

The Federal Reserve and the Cross-Section of Stock Returns*

Erica X.N. Li[†] and Francisco Palomino[‡]

Preliminary. Comments Welcome!

February 15, 2009

Abstract

We analyze the effects of monetary policy on the equity premium and the cross-section of stock returns in a general equilibrium framework. Monetary policy is conducted using an interest-rate policy rule reacting to inflation and output. The real effects of the policy are the result of product price rigidities in the production sector. The model predicts that (i) industries with lower price rigidities earn higher expected returns than industries with higher price rigidities and (ii) the difference in expected returns declines with more aggressive monetary policies. We provide an explanation for these results based on countercyclical markups. Markups of industries with low price rigidities are less variable than markups of industries with high price rigidities. When the marginal utility of consumption is high, markups in industries with high rigidities increase by more than markups in industries with low rigidities. As a result, profits of industries with low rigidities are more sensitive to policy shocks, and investors require a higher compensation for holding stocks on these industries. When the response of monetary policy to inflation is more aggressive, the markup variability reduces, and the difference in expected returns between high and low rigidity industries decreases. We find empirical evidence supporting the model's predictions.

JEL Classification: D51, E44, E52, G12.

*We thank participants at the University of Michigan Finance Brown Bag, the North American Summer Meeting of the Econometric Society 2008, and the European Central Bank and Bank of England Workshop, for helpful comments and suggestions. We are grateful to Shinwoo Kang for research assistance. All errors are our own.

[†]The University of Michigan, Ross School of Business, xuenanli@bus.umich.edu; Tel: (734) 616-3248; E-mail: xuenanli@umich.edu; <http://webuser.bus.umich.edu/xuenanli/>.

[‡]The University of Michigan, Ross School of Business, Ann Arbor, MI 48109; Tel: (734) 615-4178; E-mail: fpal@bus.umich.edu; <http://webuser.bus.umich.edu/fpal/>.

1 Introduction

The Federal Reserve conducts monetary policy to promote effectively the goals of price stability, i.e., control inflation, and maximum employment. This mandate implies the idea that monetary policy can influence real economic activity and suggests that real returns on financial assets can be affected by the policy. Therefore, monetary policy is potentially helpful to understand asset-pricing facts. This paper provides a theoretical analysis of the effects of monetary policy on the cross-section of stock returns and presents empirical evidence supporting the predictions of the theory.

We model an economy where the effects of monetary policy on stock returns are the result of price rigidities in production. Differences in returns across stocks are explained by different degrees of price rigidity across industries, and the responsiveness of the policy to inflation.¹ The policy is conducted setting a short-term interest rate using a policy rule. This rule responds to the level of inflation and a measure of output, and is affected by policy shocks. We show analytically that stocks carry a risk premium associated to policy shocks in an economy with homogeneous price rigidity across industries. The sensitivity of the risk premium increases with the degree of price rigidity and the elasticity of intertemporal substitution of consumption and labor, and decreases with the response to inflation in the policy rule. For an economy with heterogeneous price rigidities across industries, we show that industries with high price rigidities should earn lower expected returns than industries with low price rigidities, and the difference in returns decreases with a more aggressive response to inflation and output in interest-rate policy rule.

We provide a consumption-based explanation for the policy-related differences in stock returns. Industries with low price rigidities earn higher expected returns because their profits are more correlated to aggregate consumption than industries with high rigidities. Policy shocks induce a positive correlation between consumption and inflation in the model. As a result, a policy shock that reduces inflation, decreases profits in the industry with more flexible prices by more than the reduction in profits in the industry with more price rigidities. Simultaneously, the shock increases marginal utility because aggregate consumption is low. Therefore, investors require an additional compensation for holding stocks on industries with more flexible product prices.

¹There is ample evidence of infrequent adjustments in the prices of goods and services and significant differences in the degree of price rigidity across industries. Bils and Klenow (2004) analyze 350 categories. They report a median duration of prices between 4 and 6 months and the standard deviation is around 3 months. Nakamura and Steinsson (2007) exclude price changes related to sales and adjust this duration upwards to a range between 8 and 11 months. Blinder et al. (1998) conduct surveys on firms' pricing policies and summarize different theories for the existence of price rigidities based on the nature of costs, demand, contracts, market interactions and imperfect information.

The dependence of profits on the degree of price rigidity can be understood as the result of countercyclical markups induced by the rigidity. When prices are flexible, monopolistic competitors choose a level of production and a price that ensure an optimal constant markup over the marginal cost. When a producer can not change the product price, production depends on aggregate demand. During bad times, aggregate demand is low, labor demand declines and nominal wages decrease. Since prices are sticky, real wages also decline and the difference between a unit of production and the real labor cost increases. That is, the markup increases during bad times. The opposite happens during good times, and the markup is compressed with respect to the optimal constant markup. It implies that claims on profits (stocks) earn lower expected returns than claims on labor income. Claims on labor income are riskier than claims on profits since profits are a higher fraction of total production during bad times. Monetary policy then affects asset returns because it determines the distortions in markups generated by price rigidities. When inflation is low, differences between the optimal product price and the “sticky” price are small, the variability in markups is low and investors do not require high compensations for inflation risk. On the other hand, if monetary policy is conducted in such a way that inflation is volatile, markups will be volatile and that is reflected in a high compensation for claims on profits.

When there are differences in prices rigidities across industries, markups for different industries have different sensitivities to shocks in the economy. Industries with more flexible prices have implied markups that are closer to the optimal constant markup than the markups for industries with less flexible prices. As a result, during bad times markups of rigid-price industries expand more than those of flexible-price industries. Investors effectively perceive stocks on rigid-price firms as less risky than stocks on flexible-price firms. The fraction of production that is paid off as profits in the rigid-price industry is higher than this fraction for the flexible-price industry, when the marginal utility of investors is high. When monetary policy implies low inflation, the distortions caused by price rigidities in the two industries are small and, therefore, differences in expected returns in the two industries are small too.

Our theoretical results are complemented with empirical evidence supporting the predictions of the model. We sort industries into 10 deciles on price rigidity and form 10 portfolios using firms within the same deciles. We then form a hedge portfolio, defined as the price rigidity portfolio, that longs the portfolio with lowest price rigidity and shorts the portfolio with highest price rigidity. For the sample period from 1970 to 2006, we find that the price rigidity portfolio earns positive abnormal returns on average and this return is not explained by the market, size, book-to-market, and momentum factors. In addition we find that the average return of the price rigidity portfolio

is much higher from 1970 to 1979, than from 1980 to 2006. This finding is also consistent with the model’s predictions since there is evidence of a significantly more aggressive response to inflation in monetary policy after 1980 than during the 70’s.

The paper is organized as follows. Section 2 describes the economic model. Section 3 presents the stock-pricing implications of the model. For comparison purposes we present results for three different economies: an economy with flexible prices and economies with homogeneous and heterogeneous price rigidities across industries, respectively. Section 4 presents the empirical evidence and Section 5 conclude. The appendix contains all proofs.

2 The Model

We model a production economy where households derive utility from the consumption of a basket of two goods and disutility from supplying labor for the production of these goods. The two goods are produced in two different industries characterized by monopolistic competition and nominal price rigidities. We allow for heterogenous degrees of price rigidity in the two industries to learn about the effects of different rigidities on the cross section of stock returns.

Nominal rigidities generate real effects of monetary policy. When some producers are not able to adjust prices optimally, inflation generates distortions in relative prices that affect production decisions. Since inflation is determined by monetary policy, different policies have different implications for real activity. We model monetary policy as an interest-rate policy rule that reacts to inflation and deviations of output from a target.

2.1 Households

Households have preferences on the consumption of a final good, C and the supply of labor, N . Their preferences are represented by the utility function

$$\mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\gamma}}{1-\gamma} - \frac{N_t^{1+\omega}}{1+\omega} \right) \right]. \quad (1)$$

The final good is a basket of two goods produced in two industries. We will refer to these industries by $I = \{H, L\}$ where H and L are the industries with high and low price rigidities, respectively.

The consumption of each industry's good is C_I and the final good (basket) is defined as

$$C_t = \left[\varphi^{1/\theta} C_{H,t}^{\frac{\theta-1}{\theta}} + (1-\varphi)^{1/\theta} C_{L,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \quad (2)$$

where φ is the weight of industry H in the basket and $\theta > 1$ is the elasticity of substitution between industry goods. Each industry good is a Dixit-Stiglitz aggregate of a continuum of differentiated goods, defined as

$$C_{I,t} = \left[\int_0^1 C_{I,t}(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \quad (3)$$

where the elasticity of substitution across differentiated goods is the same as across industries.

Households provide labor, $N_{I,t}(i)$, for the production of differentiated goods in each industry, such that the total labor supply is

$$N_t = \left[\varphi^{-\omega} \int_0^1 N_{H,t}(i)^{1+\omega} di + (1-\varphi)^{-\omega} \int_0^1 N_{L,t}(i)^{1+\omega} di \right]^{1/(1+\omega)}.$$

The intertemporal budget constraint faced by households is

$$\mathbb{E} \left[\sum_{t=0}^{\infty} M_{0,t}^{\$} P_t C_t \right] \leq \mathbb{E} \left[\sum_{t=0}^{\infty} M_{0,t}^{\$} \sum_{j \in I} \left(\int_0^1 w_{j,t}(i) N_{j,t}(i) di + P_t \int_0^1 \Psi_{j,t}(i) di \right) \right], \quad (4)$$

where $M_{0,t}^{\$} > 0$ is the nominal pricing kernel that discounts nominal cash flows at time t to time 0, P_t is the price of the final good, $w_{I,t}(i)$ is the nominal wage earned from the production of good i in industry I and $\Psi_{I,t}(i)$ is the real profit for the producer of the differentiated good i in industry I .²

The maximization of (1) subject to (4) implies

$$M_{t,t+n}^{\$} = \beta^n \left(\frac{C_{t+n}}{C_t} \right)^{-\gamma} \left(\frac{P_{t+n}}{P_t} \right)^{-1},$$

²In the derivation of the budget constraint we use the fact that the minimum cost of the final good consumption satisfies $P_t C_t = P_{H,t} C_{H,t} + P_{L,t} C_{L,t}$, where $P_{I,t}$ is the price of the good produced in industry I , and the minimum production cost of the industry good is $P_{I,t} C_{I,t} = \int_0^1 P_{I,t}(i) C_{I,t}(i) di$, where $P_{I,t}(i)$ is the price of the good produced by firm i in industry I .

which is the intertemporal marginal rate of substitution of consumption in nominal terms, and

$$\frac{w_{I,t}(i)}{P_t} = \varphi_I^{-\omega} N_{I,t}(i)^\omega C_t^\gamma, \quad (5)$$

which is the intratemporal marginal rate of substitution between labor and consumption. This equation provides us with real wages once we determine the levels of labor and production from the production problem.

2.2 Firms

The production of differentiated goods is characterized by monopolistic competition and price rigidities in two different industries. Producers have market power to set the price of their differentiated goods within a Calvo (1983) staggered price setting. At each point of time, the producer is unable to change the price with some positive probability. We allow for different probabilities across industries to capture heterogeneous degrees of price rigidities.

The probability of not changing the price of a differentiated good at a particular time in industry I is α_I . When the producer is able to set a new price for the differentiated good, the price is set such that it maximizes the present value of expected profits over time. The maximization problem is

$$\max_{P_{I,t}(i)} \mathbb{E}_t \left[\sum_{T=t}^{\infty} \alpha_I^{T-t} M_{t,T}^{\$} (P_{I,t}(i) Y_{I,T|t}(i) - w_{I,T|t}(i) N_{I,T|t}(i)) \right]$$

subject to a demand function³ and the production function

$$Y_{I,T|t}(i) = A N_{I,T|t}(i), \quad (6)$$

where $Y_{I,T|t}(i)$ is the level of output of firm i in industry I at time T , when the last time that the price was reset was at t . We assume constant labor productivity, A , to isolate the effects of price rigidities from changes in productivity.

The solution to the firm's problem implies that the price is set as an average of expected marginal costs adjusted by a markup. Appendix A shows that this solution can be written in terms of aggregate output, inflation and a relative price. Aggregate output, Y_t , is the total production of

³This function is $P_{I,t} = P_{I,t}(i) \left(\frac{Y_{I,t}(i)}{Y_{I,t}} \right)^{1/\theta}$. See appendix A for details on its derivation.

the final good. When prices are perfectly flexible, the assumption of constant productivity implies a constant aggregate output, Y^f . We denote deviations in aggregate output from the flexible-price output, or “output gap”, by

$$x_t \equiv \log Y_t - \log Y^f.$$

Inflation in industry I is $\pi_{I,t} \equiv \log P_{I,t+1} - \log P_{I,t}$ and the relative price between the two industry goods is

$$p_{R,t} \equiv \log P_{H,t} - \log P_{L,t}.$$

The profit maximization problem implies a relation between inflation in each industry, the output gap and the relative price given by

$$\pi_{I,t} = \kappa_I x_t + \kappa_I \zeta^{-1} \varphi_{-I} p_{R,t} + \beta \mathbb{E}_t[\pi_{I,t+1}], \quad (7)$$

where $\varphi_{-H} \equiv -(1 - \varphi)$ and $\varphi_{-L} = \varphi$. The sensitivity of inflation in one industry to the output gap is $\kappa_I \equiv \frac{(1-\alpha_I\beta)(1-\alpha_I)}{\alpha_I} \zeta$ where $\zeta \equiv \frac{\omega+\gamma}{1+\theta\omega}$. In addition, inflation in one industry also depends on expectations of future inflation in that industry.

