total number of firms in the industry $N$.

**Lemma 2** *For a given total number of firms $N$, there exists a unique equilibrium number of public firms, $n^*(N)$, which is given as follows:*

*(i) If $F(1) < B_{IPO}(N,0)$ and $F(N) \geq B_{IPO}(N,N-1)$, there are both private and public firms, i.e. $0 < n^*(N) < N$, and $n^*(N) = \min\{n \in \mathbb{N}_0 : B_{IPO}(N,n) \leq F(n+1)\}$.*

*(ii) If $F(1) \geq B_{IPO}(N,0)$, all firms remain private, i.e., $n^*(N) = 0$.*

*(iii) If $F(N) < B_{IPO}(N,N-1)$, all firms go public, i.e., $n^*(N) = N$.*

Because the benefit of IPO strictly decreases as the number of public firms increases, there is a unique equilibrium number of IPOs.[18] The intuition behind Lemma 2 is illustrated in Figure 2. (In this figure, the cost and benefit of IPO are depicted as continuous linear functions of $n$ for illustrative purposes only.) Consider the two boundary cases first. If the benefit of going public does not exceed the cost of doing so even for a single firm going public (case ii), there are no IPOs. If the benefit of going public exceeds the cost of going public even for the last private firm in the industry (case iii), all firms go public. In all other situations (case i), $0 < n^* < N$ firms go public, while others remain private.

Lemma 2 has an interesting implication. Except for the boundary cases (ii) and (iii), some firms go public while others stay private even though all firms are ex-ante identical, and this is the case even if the cost of going public, $F(n)$, **is independent of the number of IPOs.** This is because as more firms go public, the benefit of going public declines and, at some point, it falls below the cost

[16] The solution of a model involving heterogenous firms appeared in previous versions of this paper and is available upon request.

[17] That is, we break the ties by assuming that when a firm is indifferent between staying private and going public, it stays private.

[18] Note that whereas the equilibrium number of IPOs is unique, the equilibrium set of firms that go public is not unique and depends.on the sequence in which firms consider whether to go public or not. Since all firms are assumed to be ex-ante identical, our model is silent on *which* of them will go public.

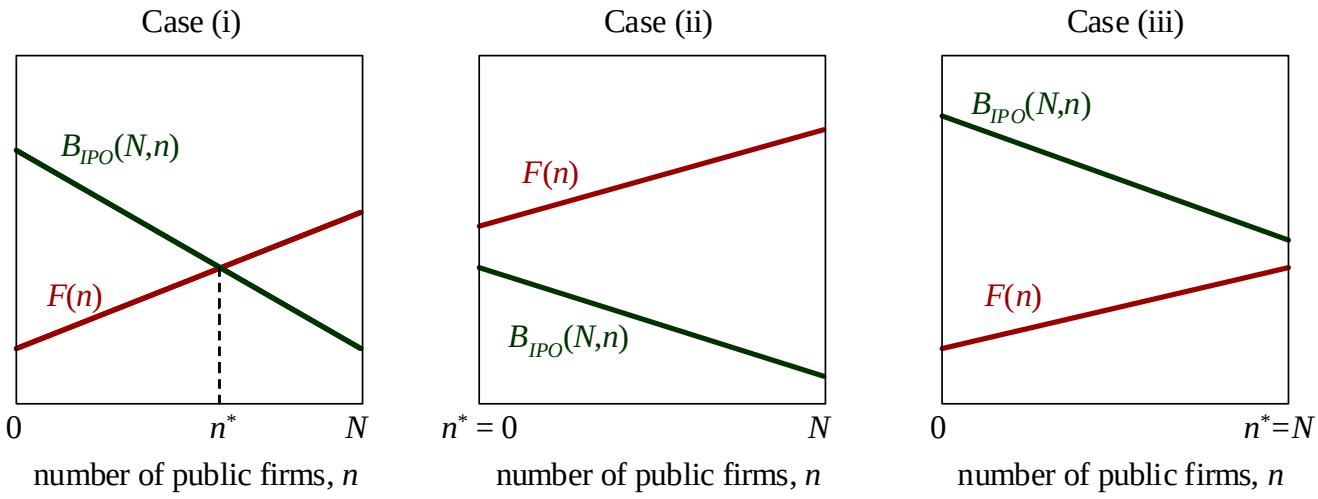


Figure 2: The cost and benefit of going public when the total number of firms fixed.

of IPO. This result is somewhat similar to other corporate finance settings in which ex-ante identical firms choose different strategies in equilibrium (e.g., Maksimovic and Zechner's (1991) for the case of capital structure, Adam, Dasgupta and Titman (2007) for the case of hedging, and De Meza (1986) for the case of production technology choices).

Note that the fact that the IPO benefit is always positive implies that all firms would go public if doing so were costless. This, however, does not mean that in an equilibrium in which all firms go public, they are better off than in an equilibrium in which all firms remain private. The reason is that private firms' production levels are closer to the monopoly level, which could result in higher equilibrium values despite the lost benefit of diversification. This result is similar to that of strategic debt in Brander and Lewis (1986), who show that under quantity competition firms choose positive debt levels despite the fact that they would be better off if all firms were all-equity financed.

**Proposition 4** *(i) For a given total number of firms $N$, the equilibrium number of public firms, $n^*(N)$, is increasing in risk aversion $r$ and in demand uncertainty $\sigma^2$.*

*(ii) The equilibrium number of public firms, $n^*(N)$, is decreasing in the total number of firms $N$.*

The first result follows from the fact that the benefit of going public is increasing in risk aversion and in demand uncertainty. The second result is due to the fact that the benefit of IPO decreases as the total number of firms increases. Intuitively, with more firms in the industry, each firm's residual demand and value are lower and the cost of IPO is more difficult to justify. In the next subsection, we consider the first stage of the game, in which firms decide to enter the industry.

Endogenizing entry is crucial for at least two reasons. First, the number of firms in an industry corresponds to the number of products in a broadly defined product market. Thus, an exogenous increase in the number of firms would lead to an additional mass of consumers, leading to higher equilibrium prices. This counterintuitive effect is absent when the number of firms in the industry is determined endogenously. Second, and equally importantly, our ultimate goal is to examine the effect of the degree of competitive interaction in an industry $\gamma$ on firms' propensity to go public and on post-IPO changes in IPO firms' rivals' operating strategies and resulting equilibrium values. However, it is impossible to analyze the comparative statics of the model with respect to $\gamma$ while keeping the number of firms constant. The reason is that an increase in $\gamma$ results in lower equilibrium prices and firm values even if $\gamma$ had no effect on firms' output market strategies (see (1)). Endogenizing the number of firm allows to examine the effects of $\gamma$ while holding firm values constant.

### 2.3 Stage 1: The decision to enter the industry

In this stage, private firms enter the industry as long as entry yields positive expected utility for their owners. There is a fixed cost of entry $E$, which we assume, for simplicity, to be identical across all potential entrants. To avoid trivial scenarios, we assume that the entry cost is low enough to allow at least one entry by a private firm, i.e., $V^*_{pri}(1,0) > E$. It follows from (8) and (10) that this condition can be written as

$$E < \frac{(\mu - c)^2}{4(\beta + \frac{1}{2}r\sigma^2)}. \tag{17}$$

Following a large body of industrial organization literature, in what follows we treat the total number of firms $N$ and the number of public firms $n$ as continuous variables.[19] Thus, the benefit of IPO defined in (11) becomes

$$B_{IPO}(N,n) = V^*_{pub}(N,n) - V^*_{pri}(N,n). \tag{18}$$

Furthermore, to ensure equilibrium existence and uniqueness, we assume that the cost of going public, $F(n)$, is continuous and strictly increasing. The latter means that the marginal cost faced by underwriting banks is strictly increasing.[20]

[19] See, for example, Ruffin (1971), Okuguchi (1973), Dixit and Stiglitz (1977), Loury (1979), von Weizsäcker (1980), and Mankiw and Whinston (1986)). See Suzumura and Kiyono (1987) for a discussion of the effect of departure from continuous number of firms on equilibrium conditions. Seade (1980) defends the practice of treating the number of firms as continuous variable by arguing that it is always possible to use continuous differentiable variables and restrict attention to integer realizations of these variables.

