Liquidity and Feasible Debt Relief[*]

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Abstract

This paper analyzes the determinants of secondary debt market liquidity, identifying conditions under which trading in competitive markets results in sufficient ownership concentration to induce ex post efficient debt relief. The feasibility of debt relief is path-dependent, hinging upon interim economic conditions. Secondary debt markets are likely to freeze during recessions, precisely when trading has high social value. This is due to three factors: severe free-riding reduces profits of large bondholders; uninformed small bondholders are reluctant to sell due to high informational sensitivity of debt; and large investors are more likely to face wealth constraints. However, secondary markets need not freeze during recessions since high liquidity demand of uninformed bondholders increases their willingness to trade. Additionally, broader liquidity shocks during recessions increase the equilibrium stake held by large investors, promoting debt relief.

1 Introduction

As argued by Shleifer (2003), the sale of debt to a large number of dispersed lenders can deter borrowers from requesting debt relief despite their having the ability to pay. However, it has also been argued widely-held debt interferes with the provision of debt relief for legitimately distressed borrowers. For example, Gilson, John and Lang (1990) find that broad ownership inhibits restructuring of corporate debt. Bolton and Jeanne (2007) argue that dispersed ownership makes it more difficult to achieve sovereign debt relief. Finally, a large number of commentators, e.g. Eggert (2007), have pointed to the diffuse ownership of mortgages arising from securitization as a principle

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factor preventing debt relief for distressed homeowners. Consistent with this view, Piskorski, Seru and Vig (2008) find that securitized loans have a higher foreclosure rate than loans held on a bank's balance sheet, ceteris paribus.

This raises a natural question: If debt restructuring is truly efficient ex post, won't the market bring this about? That is, why doesn't trading of debt claims in secondary markets bring about sufficiently concentrated ownership such that debt relief becomes feasible? This is not simply an academic question given that market-based TARP Investment Companies are the centerpiece of the Geithner Plan for resolving the credit crisis. In fact, some commentators contend the Geithner Plan does not go far enough in relying upon the market. For example, Puchala and Goldhill (2009) argue, "Unfortunately, early indications are that the scale and true risk of private capital will be very limited, putting private investors in a passive role in those elements where they can make the greatest contribution-valuation and work-out management." Unfortunately, this debate falls into a theoretical void given that we lack a benchmark model for understanding the liquidity of secondary debt markets and the interplay between those markets and the debt relief process. The objective of our paper is to fill this void.

The model opens with an agent selling debt in a primary market. Following Hart and Moore (1994), the agent has the ability to withdraw essential human capital from the underlying project at an implementation stage. The prospect of strategic default induces dispersed debt ownership in the primary debt market, since atomistic lenders rebuff requests for debt relief. The focus of the analysis is on equilibrium in the secondary debt market where the atomistic bondholders have the option to trade with each other, as well as an anonymous large investor. We are particularly interested in conditions under which the ownership structure of the debt can efficiently shift from dispersed to concentrated in light of the potential for costly insolvency.

The model of the secondary debt market is in the spirit of the seminal work of Kyle (1985), making three critical departures. First, we analyze a concave debt claim. Second, there is a feedback effect from ownership structure to fundamental asset value since the former influences incentives for

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debt relief. Finally, we depart from the standard noise-trader assumption of Kyle and consider the trading incentives of uninformed atomistic bondholders facing preference shocks that bias them towards liquidating the bond positions they acquired in the primary market. That is, in our model atomistic bondholders trade-off liquidity preference against adverse selection costs in making their trading decisions. As shown below, each model feature plays an important part in explaining secondary market liquidity and the probability of achieving debt relief.

The model delivers a number of important insights regarding factors conducive to trade in secondary debt markets, concentrated ownership, and debt relief. The feasibility of debt relief ex post depends on interim economic conditions. In a weak interim economy, the probability of concentrated debt ownership is reduced by three factors: high free-riding costs lower the profits of the large bondholder; uninformed bondholders are reluctant to liquidate due to high informational sensitivity of debt in bad states; and large investors are more likely to face binding wealth constraints. In fact, we find that the wealth constraint channel has the potential to induce a large discontinuous reduction in the probability of debt relief. In this way, the model can explain the seizing up of secondary debt markets at precisely the points in time where debt trading has the highest social value.

However, the model also shows that secondary debt markets need not become illiquid during bad times. As recent events illustrate, recessions and panics are associated with strong liquidity demand. This liquidity demand can induce debt trading in cases where adverse selection might have otherwise caused atomistic investors to hold onto their claims. In addition to intensifying the demand for liquidity, bad economic times are also likely to broaden liquidity demand. We show that broader liquidity shocks also promote concentrated debt ownership, and debt relief, since they provide better camouflage for large informed investors. More generally, these results show that liquidity shocks can have a beneficial effect in promoting trade. Intuitively, liquidity shocks induce trade that is not information-motivated. It follows that attempts to prop up distressed bondholders may have an unintended side-effect, reducing the supply of liquidity in secondary debt markets.

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The model also delivers some subtle comparative static insights. For example, the probability of debt relief can be non-monotone in the breadth of liquidity shocks. In general, broader liquidity shocks increase the likelihood of debt relief by increasing the equilibrium stake held by the large investor. However, an increase in debt stake also exacerbates free-ridership as atomistic lenders anticipate that the larger stake will induce debt relief in more states of nature ex post. When the latter effect dominates, the probability of debt relief actually declines in the breadth of liquidity shocks.

Another subtle set of insights delivered by the model concerns correlation between liquidity shocks and the real economy. We find that the higher probability of liquidity shocks during recessions may encourage liquidation by uninformed bondholders. Such bondholders are exposed to adverse selection whenever order flow is not fully revealing. Intermediate levels of order flow confound market makers, and lead them to set pooling prices, since such order flows can arise from either: entry by the large investor cum liquidity shock or no entry and no shock. When the probability of a liquidity shock is high, market makers assign a higher probability to the former combination being responsible for intermediate order flow. The equilibrium price of debt in non-revealing states rises, encouraging liquidation by uninformed bondholders.

We turn now to related literature. A variant of the free-rider problem at the heart of our model was first analyzed by Grossman and Hart (1980) in the context of hostile takeovers in equities markets. Closer to our model is that developed by Shleifer and Vishny (1986), who analyze the interplay between free-ridership and the endogenous ownership structure of equity. They show that with public trading, a large investor will never find it profitable to acquire a stake consistent with takeover. Our model is also similar to those of Maug (1998) and Mello and Repullo (2004), who develop noise-trader models to analyze the incentive to implement value-enhancing takeovers with endogenous equity ownership structure. The key difference between the models is that we analyze debt, debt relief, and consider endogenous liquidation decisions by uninformed investors.

Dooley (2000), Shleifer (2003) and Bolton and Jeanne (2007) share our argument that dispersed

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debt may arise as an ex ante efficient response to borrower moral hazard. Gertner and Scharfstein (1991) analyze the use of exchange offers to overcome free-riding by atomistic lenders. They show a necessary condition for a successful exchange offer is that the new claim have higher priority. We do not explicitly analyze exchange offers, although one can view the default costs in our model as being a reduced-form representation of the costs of making an exchange offer. As argued by Gilson (1991), exchange offers are not costless, although they are less costly for corporations than the Chapter 11 process. Another reason for ruling out exchange offers is that it might be optimal ex ante to rule out such offers via bond covenants given that exchange offers can actually be used to expropriate bondholders.

There is a voluminous literature building upon the pioneering work of Kyle (1985) and Glosten and Milgrom (1985) which analyzes trading in markets occupied by informed and noise traders. However, most of this literature is concerned with linear equity claims and does not consider the role of adverse selection in deterring trade by uninformed investors. Further, there is no feedback from ownership structure to fundamental value in those models.

Our paper also contributes to a recent literature that attempts to explain bouts of extreme illiquidity. The liquidity preference in our model can be viewed as a reduced-form representation of the recent margin-constrained investor model of Brunnermeier and Pedersen (2008), for example. A key differentiating factor is that our model predicts that forced liquidations can actually be beneficial in that the increase in non-informational liquidations serves to promote concentrated ownership and efficient debt relief.

2   The Economic Setting

There are four dates $t \in \{t_0, t_1, t_2, t_3\}$ and two investor classes. There is a large investor I and a set of ex ante identical atomistic investors N with generic member n having measure zero. Each investor class enters the economy at t_0 endowed with one unit of the nonstorable numeraire good. The endowment can be consumed or invested. At t_0 there is a single financial asset that can be used

to transfer wealth intertemporally, a debt claim with face value normalized at 1 sold in the primary market by agent A. In addition, investor I has access to a risky nonscalable real investment, labeled project-I. That project costs 1 and generates a risky payoff at the start of t_2 equal to $r \in \{0, R\}$ where $R > 1$.

Consumption can take place at either t_0, t_2, or t_3. The large investor is patient, having utility

$$U^I = 0 \times c_0 + c_2 + c_3. \tag{1}$$

Given his preferences, investor I will invest his initial endowment in either the debt claim or his risky project. Each atomistic investor has utility

$$U^n = c_0 + c_2 + [\chi^n(1 - \tau) + (1 - \chi^n)]c_3 \tag{2}$$

where $\tau \in (0, 1)$ and χ^n is an indicator for the investor being impatient. Each investor n observes his own realized χ^n just prior to secondary market trading at t_2.

The debt claim sold by A is backed by a risky project, denoted project-A, yielding a single payoff $Y \in \{L, M, H\}$ at t_3. To capture default risk, assume

$$A1 : 0 < L < M < 1 < H.$$

As shown in Figure 1, the terminal payoffs result from a recombining binomial tree with jump probabilities equal to one-half at all nodes. In order for the project to generate a final payoff, it is essential that A implement the project at date t_1. If A quits at this point in time, the project is worthless. Once the implementation stage has been completed, A is no longer necessary and investors need only wait for Y to be realized at t_3.

The ownership structure of debt is common knowledge and F_j denotes the face value of debt outstanding at the start of period t_j. After the debt is sold in the primary market, but just prior to implementation at t_1, A has the ability to make a take-it or leave-it offer to bondholders. This offer takes the form of a request for debt relief: "If the total face value of debt is not reduced to $F_1 \leq 1$, I will quit." In this way, the model captures the strategic default problem stemming from inalienable human capital as described by Hart and Moore (1994).
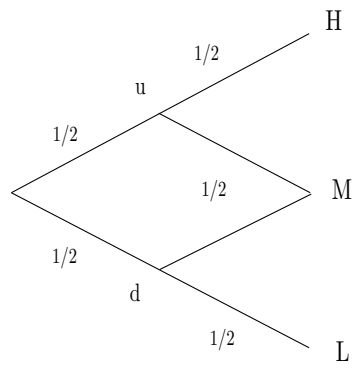
Figure 1: Project-A Payoffs

Figure 1: *This figure shows the payoffs of project A.*

Given the possibility of strategic default at the implementation stage, investor I will not buy any debt in the primary market. To see this, suppose to the contrary that I buys a fraction $s_0 \in (0,1]$ of the debt at t_0. Then A will propose $F_1 = 1 - (s_0 - \varepsilon)$ where ε is arbitrarily small. Each atomistic investor will refuse to forgive any debt, since he is not pivotal. However, investor I is pivotal and will forgive $s_0 - \varepsilon$ of his debt. Anticipating such an outcome, I will never buy any debt in the primary market. Thus, the free-rider problem in debt relief makes atomistic lenders tough, which is precisely what is necessary when confronting the threat of strategic default. However, the same free-rider problem also causes atomistic lenders to refuse to grant debt relief ex post when confronting the prospect of bona fide insolvency.