We can write the two industry equations described by (7) in terms of aggregate inflation, the output gap and the relative price. Inflation in the aggregate price index, $\pi_t \equiv \log P_{t+1} - \log P_t$, can be written in terms of industry inflations as

$$\pi_t = \varphi \pi_{H,t} + (1 - \varphi) \pi_{L,t}.$$

As a result, by adding up the two equations (weighted by the industry weights) we obtain

$$\pi_t = \bar{\kappa} x_t + b_\varphi p_{R,t} + \beta \mathbb{E}_t[\pi_{t+1}]. \quad (8)$$

where

$$\bar{\kappa} = \varphi \kappa_H + (1 - \varphi) \kappa_L \quad , \quad \underline{\kappa} = \kappa_H - \kappa_L \quad \text{and} \quad b_\varphi = -\frac{\varphi(1 - \varphi)}{\zeta} \underline{\kappa}.$$

Therefore, if the degree of price rigidities in the two industries is the same ($\underline{\kappa} = 0$), aggregate inflation does not depend on the relative price between the two industries. In order to obtain an expression for the evolution of the relative price, we can subtract one of the equations (7) from the other one and obtain

$$b_R p_{R,t} - p_{R,t-1} = \underline{\kappa} x_t + \beta \mathbb{E}_t[p_{R,t+1}], \quad (9)$$

where

$$b_R = 1 + \beta + \frac{1}{\zeta} [(1 - \varphi)\kappa_H + \varphi\kappa_L].$$

This equation describes the evolution of the relative price in terms of the output gap, the one-period lag and the expected future relative prices. Equations (8) and (9) summarize the optimality conditions for the production sector in the economy.

2.3 Monetary Authority

We model a monetary authority that sets the level of a short-term nominal interest rate. Monetary policy is described by the policy rule

$$i_t = \bar{i} + \iota_\pi \pi_t + \iota_x x_t + u_t,$$

where the one-period nominal interest rate, i_t is set responding to aggregate inflation, the output gap, and a policy shock u_t . The shock follows the process

$$u_{t+1} = \phi_u u_t + \sigma_u \varepsilon_{u,t+1}, \tag{10}$$

with $\varepsilon_u \sim \mathcal{N}(0, 1)$. Policy shocks are the only source of uncertainty in the economy and, therefore, financial assets reflect compensations only for this risk.

3 Equilibrium

We describe in this section the macroeconomic and asset pricing characteristics implied by the equilibrium of the model. For comparison purposes, we analyze two particular cases of the model before turning to the case with heterogeneous rigidities across industries. The particular cases are an economy with flexible prices and one with the same level of price rigidity across the two industries. In all cases we use the market clearing conditions $C_{I,t} = Y_{I,t}$ and $C_t = Y_t$.

3.1 Flexible-Price Economy

Production decisions are completely unlinked from inflation when prices are flexible. Aggregate output is constant, given by

$$Y^f = [\mu^{-1} A^{1+\omega}]^{1/(\omega+\gamma)},$$

where

$$\mu = \frac{\theta}{\theta - 1}$$

is the constant markup resulting from monopolistic competition.

Profit maximization implies that labor income and profits are constant shares of production. In particular, real profits in industry I are given by

$$\Psi_{I,t}^f = \frac{\varphi_I}{\theta} Y^f,$$

and the constant stock price is

$$S_{I,t}^f = \frac{\varphi_I}{\theta} \mathbb{E}_t \left[\sum_{n=0}^{\infty} M_{t,t+n} \Psi_{I,t+n}^f \right] = \frac{\varphi_I}{\theta} \frac{\beta}{1 - \beta} Y^f$$

It is clear that stock prices do not depend on policy shocks and do not involve any compensation for risk. Stock returns are equal to the real risk-free rate $r_t^f = -\log \beta$.

3.2 Homogeneous Price Rigidity Across Industries

The case of the same degree of price rigidity in the two industries ($\alpha_H = \alpha_L$) allows us to gain some insights into the effect of price rigidities on the equity premium. Since the only difference between the two industries is the degree of price rigidity, this case implies the same dynamics for the two industries. In particular, the relative price between the two industries does not play a role in equilibrium.

3.2.1 Macroeconomic dynamics

Inflation in the two industries is the same as inflation in the aggregate price index. It is given by

$$\pi_t = \bar{\pi} + \pi_u u_t,$$

where

$$\pi_u = -\frac{\kappa}{\kappa(\iota_\pi - \phi_u) + \iota_x(1 - \beta) + \gamma(1 - \beta\phi_u)(1 - \phi_u)}, \quad (11)$$

and

$$\bar{\pi} = \frac{\kappa}{\kappa(1 - \iota_\pi) - \iota_x(1 - \beta)} \left[\log \beta + \bar{i} + \frac{1}{2} \left(\frac{\gamma}{\kappa} (1 - \beta\phi_u) + 1 \right)^2 \pi_u^2 \sigma_u^2 \right]$$

where $\kappa \equiv \bar{\kappa} = \kappa_H = \kappa_L$. The output gap is

$$x_t = \frac{1}{\kappa}(1 - \beta)\bar{\pi} + \frac{1}{\kappa}(1 - \beta\phi_u)\pi_u u_t.$$

The effect of policy shocks on inflation and output decreases when monetary policy responds more aggressively to inflation and the output gap.

3.2.2 Market Price of risk

From the solutions above, we find endogenous characterization for the real pricing kernel, $m_{t,t+1} \equiv M_{t,t+1}$, given by

$$\begin{aligned} -m_{t,t+1} &= -\log \beta + \gamma \Delta y_{t+1} = -\log \beta + \gamma \Delta y^f + \gamma \Delta x_{t+1} \\ &= -\log \beta - \frac{\gamma}{\kappa}(1 - \beta\phi_u)(1 - \phi_u)\pi_u u_t + \frac{\gamma}{\kappa}(1 - \beta\phi_u)\pi_u \sigma_u \varepsilon_{u,t+1}. \end{aligned}$$

It follows that the conditional market price of risk is $\frac{\gamma}{\kappa}(1 - \beta\phi_u)\pi_u \sigma_u$. Its size decreases as the responses of monetary policy to inflation and the output gap increase.

The real one-period short-term rate, r_t , also responds to policy shocks. It is given by

$$r_t = -\log \mathbb{E}_t [M_{t,t+1}] = -\log \beta - \frac{1}{2} \left(\frac{\gamma}{\kappa}(1 - \beta\phi_u)\pi_u \sigma_u \right)^2 - \frac{\gamma}{\kappa}(1 - \beta\phi_u)(1 - \phi_u)\pi_u u_t.$$

3.2.3 Countercyclical Aggregate Markup and Stock Returns

Price rigidities in production generate time variation in the fraction of production that is distributed as labor income and profits. Appendix G.2 show that real aggregate labor income, LI_t can be written in terms of aggregate production as

$$LI_t = \frac{1}{\mu_t} Y_t,$$

where

$$\mu_t = \mu X_t^{-(\omega+\gamma)}, \tag{12}$$

can be interpreted as the time-varying markup in production, as a result of distortions in production caused by the policy shocks. The markup is more sensitive to the output gap as the elasticities of intertemporal substitution of consumption and labor, γ^{-1} and ω^{-1} , decrease. That

is, households who prefer smoother consumption and labor over time, demand a higher fraction of production paid as labor in good times (high output gap) and a lower fraction during bad times (low output gap). It implies that markups are countercyclical as a result of price rigidities.

In order to understand the implications of the countercyclical markup on stock returns, we can use the affine framework in appendix F to price claims on consumption, real labor income and real profits (stocks). In particular, we can analyze “one-period” claims which only payoff at some future time $t + n$. Therefore, claims on all future aggregate consumption, labor income and profits can be considered as portfolios of the one-period claims for all n .

Let $r_{C,t+1}^{(n)}$ be the one-period return of a claim on aggregate consumption at time $t + n$. The expected excess return of this claim over the risk-free rate r_t is

$$\mathbb{E}_t \left[r_{C,t+1}^{(n)} - r_t \right] = -\frac{1}{2} \text{var}_t \left(\Delta x_{t+1} + d_{C,t+1}^{(n-1)} \right) - \text{cov}_t \left(m_{t,t+1}, \Delta x_{t+1} + d_{C,t+1}^{(n-1)} \right),$$

where $d_{C,t+1}^{(n)}$ is the price- consumption ratio associated to the claim with payoff at time n . It can be shown that the covariance term is

$$-\text{cov}_t \left(m_{t,t+1}, \Delta x_{t+1} + d_{C,t+1}^{(n-1)} \right) = \gamma \left[\gamma + (\gamma - 1) \phi_u^{n-1} \right] \text{var}_t \left(\Delta x_{t+1} \right).$$

A similar analysis for returns on one-period labor income and profits, $r_{N,t+1}^{(n)}$ and $r_{\Psi,t+1}^{(n)}$, respectively, imply

$$-\text{cov}_t \left(m_{t,t+1}, \Delta li_{t+1} + d_{N,t+1}^{(n-1)} \right) = \gamma \left[\gamma + (1 + \omega) \phi_u^{n-1} \right] \text{var}_t \left(\Delta x_{t+1} \right),$$

and

$$-\text{cov}_t \left(m_{t,t+1}, \Delta \psi_{t+1} + d_{\Psi,t+1}^{(n-1)} \right) = \gamma \left[\gamma + (1 + \omega - \theta(\omega + \gamma)) \phi_u^{n-1} \right] \text{var}_t \left(\Delta x_{t+1} \right).$$

It can be seen from these two equations that, for all maturities n , the expected return on labor income claims is higher than the expected return on profits. The differences in the two expected returns increase as the intertemporal elasticities of consumption and labor increase. This is the result of a countercyclical markup. Stocks are less risky than claims on labor income since a higher fraction of production is paid off as labor income during bad times. In addition, more persistent policy shocks imply higher differences between the two claims.

3.3 Heterogenous Rigidities across Industries

In this section, we study the economy with two industries and different price rigidities. In order to find allocations and prices for the economy, we need to solve a system of equations that summarizes the relevant optimality conditions for households, firms and the monetary policy rule. These equations are the no-arbitrage equation for the nominal risk-free rate, the two equations that describes the optimality condition for the production sector and the policy rule for the central bank. Noticing that consumption is equal to output in equilibrium, we can summarize the equilibrium conditions as

$$e^{-i_t} = \mathbb{E}_t [\exp(\log \beta - \gamma(\Delta y^f + \Delta x_{t+1}) - \pi_{t+1})], \quad (13)$$

$$\pi_t = \bar{\kappa}x_t + b_\varphi p_{R,t} + \beta \mathbb{E}_t[\pi_{t+1}], \quad (14)$$

$$b_R p_{R,t} = \underline{\kappa}x_t + p_{R,t-1} + \beta \mathbb{E}_t[p_{R,t+1}], \quad (15)$$

$$i_t = \bar{i} + \iota_x x_t + \iota_\pi \pi_t + u_t \quad (16)$$

and
$$u_t = \phi_u u_{t-1} + \sigma_u \varepsilon_{u,t}.$$

Appendix B shows that equilibrium implies the processes for inflation, the relative price and the output gap given by

$$\pi_t = \bar{\pi} + \pi_p p_{R,t-1} + \pi_u u_t, \quad (17)$$

$$p_{R,t} = \bar{\rho} + \rho_p p_{R,t-1} + \rho_u u_t \quad (18)$$

$$\text{and } x_t = \bar{x} + x_p p_{R,t-1} + x_u u_t, \quad (19)$$

where the coefficients for all processes are characterized in the appendix.

3.3.1 Pricing Kernel and Market Price of Risk

From the equilibrium process for the output gap in equation (19), the real pricing kernel $m_{t,t+1}$ can be written in terms of the relative price and the policy shock as

$$m_{t,t+1} = \log \beta - \gamma x_p \Delta p_{R,t} + \gamma x_u (1 - \phi_u) u_t - \gamma x_u \sigma_u \varepsilon_{u,t+1}.$$

The market price of risk is therefore given by

$$\lambda = \gamma x_u \sigma_u.$$

Since monetary policy shock is the only source of risk in this economy, λ reflects the risk premium for the uncertainty on inflation. Figure 1 plots the market price of risk as a function of the response of monetary policy to inflation, using the calibrated parameters in Table 1. It can be seen that a weak response to inflation in monetary policy leads to a higher risk premium on inflation.

The real short-term rate is

$$r_t = -\log \beta - \frac{1}{2}\gamma^2 x_u^2 \sigma_u^2 + \gamma x_p \Delta p_{R,t} - \gamma x_u (1 - \phi_u) u_t.$$

3.3.2 Industry Markups and the Cross-Section of Returns

We can gain some intuition about the differences in expected returns across industries by analyzing (i) how industry markups are affected by the difference in price rigidities and (ii) the expected excess returns for claims that pay only one period in the future. It turns out that differences in the two industries can be explained in terms of the dynamics for the relative price.

