Leaders, Followers, and Risk Dynamics in Industry Equilibrium*

Murray Carlson, Engelbert J. Dockner, Adlai Fisher, and Ron Giammarino

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Abstract

We study own and rival risk in a dynamic duopoly with a homogeneous output good, stochastic industry demand, and real options to expand or contract capacity. We focus on asymmetric cost functions and sequential option exercise. In general, a competitor's options to adjust capacity reduce own-firm risk. Intuitively, improvements in the product market bring the offsetting bad news that rival expansion is nearer. Similarly, negative demand shocks are counterbalanced by an increased likelihood of competitor contraction in the near future. The magnitude of these hedging effects increases as the rival moves closer to its expansion or contraction boundaries. As a consequence, own firm and industry risk move together when contraction options are exercised and oppositely during expansions. Thus, the commonly recommended practice of using competitor or industry betas to proxy for own-firm risk should work well in certain environments, but not in others, providing testable new empirical predictions.

*Carlson, Fisher, and Giammarino: Sauder School of Business, University of British Columbia. Dockner: Vienna University of Economics and Business. We received helpful comments from Antonio Mello, Kristian Miltersen, and seminar participants at the Copenhagen School of Business, Duke University, HEC Lausanne, Mannheim University, Texas A&M, Washington University, the University of Amsterdam, the University of Bern, the University of Graz, the University of Texas at Dallas, the 2006 meetings of the American Finance Association, the 2006 UBC PH&N Summer Conference, the 2007 Meetings of the Northern Finance Association, the 2008 Corporate Finance Conference at Wilfrid Laurier University, the 2008 European Winter Finance Summit, and the 2008 North American Summer Meeting of the Econometric Society. The financial support of the Social Sciences and Humanities Research Council of Canada is gratefully acknowledged.

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JEL Classification: C23, C35.

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1 Introduction

A corporation's real options to expand output, contract, or otherwise alter production can impact its asset risk dynamics, as observed by Berk, Green, and Naik (1998), Gomes, Kogan, and Zhang (2004), and subsequent authors.[1] In practical settings, financial analysts often seek to estimate the risk of a product or corporation by using not only the historical risk of the firm, but also its industry rivals.[2] Hence, financial theory may help to shed light on practice by explaining both how competitor real options impact own-firm risk as industry conditions change, and conversely how own-firm real options affect rival risk.

In this paper, we study own and rival risk in a dynamic duopoly with a homogeneous output good, stochastic industry demand, and real options to expand or contract capacity. We focus on cases where the two firms' cost functions are sufficiently asymmetric that a natural leader and follower arise in equilibrium, so that both expansion and contraction option exercise are sequential. Emphasizing sequential exercise allows us to demonstrate how the risk dynamics of a firm and its rival may alternately move together or apart over time, depending on industry conditions and the corresponding changing weights of own and rival growth and contraction options in firm valuation equations.

We find that for both expansion and contraction, rival real options reduce own-firm risk. Consider first that the rival possesses a growth option. In this case, any good news about an improving product market will be partially offset by the implication that competitor expansion is nearer. Conversely, bad news will be counterbalanced by a decreased likelihood of rival output growth over any fixed time horizon. Hence, all else equal rival growth options reduce own firm risk. Similarly, when the rival possesses a contraction option, industry demand shocks are partially offset by inversely related movements in the likelihood of near-term rival asset sales, again reducing own firm risk. The magnitudes of the hedging effects created by rival real options depend on the industry quantity implications of the options if exer-

[1]See, for example, Aguerrevere (2008), Carlson, Fisher, and Giammarino (2004, 2006), Cooper (2006), Kogan (2004), Gomes, Kogan, and Zhang, (2004), Zhang (2005), and Novy-Marx (2008). This literature is discussed in more detail below.

[2]The use of industry competitors to proxy for own-firm risk is discussed in Brealey and Myers (2001), and Ross, Westerfield, and Jaffe (1996), for example.

cised, and increase or decrease over time as industry conditions move closer to or further from the option exercise boundaries.

To develop intuition in the simplest case possible, we first consider an industry where one firm is a "strategic dummy" with permanently fixed output, while the second firm has a single option to irreversibly expand or contract its quantity supplied to their common product market. At each instant, prices are determined by aggregate industry output and both firms receive a flow of profits. The firm with expansion and contraction options has upper and lower bounds for expansion and contraction, and risk dynamics similar to those shown in prior literature.[3] Although the strategic dummy has no real options of its own, its valuation equations and risk nonetheless reflect the dynamic output policies of its rival. In particular, the risk of the strategic dummy decreases as its rival moves towards either its expansion or contraction boundary, and immediately jumps up to a constant when the rival exercises its option.

In the more general case where both firms may expand or contract, the own-firm and rival valuation equations and betas can possess up to four real options components. Whether on an expansion or a contraction path, the leader's real option tends to be more important for both own and rival valuation equations and risk, because the follower option is further out-of-the money. On an expansion path, the dynamics of leader and follower risk follow an interesting pattern. As the leader moves closer to exercise, her own risk increases due to growth option leverage, while the follower risk decreases due to the rival hedging effect. Immediately at the instant of leader growth option exercise, the risks of the two firms jump oppositely by sufficient magnitudes such that the follower risk exceeds leader risk. Thus, on a path of increasing industry demand own- and rival-firm risk tend to move in opposite directions. By contrast, the own-firm and rival-firm effects of contraction options have the same sign, and in an environment of decreasing industry demand the leader and follower risks tend to move together. These theoretical results suggest that the commonly recommended practice of using competitor or industry betas to proxy for own-firm risk should work well in certain environments, but not in others, providing testable new empirical predictions.

Our paper builds on several areas of the literature. Berk, Green, and

[3]See, for example, Carlson, Fisher, and Giammarino (2004) and Cooper (2006).

Naik (1998) and Gomes, Kogan, and Zhang (2004) model real options by their cash flows and discount rates, abstracting from explicit consideration of the product market. Subsequent literature considers risk dynamics due to real options in a variety of homogeneous goods market structures. Carlson, Fisher, and Giammarino (2004, 2006) and Cooper (2006) study cross-sections of monopolists with varying options to expand, contract, enter, or exit. Kogan (2004) analyzes risk and return in a perfectly competitive industry with symmetric firms and investment irreversibility, while Zhang (2005) and Novy-Marx (2008) allow cross-sectional firm differences within perfectly competitive industries. Aguerevere (2008) builds on Aguerevere (2003) and Grenadier (2002) to analyze risk and return in a symmetric simultaneous-move oligopoly.[4]

Our work differs from this prior literature by permitting asymmetries in an oligopoly setting, which leads to sequential exercise leader-follower equilibria.[5] In this environment, both own firm and rival options are important to valuations and risk, but the relative importance of own- and rival- options shifts through time as industry conditions change, highlighting a new channel through which firms within the same industry may differ in their risk exposures.

Section 2 describes the general model. In Section 3, we analyze the simplest case where one firm is a strategic dummy, and show the risk-reducing effects of rival growth options. Section 4 presents the leader-follower equilibrium where firms with asymmetric costs may both expand or contract. Section 5 concludes.