The interim state for project-A, which is common knowledge at the start of t_2, is denoted $\omega \in \{d, u\}$. Conditional upon ω, project-I has success probability $\overline{\sigma}_\omega$. Thus, all other agents know that the probability of I being able to buy debt in the secondary market is $\overline{\sigma}_\omega$. The returns to the two real technologies are positively correlated with:

$$A2 : \overline{\sigma}_u \geq \overline{\sigma}_d.$$

In this way, the model captures the possibility that the wealth of deep pocketed investors is correlated with the real economy.

The large investor privately observes his realized wealth r at the start of t_2, inducing a Bayesian game. There are two types of I at time t_2 indexed by their wealth, $I_r \in \{I_0, I_R\}$. We need only concern ourselves with type I_R, since the other type is necessarily inactive. At t_2, type I_R gains access to a safe storage technology. Alternatively, he can buy debt in the secondary market.

Let σ_ω denote the probability of investor I buying debt in the secondary market given the interim state, *unconditional of his type*. For example, if I_R buys debt with probability ρ_u in state u then $\sigma_u = \rho_u \overline{\sigma}_u$.

At the start of t_2, a fraction $\gamma_\omega \leq 1/2$ of N are selected at random and become vulnerable to preference shocks. Each $n \in N$ knows whether or not he is vulnerable. Invulnerable investors in N gain access to a safe storage technology at t_2 and are time-neutral with $\chi^n = 0$. Further, the

invulnerable investors receive $1 - \gamma_\omega$ units of aggregate endowment at time t_2. For reasons that will become apparent, invulnerable investors in N are labeled *market makers* (MM below). For now it is sufficient to note that the large investor and market makers both apply a zero discount rate when assessing consumption at t_2 relative to t_3.

Vulnerable atomistic investors are labeled *atoms* below, for brevity. The atoms are *collectively* patient or impatient. If patient, atoms have $\chi^n = 0$ and are locked into their debt holdings due to transactions costs. Thus, patient atoms value debt at its fundamental long-term value. If impatient, atoms have $\chi^n = 1$, biasing them towards liquidating their portfolios at time t_2. Importantly, and in contrast to pure noise trading models, this simple setup with impatience allows us to treat the liquidation decision as an endogenous choice by uninformed bondholders. In our model, impatient atoms assess adverse selection costs before tendering their debt.

Recent events support the notion that investors place a premium on liquidity during contractions. To allow for this possibility, assume τ is contingent upon ω, with

$$A3 : \tau_d \geq \tau_u > 0.$$

The realized time preference of the atoms is private information to them. However, all agents know the state-contingent probability of atoms being impatient, π_ω. Thus, the parameter π_ω captures the arrival intensity of liquidity shocks while γ_ω captures the breadth of such shocks. To allow for the possibility that liquidity shocks hit a larger percentage of investors during recessions, we assume

$$A4 : \frac{1}{2} \geq \gamma_d \geq \gamma_u > 0.$$

We solve for the perfect Bayesian equilibrium (PBE) of the game commencing at time t_2. The game starts with nature publicly drawing the interim economic state ω. Then nature draws r and χ. Given their respective information sets, the large investor and atoms submit orders to the MM. Their respective aggregate orders are denoted x^I and x^N. There is no borrowing or short-selling. Following Kyle (1985), MM observe total order flow X, and set prices competitively as is appropriate given that there is a measure $1 - \gamma_\omega$ continuum of market makers. After the secondary market clears,
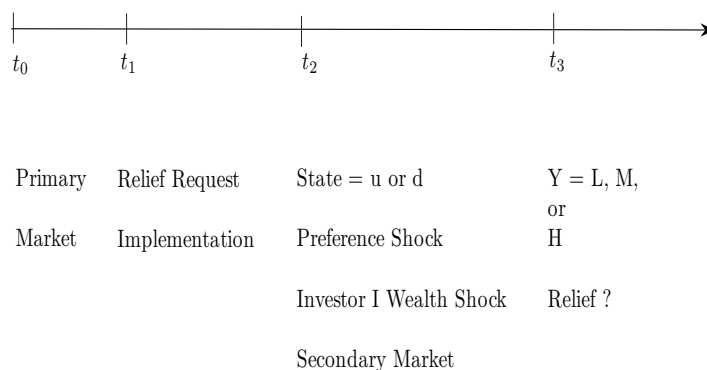
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Figure 2: Timeline

Figure 2: *This figure shows the time line.*

I enters period t_3 with an endogenous stake s in the debt claim. At this same point in time, the cash flow Y is observed. If $Y = H$, lenders are paid 1 and the shareholder A keeps $H - 1$. If $Y \in \{L, M\}$, lenders receive $Y(1 - \alpha)$ if there is no debt relief, where $\alpha \in (0, 1)$ captures deadweight costs of default. Alternatively, the pivotal investor I can forgive some debt. Iff I forgives a sufficient amount of debt such that the resulting face value $F_3 = Y$, default costs are avoided.[1] The measure zero bondholders would then be paid in full, receiving $1 - s$, leaving investor I to collect $Y - (1 - s)$.

The time line in Figure 2 summarizes the model introduced above.

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[1]It is never optimal for the large investor to forgive more debt than needed to avoid default costs.

3 Baseline Model

To fix ideas, this section presents formal results derived under a set of baseline parameters:

$$A2' \quad : \quad \overline{\sigma}_u = \overline{\sigma}_d = 1$$

$$A3' \quad : \quad \tau_d = \tau_u = \tau$$

$$A4' \quad : \quad \gamma_d = \gamma_u = \gamma.$$

After the baseline analysis, we describe how incorporating state-contingent parameter values affects equilibrium.

As argued below, the correlation structure of liquidity shocks plays an important, yet more complex, role. This section deliberately turns off this causal mechanism to focus on other factors at work in determining the feasibility of debt relief. In particular, the baseline model assumes

$$A5 : \pi_d = \pi_u = \frac{1}{2}.$$

3.1 The final period

Suppose it is time t_3 and $Y \in \{L, M\}$, implying debt relief is necessary to avoid default costs. Given a debt stake s, the large investor is willing to grant debt relief iff

$$Y - (1 - s) \geq s(1 - \alpha)Y. \tag{3}$$

The following Lemma is a useful summary of the implications of the inequality above

Lemma 1. *A necessary condition for debt relief is that the large investor holds a stake at least as large as*

$$\underline{s}(Y, \alpha) \equiv \frac{1 - Y}{1 - Y + \alpha Y}. \tag{4}$$

It is readily verified that the minimum stake $\underline{s}$ is decreasing in both Y and α, with

$$\lim_{\alpha \downarrow 0} \quad \underline{s}(Y, \alpha) \quad = \quad 1$$

$$\lim_{\alpha \uparrow 1} \quad \underline{s}(Y, \alpha) \quad = \quad 1 - Y.$$

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Lemma 1 is intuitive. If cash flow is high, debt relief is more likely since high cash flow reduces the transfer that debt relief provides to the hold-out atomistic bondholders. It is also apparent that the prospect of incurring high default costs encourages voluntary debt relief. For this reason, a highly efficient bankruptcy forum can decrease social efficiency by discouraging private workouts. This point is commonly overlooked in debates regarding bankruptcy reform.

3.2 Debt relief following expansions

Suppose it is the start of t_2 and the interim state is u. With probability one-half, debt relief will be necessary to avoid costly default. In particular, if the terminal node M is reached, a necessary condition for successful debt restructuring is that the large investor hold a stake of at least $\underline{s}(M, \alpha)$. Under what conditions will I obtain such a stake via secondary market trading? To address this question consider the trading game taking place at time t_2.

We are interested in PBE such that debt relief occurs and compute prices consistent with that conjecture. In equilibrium, impatient atoms sell a block of endogenous size $\gamma^* \in (0, \gamma]$ in aggregate. For example, if impatient atoms strictly prefer to liquidate then $\gamma^* = \gamma$. In contrast, if impatient atoms are indifferent between liquidating and holding, then it is possible to support a PBE in which only a proper subset of them liquidates, inducing $\gamma^* < \gamma$.

In equilibrium the only way for I to make weakly positive trading gains is to mask his trades by purchasing a block of size γ^* whenever he buys debt in the secondary market. Further, he must play a mixed strategy. To this end, let σ^* denote the equilibrium probability of I placing a buy order in the PBE, which is then equal to the probability of debt relief.

Table 1 depicts outcomes in the trading game.

Table 1: Baseline order flow

Buy	Shock	x^I	x^N	X	Price	Probability
Y	N	γ	0	γ	P_ω^+	$\frac{\sigma^*}{2}$

Y	Y	γ	$-\gamma$	0	P_ω^0	$\frac{\sigma^*}{2}$
N	N	$\gamma - \gamma = 0$	0	0	P_ω^0	$\frac{1-\sigma^*}{2}$
N	Y	$\gamma - \gamma = 0$	$-\gamma$	$-\gamma$	P_ω^-	$\frac{1-\sigma^*}{2}$

Since $\gamma \leq 1/2$, the MM have sufficient endowment and bondholdings to take the opposite side of any equilibrium order flow. If we step away from the standard assumption that the MM can only observe net order flow, the large investor can still confound the MM by placing offsetting buy and sell orders as shown in the table.

Since I obtains a debt stake of $\gamma^* \leq \gamma$ when he buys debt, a *necessary condition* for debt relief if M is reached via u is:

$$\gamma \geq \underline{s}(M, \alpha). \tag{5}$$

If $\gamma < \underline{s}(M, \alpha)$, debt relief is not feasible if M is reached via u. That is, if liquidity shocks are too narrow, efficient debt relief is impossible to achieve.

In all the analysis below, only three order flows occur on the equilibrium path, with

$$X \in \{-\gamma^*, 0, \gamma^*\}. \tag{6}$$

When there is positive net order flow, the equilibrium is fully revealing and the MM know the large investor is buying stake γ^*. At the opposite extreme, negative net order flow reveals that the large investor is not buying debt. In the baseline model, with $\pi_\omega = 1/2$, zero net order flow is non-revealing and the MM set price as follows:

$$P_\omega^0 = \sigma P_\omega^+ + (1 - \sigma) P_\omega^-. \tag{7}$$

At the up-node, the market makers set the following respective prices upon observing positive and negative order flows:

$$
\begin{aligned}
P_u^+ &= 1 \\
P_u^- &= \frac{1 + M(1 - \alpha)}{2}
\end{aligned}
\tag{8}
$$

We first conjecture PBE in which all impatient atoms liquidate, i.e. $\gamma^* = \gamma$. In order to confirm that the resulting trading outcomes are indeed a PBE we must verify that the large investor is playing an optimal strategy and then verify that impatient atoms prefer to liquidate.

At the up node, let $G_u(\sigma, \widehat{\gamma})$ denote the expected trading gain perceived by the large investor in the event that with probability σ he places a buy order of size $\widehat{\gamma}$, where $\widehat{\gamma}$ is the measure of liquidating impatient atoms. The gain is equal to his expected payoff at time t_3 net of the expected price paid to acquire the stake $\widehat{\gamma}$:

$$
\begin{aligned}
G_u(\sigma, \widehat{\gamma}) &\equiv \frac{1}{2}\widehat{\gamma} + \frac{1}{2}[M - (1 - \widehat{\gamma})] - \frac{1}{2}[P_u^+ + P_u^0]\widehat{\gamma} \\
&= \frac{1}{2}\left[\frac{\widehat{\gamma}(1 - \sigma)(1 - M + \alpha M)}{2} - (1 - M)\right].
\end{aligned}
\tag{9}
$$

Since the function G_u is strictly decreasing in its first argument and increasing in its second argument, a necessary condition for the large investor to enter the secondary market is that G_u be strictly positive in the limit as σ converges to zero for $\widehat{\gamma} = \gamma$. From this one obtains the following necessary condition for the large investor to enter at the up node:

$$
\gamma > \underline{\gamma}_u = 2\underline{s}(M, \alpha).
\tag{10}
$$

It follows that sufficiently broad liquidity shocks (high γ) are actually necessary for the large investor to enter the secondary debt market. That is, broad liquidity shocks have an incentive effect, a fact commonly overlooked in discussions of alleged dire consequences arising from liquidity-constrained bondholders. Since $\underline{s}$ is decreasing in α, it also follows that high bankruptcy costs promote entry by the large investor. Once again, such incentive effects are commonly overlooked.