Real labor income in industry I , $LI_{I,t}$, can be written in terms of the real value of production of that industry, $Y_{I,t}^{real}$, as

$$LI_{I,t} = \frac{1}{\mu_{I,t}} Y_{I,t}^{real},$$

where the time-varying industry markup is

$$\mu_{I,t} = \mu_t e^{-(1+\theta\omega)\varphi - I p_{R,t}},$$

and μ_t is the markup for aggregate production as in (12). It follows that the difference in markups in the two industries is

$$\frac{\mu_{H,t}}{\mu_{L,t}} = e^{(1+\theta\omega)p_{R,t}}.$$

When $p_{H,t} > p_{L,t}$, the markup is higher in industry H than in industry L . Since the aggregate output gap and the relative price are negatively autocorrelated, the markup in industry H expands more than the markup in industry L during bad times. In good times, the markup in H compresses more than the markup in L . In the more flexible industry, producers who can adjust the price will set a price that is closer to the one with the optimal flexible-price markup μ . Therefore, the markup in industry L is less sensitive than the markup in H .

Let $r_{C,I,t+1}^{(1)}$ be the one-period return of a claim on real consumption at time $t+1$ of the good produced in industry I . The expected excess return of this claim on industry H over a claim on

industry L is (up to the Jensen's inequality terms)

$$\begin{aligned}\mathbb{E}_t[r_{C,H,t+1}^{(1)} - r_{C,L,t+1}^{(1)}] &\approx -(1 - \theta)\text{cov}_t(m_{t,t+1}, \Delta p_{R,t+1}) \\ &= \gamma(1 - \theta)x_u\rho_u\text{var}_t(\Delta x_{t+1}),\end{aligned}$$

which is positive given the negative correlation between the output gap and the relative price. A claim on consumption in industry H is more risky because during bad times, the high product price in H , in comparison to the product price in L , hurts the demand of H in comparison to L .

The growth in real labor income for industry I can be written in terms of growth in aggregate labor income, Δli_t , and changes in the relative price, as

$$\Delta li_{I,t} = (1 + \omega + \gamma)\Delta x_t + \theta(1 + \omega)\varphi_{-I}\Delta p_{R,t} = \Delta li_t + \theta(1 + \omega)\varphi_{-I}\Delta p_{R,t}.$$

When the product price in industry H is higher than the product price in industry L , the value of labor income in that industry declines. It can be shown that the difference in expected excess returns for claims on one-period real labor income in the two industries is

$$\begin{aligned}\mathbb{E}_t[r_{N,H,t+1}^{(1)} - r_{N,L,t+1}^{(1)}] &\approx -\text{cov}_t(m_{t,t+1}, \Delta li_{H,t+1} - \Delta li_{L,t+1}) \\ &= -\gamma\theta(1 + \omega)x_u\rho_u\text{var}_t(\Delta x_{t+1}).\end{aligned}$$

This expected excess return is positive. Workers in industry H demand a higher return in their labor income because, during bad times, markups are higher in this industry and the fraction of production that they obtain is lower than the fraction obtained by workers in L .

Finally, growth in real profits in industry I can be written in terms of growth in aggregate profits, $\Delta\psi_t$ and changes in the relative price, as

$$\Delta\psi_{I,t} = \Delta\psi_t + \varphi_{-I}(1 - \theta)\theta\omega\Delta p_{R,t}.$$

When the relative price increases, the growth in real profits in the industry with more rigid product price is larger than in the industry with the more flexible price. Expected excess returns between real profits in the two industries are

$$\begin{aligned}\mathbb{E}_t[r_{\Psi,H,t+1}^{(1)} - r_{\Psi,L,t+1}^{(1)}] &\approx -\text{cov}_t(m_{t,t+1}, \Delta\psi_{H,t+1} - \Delta\psi_{L,t+1}) \\ &= -\gamma(1 - \theta)\theta\omega\text{cov}_t(\Delta x_{t+1}, \Delta p_{R,t+1}),\end{aligned}$$

which is negative, given the negative correlation between output gap and relative prices in equilibrium. The expected excess returns on real profits in L are higher than those in H because the markup in L is lower than the markup in H during bad times, that is, profits in industry L tend to decline more than profits in industry H during bad times.

Notice that the changes in the relative price can also be written in terms of industry inflations as

$$\Delta p_{R,t} = \pi_{H,t} - \pi_{L,t}.$$

It follows that compensations for risk in one industry are higher than in the other one as long as inflation in that industry covaries more with aggregate consumption than inflation in the other industry. It can be shown using equation (7) that inflation in the industry with low price rigidity is more sensitive to the aggregate output gap than inflation in the industry with high price rigidity.⁴ Intuitively, inflationary shocks have larger negative effects on the profits of the industry with low price rigidities and, as a result, economic agents demand high compensations for claims on these profits.

3.3.3 Numerical Exercise

We analyze the implications on expected excess returns for stocks in the two industries relying on a numerical solution and comparative statics. The details of the numerical procedure are presented in appendix E. The purpose of this exercise is to see whether the expected excess return implied by the stock of the industry with a low price rigidity is higher than that implied by the stock in the industry with high price rigidity. The comparative statics allow us to see the implications on the difference in expected returns of policies with different responses to inflation and the output gap.

Given the equilibrium processes for inflation, the relative price, and the output gap inequations (17)-(19), we obtain stock prices and expected returns for both industries using a recursive approach. The real value of industry I can be written recursively as⁵

$$V_I(p_{R,t}, u_t) = \Psi_{I,t}(p_{R,t}, u_t) + \mathbb{E}_t [M_{t,t+1} V_I(p_{R,t+1}, u_{t+1})], \quad (20)$$

where the state variables are the current period's relative price and the policy shock $(p_{R,t}, u_t)$. The first two terms summarize the real profit of industry I and the last term is the continuation

⁴Appendix D shows the equilibrium process for inflation in the two industries.

⁵This value reflects the stock price plus the current period profits.

value.

Expected real stock returns are

$$\mathbb{E}[r_{\Psi,I,t+1}] = \mathbb{E} \left[\log \left(\frac{V_I(p_{R,t+1}, u_{t+1})}{V_I(p_{R,t}, u_t) - \Psi_{I,t}(p_{R,t}, u_t)} \right) \right],$$

for $I = \{H, L\}$. Table 1 shows the parameter values used in the exercise.

Figure 2 plots the differences in expected returns between the low and high rigidity industries for claims on consumption, labor income and profits, for different parameter values. The difference in expected returns for claims on profits increase as the elasticities of consumption and labor decrease, the price rigidity in industry H increases and the persistence of the policy shock increases. More aggressive responses to inflation and the output gap in the policy rule reduce the difference in expected returns.

Figure 3 shows impulse responses to a positive policy shock. This shock represents bad news for the economy since it induces a negative output gap. Simultaneously, it increases the relative price, production in the industry with the more sticky price is negatively affected while production in the one with more flexible price is positively affected. The value of claims on consumption and labor decline, and the claims in industry H are more negatively affected. However, the values of the claims in labor income in the two industries are less affected than the values of the respective claims in consumption, reflecting expanded markups in the two industries. Since the expansion in markups in industry H is larger than in L , profits in L are more negatively affected than profits in H , resulting in higher expected returns on a stock in industry L over the expected return for industry H .

3.3.4 A Different Source of Uncertainty: Supply Shocks

In this section we analyze the differences in expected returns between industries with high and low price rigidities that result from the existence of supply shocks. We define supply shocks as a source of uncertainty affecting firm decisions. They can be seen as a source of time-variation in firm taxes or time-variation in markups. The time variation of markups can be the result of time-varying elasticity of substitution across goods. Incorporating this shock to the model amounts to modify

equations (14)-(16) to incorporate the supply shock, and obtain

$$\pi_t = \bar{\kappa}x_t + b_\varphi p_{R,t} + \beta \mathbb{E}[\pi_{t+1}] + \epsilon_t, \quad (21)$$

$$b_R p_{R,t} = \underline{\kappa}x_t + p_{R,t-1} + \beta \mathbb{E}_t[p_{R,t+1}] + b_\epsilon \epsilon_t \quad (22)$$

$$\text{and } i_t = \bar{i} + \iota_\pi \pi_t + \iota_x x_t, \quad (23)$$

respectively, where $b_\epsilon = \frac{1-2\varphi}{\varphi(1-\varphi)}$ and the supply shock, ϵ_t , follows the process

$$\epsilon_t = \phi_\epsilon \epsilon_{t-1} + \sigma_\epsilon \varepsilon_{\epsilon,t}.$$

This shock generates equilibrium processes for inflation, the relative price and the output gap that depend linearly on the shock. The derivation of the equilibrium is presented in appendix C. One important difference between the effect of this shock in comparison to the policy shock is that, while policy shocks generate an positive correlation between the output gap and inflation, supply shocks generate a negative correlation.

Figure 4 shows the difference in expected returns between industries with low and high price rigidities, for different parameter values. In general, the results are very similar to those obtained with policy shocks. However, there is a notable difference related to the effect on the excess return when the reaction to inflation in the policy rule increases. Supply shocks induce a negative correlation between output gap and inflation, creating a trade-off between inflation stabilization and output stabilization. For that reason, a higher reaction to inflation implies more distortions in output and it increases the differences between expected returns of industries with low and high rigidities. For policy shocks, a higher response to inflation reduces the expected excess return given that stabilizing inflation also stabilizes output across industries.

4 Empirical Results

We test the predictions of the model using the data of publicly traded firms. The stock market data is from the Center for Research in Security Prices (CRSP). The price rigidities for individual industries are from Bils and Klenow (2004), which provides the monthly frequency of price changes for 350 categories of consumer goods and services comprising around 70% of consumer expenditures from 1995 to 1997. Using the 49-industry classification from Kenneth French's web site, we obtain

the frequencies of price changes for 31 industries, used as our proxy for price rigidity.⁶ Table 2 lists the summary statistics of the price rigidities for 31 industries.

We sort industries into 10 deciles according to their price rigidities in descending order. Firms within the industries of the same decile are used to form both value-weighted and equal-weighted portfolios. We then run Carhart four-factor model for each of the 10 portfolios and the hedge portfolio, defined as the price-rigidity portfolio, that longs the portfolio with the lowest price rigidity (decile 10) and shorts the portfolio with the highest price rigidity (decile 1). Tables 3 and 4 present the regression results for two sample periods: 1970 – 1980 and 1980 – 2006. The selection of the two periods was based on Clarida, Galí and Gertler (2000). They find that the response of the short-term interest rate to inflation is significantly stronger after 1980 than for the 1970 – 1980 period. The model predicts that profits of industries with low price rigidity earn higher expected returns than industries with high price rigidities. This difference decreases with the response of the interest rate to inflation.

Table 3 shows the regression results using the Carhart four-factor model and the data for the first sample period. For value-weighted returns, portfolio 10 (firms with lowest price rigidity) earns 77 basis points more than portfolio 1 (firms with highest price rigidity) monthly, controlling for market, size, book-to-market, and momentum factors. The difference increases to 117 basis points for equal-weighted portfolios. The t-stats are 2.32 and 2.85, respectively. Therefore, industries with low price rigidities earn significantly higher returns than industries with high price rigidities from 1970 to 1980.

Table 4 shows the results for the second period. For value-weighted returns, portfolio 10 earns 3.9 basis points more than portfolio 1, controlling for market, size, book-to-market, and momentum factors. And 2.4 basis points for equal-weighted portfolios. The t-stats are 0.12 and 0.07, respectively. Although industries with low price rigidities still earn higher average returns, the difference is much smaller after 1980 compared to that during the 1970's and is not statistically significant.

In summary, the empirical results provide strong support for the predictions of the model. A weak response of the central bank to inflation increases expected excess returns and industries with high price rigidities earn higher expected returns than industries with low price rigidities.

⁶The frequency of price changes for a particular industry is the average of the frequencies of price changes of consumer goods and services within this industry.

5 Conclusions

This paper provides a theoretical framework for the analysis of the effects of monetary policy on stock returns. We use this framework to analyze the implications of monetary policy on the equity premium and the cross section of returns. Monetary policy has effects on stock returns because firms are not able to adjust their product prices every period. This nominal rigidity generates an equity premium for inflation risk, which depends on the elasticities of substitution of consumption and labor, the degree of price rigidity and the reaction of the policy to inflation and output. In the cross section, expected returns are higher for industries with more flexible product prices. Countercyclical markups for these industries are less sensitive to inflation risk and, as result, their profits are more sensitive to this risk. Therefore, investors require an additional compensation for holding stocks on these industries.

We find empirical evidence supporting the model predictions. The return difference between low and high price rigidity industries is positive and significant for a period in the US monetary policy characterized by a weak response to inflation. This difference in returns can not be explained by market, value, size and momentum factors. The theoretical approach suggests a potential role for relative prices across industries and/or industry-specific inflation to explain this difference.

Table 1: Baseline parameter values.

Parameter	Description	Value
β	Subjective discount factor	0.974
γ	Inverse of EIS of consumption	0.8
ω	Inverse of EIS of labor	0.4
α_H	Price rigidity in industry H	0.5
α_L	Price rigidity in industry L	0.05
θ	Elasticity of substitution of goods	1.2
ϕ_u	Autocorrelation of policy shock	0.5
σ_u	Conditional volatility of policy shock	0.05
\bar{t}	Constant in the policy rule	0.029
ι_π	Response to inflation in the policy rule	1.1
ι_x	Response to output gap in the policy rule	0

Table 2: **Summary Statistics**

This table reports the average frequencies of price changes and the standard deviation for products in each industry. We divide firms into 49 industries according to the classification from Ken French's web site.