[20] Since the slope of $F(n)$ is allowed to be arbitrarily close to zero, this assumption is not very restrictive. Since $n$ is the number of IPOs in a single industry, whereas the slope of $F(n)$ may depend on the number of IPOs in the overall economy, it is possible that empirically the slope of $F(n)$ is indeed small.

Let $N^*$ be the equilibrium number of all firms in the industry, and let $n^* \equiv n^*(N^*)$ be the equilibrium number of public firms. Depending on the model parameters, one of the following three situations arises in equilibrium.

(i) There are public as well as private firms in the industry, i.e., $0 < n^* < N^*$. In this case, the equilibrium $(N^*, n^*)$ is characterized by the following conditions:

$$B_{IPO}(N^*, n^*) = F(n^*), \tag{19}$$

$$V^*_{pri}(N^*, n^*) = E. \tag{20}$$

Condition (19) stipulates that the benefit of IPO is equal to its cost, in which case, no firm has incentive to change its mode of incorporation. Condition (20) ensures that the value of incumbent firms equals the entry cost, and thus, no firm has incentive to enter or exit the industry.

(ii) There are only private firms in the industry, i.e., $n^* = 0$. In this case the equilibrium conditions are:

$$B_{IPO}(N^*, 0) \leq F(0), \tag{21}$$

$$V^*_{pri}(N^*, 0) = E. \tag{22}$$

Condition (21) states that the benefit of going public does not exceed the cost of doing so even for a single firm going public. Condition (22) equates the value of incumbent firms to the entry cost, ensuring that no further entries or exits take place.

(iii) There are only public firms in the industry, i.e., $n^* = N^*$. In this case, the equilibrium conditions are:

$$B_{IPO}(N^*, N^*) > F(N^*), \tag{23}$$

$$V^*_{pub}(N^*, N^*) - F(N^*) = E. \tag{24}$$

Condition (23) stipulates that even the last private firm in the industry has incentive to go public. Condition (24) again equates the value of incumbent firms to the entry cost, which guarantees that no firm has incentive to enter or exit the industry.

Before we formally characterize the equilibrium industry structure, it is useful to define

$$B^*_{IPO} = r\sigma^2 E \frac{2\beta\left(2\beta + r\sigma^2\right) - \gamma^2}{\left(2\beta + r\sigma^2\right)\left(2\beta - \gamma\right)^2}, \tag{25}$$

$$\text{and } \hat{n} = \frac{\left(2\beta - \gamma\right)\left(\mu - c\right)\sqrt{\left(\beta + r\sigma^2/2\right)/E}}{\left(2\beta - \gamma + r\sigma^2\right)\gamma} - \frac{2\beta - \gamma}{\gamma}, \tag{26}$$

where $B^*_{IPO}$ is the equilibrium benefit of IPO when there exist private firms (whose equilibrium values are then equal to the entry cost, i.e., $V^*_{pri}(N^*, n^*) = E$), and where $\hat{n}$ is the number of firms such that

if there are $\hat{n}$ firms in the industry all of which are public, the value of a marginal private firm equals the entry cost, i.e., $V^*_{pri}(\hat{n}, \hat{n}) = E$.

**Proposition 5** *There exists a unique equilibrium* $(N^*, n^*)$ *which can be characterized as follows:*

*(i) If* $F(0) < B^*_{IPO} < F(\hat{n})$, *then*

$$n^* = F^{-1}(B^*_{IPO}) > 0, \tag{27}$$

$$\text{and } N^* = \frac{(\mu - c)\sqrt{(\beta + r\sigma^2/2)/E} - 2\beta + \gamma - r\sigma^2}{\gamma} - \frac{r\sigma^2}{2\beta - \gamma} n^* > n^*. \tag{28}$$

*(ii) If* $F(0) \geq B^*_{IPO}$, *then*

$$n^* = 0, \tag{29}$$

$$\text{and } N^* = \frac{(\mu - c)\sqrt{(\beta + r\sigma^2/2)/E} - 2\beta + \gamma - r\sigma^2}{\gamma}. \tag{30}$$

*(iii) If* $B^*_{IPO} \geq F(\hat{n})$, *then*

$$n^* = N^*, \tag{31}$$

$$\text{and } N^* \text{ solves } \frac{\beta(\mu - c)^2}{(\gamma N^* + 2\beta - \gamma)^2} - F(N^*) = E. \tag{32}$$

Case (i) corresponds to the interior equilibrium, in which some firms go public while others remain private, and the equilibrium cost of IPO, $F(n^*)$, equals the equilibrium IPO benefit, $B_{IPO}(N^*, n^*) = B^*_{IPO}$. Interestingly, the equilibrium benefit of IPO, $B^*_{IPO}$, is independent of $F(\ )$. This may appear counterintuitive because higher IPO costs result in fewer IPOs, which leads, ceteris paribus, to a larger equilibrium IPO benefit. What also happens in equilibrium, however, is that a lower number of public (more aggressive) firms results in more entries into the market, which, in turn, reduces the benefit of going public. In other words, higher IPO costs reduce the number of public firms $n^*$, while increasing the total number of firms $N^*$, in such a way that the equilibrium benefit of IPO, $B^*_{IPO}$, remains unchanged.

Case (ii) corresponds to the equilibrium, in which there are no public firms because the IPO benefit does not exceed the IPO cost even for a single firm going public. Finally, case (iii) corresponds to the other corner equilibrium, in which all firms go public because the IPO benefit exceeds the IPO cost for all firms in the industry.

## 3 Going public and competitive interaction

The next proposition characterizes the effect of the degree of competitive interaction among industry rivals on the equilibrium number and proportion of public firms in the industry.

**Proposition 6** *If $n^* < N^*$, the equilibrium number of public firms $n^*$ is increasing, whereas the equilibrium number of all firms $N^*$ is decreasing in the degree of competitive interaction $\gamma$. Therefore, as the degree of competitive interaction increases, the equilibrium proportion of public firms $n^*/N^*$ increases.*

Consider first the case in which not all firms are public, i.e., $n^* < N^*$. Recall that the benefit of IPO is partially due to a public firm's commitment to larger output, which reduces its rivals' equilibrium output and thereby increases the residual demand for the firm's product. As a result, the benefit of IPO is greater when the effect of rivals' output decisions on the firm's demand is stronger, i.e., when the competitive interaction among firms $\gamma$ is stronger.[21] Therefore, as competitive interaction intensifies, the equilibrium benefit of going public $B^*_{IPO}$ and the equilibrium number of public firms $n^*$ increase. At the same time, stronger competitive interaction together with larger number of public (more aggressive) firms reduces firm values, resulting in fewer firms in the industry. In other words, as the degree of competitive interaction $\gamma$ increases, the number of public firms increases, the total number of firms declines, and therefore, the proportion of public firms increases. Eventually, as $\gamma$ becomes sufficiently high, all firms become public provided that the cost of going public is low enough. After that, the total number of firms, all of which are public, continues to decline as $\gamma$ increases.

The comparative statics outlined in Proposition 6 lead to the following cross-sectional (inter-industry) prediction:

> **Empirical Prediction 3a** *The proportion of public firms is expected to be larger in industries in which the degree of competitive interaction among firms is higher, ceteris paribus.*

To gauge the degree of competitive interaction within an industry, one may use the "Competitive Strategy Measure," proposed by Sundaram, John and John (1996), which estimates the responsiveness of a firm's profit to changes in its product market rivals' strategies and is based on the correlation between the period-to-period change in firm's rivals' combined sales and the ratio of the change in the firm's profit and the change in its sales.[22] However, since the types of competitive interaction among

[21] This becomes obvious when we write the demand for firm $i$'s product as $q_i = \alpha_i/\beta - p_i/\beta - (\gamma/\beta)\sum_{j\neq i} q_j$, which leads to $|\partial q_i/\partial q_j| = \gamma/\beta$.