If $\gamma \leq \underline{\gamma}_u$, the equilibrium entails $\sigma_u^* = 0$ and there is zero probability of debt relief. Since there is no informed trading in this case, the impatient atoms always liquidate and $\gamma_u^* = \gamma$. The remaining market-making atoms would then take the opposite side of the trade.

The rest of this subsection considers the remaining case where $\gamma \geq \underline{\gamma}_u$. In this case, there is a unique $\widetilde{\sigma}_u \in (0, 1)$ satisfying the large investor's indifference condition $G_u(\widetilde{\sigma}_u, \gamma) = 0$, which must

be satisfied given that he plays a mixed strategy. In particular,

$$\widetilde{\sigma}_u = 1 - \frac{2\underline{s}(M,\alpha)}{\gamma}. \tag{11}$$

Throughout the paper, the variable $\widetilde{\sigma}_\omega$ captures endogenous investment incentives for the large investor. Differentiating equation (11) reveals that the trading intensity of the large investor should increase in the breadth of the liquidity shock and bankruptcy costs. In fact, the following signed comparative statics always hold, even when we move away from the baseline assumptions:

$$\frac{\partial \widetilde{\sigma}_\omega}{\partial \gamma} > 0 \tag{12}$$
$$\frac{\partial \widetilde{\sigma}_\omega}{\partial \alpha} > 0.$$

These comparative statics illustrate once again that broad liquidity shocks and higher bankruptcy costs have incentive effects which serve to promote debt market intervention by large investors.

There exists a PBE with $\sigma_u^* = \widetilde{\sigma}_u$ iff each impatient atom prefers to sell his debt. This will hold if the expected value captured by selling is larger than the payoff to unilaterally deviating by holding onto the debt. If the large investor buys and forgives debt with probability σ, impatient atoms are willing to liquidate iff:

$$\sigma P_\omega^0 + (1-\sigma)P_\omega^- \geq (1-\tau)P_\omega^0 \Leftrightarrow \tau \geq \underline{\tau}(\sigma, P_\omega^+, P_\omega^-) \equiv \frac{\sigma(1-\sigma)(P_\omega^+ - P_\omega^-)}{P_\omega^- + \sigma(P_\omega^+ - P_\omega^-)}. \tag{13}$$

The function $\underline{\tau}$ captures the degree of adverse selection facing impatient atoms. This induces a simple trading rule: liquidate iff $\tau \geq \underline{\tau}$. Impatient atoms are always willing to trade for limiting values of σ since

$$\lim_{\sigma \downarrow 0} \underline{\tau}(\sigma, P_\omega^+, P_\omega^-) = \lim_{\sigma \uparrow 1} \underline{\tau}(\sigma, P^+, P^-) = 0.$$

Intuitively, impatient atoms are reluctant to trade due to fear of selling at too low a price given that the large investor has private information regarding his strategy. In particular, price is below fundamental value in the second row of Table 1 in the sense that a hold-out atom would capture a payoff of one if he deviated, which is greater than the pooling price P_ω^0. If $\sigma = 0$, there is zero possibility of reaching that row. If $\sigma = 1$ that row can be reached, but the market makers set prices

equal to fundamental value since there is no possibility of their confounding the second and third rows.

The function $\underline{\tau}(\cdot, P_\omega^+, P_\omega^-)$ reaches a unique maximum at an interior point denoted $\sigma_\omega^{\max}$. Further,

$$\pi_\omega = \frac{1}{2} \Rightarrow \sigma_\omega^{\max}(P_\omega^+, P_\omega^-) \equiv \frac{-P_\omega^- + \sqrt{P_\omega^- P_\omega^+}}{P_\omega^+ - P_\omega^-} \in (0, 1). \tag{14}$$

For sufficiently high values of τ impatient atoms are willing to liquidate regardless of σ, so that $\widetilde{\sigma}_u$ is an equilibrium outcome. Formally

$$\tau \geq \underline{\tau}(\sigma_u^{\max}, P_u^+, P_u^-) \Rightarrow (\sigma_u^*, \gamma_u^*) = (\widetilde{\sigma}_u, \gamma). \tag{15}$$

Figure 3 depicts the remaining scenario where $\tau < \underline{\tau}(\sigma^{\max}, P^+, P^-)$. The subscripts are omitted in the figure because the analysis is also relevant at the down node. There are two σ values satisfying $\underline{\tau}(\sigma, P^+, P^-) = \tau$, say σ_1^ω and σ_2^ω. Checking the trading condition it follows that we can support the following PBE in which all impatient atoms liquidate:

$$\widetilde{\sigma}_u \quad \leq \quad \sigma_1^u \Rightarrow (\sigma_u^*, \gamma_u^*) = (\widetilde{\sigma}_u, \gamma) \tag{16}$$

$$\widetilde{\sigma}_u \quad \geq \quad \sigma_2^u \Rightarrow (\sigma_u^*, \gamma_u^*) = (\widetilde{\sigma}_u, \gamma).$$

The only remaining case is $\widetilde{\sigma}_u \in (\sigma_1^u, \sigma_2^u)$. In this case, $\widetilde{\sigma}_u$ cannot be sustained as a PBE since the impatient atoms are unwilling to liquidate. Here we can support a PBE in which the impatient atoms play a "mixed strategy" in which only a proper subset of them liquidate. The reduction in the size of their block trade decreases trading gains for the large investor, which induces him to reduce σ to σ_1^u, at which point the atoms are willing to mix. In this case, the low liquidity demand of the atoms reduces the probability of debt relief since $\sigma_u^* = \sigma_1^u < \widetilde{\sigma}_u$. In this equilibrium, the atoms liquidate a block of size $\gamma_u^* = \gamma_u^{mix} < \gamma$ such that

$$G_u(\sigma_1^u, \gamma_u^{mix}) = 0. \tag{17}$$

The following proposition summarizes the results derived in this subsection.
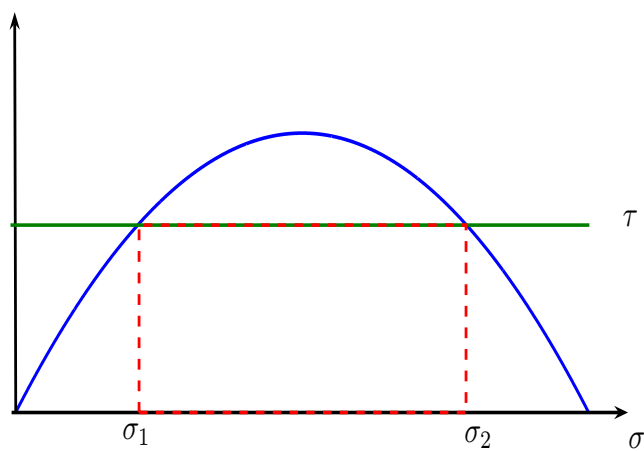
Figure 3: The Liquidation Decision

Figure 3: *This figure shows the liquidation decision of impatient bondholders at time t_2.*

Proposition 1 (Up Node). *If $\gamma \leq \underline{\gamma}_u$, there is never any debt relief ($\sigma^* = 0$) and all impatient atoms liquidate ($\gamma^* = \gamma$). If $\gamma > \underline{\gamma}_u$*

$$\tau \;\geq\; \underline{\tau}(\widetilde{\sigma}_u, P_u^+, P_u^-) \Rightarrow (\sigma_u^*, \gamma_u^*) = (\widetilde{\sigma}_u, \gamma)$$

$$\tau \;<\; \underline{\tau}(\widetilde{\sigma}_u, P_u^+, P_u^-) \Rightarrow (\sigma_u^*, \gamma_u^*) = (\sigma_1^u, \gamma_u^{mix}).$$

Having evaluated the nature of equilibrium under the baseline parameters, it is now easy to sketch comparative static effects. As argued above, it may be reasonable to assume γ_u is low, i.e. the breadth of liquidity shocks is low during good times. If the initial equilibrium features $\sigma_u^* = \sigma_1^u$, perturbing γ_u has no effect on σ_u^*. If instead, $\sigma_u^* = \widetilde{\sigma}_u$, a local reduction in γ reduces σ^*. That is, low breadth of liquidity shocks during good times reduces the probability of debt relief.

Consider next the effect of state-contingent liquidity preferences. As argued above, it may be reasonable to assume that τ_u is low. In Figure 3, this effect would be captured by shifting down the horizontal τ line. If the initial equilibrium features $\sigma_u^* = \sigma_1^u$, reducing τ_u reduces σ_u^*. Putting this result together with the prior one, we arrive at the more general conclusion that low liquidity demand during good times serves to decrease the probability of debt relief, ceteris paribus.

Recall, the baseline model assumes the large investor is never wealth constrained. Suppose instead $\overline{\sigma}_u < 1$. If σ^* described in Proposition 1 is less than $\overline{\sigma}_u$, the wealth constraint has no effect on the equilibrium. If instead $\sigma^* > \overline{\sigma}_u$, the equilibrium entry probability shifts to $\overline{\sigma}_u$ provided $\tau \leq \underline{\tau}(\overline{\sigma}_u, P_u^+, P_u^-)$. Finally, if $\sigma^* > \overline{\sigma}_u$ and $\tau > \underline{\tau}(\overline{\sigma}_u, P_u^+, P_u^-)$, the equilibrium entry probability jumps down to σ_1^u. This last scenario shows that wealth constrained large investors can dramatically reduce the probability of market entry and debt relief. For example, suppose that in the absence of a wealth constraint $\sigma^* > \sigma_2$. Suppose then one assumes $\overline{\sigma}_u \in (\sigma_1^u, \sigma_2^u)$. In this case, the wealth constraint would cause a drastic fall in σ from $\sigma^* > \sigma_2$ to σ_1^u. The absence of wealth constraints offer one mechanism, amongst other discussed below, consistent with the existence of relatively liquid debt markets during expansions.

Consider finally comparative statics performed on the bankruptcy cost parameter α. As shown

above, higher bankruptcy costs increase the profitability of entry and $\widetilde{\sigma}_u$. It follows that the equilibrium probability of debt relief is weakly increasing in α. This again illustrates a beneficial incentive effect arising from the prospect of costly bankruptcy.

3.3 Unconditional debt relief following contractions

Suppose next that node d is reached. This subsection evaluates conditions under which a PBE can be supported such that restructuring will take place with positive probability regardless of which $Y \in \{L, M\}$ is realized, i.e. at *both* terminal nodes. In this case, the market makers would set the following prices upon observing positive and negative order flows, respectively:

$$
\begin{aligned}
P_{db}^+ &= 1 & (18) \\
P_{db}^- &= \frac{(L+M)(1-\alpha)}{2}.
\end{aligned}
$$

Upon observing zero net order flow, the market makers set the pooling price in equation (7).