Industry Number	Industry	Number of Products	Avg. Freq.	STD of Freq.
2	Food Product	81	34.27	11.97
3	Candy and Soda	9	27.39	8.92
4	Beer and Liquor	4	17.43	3.19
5	Tobacco Product	3	20.07	2.92
6	Recreation	12	23.15	8.34
7	Entertainment	6	11.12	6.03
8	Printing and Publishing	7	9.53	4.65
9	Consumer Goods	54	19.54	6.48
10	Apparel	43	32.72	11.17
11	Healthcare	5	6.76	2.71
12	Medical Equipment	3	8.10	2.94
13	Pharmaceutical Products	3	14.77	1.76
14	Chemicals	3	19.43	10.62
16	Textiles	1	17.00	N/A
17	Construction Materials	8	12.40	4.47
21	Machinery	4	26.25	10.61
22	Electrical Equipment	1	19.40	N/A
23	Automobiles and Trucks	6	26.18	11.13
30	Petroleum and Natural Gas	8	56.45	20.81
31	Utilities	4	30.30	28.45
32	Communication	4	12.60	5.53
33	Personal Services	47	8.64	5.46
34	Business Services	1	10.00	N/A
35	Computer Hardware	4	18.90	13.02
36	Computer Software	1	16.50	N/A
37	Electric Equipment	2	13.45	6.15
39	Business Supplies	2	9.25	3.04
41	Transportation	11	21.05	22.84
44	Restaurants, Hotels, Motels	8	11.63	10.77
45	Banking	3	12.77	13.80
46	Insurance	2	12.50	4.24

Table 3: Performance-Attribution Regressions for Portfolios with Different Price Rigidities

This table reports the Fama-French-Carhart four-factor regression: $R_t = \alpha + \beta_1 * RMRF_t + \beta_2 * SMB_t + \beta_3 * HML_t + \beta_4 * Momentum_t + \epsilon_t$, where R_t is the excess return relative to risk-free rate of the 10 decile portfolios and the hedge portfolio at month t , α is the monthly abnormal return, $RMRF_t$ is the excess return of value-weighted market portfolio, and SMB_t , HML_t , and $Momentum_t$ are the month t returns on the zero-investment factor-mimicking portfolios that capture size, book-to-market, and momentum effects, respectively.

Sample period: 1970 – 1980

Portfolio	Value-weighted						Equal-weighted					
	α	$RMRF$	SMB	HML	$Momentum$	$Adj R^2$	α	$RMRF$	SMB	HML	$Momentum$	$Adj R^2$
1	0.33	1.07	0.24	0.06	-0.03	0.96	0.44	0.94	1.17	0.37	-0.08	0.96
2	2.98	43.1	6.72	1.54	-1.1	0.73	4.24	40.81	35.17	9.98	-2.91	0.82
3	0.38	0.99	0.09	-0.17	0.08	0.88	0.64	0.93	1.34	0.22	-0.05	0.82
4	1.26	14.72	0.9	-1.58	0.97	0.88	4.6	30.02	29.91	4.51	-1.24	0.69
5	0.35	1.17	0.15	0.26	-0.06	0.83	0.5	1.02	1.22	0.48	-0.14	0.81
6	1.64	24.12	2.14	3.41	-1.11	0.83	2.6	23.6	19.57	6.91	-2.66	0.81
7	0.68	0.79	0.08	-0.03	-0.02	0.86	0.98	1.06	1.35	0.32	-0.01	0.75
8	3.8	19.85	1.46	-0.44	-0.47	0.88	5.61	27.36	24.11	5.21	-0.14	0.84
9	0.62	1.04	-0.19	-0.48	0.03	0.87	0.52	1.02	1.13	0.25	0	0.85
10	2.95	22.06	-2.82	-6.39	0.6	0.88	3.35	29.2	22.46	4.46	0.1	0.84
H-L	0.5	1.01	-0.04	-0.12	0	0.88	0.73	1.02	0.85	0.3	-0.06	0.84
	2.78	25.2	-0.62	-1.8	-0.03	0.87	5.92	37.24	21.51	6.93	-1.91	0.85
	0.38	1.07	0	0.19	0.03	0.87	0.44	0.95	1.06	0.36	-0.09	0.85
	1.95	24.81	0.04	2.8	0.58	0.87	2.65	25.52	19.64	6	-1.95	0.87
	0.59	0.96	0.22	0.2	-0.15	0.87	0.56	0.99	1.07	0.35	-0.02	0.87
	3.01	21.74	3.4	2.83	-2.79	0.86	4.49	35.69	26.73	7.89	-0.52	0.82
	0.65	0.85	0.11	-0.08	-0.18	0.86	0.51	0.86	1.04	0.36	-0.14	0.82
	3.53	20.55	1.79	-1.21	-3.53	0.69	3.2	23.98	20.09	6.25	-3.29	0.52
	1.1	0.97	-0.34	-0.02	0.16	0.36	1.61	1.09	0.62	0.03	0.32	0.52
	3.94	15.5	-3.82	-0.22	2.07	0.36	4.04	12.27	4.82	0.23	2.96	0.3
	0.77	-0.1	-0.58	-0.08	0.19	0.36	1.17	0.15	-0.55	-0.34	0.4	0.3
	2.32	-1.4	-5.47	-0.7	2.12	0.36	2.85	1.68	-4.13	-2.28	3.59	0.3

Table 4: Performance-Attribution Regressions for Portfolios with Different Price Rigidities

This table reports the Fama-French-Carhart four-factor regression: $R_t = \alpha + \beta_1 * RMRF_t + \beta_2 * SMB_t + \beta_3 * HML_t + \beta_4 * Momentum_t + \epsilon_t$, where R_t is the excess return relative to risk-free rate of the 10 decile portfolios and the hedge portfolio at month t , α is the monthly abnormal return, $RMRF_t$ is the excess return of value-weighted market portfolio, and SMB_t , HML_t , and $Momentum_t$ are the month t returns on the zero-investment factor-mimicking portfolios that capture size, book-to-market, and momentum effects, respectively.

Sample period: 1980 – 2006

Portfolio	Value-weighted						Equal-weighted					
	α	$RMRF$	SMB	HML	$Momentum$	$Adj R^2$	α	$RMRF$	SMB	HML	$Momentum$	$Adj R^2$
1	0.36	1.06	0.21	-0.1	-0.03	0.98	0.42	0.92	0.93	0.34	-0.07	0.91
2	4.01	45.26	5.62	-2.37	-0.82	0.97	3.42	28.37	18.14	5.82	-1.51	0.9
3	0.54	0.95	0.1	-0.45	-0.25	0.97	0.76	0.99	1.08	0.03	-0.07	0.9
4	2.85	19.17	1.22	-5.03	-3.6	0.94	4.85	24.11	16.63	0.37	-1.29	0.73
5	0.37	1.19	0.55	0.05	-0.11	0.94	0.24	1.01	1.19	0.52	-0.03	0.73
6	1.15	14.31	4.16	0.35	-0.93	0.96	0.85	13.71	10.2	3.88	-0.3	0.9
7	0.37	0.92	-0.23	0.19	0.15	0.96	0.59	1.02	0.93	0.34	-0.01	0.9
8	2.32	22.02	-3.51	2.56	2.52	0.96	4.07	26.83	15.36	4.98	-0.19	0.81
9	1.11	0.99	0.29	-0.35	0.24	0.96	1.23	1.11	1.38	0.08	-0.03	0.81
10	4.39	15.01	2.81	-2.89	2.63	0.97	4.62	15.96	12.44	0.63	-0.29	0.84
H-L	0.73	1.07	-0.17	-0.78	-0.14	0.97	0.72	1.09	1.23	-0.19	-0.19	0.84
	3.53	19.79	-2.03	-8	-1.89	0.95	3.03	17.6	12.49	-1.67	-2.21	0.87
	0.49	1.06	-0.01	-0.19	-0.03	0.95	0.51	0.96	1.03	0.15	-0.02	0.87
	2.86	23.93	-0.13	-2.37	-0.48	0.97	2.97	21.34	14.33	1.83	-0.35	0.88
	0.55	1.13	0.06	0.14	-0.05	0.97	0.39	1.06	0.95	0.48	-0.04	0.88
	3.48	27.55	0.93	1.82	-0.95	0.94	2.43	25.26	14.19	6.25	-0.74	0.89
	0.52	0.99	-0.05	-0.24	0.1	0.94	0.59	0.98	0.93	0.21	-0.03	0.89
	2.81	20.38	-0.63	-2.77	1.51	0.7	4	25.14	15.05	2.95	-0.56	0.41
	0.4	0.87	-0.27	0.47	0.05	0.7	0.45	0.85	0.51	0.6	0.14	0.41
	1.4	11.8	-2.29	3.5	0.44	0.29	1.19	8.63	3.24	3.36	0.99	0.1
	0.04	-0.18	-0.48	0.57	0.07	0.29	0.02	-0.07	-0.43	0.26	0.2	0.1
	0.12	-2.24	-3.65	3.81	0.62	0.29	0.07	-0.7	-2.77	1.47	1.52	0.1

Figure 1: Market Price of Risk

The figure plots the market price of risk as a function of the response of monetary policy to inflation, using the calibrated parameters in Table 1.

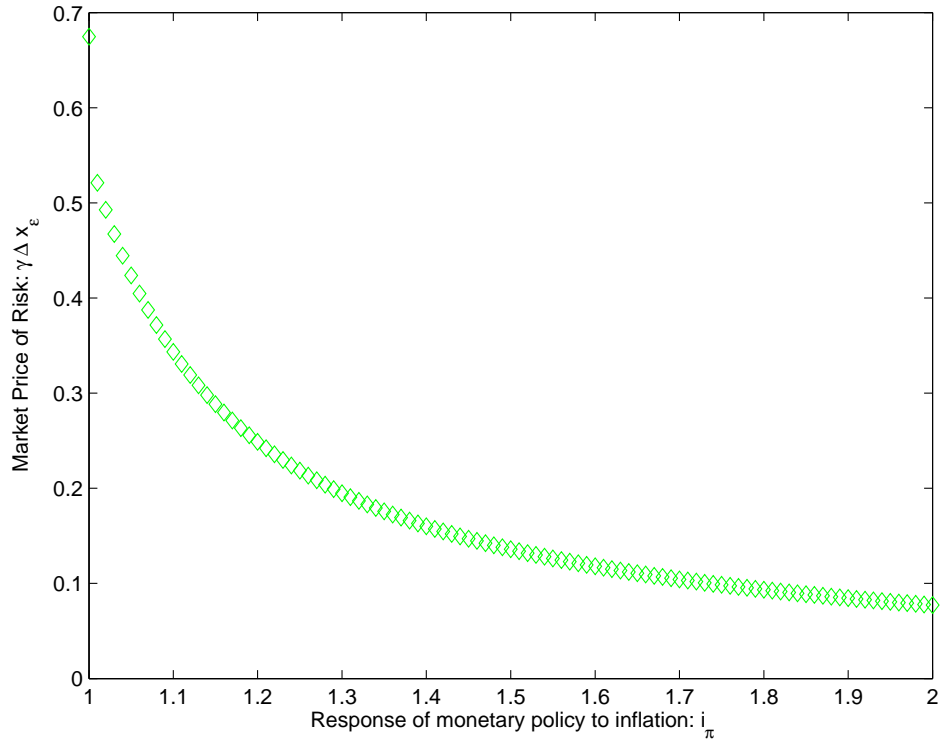


Figure 2: Expected Return Differences between Low and High Rigidity Industries (Policy Shocks)

The figure plots the differences in expected returns for claims on real consumption, real labor income and real profits between low and high rigidity industries as a function of different values for different model parameters, for an economy affected by monetary policy shocks.