[22] Lyandres (2006) proposes an adjustment to this measure that accounts for shocks to firms' profit functions.

product market rivals vary across industries, a single measure may not be ideal for estimating the extent of interaction in many diverse industries. Thus, it may be beneficial to examine the relation between changes in the degree of interaction in a given industry and changes in firms' propensity to go public over time in the spirit of the anecdotal evidence of the evolution of the investment banking industry, discussed in the introduction. While the model is static, its underlying logic leads to the following time-series prediction:

> **Empirical Prediction 3b** *The proportion of public firms in an industry is expected to increase (decrease) when the degree of competitive interaction among firms intensifies (weakens).*

In our setting, the fundamental benefit of going public stems from allowing private firms' owners to better diversify firm-specific risk in capital market. Consequently, one would expect the benefit of going public to be lower for entrepreneurs who are able to reduce profit variability at the firm level by means of financial or operational hedging or corporate diversification. In the following section, we examine whether this is indeed the case and, equally importantly, whether our main result regarding the effect of competitive interaction on firms' propensity to go public continues to hold in the presence of hedging. We also examine the effects of hedging on public and private firms' product market strategies and values. Although we concentrate on financial hedging, our results extend to any activity aimed at reducing profit variability at the corporate level.

# 4 Going public in the presence of hedging

In this section, we extend the model by allowing firms to (imperfectly) hedge the variability of profits and, by doing so, achieve benefits similar to those of an IPO. Because in our model public firm's value is not a function of profit variability, only private firms have incentives to hedge.[23] We assume that each (private) firm chooses the optimal hedging contract together with the optimal output level in the third stage of the game, as illustrated in Figure 3.

## 4.1 Market equilibrium in the presence of hedging

Suppose that firm $i$ is able to buy (or short sell) $h_i$ units of a hedging asset whose terminal value, $\xi_i$, is normally distributed and correlated with the firm's demand shock, $\alpha_i$. As is common in the literature

[23] In reality, public firms have incentives to hedge as well (e.g., Nance, Smith and Smithson (1993)) but these are exogenous to our model and outside the scope of our study. In our model's framework, public firms would have an incentive to hedge if the diversification achieved by going public were imperfect.

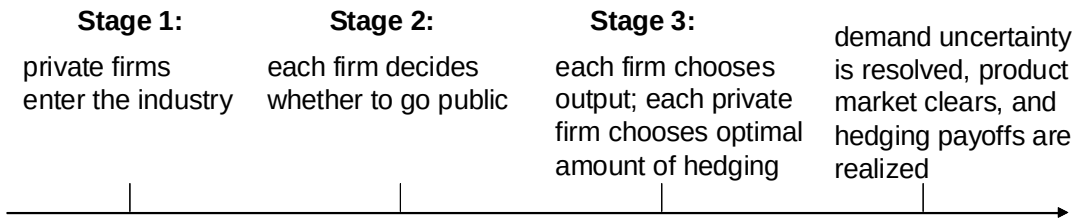


Figure 3: The sequence of events in the presence of hedging.

(e.g., Froot, Scharfstein and Stein (1993), Almeida, Campello and Weisbach (2004)), we assume that the hedging asset is fairly priced in the sense that its spot price equals its expected terminal value $\mathbb{E}\xi_i$. For simplicity, we assume that the correlation coefficients between the demand shocks, $\alpha_i$, and the hedging asset prices, $\xi_i$, are identical across all firms, and we denote them as $\rho$. We further assume, without loss of generality, that $\rho < 0$ and that $\mathbb{V}\mathtt{ar}(\xi_i) = \sigma^2$ for any $i$.[24,25]

Taking outputs as given, when choosing the optimal amount of hedging, firm $i$ faces the following problem:

$$\max_{h_i} \mathbb{E}u\left(\pi_i(\mathbf{q}) + h_i\left(\xi_i - \mathbb{E}\xi_i\right)\right). \tag{33}$$

Because the cash flow $\pi_i(\mathbf{q}) + h_i(\xi_i - \mathbb{E}\xi_i)$ is normally distributed, the firm's utility is still given by the mean-variance criterion. Because hedging affects only the variance and not the expected value, the problem can be equivalently written as

$$\min_{h_i} \mathbb{V}\mathtt{ar}\left(\pi_i(\mathbf{q}) + h_i\left(\xi_i - \mathbb{E}\xi_i\right)\right), \tag{34}$$

i.e., the optimal hedging contract is such that it minimizes profit variance, conditional on output choice. The next result characterizes the optimal hedging contract and the resulting private firm value.

**Lemma 3** *For a given output level $q_i$, the optimal amount of hedging of private firm $i$ is $h_i^*(q_i) = q_i|\rho|$, and it results in the following firm value:*

$$V_i(\mathbf{q}) = \mathbb{E}\pi_i(\mathbf{q}) - \frac{r}{2}\left(1-\rho^2\right)\mathbb{V}\mathtt{ar}\left(\pi_i(\mathbf{q})\right). \tag{35}$$

As one would expect, the more aggressive the firm's product market strategy (the higher the output level $q_i$) or the stronger the correlation $|\rho|$, the larger the quantity of the hedging asset the firm buys.

[24] Negative $\rho$ means that a firm takes a long position in the hedging asset, i.e., optimal $h_i$ is positive. Positive $\rho$ would result in the firm taking a short position in the hedging asset, i.e., optimal $h_i$ would be negative, and all other results would remain unchanged.

[25] Higher (lower) $\mathbb{V}\mathtt{ar}(\xi_i)$ would result in lower (higher) optimal $h_i$, but it would have no effect on the optimal hedging contract payoff or any subsequent results.

Importantly, the optimal hedging contract reduces profit variance by fraction $\rho^2$. Hence, we refer to the magnitude of the correlation coefficient between demand shock and the value of the hedging asset, $|\rho|$, as hedging efficacy.[26] Lemma 3 leads to the following result.

**Corollary 1** *(i) For any given industry structure $(N, n)$, the equilibrium outputs and firm values can be characterized as in Lemma 1 with $\sigma^2$ replaced by $\left(1-\rho^2\right)\sigma^2$.*

*(ii) There exists a unique equilibrium $(N^*, n^*)$ which can be characterized as in Proposition 5 with $\sigma^2$ replaced by $\left(1-\rho^2\right)\sigma^2$.*

It follows from Corollary 1 that unless hedging is perfect, private firms' outputs and values are lower than those of public firms. This is because in the context of our model, imperfect hedging only mitigates risk, whereas IPO-enabled diversification eliminates it entirely. Only when hedging is perfect (i.e., $|\rho| = 1$), it has the same effect on the firm's strategy and value as going public does, i.e., a private firm's output and value are the same as those of a public firm.

In the next section, we analyze the effect of hedging and its efficacy on the equilibrium proportion of firms that choose to go public. In addition, we examine whether the relationship that we established between the degree of competitive interaction and firms' propensity to go public remains intact when firms are allowed to hedge.

## 4.2 Going public, hedging, and competitive interaction

The effect of hedging efficacy on the equilibrium industry structure is summarized in the following proposition. For conciseness, we restrict our analysis to the interior equilibrium, i.e., to the situation in which both types of firms exist.

**Proposition 7** *If $n^* < N^*$, as hedging efficacy $|\rho|$ increases, the equilibrium number of public firms $n^*$ decreases. If furthermore $E > (\mu - c)^2 / 16\left(\beta + \frac{1}{2}\left(1-\rho^2\right) r\sigma^2\right)$, the equilibrium number of all firms $N^*$ increases and, therefore, the equilibrium proportion of public firms $n^*/N^*$ decreases.*

As hedging efficacy increases, the residual uncertainty faced by private firms decreases and so does the benefit of IPO-enabled diversification. As the benefit of going public declines, so does the equilibrium number of firms that go public.