[4]Other related work includes Hackbarth and Morellec (2008) who study risk dynamics in a merger setting; Carlson, Fisher, and Giammarino (2008) and Kuehn (2008), who discuss the impact of investment commitment on risk; Gala (2008), and Gourio (2008), who extend the literature on the cross-section of returns in general equilibrium; and Pastor and Veronesi (2008) who discuss the impact of technological innovation on asset price dynamics.

[5]Boyer, Lassere, Mariotti and Moreaux (2004) and Pawlina and Kort (2002) discuss a broader set of sequential and simultaneous-move equilibria in asymmetric oligopoly settings, but do not discuss risk dynamics. When asymmetries are small enough, preemptive equilibria may exist. See also the discussion in Back and Paulson (2008).

2 The Asymmetric Duopoly Model

We present a model in which two strategically interacting firms invest in productive capacities or sell surplus assets and compete in output levels.

2.1 Industry Demand, Production Technologies, and Capital Accumulation

We consider an industry in which two firms produce a homogenous product. Denote the industry output rate at instant t by $Q_t = Q_t^1 + Q_t^2$, where Q_t^1 and Q_t^2 are the output levels of firm one and two, respectively. The industry output price is determined by the iso-elastic inverse demand curve

$$P_t = X_t Q_t^{\gamma-1}, \tag{1}$$

where $0 < \gamma < 1$, and X_t is an exogenous state variable that represents the level of industry-wide demand. The dynamics of X_t are specified by

$$dX_t = gX_t dt + \sigma X_t dW_t, \tag{2}$$

where dW_t is the increment of a Wiener process, g is the constant drift, and σ^2 the constant variance.

Firm i produces output at time t using installed capital K_t^i where $i \in \{1, 2\}$. Any capital level K^i is associated with a maximum output level $Q^i\left(K^i\right)$, which implies that $Q_t^i \leq Q^i\left(K_t^i\right)$. For simplicity, capital levels take one of three discrete values: $K_t^i \in \{\kappa_0, \kappa_1, \kappa_2\}$, where $\kappa_0 < \kappa_1 < \kappa_2$. Costs of production for firm i at date t are given by the increasing function $F_t^i = f\left(K_t^i\right)$. This cost structure emphasizes the impact of operating leverage, since total expenditures depend only on the installed capital level K^i (as with, for example, overhead). Given the three possible capital levels, there are also three possible levels of fixed operating costs: $F_t^i \in \{f_0, f_1, f_2\}$, where $f_0 < f_1 < f_2$.

To move from one capital state to another, the firm may incur costs or generate revenues, either from buying or selling the productive asset, or from pure adjustment costs. To capture this idea in a general way, we specify for each firm a matrix of discrete transition costs:

$$\Lambda^i \equiv \begin{bmatrix} 0 & \lambda_{01}^i & \lambda_{02}^i \\ \lambda_{10}^i & 0 & \lambda_{12}^i \\ \lambda_{20}^i & \lambda_{21}^i & 0 \end{bmatrix}.$$

The instantaneously incurred lump-sum cost for firm i to move from capital level κ_m to κ_n is given by λ^i_{mn}. The only source of heterogeneity between firms in our model is the difference between transition costs, i.e., $\Lambda^1 \neq \Lambda^2$. We assume as an initial condition that at date zero, each firm is endowed with $K^i_0 = \kappa_1$ units of capital.

We finally define indicator variables D^{imn}_t that take the value one at the instant when firm i switches from capital level κ_m to κ_n, and zero elsewhere.[6] For convenience, we denote by D^i_t the matrix of investment decisions D^{imn}_t.

2.2 Output, Investment Strategies, and Equilibrium

The economy described above is a dynamic game between firms 1 and 2. At each instant, the managers of the two firms choose output rates Q^i_t and make investment decisions D^i_t knowing the complete history of the game denoted by $\Phi_t = \left(\left[Q^1_s, Q^2_s, K^1_s, K^2_s\right]_{s<t}, [X_s]_{s\leq t} \right)$, which is common to both managers.

We define the payoff to firm i as the present value of the expected discounted future cash flows. The cash flows at time t derive either from revenues in excess of fixed costs $\pi^i_t \equiv P_t Q^i_t - F^i_t$ or from lumpy investment costs related to the decision D^i_t. We assume the absence of agency conflicts, so that manager i maximizes the value function

$$V^i_t \equiv E_t \int_t^\infty e^{-r(s-t)} \frac{M_{t+s}}{M_t} \left[\pi^i_{t+s} ds + \mathbf{1}' \left(D^i_{t+s} * \Lambda^i \right) \mathbf{1} \right], \tag{3}$$

where $M_0 = 1$ and $dM_t = \frac{\mu - r}{\sigma} M_t dW_t$ represent the pricing kernel, $\mathbf{1}' = [1, 1, 1]$, and $*$ represents element-by-element multiplication.

Given the Markov structure of this environment, it is natural to restrict attention to Markov strategies. Manager i can then take actions Q^i_t and D^i_t that depend only on the most recently observed values of the payoff relevant state variables X_t and $K_{t-} \equiv \left(K^1_{t-}, K^2_{t-}\right)$, where $K^i_{t-} \equiv \lim_{s\uparrow t} K^i_s$. A pure strategy Markov-perfect equilibrium (MPE) of the game is a pair, $i = 1, 2$, of vector-valued functions $\left[Q^i, D^i\right](K_{t-}, X_t)$. In an equilibrium strategy

[6] Formally, we can define $D^{im}_t \equiv d1_{\{K^i_t = \kappa^m\}}$, which represents the decision of firm i to enter or leave state m. The variable D^{im}_t thus takes the value 1 at the instant firm i enters state m and takes the value -1 at the instant the firm leaves state m. It takes the value 0 everywhere else. The equation $D^{imn}_t \equiv \left|D^{im}\right| D^{in} \left[\left(D^{in} - D^{im} + 2\right)/4\right]$ then gives a family of indicator variables with the desired properties.

pair, each must maximize the value function (3) in every state (K_{t-}, X_t), conditional on the equilibrium strategy of the rival.

It is straightforward to show that any MPE must have quantity choices equal to static Cournot equilibrium output levels. In our setting, value maximizing firms always endogenously produce at their capacity limits due to zero marginal costs and sufficiently low price elasticity. Hence, any MPE strategy requires $Q_t^i = Q^i\left(K_t^i\right)$. The instantaneous profit functions $\pi_t^i = \pi^i\left(K_t, X_t\right) = X_t\left[Q^1\left(K_t^1\right) + Q^2\left(K_t^2\right)\right]^{\gamma-1} Q^i\left(K_t^i\right) - F_t^i$ are thus fully determined by the current capital levels K_t^1 and K_t^2 and the value of the state variable X_t.