Following the same steps as in the preceding subsection, we compute the large investor's expected gain to buying debt at the down node if debt relief would be subsequently granted at both nodes L and M

$$
\begin{aligned}
G_{db}(\sigma, \widehat{\gamma}) &= \frac{1}{2}[L - (1 - \widehat{\gamma})] + \frac{1}{2}[M - (1 - \widehat{\gamma})] - \frac{1}{2}[P_d^+ + P_d^0]\widehat{\gamma} & (19) \\
&= \frac{1}{2}\left[\frac{\widehat{\gamma}(1 - \sigma)[2 - (L + M)(1 - \alpha)]}{2} - (2 - M - L)\right].
\end{aligned}
$$

Since G_{db} is strictly decreasing in its first argument and increasing in its second, a necessary condition for debt relief is that this value is strictly positive as σ converges to zero for $\widehat{\gamma} = \gamma$. Thus, we arrive at the following necessary condition for entry by the large investor under unconditional debt relief

$$
\gamma > \underline{\gamma}_{db} \equiv \frac{2(2 - L - M)}{2 - (L + M)(1 - \alpha)}. \tag{20}
$$

If condition (20) is not satisfied, one cannot rule out entry at the down node. This is because the large investor may still then find it profitable to enter the debt market in order to implement

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conditional debt relief whereby he forgives debt if node M is reached but not if node L is reached. Conditional debt relief is analyzed in the next subsection.

For the remainder of this subsection it is assumed that condition (20) is satisfied. In this case, there is a unique $\widetilde{\sigma}_{db} \in (0,1)$ satisfying the large investor's mixing condition $G_{db}(\widetilde{\sigma}_{db}) = 0$. In particular,

$$\widetilde{\sigma}_{db} = 1 - \frac{2(2 - L - M)}{\gamma[2 - (L + M)(1 - \alpha)]}. \tag{21}$$

If $\tau \geq \underline{\tau}(\widetilde{\sigma}_{db}^{\max}, P_{db}^{+}, P_{db}^{-})$, the impatient atoms are always willing to liquidate and the PBE entails with $\sigma_{d}^{*} = \widetilde{\sigma}_{db}$. If instead $\tau < \underline{\tau}(\widetilde{\sigma}_{db}^{\max}, P_{db}^{+}, P_{db}^{-})$ we return to Figure 3, with σ_{1}^{db} and σ_{2}^{db} denoting the two σ values at which the atoms are just indifferent between liquidating and holding. If $\widetilde{\sigma}_{db} \notin (\sigma_{1}^{db}, \sigma_{2}^{db})$, the impatient atoms still prefer to liquidate and $\sigma_{d}^{*} = \widetilde{\sigma}_{db}$. Finally, if $\widetilde{\sigma}_{db} \in (\sigma_{1}^{db}, \sigma_{2}^{db})$, $\widetilde{\sigma}_{db}$ cannot be sustained as a PBE since the impatient atoms are unwilling to liquidate. Here we can support a PBE in which the impatient atoms liquidate a block of size $\gamma_{d}^{*} = \gamma_{db}^{mix} < \gamma$ such that

$$G_{db}(\sigma_{1}^{db}, \gamma_{db}^{mix}) = 0.$$

Again, unwillingness of the impatient atoms to trade at $\widetilde{\sigma}_{db}$ induces a reduction in the equilibrium probability of debt relief to $\sigma_{1}^{db} < \widetilde{\sigma}_{db}$.

The following proposition summarizes the results derived in this subsection.

Proposition 2 (Down Node). *If $\gamma \leq \underline{\gamma}_{db}$, the large investor will never implement unconditional debt relief. If $\gamma > \underline{\gamma}_{db}$, the large investor implements conditional debt relief as follows*

$$\tau \;\; \geq \;\; \underline{\tau}(\widetilde{\sigma}_{db}, P_{db}^{+}, P_{db}^{-}) \Rightarrow (\sigma_{d}^{*}, \gamma_{d}^{*}) = (\widetilde{\sigma}_{db}, \gamma)$$

$$\tau \;\; < \;\; \underline{\tau}(\widetilde{\sigma}_{db}, P_{db}^{+}, P_{db}^{-}) \Rightarrow (\sigma_{d}^{*}, \gamma_{d}^{*}) = (\sigma_{1}^{db}, \gamma_{db}^{mix}).$$

Comparison of Propositions 1 and 2 allows one to evaluate whether fully ex post efficient debt relief is more or less likely if the interim state is down rather than up. Under the baseline parameters, there are three factors making debt relief more likely if the interim state is up. First, the free-ridership capitalized into bond prices is less severe at the up node, increasing the gains to entry. To

see this, note that the entry condition in (20) is more stringent than that stipulated in (10). For this same reason, conditional upon entry the large investor grants debt relief with higher probability if the interim node is up since

$$\widetilde{\sigma}_{db} < \widetilde{\sigma}_u. \tag{22}$$

In addition to the large investor's incentives being more powerful at the up node, small uninformed bondholders are more willing to sell at that node. That is, the perceived adverse selection problem is less severe for the atoms at the up node. To see this formally, note that

$$\underline{\tau}(\sigma, P_u^+, P_u^-) < \underline{\tau}(\sigma, P_{db}^+, P_{db}^-) \quad \forall \quad \sigma \in (0,1).$$

Moving away from the baseline parameters, the relative magnitudes of σ_d^* and σ_u^* is ambiguous. In particular, high values of τ at the down node serve to encourage liquidation by the atoms, weakly increasing σ_d^*. In addition, high values of γ at the down node increase the trading profits of the large investor, resulting in a strict increase in $\widetilde{\sigma}_{db}$ and a weak increase in σ_{db}^*. Conversely, a low value of $\overline{\sigma}_d$ could induce a large fall in the equilibrium relief probability σ_d^*.

3.4   Conditional debt relief following contractions

Suppose again node d is reached and suppose also that

$$\underline{s}(L, \alpha) > \gamma \geq \underline{s}(M, \alpha).$$

Only if γ falls into that interval is it possible for investors to anticipate that debt relief will occur if M is realized but not if L is realized. By way of contrast, it can never be the case that investors anticipate restructuring at L but not at M since any γ inducing relief at L necessarily induces relief at M.

Continuing the analysis of the case in which debt relief is granted if M is reached via d, the market makers set the following prices upon observing positive and negative order flows:

$$
\begin{aligned}
P_{dm}^+ &= \frac{1 + L(1 - \alpha)}{2} \\
P_{dm}^- &= \frac{(L + M)(1 - \alpha)}{2}.
\end{aligned}
\tag{23}
$$

Following the same steps as in the preceding subsection, one can compute the large investor's expected gain to buying debt at the down node conjecturing that relief would be granted at M

$$
\begin{aligned}
G_{dm}(\sigma, \widehat{\gamma}) &= \frac{1}{2}\widehat{\gamma}L(1 - \alpha) + \frac{1}{2}[M - (1 - \widehat{\gamma})] - \frac{1}{2}[P_{dm}^{+} + P_{dm}^{0}]\widehat{\gamma} \qquad (24) \\
&= \frac{1}{2}\left[\frac{\widehat{\gamma}(1 - \sigma)(1 - M + \alpha M)}{2} - (1 - M)\right] = G_{u}(\sigma, \widehat{\gamma}).
\end{aligned}
$$

Since G_{dm} is strictly decreasing in its first argument and increasing in its second, a necessary condition for entry is that this value is strictly positive as σ converges to zero for $\widehat{\gamma} = \gamma$. Thus, we arrive at the following two necessary conditions for debt market entry with conditional debt relief

$$
(i) \quad \gamma \; > \; \underline{\gamma}_{dm} \qquad\qquad\qquad (25)
$$

$$
and
$$

$$
(ii) \quad \gamma \; < \; \underline{s}(L, \alpha)
$$

$$
\underline{\gamma}_{dm} \; = \; 2\underline{s}(M, \alpha)
$$

$$
\underline{\gamma}_{u} \; = \; \underline{\gamma}_{dm} < \underline{\gamma}_{db}.
$$

Satisfaction of the two necessary conditions stated in (25) requires

$$
\underline{s}(L, \alpha) > 2\underline{s}(M, \alpha) \Leftrightarrow \alpha[(M - L) - L(1 - M)] < (1 - L)(1 - M). \qquad (26)
$$

If condition (26) is violated, there is no possibility for conditional debt relief. When condition (26) is violated, any γ satisfying the first entry condition (i) would also induce the large bondholder to grant debt relief at node L as well as node M. Therefore, when condition (26) is violated, the only possible equilibrium features unconditional debt relief, as described in Proposition 2. If instead condition (26) is satisfied, but $\gamma \leq \underline{\gamma}_{dm} < \underline{\gamma}_{db}$, debt market entry is never profitable for the large investor and equilibrium entails $(\sigma_{d}^{*}, \gamma_{d}^{*}) = (0, \gamma)$.

It is worth noting that the threshold γ values for entry into the secondary debt markets are the same at the up and down node, provided the latter entails a PBE with conditional debt relief. Intuitively, entry incentives are in this case the same at the up and down nodes because the extent of the free-rider problem is also the same.

For the remainder of this subsection it is assumed that condition (25) is satisfied. In this case, there is a unique $\widetilde{\sigma}_{dm} \in (0,1)$ such that $G_{dm}(\widetilde{\sigma}_{dm}) = 0$. In particular,

$$\widetilde{\sigma}_{dm} = 1 - \frac{2\underline{s}(M,\alpha)}{\gamma} = \widetilde{\sigma}_u. \tag{27}$$

Again, we see that the large investor's incentives to trade on the secondary debt market are equal between the up and down node provided the latter entails only conditional debt relief.

The equivalence between the up and down nodes breaks down once one considers the trading incentives of the atomistic investors, however. To see this, note that

$$\underline{\tau}(\sigma, P_u^+, P_u^-) < \underline{\tau}(\sigma, P_{dm}^+, P_{dm}^-) < \underline{\tau}(\sigma, P_{db}^+, P_{db}^-) \quad \forall \quad \sigma \in (0,1). \tag{28}$$

That is, the adverse selection problem perceived by the atoms is more severe at the down node than at the up node. Intuitively, the debt security is more informationally sensitive at the down node, inducing greater reluctance to trade for uninformed investors.

We turn finally to pinning down the PBE at the down node when (25) is satisfied. If $\tau \geq \underline{\tau}(\widetilde{\sigma}_{dm}^{\max}, P_{dm}^+, P_{dm}^-)$, the impatient atoms are always willing to liquidate and the PBE entails with $\sigma_d^* = \widetilde{\sigma}_{dm}$. If instead $\tau < \underline{\tau}(\widetilde{\sigma}_{dm}^{\max}, P_{dm}^+, P_{dm}^-)$ we return to Figure 3, with σ_1^{dm} and σ_2^{dm} denoting the two σ values at which the atoms are just indifferent between liquidating and holding. If $\widetilde{\sigma}_{dm} \notin (\sigma_1^{dm}, \sigma_2^{dm})$, the impatient atoms still prefer to liquidate and $\sigma_d^* = \widetilde{\sigma}_{dm}$. Finally, if $\widetilde{\sigma}_{dm} \in (\sigma_1^{dm}, \sigma_2^{dm})$, $\widetilde{\sigma}_{dm}$ cannot be sustained as a PBE since the impatient atoms are unwilling to liquidate. Here we can support a PBE in which the impatient atoms liquidate a block of size $\gamma_d^* = \gamma_{dm}^{mix} < \gamma$ such that

$$G_{dm}(\sigma_1^{dm}, \gamma_{dm}^{mix}) = 0.$$

Again, unwillingness of the impatient atoms to trade at $\widetilde{\sigma}_{dm}$ induces a reduction in the equilibrium probability of debt relief.

The following proposition summarizes the results derived in this subsection.