The effect of hedging efficacy on the total number of firms is more subtle. Recall part (i) of Proposition 2, which stipulates that, for a given industry structure $(N, n)$, private firm value may

[26] For more discussion of hedging efficacy, see Chod, Rudi and Van Mieghem (2007).

be increasing or decreasing in demand uncertainty. (For a given output vector, private firms always benefit from lower demand uncertainty. However, lower demand uncertainty also increases private firms' outputs, which may result in lower profits by bringing the total industry output further from the monopoly level.) Because the effect of hedging is similar to that of reducing demand uncertainty, it may, for a given industry structure, adversely affect private firms by exacerbating their overproduction relative to the monopoly level. If this is the case, more effective hedging reduces the equilibrium number of private firms in an industry, and thus, its effect on the proportion of public firms depends on model parameters. Condition

$$E > \frac{(\mu - c)^2}{16\left(\beta + \frac{1}{2}(1-\rho^2)\, r\sigma^2\right)} \tag{36}$$

ensures that more effective hedging benefits private firms and increases their equilibrium number so that the total number of firms in the industry increases. (The relatively high entry cost means that the number of firms in the industry is relatively low and their production above the monopoly level is relatively small.) The increasing number of firms in the industry combined with the decreasing number of public firms means that as hedging efficacy increases, the proportion of public firms declines. This result leads to the following empirical prediction.

> **Empirical Prediction 4** *The proportion of public firms is expected to be inversely related to the availability of hedging opportunities in industries characterized by relatively high barriers to entry, ceteris paribus.*

As mentioned above, we would expect other firm-level risk-reducing activities such as operational hedging or corporate diversification to impact the propensity to go public in a similar fashion. The next proposition examines the relation between the extent of competitive interaction in an industry and the equilibrium number and proportion of public firms in the presence of hedging.

**Proposition 8** *If $n^* < N^*$, the equilibrium number of public firms $n^*$ is increasing, whereas the equilibrium number of all firms $N^*$ is decreasing in the degree of competitive interaction $\gamma$. Therefore, as the degree of competitive interaction increases, the equilibrium proportion of public firms $n^*/N^*$ increases.*

Proposition 8 simply stipulates that the our main result regarding the effect of the degree of competitive interaction on the number and proportion of public firms in an industry, remains intact when hedging is allowed. The extended model presented in this section makes two contributions. First, it shows that as long as hedging is imperfect, going public can still be beneficial, and the availability of hedging does not alter the positive relation between the propensity to go public and the extent of

competitive interaction among firms. Second, it shows that in industries with relatively high barriers to entry the equilibrium number and proportion of public firms in an industry decrease with the effectiveness of hedging (or any other means of reducing profit variability).

# 5 Conclusions

This paper presents a stylistic model of the decision to go public in the presence of product market competition. Because going public facilitates risk sharing, public firms are less concerned with idiosyncratic profit variability than otherwise similar private firms. In a Cournot-like competitive setting with uncertain demand, lower risk-aversion of a public firm results in a more aggressive output strategy, which, in turn, reduces the equilibrium aggressiveness of the firm's competitors. This *strategic* benefit of IPO, and thus, firms' incentives to go public and the equilibrium proportion of public firms in an industry increase with the degree of competitive interaction among firms.

In addition, we analyze firms' incentives to go public when they are able to reduce profit variability through financial hedging. Similar to going public, hedging mitigates the effect of risk on a firm's product market strategy, and, thus, results in greater product market aggressiveness. Therefore, in the presence of product market competition, hedging has a strategic benefit similar to that of an IPO. Importantly, we show that the availability of hedging reduces, but does not eliminate, the incentives to go public. Other tactics of reducing profit variability such as operational hedging or corporate diversification would impact the firm's incentives to go public in a similar fashion.

Our model generates several testable empirical predictions. First, going public is expected to increase the IPO firm's market share, and more so within industries characterized by strong competitive interaction. Second, going public is expected to adversely affect the values of the IPO firm's product market rivals, and more so within industries characterized by strong competitive interaction. Third, the proportion of public firms is expected to be higher in industries characterized by higher degree of competitive interaction. Moreover, the number of IPOs in an industry is expected to be higher in periods of intensifying competitive interaction in this industry. Finally, the proportion of public firms is expected to be smaller in industries in which barriers to entry are relatively high and more effective hedging opportunities are available.

The model presented here is a first step in examining various industrial organization aspects of the decision to go public. Strategic considerations may have a significant effect not only on the decision whether to perform an IPO or not, but also on its timing, which cannot be captured within our static model. Because the existing models examining IPO timing (e.g., Alti (2005), Benninga, Helmantel and Sarig (2005) and Pástor and Veronesi (2005)) abstract from strategic interaction among firms in

output markets, constructing a dynamic model of industry equilibrium that treats an IPO as a real option and examining the effects of competitive interaction on the optimal timing of going public is an interesting avenue for future research.

Another interesting direction of future inquiry would be a comparison of optimal capital structures of private and public firms in the presence of product market competition. As shown in the limited liability literature (e.g., Brander and Lewis (1986), Maksimovic (1988), Lyandres (2006)), debt has a strategic advantage, which is increasing in the degree of competitive interaction in an industry. However, existing models of strategic debt choices assume risk-neutrality of decision makers, and thus, are not well suited to describe the behavior of closely-held private firms. It is not immediately clear how strategic debt choices of private firms would compare to those of public firms. In the absence of strategic considerations, private firms are likely to choose lower debt levels than public firms if private equity financing is available (Chen, Miao and Wang (2008)). However, because issuing debt is a substitute to going public in committing a firm to a more aggressive product market strategy, private firms may be willing to issue more debt in industry equilibrium. Examining the effects of product market competition on the optimal capital structure choices of private and public companies is a work in progress.

Strategic considerations described in this paper are just one example of the links between corporate finance and industrial organization. While many corporate finance decisions in the presence of product market competition have been examined theoretically, empirical literature bridging corporate finance and industrial organization is more limited. In the context of IPOs, Chemmanur, He and Nandy (2007) examine the effects of firms' market shares, sales growth, total factor productivities, and industry concentration measures on firms' propensity to go public. A more detailed empirical investigation of the relation between the propensity of firms to go public and the nature and extent of competition in their product markets may provide an important contribution to this literature.

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# Appendix

### Proof of Lemma 1

Plugging (2) into (5), we obtain

$$V_i(\mathbf{q}) = \begin{cases} \mu q_i - \beta q_i^2 - \gamma q_i \sum_{j \neq i} q_j - c q_i & \text{if firm } i \text{ is public,} \\ \mu q_i - \beta q_i^2 - \gamma q_i \sum_{j \neq i} q_j - c q_i - \frac{r}{2} q_i^2 \sigma^2 & \text{if firm } i \text{ is private.} \end{cases} \tag{37}$$

First we prove, by contradiction, that among pure-strategy equilibria, only symmetric ones can exist, in which all public firms produce the same quantity and all private firms produce the same quantity. Suppose there are two public firms, $i$ and $j$, which produce different equilibrium quantities, $q_i \neq q_j$. Since $q_i$ maximizes firm $i$'s objective function in (37), it must satisfy the first-order condition:

$$\mu - 2\beta q_i - \gamma q_j - \gamma \sum_{k \neq i,\ k \neq j} q_k - c = 0. \tag{38}$$

The fact that $q_i \neq q_j$ together with (38) gives

$$\mu - 2\beta q_j - \gamma q_i - \gamma \sum_{k \neq i,\ k \neq j} q_k - c \neq 0.$$

Hence, $q_j$ does not satisfy the first-order condition for maximizing firm $j$'s objective and thus cannot be firm $j$'s equilibrium output. Therefore, our premise that $q_i \neq q_j$ is false, and outputs of all public firms in any equilibrium, if it exists, must be equal. It can be analogously shown that outputs of all private firms in any equilibrium must be identical as well.