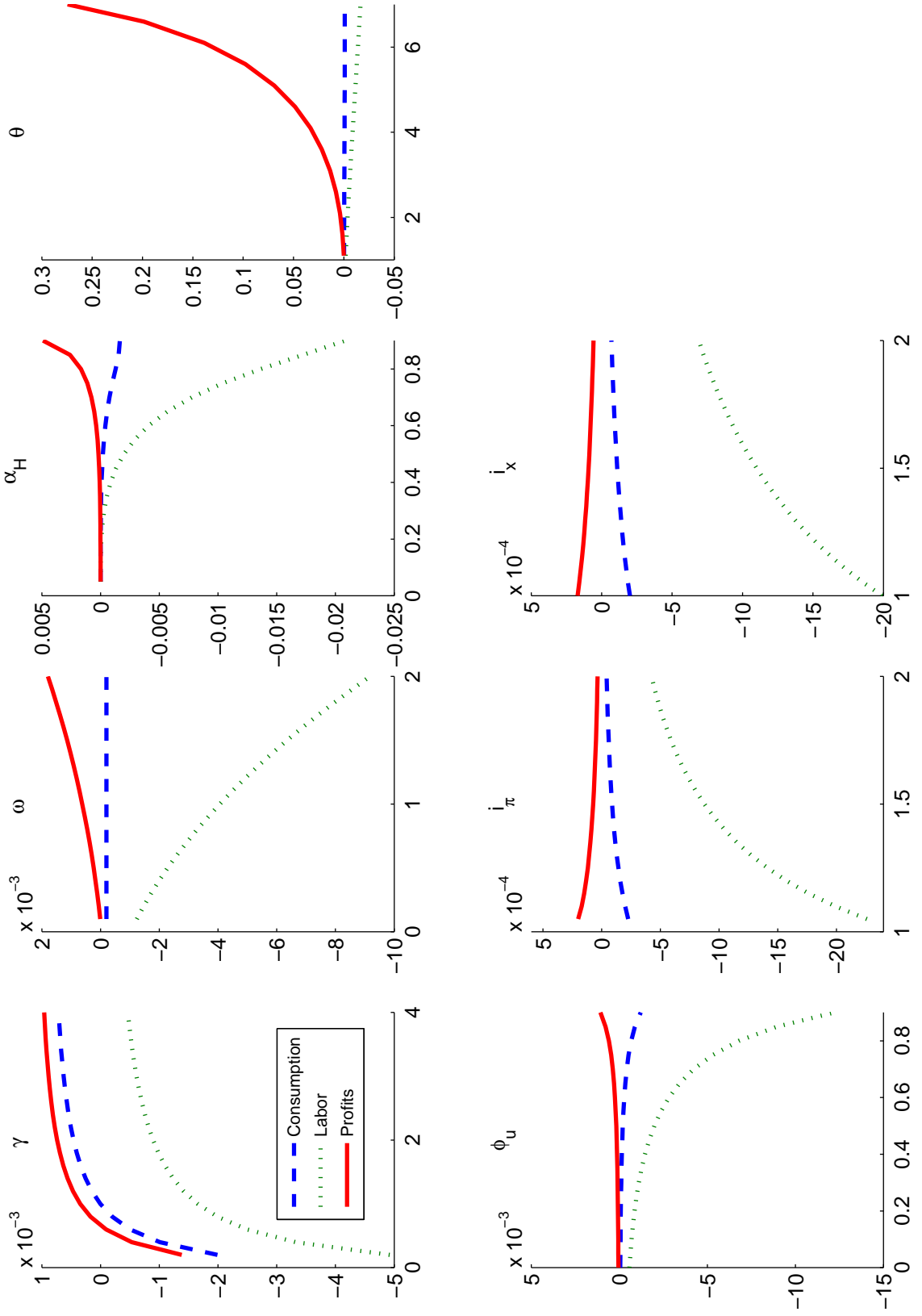


Figure 3: Impulse Responses to a Positive Policy Shock

The figure plots impulse responses for different macroeconomic variables, the one period real interest rate and the value of claims to real consumption, real labor income and real profits. “All”, “High” and “Low” refer to the aggregate economy, the industry with high price rigidity and the industry with low price rigidity, respectively.

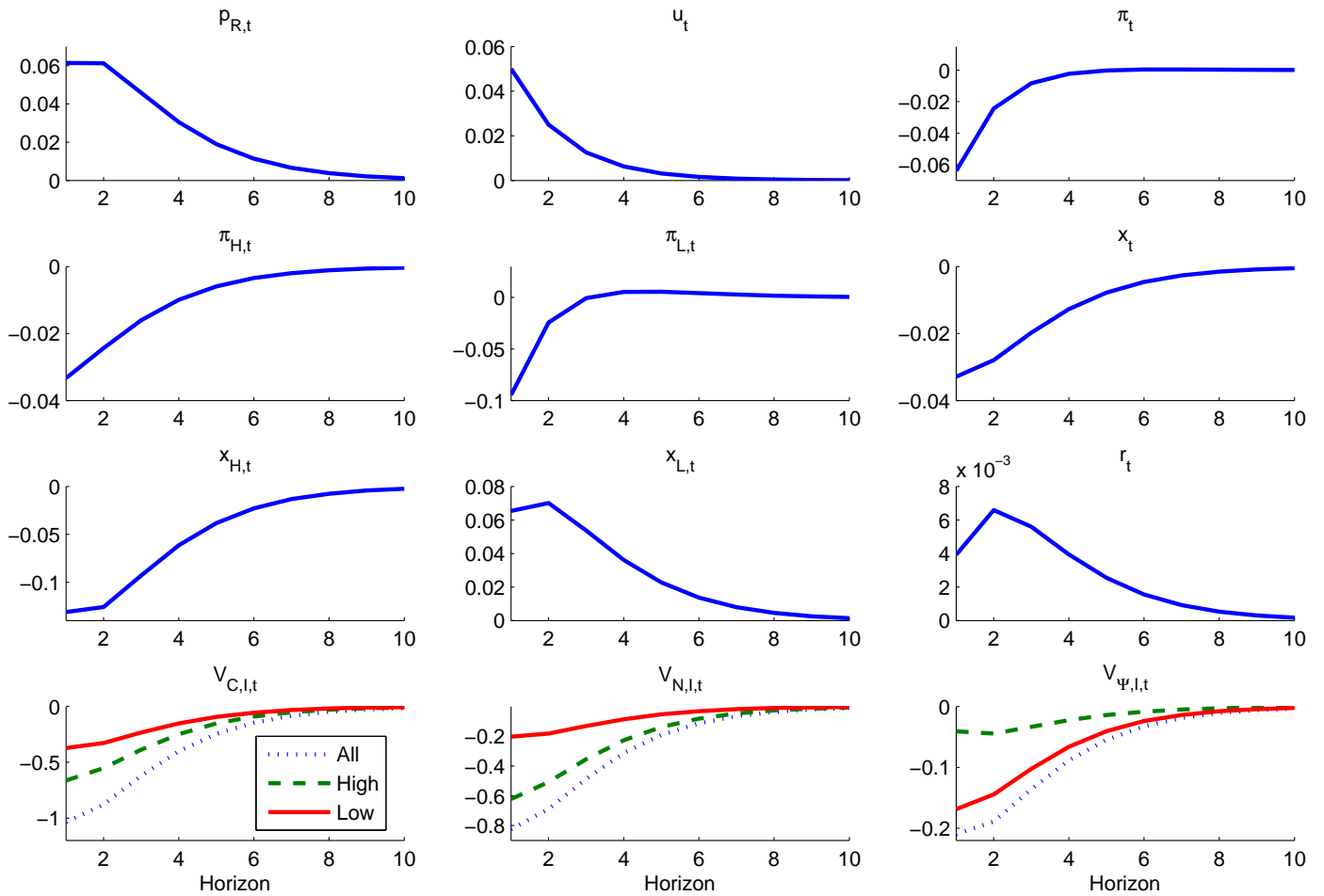
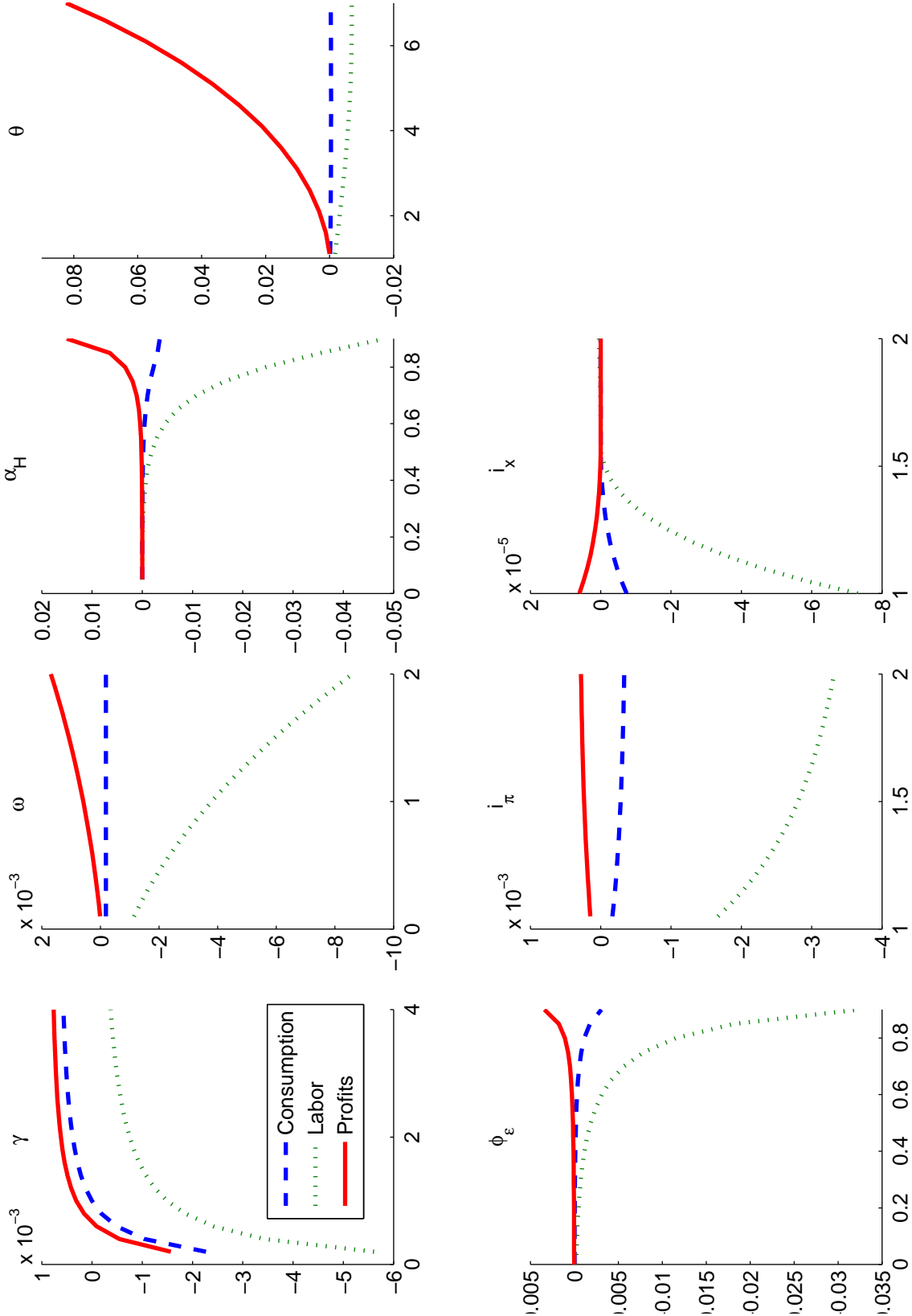


Figure 4: Expected Return Differences between Low and High Rigidity Industries (Supply Shocks)

The figure plots the differences in expected returns for claims on real consumption, real labor income and real profits between low and high rigidity industries as a function of different values for different model parameters, for an economy affected by supply shocks.



References

- Bils, Mark and Peter J. Klenow. 2004. "Some Evidence on the Importance of Sticky Prices." *Journal of Political Economy* 112(5):947–985.
- Blinder, Alan S., Elie R.D. Canetti, David E. Lebow and Jeremy B. Rudd. 1998. *Asking About Prices: A New Approach to Understanding Price Stickiness*. First ed. New York: Russell Sage Foundation.
- Calvo, Guillermo. 1983. "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics* 12:383–398.
- Clarida, Richard, Jordi Galí and Mark Gertler. 2000. "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory." *The Quarterly Journal of Economics* pp. 147–180.
- Dai, Qiang and Kenneth J. Singleton. 2000. "Specification Analysis of Affine Term Structure Models." *The Journal of Finance* 55(5):1943–1978.
- Duffie, Darrell and Rui Kan. 1996. "A Yield-Factor Model of Interest Rates." *Mathematical Finance* (6):379–406.
- Lettau, Martin and Jessica Wachter. 2007. "Why is Long-horizon Equity Less Risky? A Duration-based Explanation of the Value Premium." *Journal of Finance* 62(1):55–92.
- Nakamura, Emi and Jón Steinsson. 2007. "Five Facts About Prices: A Reevaluation of Menu Cost Models." Working paper, Columbia University.

Appendix

A Profit Maximization under Price Rigidities

Consider the Dixit-Stiglitz aggregate (3) as a production function, and a competitive “producer” of the industry good facing the problem

$$\max_{\{C_{I,t}(i)\}} P_{I,t} C_{I,t} - \int_0^1 P_{I,t}(i) C_{I,t}(i) di$$

subject to (3). Solving the problem, we find the demand function

$$P_{I,t} = P_{I,t}(i) \left(\frac{C_{I,t}(i)}{C_{I,t}} \right)^{-1/\theta} \quad (24)$$

Since the production is competitive, profits are zero, meaning that

$$P_{I,t} C_{I,t} = \int_0^1 P_{I,t}(i) C_{I,t}(i) di = \int_0^1 P_t(i) C_t \left(\frac{P_t(i)}{P_t} \right)^{-\theta} di.$$

Solving for $P_{I,t}$, it follows that

$$P_{I,t} = \left[\int_0^1 P_{I,t}(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}} \quad (25)$$

Similarly, the aggregate price index can be written in terms of the price index for the two sectors as

$$P_t = [\varphi P_{H,t}^{1-\theta} + (1-\varphi) P_{L,t}^{1-\theta}]^{1/(1-\theta)},$$

the demand function for each differentiated good in sector S is given by

$$C_{I,t}(i) = \left(\frac{P_{I,t}(i)}{P_{I,t}} \right)^{-\theta} C_{I,t}$$

and the demand function for each sector good is

$$C_{I,t} = \varphi_I \left(\frac{P_{I,t}}{P_t} \right)^{-\theta} C_t, \quad (26)$$

where $\varphi_H = \varphi$ and $\varphi_L = 1 - \varphi$. Notice that these relations imply that consumption in both sectors is related by

$$C_{H,t} = \frac{\varphi}{1 - \varphi} \left(\frac{P_{H,t}}{P_{L,t}} \right)^{-\theta} C_{L,t}.$$

Therefore, when prices are flexible, prices of the sector goods are the same and consumptions in the two sectors are proportional.