To aid future exposition, it is convenient to define the capital dependent *revenue factors*

$$\begin{aligned} R_{mn}^1 &\equiv \left[Q^1\left(\kappa_m\right) + Q^2\left(\kappa_n\right)\right]^{\gamma-1} Q^1\left(\kappa_m\right), \\ R_{mn}^2 &\equiv \left[Q^1\left(\kappa_m\right) + Q^2\left(\kappa_n\right)\right]^{\gamma-1} Q^2\left(\kappa_n\right), \end{aligned}$$

where $m, n \in \{0, 1, 2\}$ index the capital levels of firms 1 and 2, respectively. We can then conveniently write the profit function of each individual firm i as $\pi^i\left(K_t^1 = \kappa_m, K_t^2 = \kappa_n, X_t\right) = X_t R_{mn}^i - F_t^i$.

Given the simplification of the instantaneous output choices Q_t^i, we can henceforth focus attention on the dynamic game of option exercise involving the capital levels K_t^i and the investment decisions D_t^i. Any Markov strategy can be summarized by a set of exercise boundaries that for each player i and each capital state K_{t-} specify regions of the state variable X_t at which player i will change his capital level to a new state. We can use standard techniques of backward induction to derive MPE of the dynamic game.

In order to focus attention on the marginal impact of a firm's decisions on its own value, standard practice separates valuation equations into (1) the present value of the existing operations of the firm as a going-concern, and (2) the incremental value of any real options. Following this logic, in our settings we can consider the decomposition

$$V^i\left(K_t, X_t\right) = V_A^i\left(K_t, X_t\right) + V_F^i\left(K_t^i\right) + V_O^i\left(K_t, X_t\right), \tag{4}$$

where V_A^i denotes the value of revenues generated by assets-in-place, V_F^i is the present value of fixed costs, and V_O^i is the value of real options. In this equation, the first two components aggregate to going-concern value, and the last component reflects the value of current or future actions to modify operations.

For a monopolist, the impact of own-firm decisions on own-firm value and risk are fully reflected in decompositions such as (4). In a duopoly there are consequences of firm actions on rivals value and risk. In the next section we examine specific examples that allow a richer decomposition that identifies what we will refer to as "own" and "industry" effects.

3 Rival Growth Options and Risk

This section considers the simplest case of the general model developed in Section 2. Specifically, we assume that one rival is *flexible*, and begins with one option to either expand or contract, while the other rival is *inflexible* and has no ability to change its capital level. This allows us to add to the standard decomposition (4) by recognizing that in competitive environments (either oligopolistic or perfectly competitive) rival actions also have important impacts on firm value. In this paper, we focus on cross-firm valuation and risk effects that arise due to product market interactions. When product markets are shared, one firm's actions to either expand or contract output alter the output price dynamics of the industry, and hence contribute to valuation and risk dynamics of competitors.[7]

This scenario is useful because it allows us to isolate the two sources of real option risk that can occur in a real options duopoly. The flexible firm has risk that depends only on its own real option and leverage. The presence of an inflexible rival creates a permanent dampening of risk, but has no dynamic impact on the flexible firm risk loadings. As in the monopoly case explored by CFG (2004, 2006), the flexible firm thus has an own-option-leverage risk component but no independent source of dynamic industry risk.

By contrast, the inflexible firm offers the polar opposite case. The inflexible firm has no own-option-leverage component in its risk loadings, but nonetheless, it is exposed to dynamic risk due to the investment decisions of its rival. Thus, the inflexible firm stock price is exposed only to dynamic industry risk and has no independent source of own-option-leverage.

[7]Similar cross-firm real option effects could also be transmitted through factor input markets.

This example highlights that the presence of a rival is risk-reducing. Intuitively, when a competitor is near an option exercise boundary, its investment decisions act as a natural hedge against variations in the exogenous state variable. For example, when a rival is near an expansion boundary, good news about demand going up will be offset by the bad news that the competitor is closer to expanding, which will drive output prices down. A similar effect also hedges risk near competitor contraction boundaries. Future sections consider cases where both own-firm and rival-firm options are present, and we find that the presence of a rival may tend either to reinforce or to offset the own-firm effect depending on the circumstance.

To achieve a specification where one firm is flexible and the other inflexible, we set the capital adjustment costs to

$$\Lambda^1 \equiv \begin{bmatrix} 0 & -\infty & -\infty \\ S & 0 & -I \\ -\infty & -\infty & 0 \end{bmatrix} \qquad \Lambda^2 \equiv \begin{bmatrix} 0 & -\infty & -\infty \\ -\infty & 0 & -\infty \\ -\infty & -\infty & 0 \end{bmatrix},$$

where $S, I > 0$. Firm 1, the flexible firm, thus begins at capital level κ_1 and has a single option to change capacity, either by expanding to κ_2 or contracting to κ_0. If it expands, it pays the investment cost I and if it contracts it receives the salvage value S. Once firm 1 either expands or contracts, it has no further options to change capacity. Firm 2 begins at capital level κ_1 and has no options to change capacity.

We now examine the exercise decision of firm 1 (the flexible firm). Taking the operating decisions of firm 2 as given, the flexible firm policy is described by a critical demand level X_E at which the firm will expand, and a demand level $X_C < X_E$ at which it will contract. The value of the flexible firm is summarized as our first result. Let us denote by $V^1_{AM}(K_t, X_t)$ the value of the assets in place of firm 1 and by $V^1_{OM}(K_t, X_t)$ the investment option value due only to the firm's own actions. We then show:

Proposition 1: *The value of firm 1, the flexible firm, is the sum of the value of the assets in place net of the present value of fixed costs and the option value:*

$$\begin{aligned} V^1(K_t, X_t) &= \frac{R^1_{11} X_t}{\delta} - \frac{f_1}{r} + B^1_1 X_t^{\nu_1} + B^1_2 X_t^{\nu_2}, \\ &= V^1_{AM}(K_t, X_t) + V^1_F(K_t) + V^1_{OM}(K_t, X_t), \end{aligned}$$

where B_1^1 and B_2^1 are positive constants determined by the boundary conditions, and $\nu_1 > 1$ and $\nu_2 < 0$ are constants given in the Appendix.

As in standard real option models firm value is given by the value of the assets in place adjusted for the fixed costs and the option value. While both values are only related to firm 1 own action they do reflect the initial capacity and output of both firms in the industry. Since firm 1 is the flexible firm the value of the assets in place and the option value is only driven by its own actions. There is no value arising from the competitors actions. Consistent with Leahy (1993) the flexible firm acts like a myopic firm who's valuation is driven only by its own actions.

The value of the inflexible firm derives only from the assets in place since there are no options available. The value of the assets in place, however, reflect the fact that there will be revenue changes every time the flexible firm adjusts its capacity.

Proposition 2. *The value of firm 2, the inflexible firm, is entirely determined by the value of the assets in place net of the present value of fixed costs:*

$$\begin{aligned} V^2(K_t, X_t) &= \frac{R_{11}^2 X_t}{\delta} - \frac{f_1}{r} + B_1^2 X_t^{\nu_1} + B_2^2 X_t^{\nu_2} \\ &= V_{AM}^2(K_t, X_t) + V_F^2(X_t) + V_{AC}^2(K_t, X_t), \end{aligned}$$

where $B_1^2 \leq 0$ and $B_2^2 \geq 0$ are constants determined by the boundary conditions.