If firms $1, ..., n$ are public and firms $n+1, ..., N$ are private, the equilibrium output vector $q^*(N, n)$ defined in (6) is given by the first-order conditions:

$$\begin{aligned} \frac{\partial V_i(\mathbf{q})}{\partial q_i} = 0 \quad &\Leftrightarrow \quad \mu - 2\beta q_i - \gamma \sum_{j \neq i} q_j - c = 0 \text{ for } i = 1, ...n, \\ \frac{\partial V_i(\mathbf{q})}{\partial q_i} = 0 \quad &\Leftrightarrow \quad \mu - \left(2\beta + r\sigma^2\right) q_i - \gamma \sum_{j \neq i} q_j - c = 0 \text{ for } i = n+1, ...N. \end{aligned} \tag{39}$$

Since each firm's objective is strictly concave in its output, the first-order conditions are sufficient, i.e., any $\mathbf{q}$ that satisfies (39) is an equilibrium. Taking advantage of the equilibrium symmetry, we denote the equilibrium output of all private (public) firms as $q^*_{pri}(N, n)$ ($q^*_{pub}(N, n)$). Thus, the equilibrium conditions in (39) can be written as

$$\begin{aligned} \mu - 2\beta q^*_{pub}(N, n) - \gamma \left[(n-1) q^*_{pub}(N, n) + (N-n) q^*_{pri}(N, n)\right] - c = 0, \\ \mu - \left(2\beta + r\sigma^2\right) q^*_{pri}(N, n) - \gamma \left[n q^*_{pub}(N, n) + (N-n-1) q^*_{pri}(N, n)\right] - c = 0. \end{aligned} \tag{40}$$

It is straightforward to show that the system (40) has a unique solution which is given in (7) and (8). Plugging (7) and (8) into (37) produces (9) and (10).■

**Proof of Proposition 1**

(i) The first result follows directly from Lemma 1.

Differentiating $q^*_{pub}(N,n)$ and $q^*_{pri}(N,n)$ in (7) and (8) with respect to $\gamma$ gives

$$\frac{\partial q^*_{pub}(N,n)}{\partial \gamma} = \frac{-(\mu-c)\left[(N-1)\left(4\beta(\beta-\gamma)+\gamma^2\right)+2r\sigma^2\left((N-2)(\beta-\gamma)+n\beta-\gamma\right)+(n-1)r^2\sigma^4\right]}{\left((2\beta-\gamma)(2\beta-\gamma+N\gamma+r\sigma^2)+n\gamma r\sigma^2\right)^2} < 0,$$

$$\frac{\partial q^*_{pri}(N,n)}{\partial \gamma} = \frac{-(\mu-c)\left[(2\beta-\gamma)((N-1)(2\beta-\gamma)+nr\sigma^2)+n\gamma r\sigma^2\right]}{\left((2\beta-\gamma)(2\beta-\gamma+N\gamma+r\sigma^2)+n\gamma r\sigma^2\right)^2} < 0.$$

(ii) Given that

$$\frac{q^*_{pub}(N,n)}{q^*_{pri}(N,n)} = 1+\frac{r\sigma^2}{2\beta-\gamma},$$

we have $\frac{\partial}{\partial\gamma}\left(\frac{q^*_{pub}(N,n)}{q^*_{pri}(N,n)}\right) > 0$. Therefore, for a given $(N,n)$, the market share of public firms is increasing in $\gamma$.

(iii) The result is obtained by differentiating (7) and (8) with respect to $N$.

(iv) Using (7) and (8), it is straightforward to show that $q^*_{pub}(N,n+1) > q^*_{pri}(N,n)$ whereas $q^*_{pub}(N,n+1) < q^*_{pub}(N,n)$ and $q^*_{pri}(N,n+1) < q^*_{pri}(N,n)$. A private firm's market share when there are $n$ public firms is

$$M_{pri}(N,n) = \frac{q^*_{pri}(N,n)}{nq^*_{pub}(N,n)+(N-n)q^*_{pri}(N,n)} = \frac{2\beta-\gamma}{2N\beta-N\gamma+nr\sigma^2}.$$

A public firm's market share when there are $n+1$ public firms is

$$M_{pub}(N,n+1) = \frac{q^*_{pub}(N,n+1)}{(n+1)q^*_{pub}(N,n+1)+(N-n-1)q^*_{pri}(N,n+1)} = \frac{2\beta-\gamma+r\sigma^2}{2N\beta-N\gamma+nr\sigma^2+r\sigma^2}.$$

It is easy to verify that $M_{pub}(N,n+1)/M_{pri}(N,n)$ increases in $\gamma$.■

**Proof of Proposition 2**

(i) It follows directly from (9) and (10) that $V^*_{pub}(N,n) > V^*_{pri}(N,n)$**.** Public firm's value, $V^*_{pub}(N,n) = \beta\left(q^*_{pub}(N,n)\right)^2$, is increasing in $r$ and in $\sigma$ because $q^*_{pub}(N,n)$ is (see Proposition 1). Differentiating private firm's value with respect to $r$ gives

$$\frac{\partial V^*_{pri}(N,n)}{\partial r} = \frac{1}{2}(\mu-c)^2(2\beta-\gamma)^2\sigma^2\frac{(2\beta-\gamma)(N\gamma-2\beta-\gamma)-4n\beta\gamma-r\sigma^2(2\beta+n\gamma-\gamma)}{\left((2\beta-\gamma)(2\beta-\gamma+N\gamma+r\sigma^2)+n\gamma r\sigma^2\right)^3}. \quad (41)$$

Thus, $\frac{\partial V^*_{pri}(N,n)}{\partial r} > 0$ for a sufficiently large $N$, and $\frac{\partial V^*_{pri}(N,n)}{\partial r} < 0$ for a sufficiently large $\sigma$. The proof of nonmonotonicity $V^*_{pri}(N,n)$ with respect to $\sigma$ is analogous.

(ii) Firm values $V^*_{pub}(N,n) = \beta\left(q^*_{pub}(N,n)\right)^2$ and $V^*_{pri}(N,n) = \left(\beta+\frac{1}{2}r\sigma^2\right)\left(q^*_{pri}(N,n)\right)^2$ are decreasing in $\gamma$ because $q^*_{pub}(N,n)$ and $q^*_{pri}(N,n)$ are (see part (i) of Proposition 1). The relative benefit of being public,

$$\frac{V^*_{pub}(N,n)}{V^*_{pri}(N,n)} = \frac{2\beta}{2\beta+r\sigma^2}\left(\frac{q^*_{pub}(N,n)}{q^*_{pri}(N,n)}\right)^2$$

is increasing in $\gamma$ because $\frac{q^*_{pub}(N,n)}{q^*_{pri}(N,n)}$ is increasing in $\gamma$ (see part (ii) of Proposition 1).

(iii) The result follows directly from (9), (10) and part (iii) of Proposition 1.

(iv) The result that for a given $N$, the equilibrium value of either type of firm decreases with $n$ follows directly from (9), (10) and part (iv) of Proposition 1. Using, Lemma 1, we have

$$\begin{aligned}\frac{V^*_{pub}(N,n)}{V^*_{pub}(N,n+1)} &= \left(\frac{(2\beta-\gamma)\left(2\beta-\gamma+N\gamma+r\sigma^2\right)+(n+1)\gamma r\sigma^2}{(2\beta-\gamma)(2\beta-\gamma+N\gamma+r\sigma^2)+n\gamma r\sigma^2}\right)^2, \text{ and}\\ \frac{V^*_{pri}(N,n)}{V^*_{pri}(N,n+1)} &= \left(\frac{(2\beta-\gamma)\left(2\beta-\gamma+N\gamma+r\sigma^2\right)+(n+1)\gamma r\sigma^2}{(2\beta-\gamma)(2\beta-\gamma+N\gamma+r\sigma^2)+n\gamma r\sigma^2}\right)^2.\end{aligned}$$