Proposition 3 (Down Node). *If $\gamma \leq \underline{\gamma}_{dm}$, neither conditional nor unconditional debt relief ($\sigma^* = 0$) will be granted and all impatient atoms liquidate ($\gamma^* = \gamma$). If condition (26) is violated or*

$\gamma \geq \underline{s}(L, \alpha)$, *only unconditional debt relief is credible, following Proposition 2. If* $\gamma \in (\underline{\gamma}_{dm}, \underline{s}(L, \alpha))$, *the large investor implements conditional debt relief as follows*

$$\tau \quad \geq \quad \underline{\tau}(\widetilde{\sigma}_{dm}, P_{dm}^+, P_{dm}^-) \Rightarrow (\sigma_d^*, \gamma_d^*) = (\widetilde{\sigma}_{dm}, \gamma)$$

$$\tau \quad < \quad \underline{\tau}(\widetilde{\sigma}_{dm}, P_{dm}^+, P_{dm}^-) \Rightarrow (\sigma_d^*, \gamma_d^*) = (\sigma_1^{dm}, \gamma_{dm}^{mix}).$$

Considering still the baseline parameters, it follows from condition (28) that even with conditional debt relief at the terminal node, σ_d^* can fall below σ_u^* due to the fact that severe adverse selection diminishes incentives for uninformed investors to trade at the down node. More, generally this result highlights that debt relief is path-dependent, with debt relief following contractions often being more difficult to implement due to the illiquidity of the debt market during contractionary phases of the business cycle.

However, the secondary debt market need not be illiquid during downturns since high liquidity demand (τ) during such periods encourages liquidation by uninformed investors. Further, broad liquidity shocks (γ) during downturns increase the return to acquiring large debt stakes.

4   Correlated Liquidity Shocks

This section allows the liquidity shock of atomistic investors to be correlated with the production technologies. This captures the fact that liquidity needs are likely to depend on the state of the economy, for example via endowment shocks. For simplicity of exposition, this section returns to the baseline assumptions regarding the other parameters, as stated in $A2' - A4'$. Thus, the only state-contingency in underlying parameters is due to

$$A5' : 1 > \pi_d > \pi_u > 0.$$

We first note that the analysis of the minimum stake of the large investor required for debt relief ex post is identical to the baseline analysis, since the correlation of liquidity shocks with the payoffs from the investment projects at time t_2 has no effect on the analysis of debt relief once the final period is reached.

4.1 Debt relief following expansions

We begin our analysis again by considering the start of t_2 when the state is u. As in the previous section, debt relief will be necessary to avoid costly default if the terminal node M is reached. We therefore wish to determine under what condition the large investor obtains a debt stake of at least $\underline{s}(M, \alpha)$, the minimum stake inducing debt relief if M occurs. To address that question, consider the trading game taking place at time t_2.

Again, we conjecture a PBE in which impatient atoms liquidate a block of size $\gamma^* \leq \gamma$. In equilibrium, the large investor camouflages his trade by submitting a buy order of γ^* with probability σ^*.

As in the previous section we consider a PBE such that debt relief occurs and therefore compute prices consistent with that conjecture. Table 2 depicts outcomes in the trading game and their probabilities.

Table 2: Order flow: correlated liquidity shocks

Buy	Shock	x^I	x^N	X	Price	Probability
Y	N	γ	0	γ	P_ω^+	$\sigma^*(1 - \bar{\pi}_u)$
Y	Y	γ	$-\gamma$	0	P_ω^0	$\sigma^* \bar{\pi}_u$
N	N	$\gamma - \gamma = 0$	0	0	P_ω^0	$(1 - \sigma^*)(1 - \bar{\pi}_u)$
N	Y	$\gamma - \gamma = 0$	$-\gamma$	$-\gamma$	P_ω^-	$(1 - \sigma^*)\bar{\pi}_u$

In contrast to Table 1, the probabilities of the various outcomes now take into account that liquidity shocks may be correlated with the evolution of production technology A.

Again, only three order flows occur on the equilibrium path. When there is positive or negative net order flow, the equilibrium is fully revealing. In all cases considered in this section, the prices at the revealing order flows are identical to those presented in the preceding section. However, correlation in liquidity shocks does change the nature of updating and hence price-setting by the

MM. In particular:

$$P_\omega^0 = \beta(\sigma, \pi) P_\omega^+ + [1 - \beta(\sigma, \pi)] P_\omega^-. \tag{29}$$

$$\beta(\sigma, \pi) \equiv \frac{\sigma \pi}{1 + 2\sigma\pi - \sigma - \pi}.$$

Here β measures the market makers' updated probability of the large investor buying given that observed order flow is zero. The updated probability has the following properties worth noting

$$\lim_{\sigma \downarrow 0} \beta(\sigma, \pi) = \lim_{\pi \downarrow 0} \beta(\sigma, \pi) = 0$$

$$\lim_{\sigma \uparrow 1} \beta(\sigma, \pi) = \lim_{\pi \uparrow 1} \beta(\sigma, \pi) = 1$$

$$\beta_\sigma(\sigma, \pi) = \frac{\pi(1 - \pi)}{(1 + 2\sigma\pi - \sigma - \pi)^2} > 0$$

$$\beta_\pi(\sigma, \pi) = \frac{\sigma(1 - \sigma)}{(1 + 2\sigma\pi - \sigma - \pi)^2} > 0.$$

To verify that a PBE exists in which the large investor buys debt with positive probability, we redefine his expected trading gain function G_u as follows:

$$\begin{aligned} G_u(\sigma, \pi, \widehat{\gamma}) &= \frac{1}{2}\widehat{\gamma} + \frac{1}{2}[M - (1 - \widehat{\gamma})] - [(1 - \pi_u)P_u^+ + \pi_u P_u^0]\widehat{\gamma} \\ &= \frac{\widehat{\gamma}\pi(1 - \beta)(1 - M + \alpha M) - (1 - M)}{2}. \end{aligned} \tag{30}$$

Since G_u is again strictly decreasing in σ and increasing in $\widehat{\gamma}$, a necessary condition for debt relief is that it be strictly positive in the limit as σ converges to zero for $\widehat{\gamma} = \gamma$. Rearranging terms, one obtains the following entry condition at the up node:

$$\gamma > \underline{\gamma}_u = \underline{s}(M, \alpha)\pi_u^{-1}. \tag{31}$$

It is interesting to observe that the γ needed for entry is in fact inversely related to the probability of the liquidity shock. Thus, entry into the secondary debt market is promoted by broad liquidity shocks (high γ) and higher arrival intensity of those shocks (high π). As argued above, both γ and π are arguably lower during expansions, serving to deter entry by large investors, all else equal.

Assuming that (31) is indeed satisfied, there is a unique $\widetilde{\sigma}_u \in (0,1)$ satisfying the large investor's mixing condition $G_u(\widetilde{\sigma}, \pi_u, \gamma) = 0$. In particular,

$$\widetilde{\sigma}_u = \frac{\gamma \pi_u (1 - M + \alpha M) - (1 - M)}{\gamma \pi_u (1 - M + \alpha M) - (1 - M) + \pi_u (1 - \pi_u)^{-1}(1 - M)}. \tag{32}$$

Since the focus of this section is the role of correlated liquidity shocks, it is worth noting that the large investor's incentives, as captured by $\widetilde{\sigma}_u$ are actually non-monotone in π_u. In particular:

$$\pi_u < \pi_u^{\max} \Rightarrow \frac{\partial \widetilde{\sigma}_u}{\partial \pi_u} > 0$$

$$\pi_u > \pi_u^{\max} \Rightarrow \frac{\partial \widetilde{\sigma}_u}{\partial \pi_u} < 0$$

$$\pi_u^{\max} \equiv \sqrt{\frac{1 - M}{\gamma(1 - M + \alpha M)}}.$$

Intuitively, the large investor dislikes extreme arrival intensities of the liquidity shock. To see this, note that if $\pi = 0$, the large investor has no camouflage for his trades. At the opposite extreme, when $\pi = 1$ the MM set $\beta = 1$ and revise prices upward to such an extent that no profits can be earned by an informed trader.

We turn next to the trading incentives of the atoms. With correlated liquidity shocks, one must redefine the hurdle $\underline{\tau}$ taking into account the new pricing rule used by the MM. Impatient atoms are willing to liquidate iff:

$$\sigma P_\omega^0 + (1 - \sigma) P_\omega^- \geq (1 - \tau)[\sigma P_\omega^+ + (1 - \sigma) P_\omega^-] \Leftrightarrow \tau \geq \underline{\tau}(\sigma, \pi, P_\omega^+, P_\omega^-) \equiv \frac{\sigma(1 - \beta)(P_\omega^+ - P_\omega^-)}{P_\omega^- + \sigma(P_\omega^+ - P_\omega^-)}. \tag{33}$$

It is worth noting that, holding all else constant, atoms' incentive to liquidate is increasing in π since

$$\frac{\partial \underline{\tau}}{\partial \pi} = \frac{\partial \underline{\tau}}{\partial \beta} \frac{\partial \beta}{\partial \pi} < 0. \tag{34}$$

Intuitively, high arrival intensity of the liquidity shock induces more favorable pricing from the perspective of the uninformed investors since $\beta_\pi > 0$. Since π is likely to be lower during expansions, this factor would tend to reduce the probability of debt relief occurring after good interim states.

Having characterized the incentives of the large investor and impatient atoms, as summarized by the redefined $\widetilde{\sigma}_u$ and $\underline{\tau}$ respectively, the PBE for correlated shocks at the up node is identical

in form to that specified in Proposition 1. Summarizing, this analysis of correlated shocks suggests that lower π values during expansions serves to: discourage entry by the large investor; discourage liquidation by uninformed bondholders; and has an ambiguous effect on $\widetilde{\sigma}_u$.

4.2 Unconditional debt relief following contractions

Consider next equilibrium at the down node. Following the same steps as in the preceding subsection, we begin by computing the large investor's expected gain to buying debt at the down node under a conjectured PBE in which debt relief is granted at both possible terminal nodes. We redefine the trading gain function accounting for the new price setting equation with correlated shocks as given in equation (29):

$$
\begin{aligned}
G_{db}(\sigma, \pi_d, \widehat{\gamma}) &= \frac{1}{2}[L - (1 - \widehat{\gamma})] + \frac{1}{2}[M - (1 - \widehat{\gamma})] - [(1 - \pi_d)P_{db}^+ + \pi_d P_{db}^0]\widehat{\gamma} \qquad (35) \\
&= \frac{\widehat{\gamma}\pi_d(1 - \beta)[2 - (L + M)(1 - \alpha)] - (2 - M - L)}{2}.
\end{aligned}
$$

Since G_{db} is strictly decreasing in σ and increasing in $\widehat{\gamma}$, a necessary condition for unconditional debt relief via the down node is is that G_{db} is strictly positive as σ converges to zero for $\widehat{\gamma} = \gamma$. Thus, we arrive at the following necessary condition for entry in the conjectured PBE

$$
\gamma > \underline{\gamma}_{db} \equiv \left[\frac{(2 - L - M)}{2 - (L + M)(1 - \alpha)}\right]\pi_d^{-1}. \qquad (36)
$$

Retaining our focus on the role of correlated liquidity shocks, the above condition illustrates that the high probability of liquidity shocks during contractions promotes entry by large investors into the secondary debt market.