The first part of the inflexible firm valuation equation corresponds to the present value of output assuming no real option exercise by the rival. The second component is the present value of fixed costs associated with a given level of capacity, and the third component is the present value of the revenue gain or loss due to the potential for rival firm option exercise. The portion of value contributed by rival firm options has a positive part associated with the flexible firm contraction option, and a negative part due to the rival firm option to expand in good times.

The explicit valuation formulas for the flexible and the inflexible firm allow us to derive dynamic betas for both firms following standard arguments. Beta for each firm can be calculated as the elasticity of firm value with respect to X_t, i.e.,

$$\beta^i(K_t, X_t) = \frac{\partial V^i(X)}{\partial X} \frac{X}{V^i(X)},\ i = 1, 2,$$

leading to:

Proposition 3. *The dynamic betas for the flexible and inflexible firm are:*

$$\beta^i(K_t, X_t) = 1 + \frac{f_1/r}{V^i(K_t, X_t)} + \left\{ (\nu_1 - 1) \frac{B_1^i X_t^{\nu_1}}{V^i(K_t, X_t)} + (\nu_2 - 1) \frac{B_2^i X_t^{\nu_2}}{V^i(K_t, X_t)} \right\}.$$

Consistent with the valuation equations, betas for both the flexible and inflexible firm consists of three parts prior to option exercise. By assumption revenue beta is equal to 1. The second part is operating leverage, which is always risk increasing, and the final term arises from the flexible firm growth options . Since $\nu_1 > 1$, $\nu_2 < 0$, and $B_1^1, B_2^1 \geq 0$, the flexible firm risk increases due to its own option to expand, and is reduced by its option to contract.

By contrast, since $B_1^2 \leq 0$ and $B_2^2 \geq 0$, the flexible firm risk decreases due to both the rival growth option and the rival expansion option. We note that although the structure of beta for both firms is similar, the economic interpretation is very different. The flexible firm's risk depends only on its own firm specific decisions, whereas the inflexible firm has no decisions to make and its risk is determined entirely by industry effects. The reduction in risk due to rival firm growth options is consistent with the intuition explained previously. Near the rival expansion boundary, increases in demand are accompanied by a greater likelihood that the flexible firm will add capacity, increase output, and thereby mitigate the positive impact of a demand increase. Similarly, near the contraction boundary, the likelihood that the flexible firm will reduce capacity increases when demand falls, again partially offsetting any demand shock.

This simple case is illustrated in Figure 1 where, in order to highlight the importance of industry and own effects we have assumed that the inflexible firm also does not have operating leverage and we illustrate the risk of the flexible firm with and without operating leverage. X_C is the critical level of demand at which the flexible firm shrinks and X_E is critical level at which the flexible firm expands. Starting from demand level such as X_M consider a series of increases in demand. As demand increases, operating leverage decreases but option leverage increases in importance. At X_E the flexible firm expands, extinguishes its option and replaces the option with more capacity and operating leverage. As demand increases above X_E operating leverage risk decreases. Starting from demand level X_M the same path of demand increases would lead to a decrease the risk of the inflexible firm due to the fact that further positive demand shocks will induce the flexible rival to increase capacity and dampen demand shocks. Once the flexible firm has expanded, the industry is left with two firms who are not able to offset demand shocks further so the inflexible firm's risk reflects underlying demand risk which is invariant to the level of demand. On the other hand, as demand decreases from X_M both firms experience a decrease in risk in the absence of operating leverage due to the fact that further decreases in demand could induce an offsetting decrease in capacity. With operating leverage, however, decreases in demand increase risk so the net effect could be an increase in risk.

An important point to note in this diagram is the own and rival firm risk levels can move in opposite directions. As demand increases, the flexible firm's risk increases while the inflexible firm's risk decreases. If the rival is thought of as the industry, then own and industry risk can move in opposite directions as well.

In the next two sections we generalize the results found in this section by allowing two flexible firms. In section 4.1 we consider the case where both firms have an expansion option each, while in Section 4.2 we consider the case of each firm having a contraction option.

4 Dynamic Risk in Asymmetric Industry Equilibrium

We now permit both firms to have real options to expand or contract capacity, and consider sequential exercise equilibria. We obtain analytical solutions in the two polar opposite cases: 1) when both firms have expansion options, and 2) when both firms have contraction options.

4.1 Equilibrium Exercise of Expansion Options

In this subsection we relax the assumption that there is a flexible and an inflexible firm and allow both firms to expand capacity. Option exercise in this setting is determined by equilibrium play of both firms. In principle there are three different types of equilibria when firms have asymmetric investment costs. There is sequential exercise of options (the low cost firm exercises first and the high cost firm second), and there is preemption and simultaneous exercise of options.

Although we derive equilibrium play that includes preemption and sequential exercise, we are primarily interested in the equilibrium with sequential exercise. This implies that one of the firms must act as the leader and the other one as the follower. It turns out that in a sequential exercise equilibrium the low cost firm is the leader and the high cost firm is the follower. We make use of standard dynamic programming techniques to derive optimal value functions for the different cases.

In an equilibrium in which asymmetric firms exercise their options sequentially, industry structure can be thought of as consisting of three distinct phases, a juvenile industry, an adolescent industry, and a mature industry. Figure 2 depicts the different industry stages.

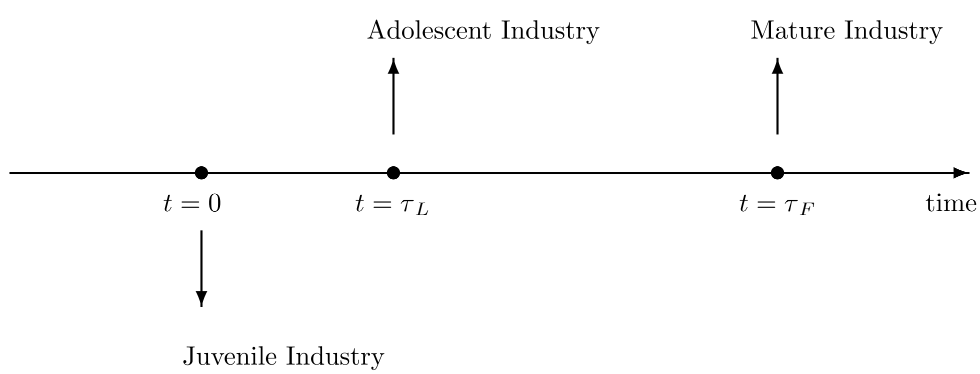

Figure 2

To achieve a specification for our general model where both firms only have a single growth option we set the capital adjustment costs to

$$\Lambda^1 \equiv \begin{bmatrix} 0 & -\infty & -\infty \\ -\infty & 0 & -I \\ -\infty & -\infty & 0 \end{bmatrix} \qquad \Lambda^2 \equiv \begin{bmatrix} 0 & -\infty & -\infty \\ -\infty & 0 & -\rho I \\ -\infty & -\infty & 0 \end{bmatrix},$$

where we assume that $\rho \geq 1$ so that the costs of firm 1 are lower than the costs of firm 2, $I^1 \leq I^2$.