Therefore,

$$\frac{\partial}{\partial\gamma}\left(\frac{V^*_{pub}(N,n)}{V^*_{pub}(N,n+1)}\right) =$$
$$2\frac{(2\beta-\gamma)\left(2\beta-\gamma+N\gamma+r\sigma^2\right)+(n+1)\gamma r\sigma^2}{(2\beta-\gamma)(2\beta-\gamma+N\gamma+r\sigma^2)+n\gamma r\sigma^2}\frac{4\beta^2-\gamma^2+2r\sigma^2\beta+N\gamma^2}{\left((2\beta-\gamma)(2\beta-\gamma+N\gamma+r\sigma^2)+n\gamma r\sigma^2\right)^2}r\sigma^2>0,$$
and
$$\frac{\partial}{\partial\gamma}\left(\frac{V^*_{pri}(N,n)}{V^*_{pri}(N,n+1)}\right) =$$
$$2\frac{(2\beta-\gamma)\left(2\beta-\gamma+N\gamma+r\sigma^2\right)+(n+1)\gamma r\sigma^2}{(2\beta-\gamma)(2\beta-\gamma+N\gamma+r\sigma^2)+n\gamma r\sigma^2}\frac{4\beta^2-\gamma^2+2r\sigma^2\beta+N\gamma^2}{\left((2\beta-\gamma)(2\beta-\gamma+N\gamma+r\sigma^2)+n\gamma r\sigma^2\right)^2}r\sigma^2>0.\blacksquare$$

**Proof of Proposition 3**

Before proving (i)-(iv), it is useful to show that

$$\frac{2\beta-\gamma+\sigma^2r}{2\beta-\gamma}\frac{2\beta+r\sigma^2}{2\beta}\left(q^*_{pri}(N,n)\right)^3<\left(q^*_{pub}(N,n+1)\right)^3. \tag{42}$$

Inequality (42) can be written as $A*D<1$ where

$$A=\frac{2\beta+r\sigma^2}{2\beta}\frac{2\beta-\gamma}{2\beta-\gamma+\sigma^2r},$$
$$D=\frac{2\beta-\gamma}{2\beta-\gamma+\sigma^2r}\left(\frac{(2\beta-\gamma)\left(N\gamma+2\beta-\gamma+\sigma^2r\right)+\gamma n\sigma^2r+\gamma\sigma^2r}{(2\beta-\gamma)(N\gamma+2\beta-\gamma+\sigma^2r)+\gamma n\sigma^2r}\right)^3.$$

Because $0<A<1$, it is enough to show that $D<1$, i.e.

$$\left(1+\frac{\gamma\sigma^2r}{(2\beta-\gamma)(N\gamma+2\beta-\gamma+\sigma^2r)+\gamma n\sigma^2r}\right)^3<1+\frac{\sigma^2r}{2\beta-\gamma}. \tag{43}$$

Since the left-hand side of (43) is decreasing in both $n$ and $N$, it is enough to prove the inequality for $n=0$ and $N=2$, i.e.

$$\left(1+\frac{\gamma\sigma^2r}{(2\beta-\gamma)(2\beta+\gamma+\sigma^2r)}\right)^3<1+\frac{\sigma^2r}{2\beta-\gamma} \tag{44}$$
$$\Leftrightarrow\left(4\beta^2-\gamma^2+2\beta\sigma^2r\right)^3<\left(2\beta-\gamma+\sigma^2r\right)(2\beta-\gamma)^2\left(2\beta+\gamma+\sigma^2r\right)^3.$$

Expanding the terms, (44) can be expressed as

$$\left(16\beta^4\sigma^2 r + \left(\beta^2-\gamma^2\right)\left(16\beta^2\sigma^2 r + 12\beta\sigma^4 r^2\right) + 36\beta^3\sigma^4 r^2 + 2\sigma^2 r\gamma^4 + 4\beta\sigma^8 r^4 + 24\beta^2\sigma^6 r^3\right)(\beta-\gamma)$$
$$+\gamma^2\sigma^8 r^4 + 2\gamma^3\sigma^6 r^3 > 0,$$

which holds because $\beta > \gamma > 0$.

(i) Using (11), we have

$$\frac{\partial B_{IPO}(N,n)}{\partial r} = \frac{\partial V^*_{pub}(N,n+1)}{\partial r} - \frac{\partial V^*_{pri}(N,n)}{\partial r}.$$

Therefore, $\frac{\partial B_{IPO}(N,n)}{\partial r} > 0$ if, and only if,

$$\frac{\partial V^*_{pub}(N,n+1)}{\partial r} > \frac{\partial V^*_{pri}(N,n)}{\partial r}. \tag{45}$$

Using (7)-(10), we obtain

$$\frac{\partial V^*_{pub}(N,n+1)}{\partial r} = \frac{2\beta\sigma^2\left(2N\beta\gamma - 2\beta\gamma - 2n\beta\gamma + \gamma^2 - N\gamma^2 + n\gamma^2\right)}{(\mu-c)(2\beta-\gamma+r\sigma^2)^2}\left(q^*_{pub}(N,n+1)\right)^3, \text{ and}$$
$$\frac{\partial V^*_{pri}(N,n)}{\partial r} = \frac{\sigma^2\left((2\beta-\gamma)(N\gamma-2\beta-\gamma) - 4n\beta\gamma - r\sigma^2(2\beta+n\gamma-\gamma)\right)}{2(\mu-c)(2\beta-\gamma)}\left(q^*_{pri}(N,n)\right)^3.$$

Thus, inequality (45) can be rewritten as

$$\frac{2\beta(2\beta-\gamma)}{(2\beta-\gamma+r\sigma^2)^2}\left(\frac{q^*_{pub}(N,n+1)}{q^*_{pri}(N,n)}\right)^3 > \frac{1}{2}\frac{(2\beta-\gamma)(N\gamma-2\beta-\gamma) - 4n\beta\gamma - r\sigma^2(2\beta+n\gamma-\gamma)}{2N\beta\gamma - 2\beta\gamma - 2n\beta\gamma + \gamma^2 - N\gamma^2 + n\gamma^2}. \tag{46}$$

We know from (42) that

$$\frac{2\beta(2\beta-\gamma)}{(2\beta-\gamma+\sigma^2 r)^2}\left(\frac{q^*_{pub}(N,n+1)}{q^*_{pri}(N,n)}\right)^3 > \frac{2\beta+r\sigma^2}{2\beta-\gamma+\sigma^2 r}.$$

Hence, to prove (46), it is enough to show that

$$\frac{2\beta+r\sigma^2}{2\beta-\gamma+\sigma^2 r} > \frac{1}{2}\frac{(2\beta-\gamma)(N\gamma-2\beta-\gamma) - 4n\beta\gamma - r\sigma^2(2\beta+n\gamma-\gamma)}{2N\beta\gamma - 2\beta\gamma - 2n\beta\gamma + \gamma^2 - N\gamma^2 + n\gamma^2},$$

which can be expanded as

$$8\beta^2(\beta-\gamma) + N\gamma\left(\beta^2-\gamma^2\right) + \gamma^3 + 2\beta\gamma^2 + \beta^2\gamma(3N-4) + 2r^2\sigma^4\beta + (n-1)r^2\sigma^4\gamma \tag{47}$$
$$+\left(N\gamma(2\beta-\gamma) + 8\beta(\beta-\gamma) + 2n\beta\gamma + 2\gamma^2 + n\gamma^2\right)r\sigma^2 > 0,$$

(47) holds because $\beta > \gamma$. This proves that $\frac{\partial B_{IPO}(N,n)}{\partial r} > 0$. The proof of $\frac{\partial B_{IPO}(N,n)}{\partial \sigma} > 0$ is analogous.

(ii) Combining (11) and (10), the benefit of IPO can be expressed as

$$B_{IPO}(N,n) = \beta\left(q^*_{pub}(N,n+1)\right)^2 - \left(\beta + \frac{1}{2}r\sigma^2\right)\left(q^*_{pri}(N,n)\right)^2. \tag{48}$$

Plugging $q^*_{pub}(N,n+1)$ and $q^*_{pri}(N,n)$ from (7) and (8), differentiating (48) with respect to $N$, and simplifying, we obtain

$$\frac{\partial B_{IPO}(N,n)}{\partial N} = \frac{(2\beta-\gamma)\gamma}{\mu-c}\left(\frac{2\beta+r\sigma^2}{2\beta-\gamma}\left(q^*_{pri}(N,n)\right)^3 - \frac{2\beta}{2\beta-\gamma+\sigma^2 r}\left(q^*_{pub}(N,n+1)\right)^3\right).$$

Thus, $\frac{\partial B_{IPO}(N,n)}{\partial N} < 0$ if, and only if,

$$\frac{2\beta+r\sigma^2}{2\beta-\gamma}\left(q^*_{pri}(N,n)\right)^3 - \frac{2\beta}{2\beta-\gamma+\sigma^2 r}\left(q^*_{pub}(N,n+1)\right)^3 < 0,$$

which we know to be true from (42).