For the remainder of this subsection it is assumed that condition (36) is satisfied. In this case, there is a unique $\widetilde{\sigma}_{db} \in (0, 1)$ satisfying the large investor's mixing condition which demands that $G_{db}(\widetilde{\sigma}_{db}) = 0$. In particular,

$$
\widetilde{\sigma}_{db} = \frac{\gamma\pi_d[2 - (L + M)(1 - \alpha)] - (2 - L - M)}{\gamma\pi_d[2 - (L + M)(1 - \alpha)] - (2 - L - M) + \pi_d(1 - \pi_d)^{-1}(2 - L - M)}. \qquad (37)
$$

Again, the large investor's incentives, as captured by $\widetilde{\sigma}_{db}$ are non-monotone in π:

$$\pi_d \ < \ \pi_{db}^{\max} \Rightarrow \frac{\partial \widetilde{\sigma}_{db}}{\partial \pi_d} > 0 \tag{38}$$

$$\pi_d \ > \ \pi_{db}^{\max} \Rightarrow \frac{\partial \widetilde{\sigma}_{db}}{\partial \pi_d} < 0$$

$$\pi_{db}^{\max} \ \equiv \ \sqrt{\frac{2 - L - M}{\gamma_d(2 - (L + M)(1 - \alpha))}}.$$

Given the non-monotonicity of $\widetilde{\sigma}_{db}$ and $\widetilde{\sigma}_u$ it is impossible to rank these two restructuring probabilities if $\pi_d > \pi_u$. However, for equal values of π the large investor has stronger incentives at the up node than at the down node, with

$$\widetilde{\sigma}_u(\pi) > \widetilde{\sigma}_{db}(\pi) \quad \forall \quad \pi \in (0, 1). \tag{39}$$

Consider next the trading incentives of the impatient atoms, with the trading condition for correlated shocks given in (33). For equal π values, the perceived adverse selection problem is less severe for the atoms at the up node. To see this formally, we note that

$$\underline{\tau}(\sigma, \pi, P_u^+, P_u^-) < \underline{\tau}(\sigma, \pi, P_{db}^+, P_{db}^-) \quad \forall \quad (\sigma, \pi) \in (0, 1) \times (0, 1). \tag{40}$$

However, it was shown in equation (34) that there is a countervailing effect encouraging trade at the down node since $\underline{\tau}$ is decreasing in π.

Having characterized the incentives of the large investor and impatient atoms, as summarized by $\widetilde{\sigma}_{db}$ and $\underline{\tau}$ respectively, the PBE for correlated shocks at the down node, with unconditional debt relief, is identical in form to that specified in Proposition 2. Summarizing, this analysis of correlated shocks suggests that higher π values during contractions serve to: encourage entry by the large investor; encourages liquidation by uninformed bondholders; and has an ambiguous effect on $\widetilde{\sigma}_{db}$.

4.3 Conditional debt relief following contractions

Consider again the scenario in which node d is reached and suppose that

$$\underline{s}(L, \alpha) > \gamma \geq \underline{s}(M, \alpha).$$

29

With correlated shocks we have the following trading gain for the large investor in a conjectured PBE with conditional debt relief:

$$
\begin{aligned}
G_{dm}(\sigma, \pi, \widehat{\gamma}) &= \frac{1}{2}[M - (1 - \widehat{\gamma})] + \frac{\widehat{\gamma}}{2}L(1 - \alpha) - [(1 - \pi)P_{dm}^+ + \pi P_{dm}^0]\widehat{\gamma} \qquad (41) \\
&= \frac{\widehat{\gamma}\pi(1 - \beta)(1 - M + \alpha M) - (1 - M)}{2} = G_u(\sigma, \pi, \widehat{\gamma}).
\end{aligned}
$$

Since the trading gain functions are equal at the up and down nodes in the present case, it follows that

$$
\begin{aligned}
\underline{\gamma}_{dm}(\pi) &= \underline{\gamma}_u(\pi) \\
\widetilde{\sigma}_u(\pi) &= \widetilde{\sigma}_{dm}(\pi) \quad \forall \quad \pi \in (0, 1).
\end{aligned}
$$

However, the higher arrival intensity of liquidity shocks at the down node would here serve to encourage entry, with

$$
\pi_d > \pi_u \Rightarrow \underline{\gamma}_{dm}(\pi_d) < \underline{\gamma}_u(\pi_u).
$$

Having characterized trading incentives for the large investor, consider next liquidation incentives for the atoms. We again consider the general trading condition for correlated shocks as given in equation (33). On one hand, there is greater intrinsic liquidity at the up node, due to lower adverse selection for uninformed bondholders, with

$$
\underline{\tau}(\sigma, \pi, P_u^+, P_u^-) < \underline{\tau}(\sigma, \pi, P_{dm}^+, P_{dm}^-) < \underline{\tau}(\sigma, \pi, P_{db}^+, P_{db}^-) \quad \forall \quad (\sigma, \pi) \in (0, 1) \times (0, 1). \qquad (42)
$$

However, it was shown in equation (34) that there is a countervailing effect encouraging trade at the down node since $\underline{\tau}$ is decreasing in π.

Having characterized the incentives of the large investor and impatient atoms, as summarized by $\widetilde{\sigma}_{dm}$ and $\underline{\tau}$ respectively, the PBE for correlated shocks at the down node, with conditional debt relief, is identical in form to that specified in Proposition 3. Summarizing, this analysis of correlated shocks suggests that higher π values during contractions serve to: encourage entry by the large investor; encourages liquidation by uninformed investors; and has an ambiguous effect on $\widetilde{\sigma}_{dm}$.

5 Bond Valuation

This section derives the value of the bond when it is issued at time t_0 and explores how it is affected by secondary debt market trading activity. Since states u and d are equally likely and there is no discounting between time t_2 and t_0, the bond value at time t_0, V, is given by

$$V = \frac{1}{2}v_u + \frac{1}{2}v_d, \tag{43}$$

where

$$
\begin{aligned}
v_\omega &= (1 - \gamma\pi)[\sigma_\omega^* P_\omega^+ + (1 - \sigma_\omega^*)P_\omega^-] + \gamma\pi[\sigma_\omega^* P_\omega^0 + (1 - \sigma_\omega^*)P_\omega^-] \tag{44} \\
&= \sigma_\omega^* P_\omega^+ + (1 - \sigma_\omega^*)P_\omega^- - \pi\gamma\sigma_\omega^*(1 - \beta_\omega)(P_\omega^+ - P_\omega^-).
\end{aligned}
$$

The preceding equation reveals that bond price is equal to fundamental value less uninformed bondholders' expected losses due to adverse selection.

To illustrate the effects of the model parameters γ, α and π on the initial bond value, V, we provide numerical simulations. Most comparative static results can be derived analytically, based on the expressions from the preceding sections. The numerical analysis initially assumes a sufficiently high value of τ so that impatient bondholders always find it optimal to sell. This allows us to isolate the effect of changing parameters on the incentives of the large investor.

We use the following base case parameters.

Table 3: Base Case Parameters

γ	π	α	$\bar{\sigma}$	M	L	τ
0.35	0.5	0.3	1	0.95	0.90	1

We begin by exploring the effect of γ on bond value. The top panel of Figure 4 plots the value of the bond at time t_0 as a function of the broadness of the liquidity shock, γ. The bottom two panels
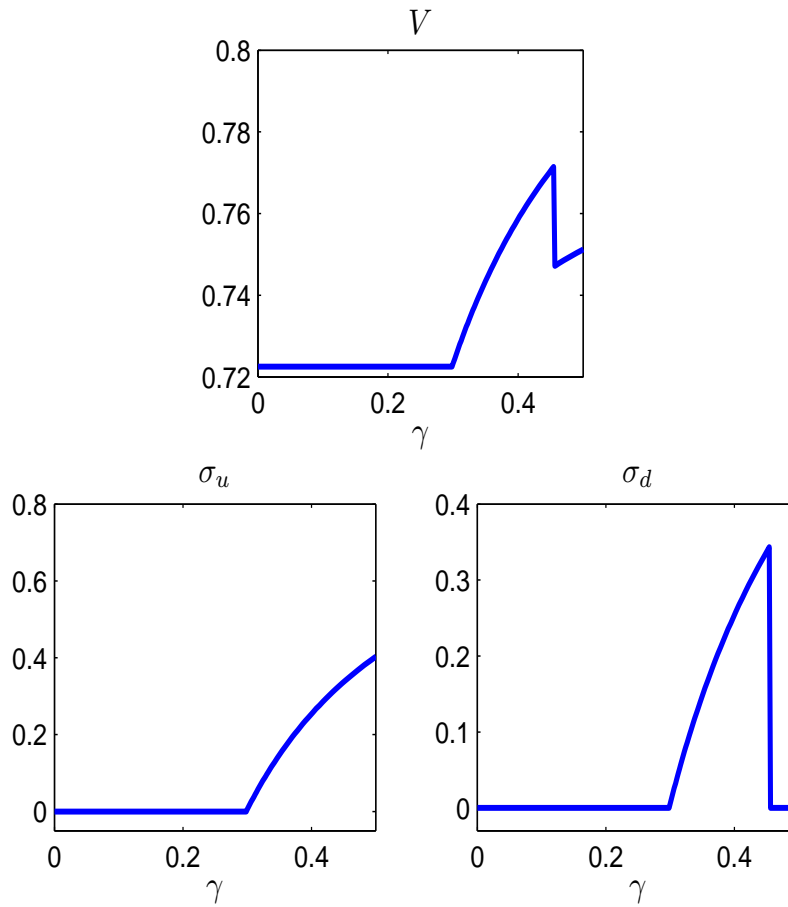
Figure 4: *This figure shows the bond value V and the probability of I trading in states u and d, σ_u and σ_d as a function of the broadness of the liquidity shock, γ.*

of Figure 4 show the corresponding probabilities of debt relief. For low values of γ, the large investor does not enter the secondary debt market. Here, the value of the bond therefore corresponds to the case without debt relief ($\sigma^* = 0$). Starting at γ approximately equal to 0.3, the liquidity in the secondary market becomes sufficiently high to make it worthwhile for I to enter. The bottom panels reveal that trading starts in both states u and d at the same critical value of γ. This is because initially restructuring only occurs in state M, independent of whether this state is reached via the down or up node. Locally, the probability of debt relief increases in γ, a fact reflected in an increasing bond value. This reveals the importance of the incentive effect causing the large investor to buy debt more aggressively when γ increases. In particular, if one holds σ constant, debt value necessarily falls with γ since this increases adverse selection costs capitalized into the initial bond price. Apparently, however, the positive incentive effect dominates.

If γ increases sufficiently, the large investor no longer trades at node d, as shown in the lower right panel. This results in a discrete reduction in the ex ante bond value. The intuition for this effect is simple. Once γ increases sufficiently, atomistic bondholders begin to anticipate a shift from conditional to unconditional debt relief. Free-ridership costs here increase discretely, inducing the large investor to drop out of the secondary market.

At the up node, the probability of debt relief increases monotonically in γ. Therefore, following the discrete drop in bond value discussed above, value again rises in γ. Interestingly, for this numerical example, bond value is maximized at the interior value $\gamma = .43$.