Based on the time line of option exercise we apply backward induction to derive the value functions for the two firms in the duopolistic industry. In a mature industry firm values equal the present value of profits minus the present value of fixed costs. In an adolescent industry the leader has already exercised his option but the follower needs to decide when optimally to exercise his option. Since both firms are operating in an imperfectly competitive output market the firm value of the leader necessarily depends on the exercise strategy of the follower. In a juvenile industry neither firm has exercised their option so that both firms have to take into account the value implications of option exercise by the leader and the follow on exercise by the follower.

To derive equilibrium behavior for our model it turns out that the value of ρ plays a crucial role. This relative investment cost difference determines which behavior corresponds to equilibrium play, i.e., sequential exercise or preemptive behavior. In deriving equilibrium play for our example we follow the approach of Pawlina and Kort (2006) who analyze equilibrium option exercise under asymmetric costs.

Proposition 4. *There exists a level of $\rho^* > 1$ such that for all $\rho \geq \rho^*$ the unique equilibrium of the investment game is that the low cost firm acts as the leader and the high cost firm acts as the follower. For $1 \leq \rho < \rho^*$ any equilibrium of the investment game results in preemptive investment, in which the leader preempts the follower. In case of symmetric costs there is no pure strategy Nash equilibrium, and the only equilibrium that exists is in mixed strategies.*

We henceforth concentrate on sequential exercise of options with firm 1 being the leader and firm 2 being the follower. The leader exercises his growth option at time τ_L or equivalently at the demand trigger X_E^1, the follower exercises at time τ_F or the demand trigger X_E^2. It holds that $\tau_L < \tau_F$ and equivalently $X_E^1 < X_E^2$. Given these two sequential exercise dates (trigger levels) the value functions of each firm consists of three components, (i) firm value in the juvenile industry ($X_t < X_E^1$), (ii) firm value in the adolescent industry ($X_E^1 \leq X_t \leq X_E^2$), and (iii) firm value in the mature industry ($X_t > X_E^2$).

We derive the corresponding firm values.

Proposition 5. *The leader's value function is*

$$V^1(K_t, X_t) = \begin{cases} \frac{R_{11}^1 X_t}{\delta} - \frac{f_1}{r} + \frac{f_2 - f_1 + rI^1}{r(\nu_1 - 1)} \left(\frac{X_t}{X_E^1}\right)^{\nu_1} & \\ + \frac{X_E^2}{\delta}\left[R_{22}^1 - R_{21}^1\right]\left(\frac{X_t}{X_E^2}\right)^{\nu_1} & X_t < X_E^1, \\ \frac{R_{21}^1 X_t}{\delta} - \frac{f_2}{r} & \\ + \frac{X_E^2}{\delta}\left[R_{22}^1 - R_{21}^1\right]\left(\frac{X_t}{X_E^2}\right)^{\nu_1} & X_t \in [X_E^1, X_E^2], \\ \frac{R_{22}^1 X_t}{\delta} - \frac{f_2}{r} & X_t > X_E^2, \end{cases} \tag{5}$$

with the optimal expansion trigger X_E^1 equal to

$$X_E^1 = \frac{\delta \nu_1 (f_2 - f_1 + rI^1)}{(\nu_1 - 1) r [R_{21}^1 - R_{11}^1]} > 0. \tag{6}$$

The follower's value function is

$$V^2(K_t, X_t) = \begin{cases} \frac{R_{11}^2 X_t}{\delta} - \frac{f_1}{r} + \frac{f_2 - f_1 + rI^2}{r(\nu_1 - 1)} \left(\frac{X_t}{X_E^2}\right)^{\nu_1} & \\ + \frac{X_E^1}{\delta}\left[R_{21}^2 - R_{11}^2\right]\left(\frac{X_t}{X_E^1}\right)^{\nu_1} & X_t \leq X_E^1, \\ \frac{R_{21}^2 X_t}{\delta} - \frac{f_1}{r} + \frac{f_2 - f_1 + rI^2}{r(\nu_1 - 1)} \left(\frac{X_t}{X_E^2}\right)^{\nu_1} & X_t \in [X_E^1, X_E^2], \\ \frac{R_{22}^2 X_t}{\delta} - \frac{f_2}{r} & X_t > X_E^2. \end{cases} \tag{7}$$

with the optimal expansion trigger X_E^2 equal to

$$X_E^2 = \frac{\delta \nu_1 (f_2 - f_1 + rI^2)}{(\nu_1 - 1) r [R_{22}^2 - R_{21}^2]} > 0. \tag{8}$$

We emphasize that the optimal expansion trigger of the leader depends only on the leader's own characteristics and actions. There is neither a direct nor an indirect influence coming from the follower. Firm value of the leader is composed of the value of the assets in place, the value of the fixed costs, the option value, and the value correction arising from the anticipated changes resulting from the follower's option exercise. Similarly, the optimal expansion trigger of the follower is determined only by the follower's fundamentals and actions, and is independent of the leader's investment strategy.

To explore the different characteristics of the leader's and follower's value functions consider the juvenile industry. It is helpful to rewrite the equations for the trigger levels as

$$\frac{X_E^1}{\delta\nu_1}[R_{21}^1 - R_{11}^1] = \frac{(f_2 - f_1 + rI^1)}{(\nu_1 - 1)r} > 0$$

and

$$\frac{X_E^2}{\delta\nu_1}[R_{22}^2 - R_{21}^2] = \frac{(f_2 - f_1 + rI^2)}{(\nu_1 - 1)r} > 0.$$

Substituting into the value functions we arrive at

$$\begin{aligned} V^1(K_t, X_t) &= \underbrace{\frac{R_{11}^1}{\delta}X_t - \frac{f_1}{r}}_{\textbf{assets in place}} + \underbrace{\frac{X_E^1}{\delta\nu_1}[R_{21}^1 - R_{11}^1]\left(\frac{X_t}{X_E^1}\right)^{\nu_1}}_{\textbf{growth option}} \\ &\quad + \underbrace{\frac{X_E^2}{\delta}[R_{22}^1 - R_{21}^1]\left(\frac{X_t}{X_E^2}\right)^{\nu_1}}_{\textbf{value adjustment}} \end{aligned}$$

and

$$\begin{aligned} V^2(K_t, X_t) &= \underbrace{\frac{R_{11}^2}{\delta}X_t - \frac{f_1}{r}}_{\textbf{assets in place}} + \underbrace{\frac{X_E^1}{\delta}[R_{21}^2 - R_{11}^2]\left(\frac{X_t}{X_E^1}\right)^{\nu_1}}_{\textbf{value adjustment}} \\ &\quad \underbrace{+\frac{X_E^2}{\delta\nu_1}[R_{22}^2 - R_{21}^2]\left(\frac{X_t}{X_E^2}\right)^{\nu_1}}_{\textbf{growth option}} \end{aligned}$$