(iii) The result is proved analogously, by differentiating the IPO benefit $B_{IPO}(N,n)$ with respect to $n$.

(iv) It follows from (9) and (10) that

$$\frac{V^*_{pub}(N,n+1)}{V^*_{pri}(N,n)} = \frac{2\beta C^2}{2\beta+r\sigma^2},$$

where

$$C = \frac{2\beta-\gamma+\sigma^2 r}{2\beta-\gamma}\frac{2N\beta\gamma-4\beta\gamma+4\beta^2+\gamma^2-N\gamma^2+2\beta\sigma^2 r+\gamma n\sigma^2 r-\gamma\sigma^2 r}{2N\beta\gamma-4\beta\gamma+4\beta^2+\gamma^2-N\gamma^2+2\beta\sigma^2 r+\gamma n\sigma^2 r}.$$

Thus, to show that $\frac{V^*_{pub}(N,n+1)}{V^*_{pri}(N,n)}$ is increasing in $\gamma$, we need to show that $C$ is increasing in $\gamma$. Differentiating $C$ with respect to $\gamma$ results in

$$\begin{aligned}\frac{\partial C}{\partial \gamma} = {} & \frac{\sigma^2 r}{(2\beta-\gamma)^2}\frac{2N\beta\gamma-4\beta\gamma+4\beta^2+\gamma^2-N\gamma^2+2\beta\sigma^2 r+\gamma n\sigma^2 r-\gamma\sigma^2 r}{2N\beta\gamma-4\beta\gamma+4\beta^2+\gamma^2-N\gamma^2+2\beta\sigma^2 r+\gamma n\sigma^2 r} \\ & + \frac{2\beta-\gamma+\sigma^2 r}{2\beta-\gamma}\frac{\gamma^2\sigma^2 r-4\beta^2\sigma^2 r-2\beta\sigma^4 r^2-N\gamma^2\sigma^2 r}{\left(2N\beta\gamma-4\beta\gamma+4\beta^2+\gamma^2-N\gamma^2+2\beta\sigma^2 r+\gamma n\sigma^2 r\right)^2},\end{aligned}$$

and, thus, $\frac{\partial C}{\partial \gamma} > 0$ if, and only if,

$$\begin{aligned}&\frac{2N\beta\gamma-4\beta\gamma+4\beta^2+\gamma^2-N\gamma^2+2\beta\sigma^2 r+\gamma n\sigma^2 r-\gamma\sigma^2 r}{(2\beta-\gamma)(2\beta-\gamma+\sigma^2 r)} > \\ &\frac{4\beta^2-\gamma^2+N\gamma^2+2\beta\sigma^2 r}{2N\beta\gamma-4\beta\gamma+4\beta^2+\gamma^2-N\gamma^2+2\beta\sigma^2 r+\gamma n\sigma^2 r}.\end{aligned} \tag{49}$$

Since the left-hand side of (49) is increasing in $n$ while its right-hand side is decreasing in $n$, it is enough to show that (49) holds for $n=0$, i.e.,

$$\begin{aligned}&\frac{2N\beta\gamma-4\beta\gamma+4\beta^2+\gamma^2-N\gamma^2+2\beta\sigma^2 r-\gamma\sigma^2 r}{(2\beta-\gamma)(2\beta-\gamma+\sigma^2 r)} > \frac{4\beta^2-\gamma^2+N\gamma^2+2\beta\sigma^2 r}{2N\beta\gamma-4\beta\gamma+4\beta^2+\gamma^2-N\gamma^2+2\beta\sigma^2 r} \\ \Leftrightarrow\ &\frac{(2\beta-\gamma)N\gamma+(2\beta-\gamma)^2+(2\beta-\gamma)\sigma^2 r}{(2\beta-\gamma)(2\beta-\gamma+\sigma^2 r)} > \frac{4\beta^2-\gamma^2+N\gamma^2+2\beta\sigma^2 r}{2N\beta\gamma-4\beta\gamma+4\beta^2+\gamma^2-N\gamma^2+2\beta\sigma^2 r} \\ \Leftrightarrow\ &\frac{N\gamma+2\beta-\gamma+\sigma^2 r}{2\beta-\gamma+\sigma^2 r} > \frac{4\beta^2-\gamma^2+N\gamma^2+2\beta\sigma^2 r}{2N\beta\gamma-4\beta\gamma+4\beta^2+\gamma^2-N\gamma^2+2\beta\sigma^2 r}\end{aligned}$$

$$\Leftrightarrow (N-2)\left(\gamma^3+N\beta\gamma^2\right)+2N\gamma^3+(\beta-\gamma)\gamma^2N^2+8\beta\gamma(\beta-\gamma)(N-1)+2\gamma(2\beta-\gamma)(N-1)\sigma^2 r > 0,$$

which is true because $\beta > \gamma > 0$ and $N \geq 2$.■

**Proof of Lemma 2**

(i) The equilibrium condition for $n^*(N)$ follows from (16). The existence and uniqueness of the equilibrium number of public firms $n^*$ follow from the fact that $B_{IPO}(N,n)$ is strictly decreasing, while $F(n)$ is non-decreasing in $n$. The facts that $F(1) < B_{IPO}(N,0)$ and $F(N) \geq B_{IPO}(N,N-1)$ ensure that $0 < n^*(N) < N$.

(ii) If $F(1) \geq B_{IPO}(N,0)$, then $n^*(N) = \min\{n \in \mathbb{N}_0 : B_{IPO}(N,n) \leq F(n+1)\} = 0$.

(iii) If $F(N) < B_{IPO}(N,N-1)$, then $\min\{n \in \mathbb{N}_0 : B_{IPO}(N,n) \leq F(n+1)\} \geq N$, and thus $n^*(N) = N$.■

**Proof of Proposition 4**

The results follow directly from Proposition 3 and Lemma 2.■

**Proof of Proposition 5**

First we consider case (i), in which both types of firms exist, i.e., $0 < n^* < N^*$. Using condition (20) and firms' equilibrium outputs and values in (7)-(10), the equilibrium benefit of IPO is $B_{IPO}(N^*,n^*) = B^*_{IPO}$. It follows from condition (19) that the equilibrium number of IPOs $n^* = F^{-1}(B^*_{IPO})$. Because $F(0) < B^*_{IPO} < F(\hat{n})$ and $F(n)$ is continuous and strictly increasing, such $n^*$ exists and is unique, and furthermore, $0 < n^* < \hat{n}$. Condition (20) together with equations (7)-(10) imply the equilibrium number of firms in the industry is given by (28). The fact that $n^* < \hat{n}$ implies that $n^* < N^*$.

Next, we consider case (ii), in which there are no public firms, i.e., $n^* = 0$. Using condition (22) and equations (7)-(10), the equilibrium benefit of IPO is $B_{IPO}(N^*,0) = B^*_{IPO}$ and the equilibrium number of firms in the industry is given by (30). Condition (21) then requires that $F(0) \geq B^*_{IPO}$.