Next we analyze how the initial bond value depends on bankruptcy costs, α. For low values of bankruptcy costs, the large investor does not grant debt relief. For $\gamma = 0.35$, debt relief only becomes incentive compatible in state M for $\alpha \geq 0.3$. From this point on, the large investor enters the secondary market in both states u and d, as can be seen from the lower panels of 5. There is only partial restructuring following state d, i.e. for low default costs it is not optimal for the large investor to grant debt relief if state L occurs. For intermediate values of α, atomistic investors anticipate that the large investor would be willing to grant debt relief ex post even in state L. This
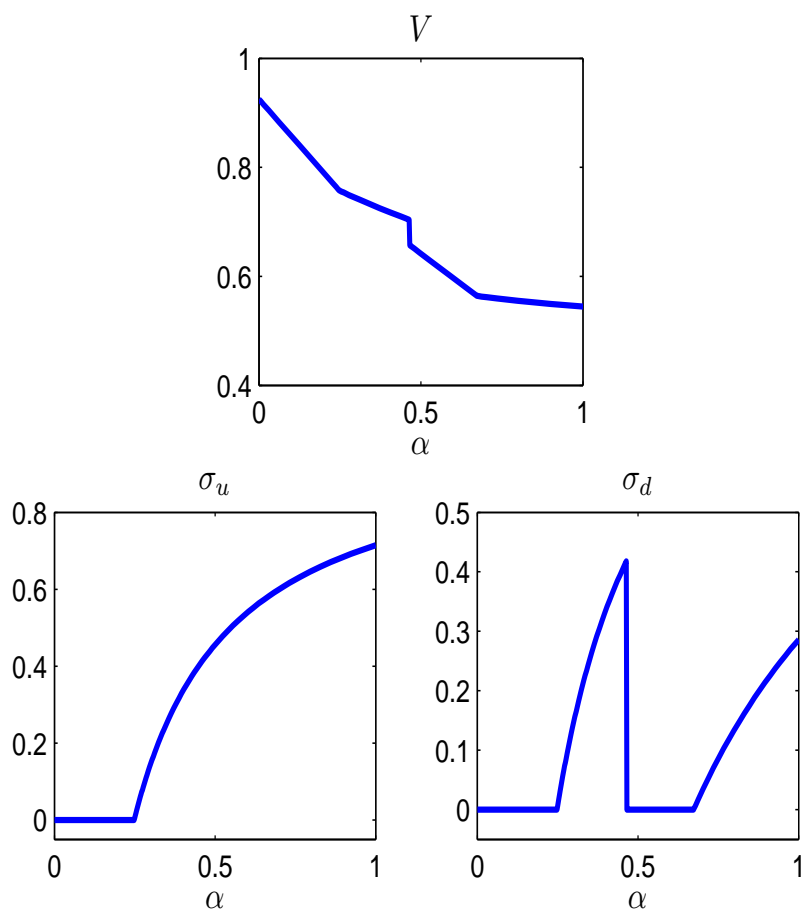
Figure 5: *This figure shows the bond value V and the probability of I trading in states u and d, σ_u and σ_d as a function of costs of financial distress, α.*

increases the cost of free-ridership, causing the large investor to drop out of the secondary debt market in state d.

However, once α increases sufficiently, profit opportunities in the secondary market for I improve sufficiently so that it again becomes optimal for him to enter the secondary market in state d, as shown in the lower right panel of Figure 5. In this range, unconditional debt relief occurs with positive probability.

The top panel of figure 5 reveals that bond value decreases monotonically in α. This is a standard prediction. However, the model shows that the impact of an increase in bankruptcy costs is mitigated due to the incentives provided to the large investor. When α exceeds 0.3, restructuring occurs in state M, no matter whether M is reached following an expansion or a contraction. Locally, the debt pricing line flattens, reflecting an increasing probability of debt relief. As α continues to increase, the large investor stops trading in state d, which leading to a discrete drop in bond value. For sufficiently high values of α, debt relief occurs both in states M and L. In this range a marginal increase in bankruptcy costs has only a small effect on debt value since restructuring occurs with high probability.

Next we consider the effect of changes in the probability of a liquidity shock, π. The initial value of the bond and the liquidity of the secondary market are illustrated in Figure 6. For low values of π, I does not enter the secondary market. For these values, the secondary market is too illiquid and the bond value therefore reflects zero debt relief. Once π exceeds a critical value, the secondary market becomes sufficiently liquid so that I trades with positive probability. The resulting efficiency gain through debt relief is reflected in a rising bond value. However, the bond value reaches a maximum for a value of π less than one. As π continues to increase towards one, it becomes increasingly difficult for the large trader to camouflage his presence. Whenever the market maker observes a net order flow of zero, he infers that there is a high probability that investor I is in the market and he will price the bond accordingly. This reduces the profit opportunities for I and as a result he reduces his trading probability. This leads to a decrease in the bond value.
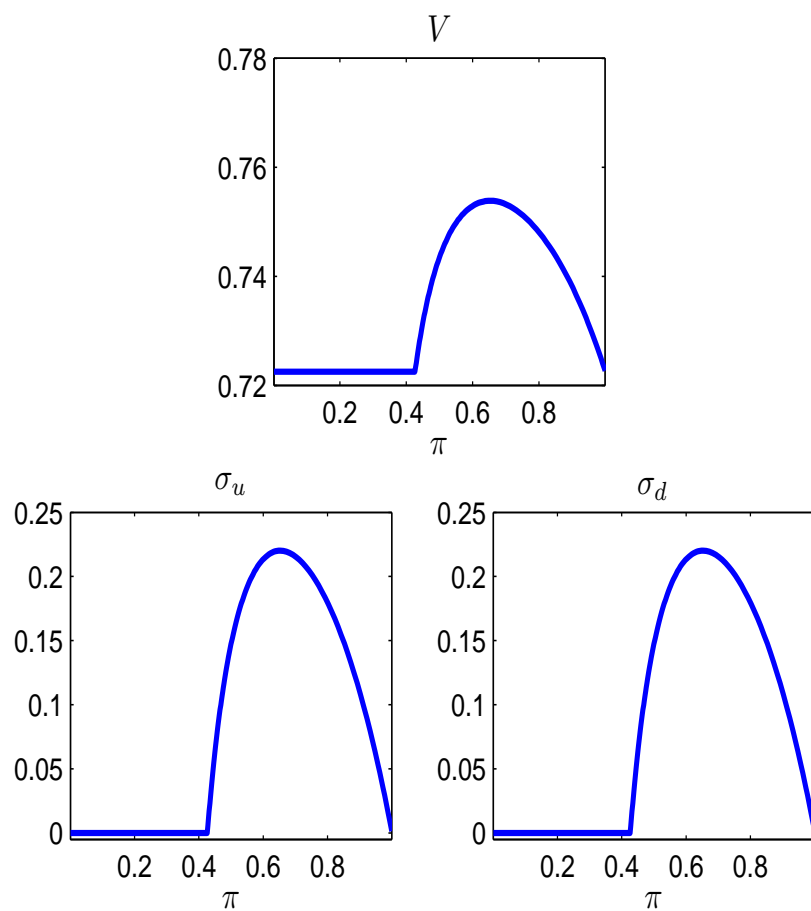
Figure 6: *This figure shows the bond value V and the probability of I trading in states u and d, σ_u and σ_d as a function of the probability of a liquidity shock, π.*
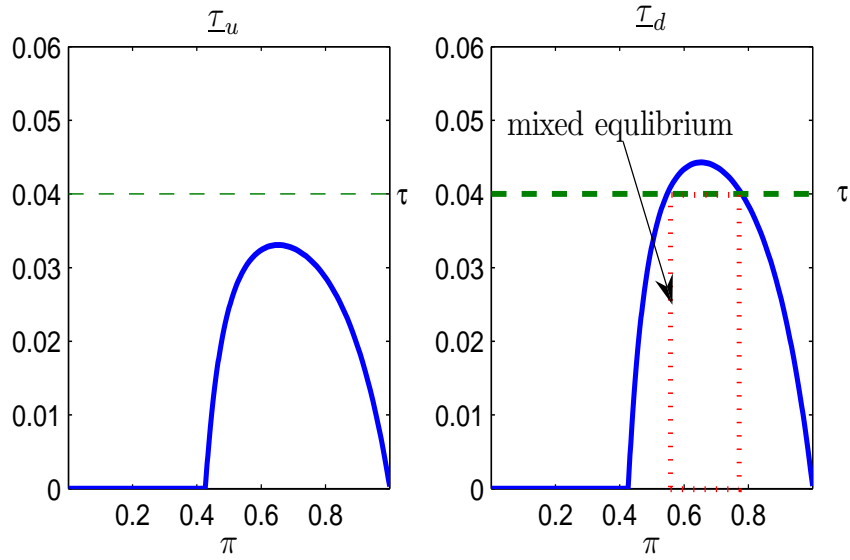
Figure 7: *This figure shows the critical impatience parameters $\underline{\tau}_u$ and $\underline{\tau}_d$ that make the impatient bondholders indifferent between selling at t_2 and holding the bond until t_3. The actual impatience parameter in states u and d is assumed to be 0.04.*

We now turn to the impatient investors' incentives to trade. The numerical analysis above assumed a sufficiently high value of τ such that impatient bondholders always found it optimal to sell. Recall that τ measures the degree of impatience, i.e. one unit of consumption at t_3 produces a utility of $1 - \tau$ whereas one unit of consumption at t_2 generates a utility value of 1. Recall also that $\underline{\tau}$ measures the degree of adverse selection in the secondary market. For the following numerical simulations we assume that the impatience parameter in a liquidity shock is $\tau = 0.04$. All the simulations allow for endogenous variations in σ.

We first demonstrate how the probability of a liquidity shock, π influences impatient bondholders' incentives to trade, taking into account endogenous variations in σ. Figure 7 reveals that, for low values of π, investor I is not in the market, and impatient bondholders sell without fearing adverse selection. For example, initially $\underline{\tau}_u = \underline{\tau}_d = 0$. As π increases, the opportunities for I to hide his trading improve and he starts trading with positive probability. This exposes uninformed bondholders to more severe adverse selection, raising both $\underline{\tau}_u$ and $\underline{\tau}_d$. Since the "mispricing" conditional on I being in the market is larger in state d than in state u, the rise in $\underline{\tau}_d$ is more pronounced than in $\underline{\tau}_u$.

Figure 7 shows that in the up state, τ always exceeds $\underline{\tau}_u$, so that all impatient atomistic bondholders rationally sell. By contrast, in state d, $\underline{\tau}_d$ first intersects τ from below as π increases. For these values of π, it is not an equilibrium for all impatient bondholders to sell. Equilibrium requires that only a fraction $\gamma^* < \gamma$ of them liquidate. As π continues to increase, $\underline{\tau}_d$ begins to decrease, reflecting an endogenous decline in σ. For π sufficiently high, the adverse selection in the secondary market is sufficiently low such that all impatient atomistic bondholders are willing to give sell orders.

A similar intuition applies for Figure 8, which considers the effect of changing γ. Consider first the bottom panels, which assume $\alpha = 0.3$. If only a small percentage of bondholders is exposed to liquidity shocks, the secondary market is not very liquid and the large investor does not enter. This means that there is no adverse selection in the secondary market and $\underline{\tau}_d$ and $\underline{\tau}_u$ are both zero.

For γ sufficiently high, I starts trading with positive probability in both states u and d, with only conditional debt relief anticipated at node d. The increase in γ intensifies the trading of the large investor, leading to increases in $\underline{\tau}_d$ and $\underline{\tau}_u$. Eventually, the uninformed bondholders are unwilling to liquidate and equilibrium entails their playing a mixed strategy. However, as γ continues to increase, the large investor drops out of the secondary market in state d, due to a discrete increase in free-ridership as atomistic bondholders anticipate a shift from conditional to unconditional debt relief. Here, $\underline{\tau}_d$ drops to zero again, since there is no more adverse selection in this state.

The top panels show $\underline{\tau}_u$ and $\underline{\tau}_d$ for higher bankruptcy cost values. The main difference occurs

in state d. Consider the top right panel of Figure 8. There is now on additional region for very high values of γ that is absent in the lower right panel where bankruptcy costs are lower. In this region, I enters the secondary market, although he will engage in debt relief in both states M and L. Comparing the top and the bottom panels of Figure 8 reveals that this region is only feasible for high bankruptcy costs.