Each value function in a juvenile industry consists of four parts that can be given the following interpretation. In both cases (leader and follower) the first part measures the value of the assets in place assuming that the firms use their initial levels of capacity forever (i.e., as if no options exist). The second part measures the value of the fixed costs. The third term in the leader's valuation is the option value that is only related to the leader's own action (option exercise). The fourth term in the leader's value function is the value adjustment of the assets in place after the follower's option exercise. This value adjustment is negative, because an increase in capacity by the rival causes the market price to drop. Since the leader's assets in place are at the capacity level κ_2 this value adjustment can be viewed as an adjustment to the value of the assets in place induced by an action of the rival firm. Denoting this value adjustment by $V^1_{AC}(K_t, X_t)$ we can rewrite the leaders value function as

$$V^1(K_t, X_t) = V^1_{AM}(K_t, X_t) + V^1_{AC}(K_t, X_t) + V^1_{OM}(K_t, X_t) + V^1_F(K_t, X_t).$$

Let us next turn to the value function of the follower. As pointed out above the first two value components of the follower's value function are the value of the assets in place and the value of the fixed costs. The third part in the follower's value is now driven by the leader's action. It is the value adjustment necessary to account for the change in the follower's revenues when the leader exercises the option and the market price drops. As with the leader's value function this is negative. The fourth part of the follower's value function is the option value that accounts for the increase in capacity and takes into account the corresponding revenue change. Taking all the four value drivers together we get

$$V^2(K_t, X_t) = V^2_{AM}(K_t, X_t) + V^2_{AC}(K_t, X_t) + V^2_{OM}(K_t, X_t) + V^2_F(K_t, X_t).$$

The value functions of the leader and follower are thus given by for $i = 1, 2$,

$$V^i(K_t, X_t) = V^i_{AM}(K_t, X_t) + V^i_{AC}(K_t, X_t) + V^i_{OM}(X_t) + V^i_F(K_t, X_t),$$

which follows immediately from the preceding discussion.

The value functions for both firms can now be used to derive risk implications.

Proposition 6. *Consider a growing industry with sequential exercise of options in which each firm has a single expansion option. Systematic firm risks for the follower and the leader over different industry stages (* $k = 1, 2$*) are given by*

$$\beta^i(K_t, X_t) = 1 + \frac{V^i_{OM}(K_t, X_t) + V^i_{AC}(K_t, X_t)}{V^i(K_t, X_t)}(\nu_1 - 1) + \frac{f_k/r}{V^i(K_t, X_t)}, \; k = 1, 2 \tag{9}$$

where $V^i_{AC}(K_t, X_t) < 0$.

We note that the growth option of the rival is again risk reducing, since $V^i_{AC}(K_t, X_t) < 0$. This result is consistent with the hedging role for competition shown in Section 3.

Figure 3 gives a graphical presentation of the hedging argument. Before the follower exercises his growth option, industry output is given by the level Q_1. Since both firms have to produce at full capacity levels price fluctuates along the supply curve Q_1. Let's suppose that demand increases by an efficient amount so that the follower finds it optimal to exercise his growth option. Option exercise results in an increase in industry output to the level Q_2. The increase in industry supply causes prices to increase less than to the level indicated by the old supply curve P^*. The new price level is P_2 instead of P^*. This dampening corresponds to the hedging effect.

Systematic firm risk in a growing oligopolistic industry is thus driven by the a firm's operating leverage, its own growth options, and the risk reducing effects of rival growth options. We note that the own firm and rival growth options have opposite effects on own-firm risk, suggesting that own firm and rival characteristics may have opposite implications for risk in some scenarios.

Figure 4 displays the evolotion of risk in a growing industry. Prior to leader exercise, the leader risk increases while the follower risk decreases. Immediately upon the exercise of the leader growth option, the leader risk drops below one and the follower risk jumps above one. Hence, the risk-ordering of the two firms reverses. The two firms' risk continues to move apart until the follower growth option is exercised, after which both firms' risk is driven only by operating leverage.

4.2 Equilibrium Exercise of Contraction Options

The risk analysis of a growing industry in which firms exercise growth options has revealed two important results. We found that industry effects arising from the strategic interactions of rival firms are risk reducing and that firm's own and industry characteristics have opposite risk implications. Moreover, we have found that equilibrium play crucially depends on the investment cost differences of the two rival firms. If the cost difference is large enough it follows that there exists a unique Nash equilibrium with sequential exercise of options. If the costs difference is not too high, there are preemptive equilibria and in case of symmetric costs ($\rho = 1$) there only exists a Nash equilibrium in mixed strategies (see Boyer et al. (2007)).

In this section we assume that each firm has a single contraction option. Firms operate with a given initial capacity κ_1 and that each has a single option to reduce capacity to a level given by $\kappa_0 < \kappa_1$. Contraction to a smaller firm size and capacity is optimal if demand is sufficiently low. To achieve a specification where both firms only have a single contraction option we set the capital adjustment costs to

$$\Lambda^1 \equiv \begin{bmatrix} 0 & -\infty & -\infty \\ S & 0 & -\infty \\ -\infty & -\infty & 0 \end{bmatrix} \qquad \Lambda^2 \equiv \begin{bmatrix} 0 & -\infty & -\infty \\ \rho S & 0 & -\infty \\ -\infty & -\infty & 0 \end{bmatrix},$$

where we assume that $0 < \rho \leq 1$, so that firm 1 has the high and firm 2 the low salvage value, $S^2 < S^1$. This implies that firm 1 has an incentive to contract earlier. Our interest again lies in equilibrium play of the two rivals and in contrast to the case of expansion option, sequential exercise is the unique equilibrium as long as there is a difference in the salvage values, i.e., $\rho < 1$.

Proposition 7. *For every $0 < \rho < 1$ there exists a unique equilibrium of the contraction game in which the high salvage value firm acts as the leader and the low salvage value firm acts as the follower.*

The last result is very interesting and establishes an asymmetric role of expansion and contraction. The result is driven by the fact that the strategic effect of a contraction is not negative, because the follower profits from a capacity contraction of the leader. That the leader has an incentive to exercise first is the consequence of the higher salvage value. Hence, there is no preemptive role for the players in the contraction game. This result was previously discussed by Murto (2004). Henceforth, we discuss the sequential contraction case only.

We again assume both firms initially have capacity κ_1. In the sequential exercise equilibrium the leader contracts first at the demand trigger X_C^1, and the follower contracts at the trigger $X_C^2 > X_C^1$. We then show:

Proposition 8. *The leader's value function is*

$$V^1(K_t, X_t) = \begin{cases} \frac{R_{11}^1 X_t}{\delta} - \frac{f_1}{r} + \frac{f_1 - f_0 + rS^1}{r(1-\nu_2)} \left(\frac{X_t}{X_C^1}\right)^{\nu_2} & \\ + \frac{X_C^2}{\delta}\left[R_{00}^1 - R_{01}^1\right]\left(\frac{X_t}{X_C^2}\right)^{\nu_2} & X_t > X_C^1, \\ \frac{R_{01}^1 X_t}{\delta} - \frac{f_0}{r} & \\ + \frac{X_C^2}{\delta}\left[R_{00}^1 - R_{01}^1\right]\left(\frac{X_t}{X_C^2}\right)^{\nu_2} & X_t \in [X_C^2, X_C^1], \\ \frac{R_{00}^1 X_t}{\delta} - \frac{f_0}{r} & X_t < X_C^2, \end{cases} \tag{10}$$

with a contraction trigger

$$X_C^1 = \frac{\delta\nu_2(f_1 - f_0 + rS^1)}{(1-\nu_2)r[R_{01}^1 - R_{11}^1]} > 0. \tag{11}$$