The profit maximization problem (6) is solved relying on a linear approximation around a “steady state”. The steady state is defined as the solution of the profit maximization problem in an economy with perfectly flexible prices. It is convenient to analyze this problem for the hypothetical flexible economy first and then show the solution for the actual economy.

$$\max_{P_{I,t}(i)} P_{I,t}(i) Y_{I,t}^f(i) - w_{I,t}(i) h_{I,t}(i)$$

subject to (24) and (6). The solution to this problem implies

$$\frac{P_{I,t}(i)}{P_t} = \mu s_{I,t}(i)$$

where the markup $\mu = \frac{\theta}{\theta - 1}$ over the real marginal cost $s_{I,t}(i) \equiv \frac{1}{P_t} \frac{\partial(w_{I,t}(i) h_{I,t}(i))}{\partial Y_{I,t}(i)}$ is the result of monopolistic power. By using the production function (6) and the marginal rate of substitution (5) we can write the real marginal production cost as

$$s_{I,t}(i) = \frac{1}{Y_{I,t}(i)} \left(\frac{Y_{I,t}(i)}{A} \right)^{1+\omega} Y_t^\gamma. \quad (27)$$

Since prices are flexible and firms are identical, $P_t(i) = P_t$, $Y_t(i) = Y_t$. As a result, production in the flexible-price economy can be written as

$$y_t^f = \log Y_t^f = -\frac{1}{\omega + \gamma} [(1 + \omega) \log A - \log \mu]. \quad (28)$$

The flexible-price output provides us with a “point” to approximate the solution to the profit maximization problem in the sticky price economy.

Denote $M_{t,T}^\$ = \beta^{T-t} \Lambda_T$, $S_{I,t} = P_t s_{I,t}$. Consider the derivative

$$\frac{\partial \Psi_{I,T|t}(i)}{\partial P_{I,t}(i)} = Y_{I,T|t}(i) \frac{1 - \theta}{P_{I,t}(i)} [P_{I,t}(i) - \mu_T S_{I,T|t}(i)].$$

Therefore, the first order condition to the profit maximization problem (6) is

$$\mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha_I \beta)^{T-t} \Lambda_T Y_{I,T|t}(i) P_{I,t}^*(i) \right] = \mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha_I \beta)^{T-t} \Lambda_T Y_{I,T|t}(i) \mu S_{I,T|t}(i) \right]. \quad (29)$$

Since all producers who change prices optimally at t face the same problem, $Y_{I,T|t}(i) = Y_{I,T|t}$, $P_{I,t}^*(i) = P_{I,t}^*$ and $S_{I,T|t}(i) = S_{I,T|t}$. Applying the Taylor expansion $a_t b_t = \bar{a} \bar{b} + \bar{b}(a_t - \bar{a}) + \bar{a}(b_t - \bar{b})$ to both sides of the equation around a steady-state with $\bar{P} = \mu \bar{S}$, we have for the left hand side of the equation

$$\begin{aligned} \mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha_I \beta)^{T-t} \Lambda_T Y_{I,T|t} P_{I,t}^* \right] &= \bar{\Lambda Y} \bar{P} \sum_{T=t}^{\infty} (\alpha_I \beta)^{T-t} + \bar{P} \mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha_I \beta)^{T-t} (\Lambda_T Y_{I,T|t} - \bar{\Lambda Y}) \right] \\ &+ \bar{\Lambda Y} (P_{I,t}^* - \bar{P}) \sum_{T=t}^{\infty} (\alpha_I \beta)^{T-t} \end{aligned}$$

and for the right hand side

$$\begin{aligned} \mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha_I \beta)^{T-t} \Lambda_T Y_{I,T|t} \mu_T S_{I,T|t}(i) \right] &= \mu \bar{\Lambda Y} \bar{S} \sum_{T=t}^{\infty} (\alpha_I \beta)^{T-t} + \mu \bar{S} \mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha_I \beta)^{T-t} (\Lambda_T Y_{I,T|t} - \bar{\Lambda Y}) \right] \\ &+ \mu \bar{\Lambda Y} \mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha_I \beta)^{T-t} (S_{I,T|t} - \bar{S}) \right]. \end{aligned}$$

Noting that the first and second terms in both sides of the equation are the same, equation (29) becomes

$$\frac{1}{(1 - \alpha_I \beta)} P_{I,t}^* = \mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha_I \beta)^{T-t} \mu S_{I,T|t} \right].$$

Since $S_{T|t} = s_{T|t} P_T$, replacing equation (27) in the equation above and re-arranging terms, we obtain

$$\frac{1}{(1 - \alpha_I \beta)} (P_{I,t}^*)^{1+\theta\omega} = \mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha_I \beta)^{T-t} \mu P_T^{1+\theta\omega} Y_T^{\omega+\gamma} A^{-(1+\omega)} \right].$$

Dividing by $\bar{P}^{1+\theta\omega}$, the equation can be written in terms of the output gap $x_t = y_t - y_t^f$ as

$$\frac{1}{(1 - \alpha_I\beta)} \left(\frac{P_{I,t}^*}{\bar{P}} \right)^{1+\theta\omega} = e^{(\omega+\gamma)x_t} \left(\frac{P_t}{\bar{P}} \right)^{1+\theta\omega} + \frac{\alpha_I\beta}{1 - \alpha_I\beta} \mathbb{E}_t \left[\left(\frac{P_{I,t+1}^*}{\bar{P}} \right)^{1+\theta\omega} \right].$$

Letting $p_{I,t}^* = \log \frac{P_{I,t}^*}{\bar{P}}$ and using the approximation $e^x \approx 1 + x$, we obtain

$$\frac{1}{(1 - \alpha_I\beta)} (1 + (1 + \theta\omega)p_{I,t}^*) = 1 + (\omega + \gamma)x_t + (1 + \theta\omega)p_t + \frac{\alpha_I\beta}{1 - \alpha_I\beta} \mathbb{E}_t [1 + (1 + \theta\omega)p_{I,t+1}^*] \quad (30)$$

that simplifies to

$$p_{I,t}^* = \frac{\omega + \gamma}{1 + \theta\omega} x_t + p_t + \frac{\alpha_I\beta}{1 - \alpha_I\beta} \mathbb{E}_t [p_{I,t+1}^* - p_{I,t}^*]. \quad (31)$$

A first order Taylor approximation of $P_{I,t} = \left[(1 - \alpha_I) (P_{I,t}^*)^{1-\theta} + \alpha_I P_{I,t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}}$ results in

$$p_{I,t} = (1 - \alpha_I)p_{I,t}^* + \alpha p_{I,t-1}.$$

It implies

$$p_{I,t}^* = \frac{\alpha_I}{1 - \alpha_I} \pi_{I,t} + p_{I,t} \quad \text{and} \quad p_{I,t+1}^* - p_{I,t}^* = \frac{1}{1 - \alpha_I} \pi_{I,t+1} - \frac{\alpha_I}{1 - \alpha_I} \pi_{I,t}.$$

Replacing these equations in equation (30) we obtain

$$\pi_{I,t} = \kappa_I x_t + \frac{\kappa_I}{\zeta} (p_t - p_{I,t}) + \beta \mathbb{E}_t [\pi_{I,t+1}],$$

where $\kappa_I \equiv \frac{(1 - \alpha_I\beta)(1 - \alpha_I)}{\alpha_I} \zeta$ and $\zeta \equiv \frac{\omega + \gamma}{1 + \theta\omega}$.

B Equilibrium - Policy Shock

$$\begin{aligned}
e^{-i_t} &= \mathbb{E}_t [\exp(\log \beta - \gamma(\Delta y^f + \Delta x_{t+1}) - \pi_{t+1})], \\
\pi_t &= \bar{\kappa}x_t + b_\varphi p_{R,t} + \beta \mathbb{E}[\pi_{t+1}], \\
b_R p_{R,t} &= \underline{\kappa}x_t + p_{R,t-1} + \beta \mathbb{E}_t[p_{R,t+1}], \\
i_t &= \bar{i} + \iota_\pi \pi_t + \iota_x x_t + u_t \\
\text{and } u_t &= \phi_u u_{t-1} + \sigma_u \varepsilon_{u,t}.
\end{aligned}$$

Where $b_\varphi = -\frac{\varphi(1-\varphi)}{\zeta}\underline{\kappa}$, $\bar{\kappa} = \varphi\kappa_H + (1-\varphi)\kappa_L$, $\underline{\kappa} = \kappa_H - \kappa_L$ and

$$b_R = 1 + \beta + \frac{1}{\zeta}[(1-\varphi)\kappa_H + \varphi\kappa_L].$$

Equation (14) can be written as

$$x_t = \frac{1}{\bar{\kappa}} [\pi_t - b_\varphi p_{R,t} - \beta \mathbb{E}_t[\pi_{t+1}]] \quad (32)$$

and its first difference as

$$\Delta x_{t+1} = \frac{1}{\bar{\kappa}} [\Delta \pi_{t+1} - b_\varphi \Delta p_{R,t+1} - \beta (\mathbb{E}_{t+1}[\pi_{t+2}] - \mathbb{E}_t[\pi_{t+1}])]. \quad (33)$$

Replacing (32) in (15) we obtain

$$b_R p_{R,t} - p_{R,t-1} = \mathbb{K} [\pi_t - b_\varphi p_{R,t} - \beta \mathbb{E}_t[\pi_{t+1}]] + \beta \mathbb{E}_t[p_{R,t+1}]. \quad (34)$$

where $\mathbb{K} = \frac{\underline{\kappa}}{\bar{\kappa}}$.

Guess solutions for inflation and the relative price of the form

$$\pi_t = \bar{\pi} + \pi_p p_{R,t-1} + \pi_u u_t \quad \text{and} \quad p_{R,t} = \bar{\rho} + \rho_p p_{R,t-1} + \rho_u u_t,$$

respectively. Replacing this solution in equation (34) and matching coefficients we obtain the

sub-system of equations

$$b_\pi \bar{\rho} = \mathbb{K}(1 - \beta)\bar{\pi} + \beta \bar{\rho}, \quad (35)$$

$$b_\pi \rho_p = 1 + \mathbb{K}\pi_p, \quad (36)$$

$$b_\pi \rho_u = \mathbb{K}(1 - \beta\phi_u)\pi_u + \beta\rho_u\phi_u, \quad (37)$$

where

$$b_\pi = b_R + b_\varphi \mathbb{K} + \beta \mathbb{K} \pi_p - \beta \rho_p.$$

To complete the system of equations, replace (33) and (16) in (13). The guessed solutions imply log-normal distributions for all variables and therefore we obtain

$$\begin{aligned} -\bar{l} - \iota_\pi \pi_t - \iota_x x_t - u_t &= \log \beta - \frac{\gamma}{\bar{K}} [(\pi_p - b_\varphi \rho_p - \beta \pi_p \rho_p)(\bar{\rho} + (\rho_p - 1)p_{R,t-1} + \rho_u u_t) \\ &\quad - (1 - \phi_u)(\pi_u - b_\varphi \rho_u - \beta \pi_p \rho_u - \beta \pi_u \phi_u) u_t] \\ &\quad - \bar{\pi} - \pi_p(\bar{\rho} + \rho_p p_{R,t-1} + \rho_u u_t) - \pi_u \phi_u u_t \\ &\quad + \frac{1}{2} \text{var}_t \left(\frac{\gamma}{\bar{K}} (\pi_u - b_\varphi \rho_u - \beta \pi_p \rho_u - \beta \pi_u \phi_u) u_{t+1} + \pi_u u_{t+1} \right). \end{aligned} \quad (38)$$

Matching coefficients we obtain the sub-system

$$\begin{aligned} -\bar{l} - \iota_\pi \bar{\pi} - \frac{\iota_x}{\bar{K}} [(1 - \beta)\bar{\pi} - (b_\varphi + \beta \pi_p)\bar{\rho}] &= \log \beta - \bar{\pi} - \frac{\gamma}{\bar{K}} (\pi_p - b_\varphi \rho_p - \beta \pi_p \rho_p) \bar{\rho} - \pi_p \bar{\rho} \\ &\quad + \frac{1}{2} \left(\frac{\gamma}{\bar{K}} (\pi_u - b_\varphi \rho_u - \beta \pi_p \rho_u - \beta \pi_u \phi_u) + \pi_u \right)^2 \phi_u^2 \bar{\rho} \end{aligned} \quad (39)$$

$$-\iota_\pi \pi_p - \frac{\iota_x}{\bar{K}} [\pi_p - (b_\varphi + \beta \pi_p)\rho_p] = \frac{\gamma}{\bar{K}} (\pi_p - b_\varphi \rho_p - \beta \pi_p \rho_p)(1 - \rho_p) - \pi_p \rho_p, \quad (40)$$

$$\begin{aligned} -\iota_\pi \pi_u - \frac{\iota_x}{\bar{K}} [(1 - \beta\phi_u)\pi_u - (b_\varphi + \beta \pi_p)\rho_u] - 1 &= -\frac{\gamma}{\bar{K}} (\pi_p - b_\varphi \rho_p - \beta \pi_p \rho_p) \rho_u \\ &\quad + \frac{\gamma}{\bar{K}} (1 - \phi_u)(\pi_u - b_\varphi \rho_u - \beta \pi_p \rho_u - \beta \pi_u \phi_u) \\ &\quad - \pi_p \rho_u - \pi_u \phi_u. \end{aligned} \quad (41)$$

The complete system is given by equations (35)-(37) and (39)-(41). This system allows us to obtain the equilibrium parameters $\{\bar{\pi}, \pi_p, \pi_u, \bar{\rho}, \rho_p, \rho_u\}$. Notice that equations (36) and (40) only depend on π_p and ρ_p . Therefore, we can use these two equations to solve for these two parameters.