Finally, Figure 9 displays the impatient bondholders' willingness to trade for different values of bankruptcy costs, α. The panel to the left displays the minimum impatience parameter τ that would induce atomistic bondholders to sell their holdings in state u. For sufficiently low bankruptcy costs, the large investor does not find it profitable to enter the secondary market. Therefore, the market makers always price the bonds correctly and uninformed bondholders are willing to sell, even if their impatience parameter is zero. As bankruptcy costs increase, I enters the secondary market with increasing probability, thus raising the degree of adverse selection in the secondary market. This makes atomistic bondholders more reluctant to liquidate their holdings. As a result higher values of the impatience parameter are needed to induce trading. Once $\underline{\tau}_u$ reaches τ, only mixed strategy equilibria are possible, with only a subset γ^* of impatient atomistic bondholders submitting sell orders.

A similar intuition applies to the right panel of Figure 9, depicting the impatient bondholders' willingness to submit sell orders in state d. For sufficiently low bankruptcy costs, the large investor does not enter the secondary market and $\underline{\tau}_d$ is zero. Then I begins to buy debt with increasing probability, and restructuring occurs in state M but not L. In this region $\underline{\tau}_d$ rises with α. As α continues to increase, atomistic bondholders anticipate a shift from conditional to unconditional restructuring. The increase in free-ridership causes the large investor to drop out of the secondary market at node d. Thus, adverse selection costs for uninformed bondholders here drops to zero. For α sufficiently high, the large investor re-enters the debt market at node d, leading to a concomitant increase in $\underline{\tau}_d$ as uninformed bondholders again perceive adverse selection risk.
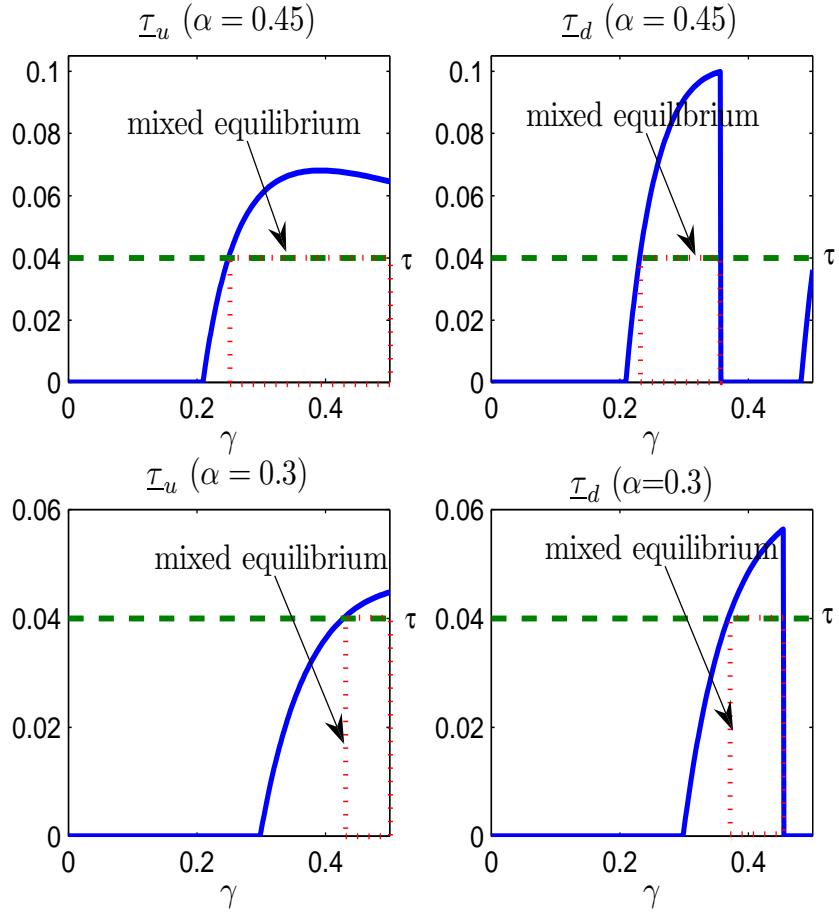
Figure 8: *This figure shows the critical impatience parameters $\underline{\tau}_u$ and $\underline{\tau}_d$ that make the impatient bondholders indifferent between selling at t_2 and holding the bond until t_3. The actual impatience parameter in states u and d is assumed to be 0.04. The panel to the left assumes costs of financial distress, α of 0.3 and the panel to the left assumes α of 0.45.*
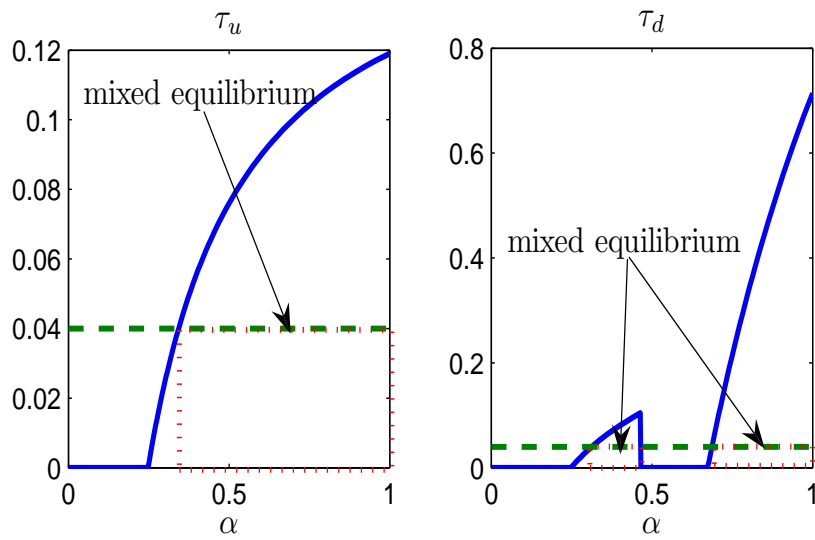
Figure 9: *This figure shows the critical impatience parameters $\underline{\tau}_u$ and $\underline{\tau}_d$ that make the impatient bondholders indifferent between selling at t_2 and holding the bond until t_3. The actual impatience parameter in states u and d is assumed to be 0.04.*

6 Concluding Remarks

The inability of dispersed debtholders to coordinate can be beneficial to avoid strategic default by the borrower. However, when the likelihood of financial distress increases, a more concentrated ownership structure becomes optimal, since it allows for efficient reorganization. The central question this paper has addressed is under what conditions the market can bring such changes in ownership structure about? We have therefore developed a framework to analyze a secondary market where endogenously determined ownership structure affects the value of the traded debt security.

The main obstacle that can prevent implementation of efficient debt ownership via the market is free ridership by small bondholders. While the cost of debt relief is only borne by the large bondholder, all bondholders enjoy the resulting efficiency gain. Thus, an investor will only acquire a large stake in the secondary market if the benefit from efficient restructuring is not fully incorporated in the purchasing price. In our framework this is possible since some investors may become subject to a liquidity shock, and thus may have a preference to sell. We identify several reasons why the free rider problem is likely to be most severe in economic downturns. First, both the likelihood of financial distress and the magnitude of the required debt relief in the financial distress states are higher in economic downturns. Thus, the cost of free ridership borne by a large bondholder is more severe. Second, in an economic contraction, the bond price becomes more information sensitive. Thus, even when investors experience a liquidity preference, they may still not wish to sell in a market in which adverse selection is high. Finally, even a potentially large investor may face wealth constraints in an economic downturn, thus preventing efficient concentration of debt ownership. Therefore, precisely in the states when concentrated debt ownership is crucial, it is particularly difficult for the market to bring such ownership changes about.

We also identify several channels which can prevent the freezing of secondary credit markets in economic downturns. First, if the economy is doing poorly, more investors may be subject to liquidity demands. We show that in general, a higher probability of investors experiencing a liquidity shock can help to generate efficient debt ownership structures. This may occur either by inducing

a more frequent entry of a large investor or by reducing the adverse selection costs of the liquidity traders, thus inducing them to enter the market. Second, in an economic downturn, the set of investors who are potentially affected by a liquidity shock may also increase. Again, this will in general lubricate the secondary market and promote efficient restructuring.

There are several directions for future research in this area. First, we believe that empirical work on the dynamics of debt ownership would be fruitful. Our knowledge of the dynamics of firms' debt structure and particularly about the dynamics of the ownership structure within classes of debt securities is rather limited. Second, we believe that several of our insights also apply to the market for securitized debt instruments. For example, recently, numerous SIVs have entered financial distress. In several cases large players, such as Goldman Sachs have or are in the process of acquiring concentrated ownership stakes in such legal entities. Although we are confident that the basic intuition of our analysis carries over to these markets, a model which carefully reflects the institutional framework of this market would be an important contribution.

Third, our analysis has dealt only with the case of one large investor. In practice there are many investors and institutions who could potentially acquire large stakes in the secondary debt market. Exploring the interactions between such investors would be an interesting research avenue.

Finally, we have considered the case in which the firm initially issues one homogenous bond to atomistic bondholders. If the firm also has bank debt or private debt outstanding, this would change the nature of the game at the restructuring stage at t_3 and thus the incentives to trade in the secondary market at t_2. We believe that a more general model of the optimal mix of debt contracts in the presence of secondary markets would provide an interesting set of questions to explore.

References

Bolton, Patrick and Olivier Jeanne, 2009,Structuring and Restructuring Sovereign Debt: The Role of a Bankruptcy Regime, *Journal of Political Economy*, forthcoming.

Bolton, Patrick and David S. Scharfstein, 1996, *Journal of Political Economy*, 104,1-25.

Brunnermeier, Markus K. and Lasse Heje Pedersen, 2008, Market Liquidity and Funding Liquidity, *Review of Financial Studies*, 1-38.

Caruana, Jaime and Laura Kodres, 2008, Liquidity in Global Markets, *Banque de France Financial Stability Review*, 65-74.

Eggert, Kourt, 2007, What Prevents Loan Modifications?, *Housing Policy Debate*, 18-2.

Gertner, Robert and David S. Scharfstein,*Journal of Finance*,46, 1189-1222.

Gilson,Stuart C., 1990, Bankruptcy, boards, banks, and blockholders: Evidence on changes in corporate ownership and control when firms default, *Journal of Financial Economics*, 27, 355-387.

Gilson, Stuart C., Kose John and Larry H.P. Lang, 1990, Troubled Debt Restructurings, *Journal of Financial Economics*, 315-353.

Glosten, Lawrence R., and Paul R. Milgrom, 1985, Bid, Ask and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders, *Journal of Financial Economics*, 14, 71100.

Grossman, Sanford J. and Oliver D. Hart, 1980, Takeover Bids, The Free-Rider Problem, and the Theory of the Corporation, *The Bell Journal of Economics*, 42-64.

Hart, Oliver and John Moore, 1994, A Theory of Debt Based on the Inalienability of Human Capital, *The Quarterly Journal of Economics*, 109, 841-879.

Kyle, Albert, 1985, Continuous Auctions and Insider Trading, *Econometrica* 53, 1315-1335.

Maug, Ernst, 1998, Large Shareholders as Monitors: Is There a Tradeoff Between Liquidity and Control?, *Journal of Finance*, 65-98.

Mello, Antonio and Rafael Repullo, 2004, Shareholder Activism is Non-Monotonic in Liquidity, *Finance Research Letters*, 2-12.

Piskorski, Tomasz, Amit Seru, and Vikrant Vig, 2008, Securitization and Distressed Loan Renegotiation: Evidence from the Subprime Mortgage Crisis, Working Paper 09-02, Chicago Booth School of Business.

Puchala, Alfred J. and David Goldhill, 2009, The Markets Solution, Conde Nast, Portfolio.com.

Shleifer, Andrei, 2003, Will The Sovereign Debt Market Survive?, *American Economic Review Papers and Proceedings*, 85-90.

Shleifer, Andrei and Robert Vishny, 1986, Large Shareholders and Corporate Control, Journal of Political Economy, 466-488.