The sign of the optimal contraction trigger (11) of the leader is determined by the properties of the revenue factors. The assumption on the price elasticity of the inverse demand function implies that

$$R_{01}^1 < R_{11}^1.$$

Hence X_C^1 is strictly positive. Rewriting the contraction trigger of the leader as

$$\frac{X_C^1}{\delta\nu_2}[R_{01}^1 - R_{11}^1] = \frac{(f_1 - f_0 + rS^1)}{(1-\nu_2)r} > 0 \tag{12}$$

and substituting this into the value function (10) gives

$$\begin{aligned} V^1(K_t, X_t) \quad = \quad & \underbrace{\frac{R_{11}^1}{\delta}X_t - \frac{f_1}{r}}_{\textbf{assets in place}} + \underbrace{\frac{X_C^1}{\delta\nu_2}[R_{01}^1 - R_{11}^1]\left(\frac{X_t}{X_C^1}\right)^{\nu_2}}_{\textbf{contraction option}} \\ & + \underbrace{\frac{X_C^2}{\delta}\left[R_{00}^1 - R_{01}^1\right]\left(\frac{X_t}{X_C^2}\right)^{\nu_2}}_{\textbf{value adjustment}}. \end{aligned}$$

The value function has four elements. The first measures the value of the assets in place starting with the initial capacity level. The second one measures the value of the fixed costs. Both values are the outcome of the firm's own actions. The third element measures the value of the contraction option. This corresponds to the positive value of a put option as indicated by ν_2. The last term measures the value adjustment arising from the contraction of the follower after the leader already has contracted. This term has a positive value because any reduction in the capacity of the rival firm causes the market price to increase which benefits the leader who already contracted. This last term, however, is not the outcome of the firm's own action but has to be attributed to the actions of the rival firms. By combining these elements we obtain

$$V^1(K_t, X_t) = V^1_{AM}(K_t, X_t) + V^1_{AC}(K_t, X_t) + V^1_{OM}(K_t, X_t) + V^1_F(K_t, X_t).$$

We similarly derive the follower's value.

Proposition 9. *The follower's value function is*

$$V^2(K_t, X_t) = \begin{cases} \frac{R^2_{11} X_t}{\delta} - \frac{f_1}{r} + \frac{f_1 - f_0 + rS^2}{r(1-\nu_2)} \left(\frac{X_t}{X^2_C}\right)^{\nu_2} & \\ \frac{X^1_C}{\delta}\left[R^2_{01} - R^2_{11}\right]\left(\frac{X_t}{X^1_C}\right)^{\nu_2} & X_t \geq X^1_C, \\ \frac{R^2_{01} X_t}{\delta} - \frac{f_1}{r} + \frac{f_1 - f_0 + rS^2}{r(1-\nu_2)} \left(\frac{X_t}{X^2_C}\right)^{\nu_2} & X_t \in [X^2_C, X^1_C], \\ \frac{R^2_{00} X_t}{\delta} - \frac{f_0}{r} & X_t < X^2_C. \end{cases} \tag{13}$$

with a contraction trigger

$$X^2_C = -\frac{\delta \nu_2 (f_1 - f_0 + rS^2)}{(1-\nu_2) r [R^2_{00} - R^2_{01}]} > 0. \tag{14}$$

To decompose the follower value we rewrite the the trigger level equation as

$$\frac{X^2_C}{\delta \nu_2}[R^2_{00} - R^2_{01}] = \frac{(f_1 - f_0 + rS^2)}{(1-\nu_2) r} > 0$$

and obtain the follower value function:

$$\begin{aligned} V^2(K_t, X_t) \quad = \quad & \underbrace{\frac{R^2_{11}}{\delta} X_t - \frac{f_1}{r}}_{\textbf{assets in place}} + \underbrace{\frac{X^2_C}{\delta \nu_2}[R^2_{00} - R^2_{01}] \left(\frac{X_t}{X^2_C}\right)^{\nu_2}}_{\textbf{contraction option}} \\ & + \underbrace{\frac{X^1_C}{\delta}\left[R^2_{01} - R^2_{11}\right]\left(\frac{X_t}{X^1_C}\right)^{\nu_2}}_{\textbf{value adjustment}}. \end{aligned}$$

This value function exhibits all the four different value elements discussed previously. As in the contraction case, the value functions again satisfy

$$V^i(K_t, X_t) = V^i_{AM}(K_t, X_t) + V^i_{AC}(K_t, X_t) + V^i_{OM}(K_t, X_t) + V^i_F(K_t, X_t).$$

Compared to the results derived for the case of expansion options the strategic effect in case of downsizing is positive $V^i_{AC}(K_t, X_t) > 0$ and hence value increasing.

The risk dynamics of the two firms in the industry follows from the valuation equations.

Proposition 10. *Systematic firm risks for both firms over the different industry stages are*

$$\beta^i(t) = 1 + \frac{V^i_{OM}(K_t, X_t) + V^i_{AC}(K_t, X_t)}{V^i(K_t, X_t)}(\nu_2 - 1) + \frac{f_k/r}{V^i(K_t, X_t)},\ k = 1, 0.$$

where $\nu_2 < 0$ and $V^i_{OM}(K_t, X_t) + V^i_{AC}(K_t, X_t) > 0$.

The last result demonstrates that the strategic effect is again risk reducing so that competition in our model is risk reducing, independent of whether we are in a growing or a shrinking industry.

5 Conclusion

In this paper we consider a duopolistic industry with firms producing a homogenous product at given capacity levels. Demand in the industry is stochastic and governed by an industry shock that follows a geometric Brownian motion. Firms produce with given capacity levels that are fixed (i.e. there is no operating flexibility) but can increase (decrease) their output with the exercise of a growth (contraction) option. Although there are no variable production costs, firms operate with fixed costs that change with the level of capacity. Given the industry structure we derive firm values and risk dynamics for individual firms and their rivals.

We find that in general rival options to adjust capacity reduce risk. This reduced risk is the consequence of a hedging effect. In case both firms operate with fixed capacity levels any profit uncertainty arises from the industry demand shock. Demand shocks directly translate into changes in the firm's cash flows. If, however, the leader who already exercised his growth option and faces fixed capacity forever, faces an change in the capacity of the follower upon the follower's option exercise, demand shocks are hedged by an output increase or decrease, depending whether we are in a growing or shrinking industry. This hedge is larger the closer the follower comes to exercising his option. As a consequence, the leader's risk is reduced and is below the market risk normalized by 1. In case of expansion options, firm own and industry characteristics have opposite risk implications. In case of a contraction option, firm own and industry characteristics have the same risk implications. These results suggest that the commonly recommended practice of using competitor or industry betas to proxy for own-firm risk should work well in certain environments, but not in others, providing testable new empirical predictions.