After some algebra manipulations we obtain

$$\begin{aligned} & \gamma\beta^2\rho_p^4 - \beta[\bar{\kappa} + \gamma(1 + \beta + b_R) + \beta\iota_x]\rho_p^3 \\ & + [\gamma(\beta + b_R + \beta(1 + b_R)) + \bar{\kappa}b_R + b_\varphi\bar{\kappa} + \iota_\pi\bar{\kappa}\beta + \iota_x\beta(1 + b_R)]\rho_p^2 \\ & - [\bar{\kappa} + \iota_\pi(\bar{\kappa}b_R + b_\varphi\bar{\kappa} + \iota_x(b_R + \beta)) + \gamma(1 + b_R + \beta)]\rho_p + \iota_\pi\bar{\kappa} + \gamma + \iota_x = 0. \end{aligned}$$

The coefficient π_p can be obtained from

$$\pi_p = \frac{b_\varphi\rho_p[\gamma(1 - \rho_p) + \iota_x]}{[\gamma(1 - \rho_p) + \iota_x](1 - \beta\rho_p) + \bar{\kappa}(\iota_\pi - \rho_p)}.$$

Using equations (37) and (41) we find ρ_u and π_u . The sensitivity of inflation to the policy shock solves

$$\begin{aligned} \pi_u &= \left[\bar{\kappa}^2(\phi_u - \iota_\pi) - (\gamma(1 - \phi_u) + \iota_x)\bar{\kappa}(1 - \beta\phi_u) + \frac{1 - \beta\phi_u}{b_\varphi - \beta\phi_u}\bar{\kappa} \right. \\ & \times \left. (\pi_p(\gamma + \bar{\kappa}) + (b_\varphi + \beta\pi_p)(\gamma(1 - \rho_p - \phi_u) + \iota_x)) \right]^{-1} \bar{\kappa}^2 \end{aligned}$$

and the sensitivity of the relative price to policy shocks is

$$\rho_u = \frac{\mathbb{K}}{b_\pi - \beta\phi_u}(1 - \beta\phi_u)\pi_u.$$

From equations (35) and (39) we find $\bar{\rho}$ and $\bar{\pi}$. The constants are

$$\begin{aligned} \bar{\pi} &= \left[1 - \iota_\pi - \frac{\iota_x}{\bar{\kappa}}(1 - \beta) + \frac{\mathbb{K}}{b_\pi - \beta}(1 - \beta) \left(\frac{\gamma}{\bar{\kappa}}(\pi_p - b_\varphi\rho_p - \beta\pi_p\rho_p) + \pi_p + (b_\varphi + \beta\pi_p)\frac{\iota_x}{\bar{\kappa}} \right) \right]^{-1} \\ & \times \left[\bar{i} + \log \beta + \frac{1}{2} \left(\frac{\gamma}{\bar{\kappa}}(\pi_u - b_\varphi\rho_u - \beta\pi_p\rho_u - \beta\pi_u\phi_u) + \pi_u \right)^2 \sigma_u^2 \right] \end{aligned}$$

and

$$\bar{\rho} = \frac{\mathbb{K}}{b_\pi - \beta}(1 - \beta)\bar{\pi}.$$

C Equilibrium - Supply Shocks

The model with supply shocks is obtained replacing equations (14) and(16) with

$$\begin{aligned}\pi_t &= \bar{\kappa}x_t + b_\varphi p_{R,t} + \beta\mathbb{E}[\pi_{t+1}] + \epsilon_t, \\ b_R p_{R,t} &= \underline{\kappa}x_t + p_{R,t-1} + \beta\mathbb{E}_t[p_{R,t+1}] + b_\epsilon\epsilon_t \\ \text{and} \quad i_t &= \bar{i} + \iota_\pi\pi_t + \iota_x x_t,\end{aligned}$$

respectively, where $b_\epsilon = \frac{1-2\varphi}{\varphi(1-\varphi)}$ and the Phillips curve shock, ϵ_t follows the process

$$\epsilon_t = \phi_\epsilon\epsilon_{t-1} + \sigma_\epsilon\varepsilon_{\epsilon,t}.$$

Equation (21) can be written as

$$x_t = \frac{1}{\bar{\kappa}} [\pi_t - b_\varphi p_{R,t} - \beta\mathbb{E}_t[\pi_{t+1}] - \epsilon_t] \quad (42)$$

Guess solutions for inflation and the relative price of the form

$$\pi_t = \bar{\pi} + \pi_p p_{R,t-1} + \pi_\epsilon \epsilon_t \quad \text{and} \quad p_{R,t} = \bar{\rho} + \rho_p p_{R,t-1} + \rho_\epsilon \epsilon_t,$$

respectively. Replacing the output gap equation (42) in equation (22) and matching coefficients we obtain the sub-system of equations

$$b_\pi \bar{\rho} = \mathbb{K}(1 - \beta)\bar{\pi} + \beta\bar{\rho}, \quad (43)$$

$$b_\pi \rho_p = 1 + \mathbb{K}\pi_p, \quad (44)$$

$$b_\pi \rho_\epsilon = \mathbb{K}(1 - \beta\phi_\epsilon)\pi_\epsilon + \beta\rho_\epsilon\phi_\epsilon + b_\epsilon - \mathbb{K}, \quad (45)$$

To complete the system of equations, difference (42) and replace (23) in (13). The guessed solutions imply log-normal distributions for all variables and therefore we obtain

$$\begin{aligned}-\bar{i} - \iota_\pi\pi_t - \iota_x x_t &= \log \beta - \frac{\gamma}{\bar{\kappa}} [(\pi_p - b_\varphi \rho_p - \beta\pi_p \rho_p)(\bar{\rho} + (\rho_p - 1)p_{R,t-1} + \rho_\epsilon \epsilon_t) \\ &+ \frac{\gamma}{\bar{\kappa}} (1 - \phi_\epsilon)[(1 - \beta\phi_\epsilon)\pi_\epsilon - 1 - (b_\varphi + \beta\pi_p)\rho_\epsilon]\epsilon_t \\ &- \bar{\pi} - \pi_p(\bar{\rho} + \rho_p p_{R,t-1} + \rho_\epsilon \epsilon_t) - \pi_\epsilon \phi_\epsilon \epsilon_t \\ &+ \frac{1}{2} \text{var}_t \left(\frac{\gamma}{\bar{\kappa}} (\pi_\epsilon - b_\varphi \rho_\epsilon - \beta\pi_p \rho_\epsilon - \beta\pi_\epsilon \phi_\epsilon - 1)\epsilon_{t+1} + \pi_\epsilon \epsilon_{t+1} \right). \quad (46)\end{aligned}$$

Matching coefficients we obtain the sub-system

$$-\bar{i} - \iota_\pi \bar{\pi} - \frac{\iota_x}{\bar{\kappa}} [(1 - \beta)\bar{\pi} - (b_\varphi + \beta\pi_p)\bar{\rho}] = \log \beta - \bar{\pi} - \frac{\gamma}{\bar{\kappa}} (\pi_p - b_\varphi \rho_p - \beta\pi_p \rho_p) \bar{\rho} - \pi_p \bar{\rho} + \frac{1}{2} \left(\frac{\gamma}{\bar{\kappa}} (\pi_\epsilon - b_\varphi \rho_\epsilon - \beta\pi_p \rho_\epsilon - \beta\pi_\epsilon \phi_\epsilon - 1) + \pi_\epsilon \right)^2 \sigma_\epsilon^2 \quad (47)$$

$$-\iota_\pi \pi_p - \frac{\iota_x}{\bar{\kappa}} [\pi_p - (b_\varphi + \beta\pi_p)\rho_p] = \frac{\gamma}{\bar{\kappa}} (\pi_p - b_\varphi \rho_p - \beta\pi_p \rho_p) (1 - \rho_p) - \pi_p \rho_p, \quad (48)$$

$$\begin{aligned} -\iota_\pi \pi_\epsilon - \frac{\iota_x}{\bar{\kappa}} [(1 - \beta\phi_\epsilon)\pi_\epsilon - (b_\varphi + \beta\pi_p)\rho_\epsilon - 1] &= -\frac{\gamma}{\bar{\kappa}} (\pi_p - b_\varphi \rho_p - \beta\pi_p \rho_p) \rho_\epsilon \\ &+ \frac{\gamma}{\bar{\kappa}} (1 - \phi_\epsilon) (\pi_\epsilon - b_\varphi \rho_\epsilon - \beta\pi_p \rho_\epsilon - \beta\pi_\epsilon \phi_\epsilon - 1) \\ &- \pi_p \rho_\epsilon - \pi_\epsilon \phi_\epsilon. \end{aligned} \quad (49)$$

Notice that the set of equations (44)-(48) and (36)-(40) are the same, implying that ρ_p and p_i for the model with Phillips curve shocks are the same as in the model with policy shocks.

From equations (45) and (49) we find that

$$\begin{aligned} \pi_\epsilon &= \left[\bar{\kappa}(\phi_\epsilon - \iota_\pi) - (\gamma(1 - \phi_\epsilon) + \iota_x)(1 - \beta\phi_\epsilon) + \frac{1 - \beta\phi_\epsilon}{b_\pi - \beta\phi_\epsilon} \mathbb{K} \right. \\ &\times \left. (\pi_p(\gamma + \bar{\kappa}) + (b_\varphi + \beta\pi_p)(\gamma(1 - \rho_p - \phi_\epsilon) + \iota_x)) \right]^{-1} \\ &\times \left[-\frac{b_\epsilon - \mathbb{K}}{b_\pi - \beta\phi_\epsilon} (\pi_p(\gamma + \bar{\kappa}) + (b_\varphi + \beta\pi_p)(\gamma(1 - \rho_p - \phi_\epsilon) + \iota_x)) - \gamma(1 - \phi_\epsilon) - \iota_x \right] \end{aligned}$$

and

$$\rho_\epsilon = \frac{1}{b_\pi - \beta\phi_\epsilon} [\mathbb{K}(1 - \beta\phi_\epsilon)\pi_\epsilon + b_\epsilon - \mathbb{K}].$$

From equations (43) and (47) we find that

$$\begin{aligned} \bar{\pi} &= \left[1 - \iota_\pi - \frac{\iota_x}{\bar{\kappa}} (1 - \beta) + \frac{\mathbb{K}}{b_\pi - \beta} (1 - \beta) \left(\frac{\gamma}{\bar{\kappa}} (\pi_p - b_\varphi \rho_p - \beta\pi_p \rho_p) + \pi_p + (b_\varphi + \beta\pi_p) \frac{\iota_x}{\bar{\kappa}} \right) \right]^{-1} \\ &\times \left[\bar{i} + \log \beta + \frac{1}{2} \left(\frac{\gamma}{\bar{\kappa}} (\pi_\epsilon - b_\varphi \rho_\epsilon - \beta\pi_p \rho_\epsilon - \beta\pi_\epsilon \phi_\epsilon - 1) + \pi_\epsilon \right)^2 \sigma_\epsilon^2 \right] \end{aligned}$$

and

$$\bar{\rho} = \frac{\mathbb{K}}{b_\pi - \beta} (1 - \beta) \bar{\pi}.$$

D Inflation in Individual Industries

We can write the inflation within industry I as a function of the state variables:

$$\pi_{I,t} = \bar{\pi}_I + \pi_{I,p}p_{R,t-1} + \pi_{I,u}u_t. \quad (50)$$

We know that the first order Taylor expansion of the relative price relation is

$$p_t - p_{I,t} = \varphi_{-I}p_{R,t}$$

and the inflation in sector I is

$$\pi_{I,t} = \kappa_I x_t + \frac{\kappa_I b_I}{\zeta} p_{R,t} + \beta \mathbb{E}_t [\pi_{I,t+1}].$$

Combined with the equilibrium conditions in Section 3, we find the coefficients for industry inflations as

$$\begin{aligned} \bar{\pi}_I &= \frac{\kappa_I}{1-\beta} \left[\bar{x} - \frac{\varphi_{-I}\bar{\rho}}{\zeta(1-\beta\rho_p)} + \frac{\beta\bar{\rho}x_p}{1-\beta\rho_p} \right]; \\ \pi_{I,p} &= \frac{\kappa_I}{1-\beta\rho_p} \left[x_p - \frac{\varphi_{-I}\rho_p}{\zeta} \right], \\ \pi_{I,u} &= \frac{\kappa_I}{1-\beta\phi_u} \left[x_u - \frac{\varphi_{-I}\rho_u}{\zeta(1-\beta\rho_p)} + \frac{\beta\rho_u x_p}{1-\beta\rho_p} \right]. \end{aligned}$$

E Numerical Solutions

We solve equation (20) for the two industries on a set of grid points of state variables $(p_{R,t}, u_t)$ using value function iteration. The unconditional distributions of relative price $p_{R,t}$ and policy shock u_t are normal with

$$\begin{aligned} \mathbb{E}(u_t) &= 0 \\ \text{var}(u_t) &= \frac{\sigma_u^2}{1-\phi_u^2} \\ \mathbb{E}(p_{R,t}) &= \frac{\bar{\rho}}{1-\rho_p} \\ \text{var}(p_{R,t}) &= \frac{\rho_u^2(1+\rho_p\phi_u)}{(1-\rho_p^2)(1-\rho_p\phi_u)} \text{var}(u_t). \end{aligned}$$

We choose Np grid points in the range of $[-3\text{var}(p_{R,t})^{1/2}, 3\text{var}(p_{R,t})^{1/2}]$ and choose Nu grid points in the range of $[-3\text{var}(u_t)^{1/2}, 3\text{var}(u_t)^{1/2}]$. Let's name the grid points as $\{p_i\}_{i=1,\dots,Np}$ and $\{p_j\}_{j=1,\dots,Nu}$. We then calculate the real values of high and low rigidity industries at these grid points as follows.