Lastly, we consider case (iii), in which all firms are public, i.e., $n^* = N^*$. Condition (24) together with equations (7)-(10) imply that the equilibrium number of firms in the industry must satisfy

$$\frac{\beta(\mu-c)^2}{(\gamma N^* + 2\beta - \gamma)^2} - F(N^*) = E. \tag{50}$$

The existence and uniqueness of $N^*$ that satisfies (50) follows from the following two facts. First, in order for at least one firm to enter the market, the following inequality has to be satisfied:

$$\frac{\beta(\mu-c)^2}{(\gamma N + 2\beta - \gamma)^2} - F(N) > E$$

at $N = 0$. Second, as $N$ increases, $\frac{\beta(\mu-c)^2}{(\gamma N+2\beta-\gamma)^2} - F(N)$ strictly decreases and approaches a negative limit as $N \longrightarrow \infty$. Finally, condition (23) requires that $B_{IPO}(N^*,N^*) \geq F(N^*)$. Using Lemma 1,

the last inequality becomes $N^* \geq \hat{n}$. Using (50), this is true if and only if

$$\frac{\beta(\mu-c)^2}{(\gamma\hat{n}+2\beta-\gamma)^2} - F(\hat{n}) \geq E$$
$$\iff B^*_{IPO} \geq F(\hat{n}),$$

which completes the proof.■

**Proof of Proposition 6**

Using (25), we have

$$\frac{dB^*_{IPO}}{d\gamma} = \frac{4\beta\left(2\beta-\gamma+\sigma^2 r\right)\sigma^2 rE}{(2\beta+r\sigma^2)(2\beta-\gamma)^3} > 0.$$

Next, we consider the effect of $\gamma$ on $n^*$ and $N^*$ within each of the three cases characterized in Proposition 5.

(i) Since $dB^*_{IPO}/d\gamma > 0$ and $F()$ is an increasing function, $dn^*/d\gamma > 0$. As a result of this and inequality (17), $dN^*/d\gamma < 0$.

(ii) In this case, $dn^*/d\gamma = 0$ and it follows from (17) that $dN^*/d\gamma < 0$.

(iii) In this case, implicitly differentiating $N^*$ given in (32) results in

$$\frac{dn^*}{d\gamma} = \frac{dN^*}{d\gamma} = -\frac{2\beta(\mu-c)^2(N^*-1)}{2\beta\gamma(\mu-c)^2 + F'(N^*)(\gamma N^*+2\beta-\gamma)^3} < 0.$$

These relationships together with the fact that $d\hat{n}/d\gamma < 0$ and the continuity of $n^*(\gamma)$ and $N^*(\gamma)$ imply the results. ■

**Proof of Lemma 3**

The objective function (34) is convex, therefore the optimal $h_i$ is given by the first order condition

$$\frac{d}{dh_i}\mathbb{V}\mathtt{ar}\left(\pi_i(\mathbf{q}) + h_i(\xi_i - \mathbb{E}\xi_i)\right) = 0,$$

which gives $h^*_i(q_i) = q_i|\rho|$. Firm's value given the optimal amount of hedging is then

$$V_i(\mathbf{q}) = \mathbb{E}\pi_i(\mathbf{q}) - \frac{r}{2}\mathbb{V}\mathtt{ar}\left(\pi_i(\mathbf{q}) + h^*_i(q_i)(\xi_i - \mathbb{E}\xi_i)\right) = \mathbb{E}\pi_i(\mathbf{q}) - \frac{r}{2}\left(1-\rho^2\right)\mathbb{V}\mathtt{ar}\left(\pi_i(\mathbf{q})\right).■$$

**Proof of Corollary 1**

While for any given output vector $\mathbf{q}$, public firm values are the same as in the base-case model, private firm values are given by (35), i.e.,

$$V_i(\mathbf{q}) = \begin{cases} \mathbb{E}\pi_i(\mathbf{q}) & \text{if firm } i \text{ is public,} \\ \mathbb{E}\pi_i(\mathbf{q}) - \frac{r}{2}\left(1-\rho^2\right)\mathbb{V}\mathtt{ar}\left(\pi_i(\mathbf{q})\right) & \text{if firm } i \text{ is private} \end{cases} \tag{51}$$

Plugging (2) and (1) into (51), we obtain

$$V_i(\mathbf{q}) = \begin{cases} \mu q_i - \beta q_i^2 - \gamma q_i \sum_{j \neq i} q_j - c q_i & \text{if firm } i \text{ is public,} \\ \mu q_i - \beta q_i^2 - \gamma q_i \sum_{j \neq i} q_j - c q_i - \left(1-\rho^2\right) \frac{r}{2} q_i^2 \sigma^2 & \text{if firm } i \text{ is private.} \end{cases} \tag{52}$$

Thus, the remainder of the proof follows the proofs of Lemma 1 and Proposition 5 with $\left(1-\rho^2\right)\sigma^2$ replacing $\sigma^2$.■

**Proof of Proposition 7**

Replacing $\sigma^2$ by $\left(1-\rho^2\right)\sigma^2$ in (25) gives the equilibrium benefit of IPO in the presence of hedging when $n^* < N^*$, i.e.,

$$B_{IPO}^* = \left(1-\rho^2\right) r\sigma^2 E \frac{2\beta\left(2\beta + \left(1-\rho^2\right) r\sigma^2\right) - \gamma^2}{\left(2\beta + \left(1-\rho^2\right) r\sigma^2\right)\left(2\beta-\gamma\right)^2}. \tag{53}$$

Suppose, without loss of generality, that $\rho > 0$. Since $n^* = F^{-1}\left(B_{IPO}^*\right)$ and $F^{-1}()$ is an increasing function, $dn^*/d\rho < 0$ if and only if $dB_{IPO}^*/d\rho < 0$. Differentiating (53), we have

$$\begin{aligned} \frac{dB_{IPO}^*}{d\rho} &= -2E\rho r\sigma^2 \frac{2\beta\left(2\beta + 2\left(1-\rho^2\right) r\sigma^2\right) - \gamma^2}{\left(2\beta + \left(1-\rho^2\right) r\sigma^2\right)\left(2\beta-\gamma\right)^2} \\ &\quad + \left(1-\rho^2\right) r\sigma^2 E \frac{\left(2\beta\left(2\beta + \left(1-\rho^2\right) r\sigma^2\right) - \gamma^2\right) 2\rho r\sigma^2 \left(2\beta-\gamma\right)^2}{\left(\left(2\beta + \left(1-\rho^2\right) r\sigma^2\right)\left(2\beta-\gamma\right)^2\right)^2}. \end{aligned}$$

Thus, $dB_{IPO}^*/d\rho < 0$ if and only if

$$\begin{aligned} 2\beta\left(2\beta + 2\left(1-\rho^2\right) r\sigma^2\right) - \gamma^2 - \left(1-\rho^2\right) \frac{\left(2\beta\left(2\beta + \left(1-\rho^2\right) r\sigma^2\right) - \gamma^2\right) r\sigma^2}{\left(2\beta + \left(1-\rho^2\right) r\sigma^2\right)} &> 0 \\ \Leftrightarrow 2\beta\left(2\beta + \left(1-\rho^2\right) r\sigma^2\right) - \gamma^2 + \left(1-\rho^2\right) r\sigma^2 \left(2\beta + \left(1-\rho^2\right) r\sigma^2\right) &> 0, \end{aligned}$$

which is true because $\beta > \gamma$.

Next, we show that if $(\mu - c) < 4\sqrt{E\left(\beta + r\sigma^2/2\right)}$, then $dN^*/d\rho > 0$. Replacing $\sigma^2$ with $\left(1-\rho^2\right)\sigma^2$ in (28), we have

$$N^* = \frac{(\mu-c)\sqrt{\left(\beta + \left(1-\rho^2\right) r\sigma^2/2\right)/E} - 2\beta + \gamma - \left(1-\rho^2\right) r\sigma^2}{\gamma} - \frac{\left(1-\rho^2\right) r\sigma^2}{2\beta-\gamma} n^*. \tag{54}$$

It is straightforward to show that if $(\mu - c) < 4\sqrt{E\left(\beta + \left(1-\rho^2\right) r\sigma^2/2\right)}$, the first term in (54) increases in $\rho$. Since $dn^*/d\rho < 0$, we have $dN^*/d\rho > 0$. ■

**Proof of Proposition 8**

The proof is analogous to the proof of Proposition 6 with $\left(1-\rho^2\right)\sigma^2$ replacing $\sigma^2$.■