1. Make an initial guess for the value of high rigidity industry, $V_H^0(p_i, u_j)$.
2. Given the equilibrium processes for the relative price and the policy shock, we know the possible values of next period state variables (p', u') with the corresponding probabilities. Therefore, we can calculate the right hand side of equation (20) and update the value function as follows:

$$V_H^1(p_i, u_j) = \frac{\varphi Y^f \exp(x(p_i, u_j))}{\varphi + (1 - \varphi) \exp[(\theta - 1)p_i]} + \mathbb{E}_t [M(p', u') V_H^0(p', u')].$$

3. Calculate the difference between V_H^0 and V_H^1 at every grid point. If the maximum of the differences is larger than a pre-decided criterion, then go back to step 2 to get the next iterated value V_H^2 using V_H^1 ; if not, we have just found the value for the high rigidity industry.
4. Repeat step 1-3 for the value of low rigidity industry V_L at the same set of grid points. The real value of the industry with low price rigidity as

$$V_L(p_i, u_j) = \frac{\varphi Y^f \exp(x(p_i, u_j))}{\varphi \exp[(1 - \theta)p_{R,t}] + (1 - \varphi)} + \mathbb{E}_t [M(p', u') V_L^0(p', u')].$$

F A General Affine Asset Pricing Framework

Affine asset pricing has been widely applied in the term structure literature. See for example Duffie and Kan (1996) or Dai and Singleton (2000). The framework has also been used recently by Lettau and Wachter (2007) for the valuation of stocks. This section describes the main features of the affine framework. It will be used in this paper to price bonds, claims on aggregate consumption and claims on labor income.

Consider the k -dimensional set of state variables \mathbf{s}_t following the autoregressive process

$$\mathbf{s}_{t+1} = \psi + \Phi \mathbf{s}_t + \Sigma^{1/2} \varepsilon_{t+1}, \quad (51)$$

where $\{\varepsilon_t\} \sim \text{IIDN}(0_{l \times 1}, \mathbb{I}_{l \times l})$ is the $l \times 1$ vector of uncertainty, Φ is a $k \times k$ matrix of autoregressive

parameters and ψ is a $k \times 1$ vector containing drift parameters. The $k \times l$ matrix $\Sigma^{1/2}$ implies the $k \times k$ conditional covariance matrix for the state variables $\Sigma = \Sigma^{1/2} (\Sigma^{1/2})^\top$.

Consider the zero-coupon instrument⁷ that pays the contingent value z_{t+n} at time $t+n$. The no arbitrage price of this claim at time t is given by

$$Q_t^{(n)} = \mathbb{E}_t[M_{t,t+n}Z_{t+n}],$$

where $M_{t,t+n} > 0$ is the pricing kernel that discount payoffs at time $t+n$ for n periods. By defining $q_{z,t}^{(n)} \equiv \log Q_t - \log Z_t$ and using the law of iterated expectations, we can write the normalized price $q_{z,t}^{(n)}$ recursively as

$$e^{q_{z,t}^{(n)}} = \mathbb{E}_t \left[M_{t,t+1} e^{\Delta z_{t+1} + q_{z,t+1}^{(n-1)}} \right],$$

where $\Delta z_{t+1} \equiv \log Z_{t+1} - \log Z_t$. We specifying processes for the pricing kernel and Δz given by

$$-\log M_{t,t+1} = \Gamma_0 + \Gamma_1^\top \mathbf{s}_t + \lambda^\top \Sigma^{1/2} \varepsilon_{t+1} \quad (52)$$

and

$$\Delta z_{t,t+1} = \Gamma_{z,0} + \Gamma_{z,1}^\top \mathbf{s}_t + \lambda_z^\top \Sigma^{1/2} \varepsilon_{t+1}. \quad (53)$$

The $k \times 1$ vector λ contains the prices of risk for the different sources of uncertainty. These specifications, a guess for the solution with the form

$$q_{z,t}^{(n)} = \mathcal{A}_n + \mathcal{B}_n^\top \mathbf{s}_t$$

and boundary conditions $\mathcal{A}_0 = 0$ and $\mathcal{B}_0 = \mathbf{0}$, allow us to find recursive representations for \mathcal{A}_n and \mathcal{B}_n given by

$$\begin{aligned} \mathcal{A}_n &= -\Gamma_0 + \Gamma_{z,0} + \mathcal{A}_{n-1} + \mathcal{B}_{n-1}^\top \psi + \frac{1}{2} (\lambda - \lambda_z - \mathcal{B}_{n-1})^\top \Sigma (\lambda - \lambda_z - \mathcal{B}_{n-1}), \\ \mathcal{B}_n &= -\Gamma_1 + \Gamma_{z,1} + \Phi^\top \mathcal{B}_{n-1}. \end{aligned}$$

Since any instrument can be written as a portfolio of zero coupon instruments, the affine framework allows us to obtain prices for any instrument. Consider the instrument which pays Z_{t+n} every

⁷We follow Lettau and Wachter (2007) and define a zero-coupon instrument as the claim with payoffs at a particular time $t+n$ and no payoffs before that.

period. The price of this instrument is

$$Q_t = \sum_{i=1}^{\infty} Q_t^{(n)}.$$

The one-period return of this instrument is

$$r_{Q,t+1} = \log \left(\frac{Q_{t+1} + Z_{t+1}}{Q_t} \right) = \log(1 + e^{q_{z,t+1}}) + \Delta z_{t+1} - q_{z,t}, \quad (54)$$

where

$$e^{q_{z,t}} = \frac{Q_t}{Z_t} = \sum_{n=1}^{\infty} e^{q_{z,t}^{(n)}}.$$

This framework will allow us to obtain prices and returns for real and nominal one-period bonds, stocks, claims on consumption and claims on labor income as the following section shows.

G Affine Pricing for State Contingent Claims

A stock for industry I is a claim to all future real profits in that industry, Ψ_I . Therefore, the price of the stock is

$$\begin{aligned} S_{I,t} &= \mathbb{E}_t \left[\sum_{s=1}^{\infty} M_{t,t+s} \Psi_{I,t+s} \right] \\ &= \mathbb{E}_t \left[\sum_{s=1}^{\infty} M_{t,t+s} \frac{P_{I,t+s}}{P_{t+s}} C_{I,t+s} \right] - \mathbb{E}_t \left[\sum_{s=1}^{\infty} M_{t,t+s} \int_0^1 \frac{w_{I,t+s}(i)}{P_{t+s}} N_{I,t+s}(i) di \right]. \end{aligned}$$

It implies that the stock price can be computed as the difference between the price of a claim to all future consumption and the price of a claim to all future labor income. We price these claims using the affine framework described in Section F in order to obtain stock returns.

Since we are interested in the analysis of the two industries, H and L , and the aggregate economy, denote the aggregate economy by $H + L$ and define

$$\varphi_{-I} = \begin{pmatrix} -(1 - \varphi), & \text{if } I = H, \\ \varphi, & \text{if } I = L, \\ 0 & \text{if } I = H + L. \end{pmatrix}$$

G.1 Claims on Future Consumption

The price-consumption ratio for zero-coupon equity in sector I can be written recursively as

$$G_{I,t}^{(n)} = \mathbb{E}_t \left[M_{t,t+1} \frac{C_{t+1}}{C_t} \left(\frac{P_{I,t+1}}{P_{t+1}} \right)^{1-\theta} \left(\frac{P_{I,t}}{P_t} \right)^{-(1-\theta)} G_{I,t+1}^{(n-1)} \right].$$

where consumption in the industry is replaced with aggregate consumption using the demand equation (26). Using the approximation for the relative price

$$p_{I,t} - p_t = \varphi_{-I} p_{R,t},$$

with $\varphi_{-H} = (1 - \varphi)$ and $\varphi_{-L} = -\varphi$, it follows that the process in equation (53) associated to this claim is

$$\Delta z_{t+1} = (1 - \theta) \varphi_{-I} \Delta p_{R,t+1} + \Delta x_{t+1}.$$

The result follows re-writing this process in terms of the state variables using equations (18) and (19).

G.2 Claims on Future Labor Income

Denote by $LI_{I,t}$ the real labor income at time t in industry I , given by

$$LI_{I,t} = \int_0^1 \frac{w_{I,t}(i)}{P_t} N_{I,t}(i) di.$$

Using equation (27), real labor income can be written as

$$LI_{I,t} = \frac{Y_t^\gamma}{A^{1+\omega}} \int_0^1 \left(\frac{Y_{I,t}(i)}{\varphi_I} \right)^{1+\omega} di.$$

Since flexible prices imply a real marginal cost given by $s_t^n = \mu^{-1}$, we find that the natural rate of output satisfies

$$\left(Y_t^f \right)^{\omega+\gamma} = \mu^{-1} A^{1+w}.$$

Substituting in the labor income equation and using the demand equation (26) we obtain

$$LI_{I,t} = \frac{1}{\mu} \frac{Y_t^{1+\omega+\gamma}}{\left(Y_t^f \right)^{\omega+\gamma}} \left(\frac{P_{I,t}}{P_t} \right)^{-\theta(1+\omega)} \int_0^1 \left(\frac{P_{I,t}(i)}{P_{I,t}} \right)^{-\theta(1+\omega)} di = LI_t^f X_t^{1+\omega+\gamma} \int_0^1 \left(\frac{P_{I,t}(i)}{P_{I,t}} \right)^{-\theta(1+\omega)} di.$$

where $LI_t^f = \frac{1}{\mu} Y_t^f$ is the labor income under flexible prices.

Decomposing the last term in the labor income equation we obtain

$$\begin{aligned} \int_0^1 \left(\frac{P_{I,t}(i)}{P_{I,t}} \right)^{-\theta(1+\omega)} di &= \int_{i \in (1-\alpha)} \left(\frac{P_{I,t}^*}{P_{I,t}} \right)^{-\theta(1+\omega)} di + \int_{i \in \alpha} \left(\frac{P_{I,t-1}(i)}{P_{I,t}} \right)^{-\theta(1+\omega)} di \\ &= (1-\alpha) e^{-\theta(1+\omega)(p_{I,t}^* - p_{I,t})} + \int_{i \in \alpha} e^{-\theta(1+\omega)(p_{I,t-1}(i) - p_{I,t})} di. \end{aligned}$$

A first order Taylor approximation results in

$$\int_0^1 \left(\frac{P_{I,t}(i)}{P_{I,t}} \right)^{-\theta(1+\omega)} di \approx (1-\alpha)[1 - \theta(1+\omega)(p_{I,t}^* - p_{I,t})] + \int_{i \in \alpha} [1 - \theta(1+\omega)(p_{I,t-1}(i) - p_{I,t})] di.$$

Replacing the approximations $p_{I,t-1} = \int_0^1 p_{I,t-1}(i) di$ and $p_{I,t} = (1-\alpha)p_{I,t}^* + \alpha p_{I,t-1}$ implies

$$\begin{aligned} \int_0^1 \left(\frac{P_{I,t}(i)}{P_{I,t}} \right)^{-\theta(1+\omega)} di &\approx 1 - (1-\alpha)\theta(1+\omega)(p_{I,t}^* - p_{I,t}) - \theta(1+\omega)(p_{I,t-1} - p_{I,t}) \\ &= 1 - \theta(1+\omega)\alpha\pi_{I,t} + \theta(1+\omega)\alpha\pi_{I,t} = 1 \end{aligned}$$

Therefore, a first order approximation to labor income is

$$LI_{I,t} = \frac{1}{\mu} Y_t^f \left(\frac{Y_t}{Y_t^f} \right)^{1+\omega+\gamma} \left(\frac{P_{I,t}}{P_t} \right)^{-\theta(1+\omega)} = e^{-\log \mu + y_t^f + (1+\omega+\gamma)x_t - \theta(1+\omega)\varphi - \log P_{R,t}}.$$

This representation for labor income can be replaced in the equation for the price-labor income ratio of a claim on the stream of future real labor incomes, given by

$$D_{I,t} = \sum_{n=1}^{\infty} D_t^{(n)} = \mathbb{E}_t \left[M_{t,t+1} \frac{LI_{I,t+1}}{LI_{I,t}} D_{I,t+1}^{(n-1)} \right].$$

Comparing this equation to the affine pricing equation in section F, Δz_{t+1} corresponds to the one-period changes in log labor income. Given the representation of labor income in terms of the output gap and the relative price we find that

$$\Delta z_{t+1} = \log \frac{LI_{I,t+1}}{LI_{I,t}} = (1+\omega+\gamma)\Delta x_{t+1} - \theta(1+\omega)\Delta p_{R,t+1}.$$

Writing Δx_{t+1} and $\Delta p_{R,t+1}$ in terms of the state variables, the result follows.