Appendix

A Proofs

A.1 Proof of Proposition One

To derive the value of the flexible firm we assume that there exist two traded assets that can be used to hedge industry demand uncertainty. Let B_t denote the price of a riskless bond with dynamics $dB_t = rB_t dt$ where $r > 0$ is the constant riskless rate of interest, and let S_t be the price of a risky asset. The price dynamics of the risky asset is given by

$$dS_t = \mu S_t dt + \sigma S_t dW_t.$$

The risky asset S_t and the industry demand shock X_t are perfectly correlated. Hence, we can use the securities B_t and S_t, to construct a portfolio of the bond and the asset S_t that perfectly replicates the industry shocks X_t and derive its risk neutral measure. Demand dynamics under risk neutral measure are given by

$$dX_t = (r - \delta)X_t dt + \sigma X_t d\hat{W}_t, \tag{15}$$

where $\delta \equiv \mu - g > 0$. All the valuations in this paper are based on the risk neutral measure (15).

Under the risk neutral measure (15) the value of the flexible firm needs to satisfy the valuation equation

$$\frac{1}{2}\sigma^2 X^2 V^1_{XX} + (r - \delta)XV^1_X - rV^1 + XR^1_{11} - f_1 = 0. \tag{16}$$

with the boundary conditions

$$\begin{aligned} V^1(X_E) &= \frac{R^1_{21}X_E}{\delta} - I - \frac{f_2}{r} \\ V^1(X_C) &= \frac{R^1_{01}X_C}{\delta} + S - \frac{f_0}{r} \\ V^1_X(X_E) &= \frac{R^1_{21}}{\delta} \\ V^1_X(X_C) &= \frac{R^1_{01}}{\delta}. \end{aligned}$$

The first two equations are the value matching conditions and specify that the option value at the critical boundaries are exactly equal to the present value of the incremental revenues net of adjustment costs. The last two equations are the smooth pasting conditions which are necessary for value maximization. This system of equations has no convenient analytical solution for X_E and X_C due to its nonlinearity.

Using standard techniques the solution to equation (16) is given by

$$V^1(K_t, X_t) = \frac{R^1_{11} X_t}{\delta} - \frac{f_1}{r} + B^1_1 X_t^{\nu_1} + B^1_2 X_t^{\nu_2},$$

where B^1_1 and B^1_2 solve

$$\begin{aligned}
(1-\nu_1)B^1_1 X_E^{\nu_1} + (1-\nu_2)B^1_2 X_E^{\nu_2} &= -I - \frac{f_2 - f_1}{r}, \\
(1-\nu_1)B^1_1 X_C^{\nu_1} + (1-\nu_2)B^1_2 X_C^{\nu_2} &= S - \frac{f_0 - f_1}{r}.
\end{aligned}$$

For given positive values of X_E and X_C these solution satisfies $B^1_1, B^1_2 > 0$. The values X_E and X_C can only be derived numerically. But it holds that $X_E > X_C$.

It is therefore straightforward to conclude that the parameters given in the proposition must satisfy

$$\frac{1}{2}\sigma^2 \nu(\nu - 1) + (r - \delta)\nu - r = 0,$$

with roots given by

$$\nu_{1,2} = \frac{1}{2} - \frac{r-\delta}{\sigma^2} \pm \sqrt{\left(\frac{1}{2} - \frac{r-\delta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}.$$

A.2 Proof of Proposition Two

Using the risk neutral dynamics (15) the value of the inflexible firm has to satisfy the valuation equation

$$\frac{1}{2}\sigma^2 X^2 V^2_{XX} + (r-\delta) X V^2_X - r V^2 + X R^2_{11} - f_1 = 0. \tag{17}$$

with the boundary conditions

$$\begin{aligned}
V^2(X_E) &= \frac{R^2_{21} X_E}{\delta} - \frac{f_1}{r} \\
V^2(X_C) &= \frac{R^2_{01} X_C}{\delta} - \frac{f_1}{r},
\end{aligned}$$

for given trigger levels X_E and X_C. The boundary conditions are two value matching conditions. Solving the valuation equation using the value matching conditions results in the value of the flexible firm given by

$$V^2(K_t, X_t) = \frac{R^2_{11} X_t}{\delta} - \frac{f_1}{r} + B^2_1 X_t^{\nu_1} + B^2_2 X_t^{\nu_2},$$

where B^2_1 and B^2_2 are the solutions to the equations

$$\begin{aligned} B^2_1 X_E^{\nu_1} + B^2_2 X_E^{\nu_2} &= \frac{R^2_{21} - R^2_{11}}{\delta} X_E - \frac{f_2 - f_1}{r} \\ B^2_1 X_C^{\nu_1} + B^2_2 X_C^{\nu_2} &= \frac{R^2_{01} - R^2_{11}}{\delta} X_C - \frac{f_0 - f_1}{r}. \end{aligned}$$

For this equation system it is easy to show that $B^1_2 < 0$ and $B^2_2 > 0$.

A.3 Proof of Proposition Three

Follows from the definition of the beta as an elasticity and the value function.

A.4 Proof of Proposition Four

We follow closely Pawlina and Kort (2006). The sequential exercise of growth options only occurs if firm 2, the follower, does not have an incentive to be the leader. Let $V^2_F(K_t, X_t)$ be the value function of firm 2, when it acts as the follower, and let $V^2_L(K_t, X_t)$ be the value function when firm 2 acts as the leader. In the adolescent industry, when the leader already exercised his option these the value function of firm 2 when it acts as the follower becomes

$$V^2_F(K_t, X_t) = \begin{cases} \frac{R^2_{21} X_t}{\delta} - \frac{f_1}{r} + \frac{f_2 - f_1 + r\rho I}{r(\nu_1 - 1)} \left(\frac{X_t}{X_E^{2,F}} \right)^{\nu_1} & X_t \leq X_E^{2,F} \\ \frac{R^2_{22} X_t}{\delta} - \frac{f_2}{r} - \rho I & X_t > X_E^{2,F}, \end{cases} \tag{18}$$

where $X_E^{2,F}$ is the investment trigger when firm 2 acts as the follower. Now assume, instead, that firm 2 acts as the leader. The value function of firm 2 at the time when it invests as the leader becomes

$$V^2_L(K_t, X_t) = \begin{cases} \frac{R^2_{12} X_t}{\delta} - \frac{f_2}{r} - \rho I + \frac{X_E^{1,F} [R^2_{22} - R^2_{12}]}{\delta} \left(\frac{X_t}{X_E^{2,F}} \right)^{\nu_1} & X_t \leq X_E^{1,F} \\ \frac{R^2_{22} X_t}{\delta} - \frac{f_2}{r} - \rho I & X_t > X_E^{1,F}, \end{cases} \tag{19}$$

where $X_E^{1,F}$ is the investment trigger of firm 1 when firm 1 acts as the follower. Firm 2 does not have an incentive to be the leader if and only if[8]

$$G(X_t, \rho) \equiv V_L^2(X_t, \rho) - V_F^2(X_t, \rho) \leq 0.$$

The value functions (18) and (19) satisfy the properties:

$$\begin{aligned} &V_F^2(K_t, X_t) \text{ is strictly convex in } X_t \text{ for all } X_t \leq X_E^{2,F} \\ &V_L^2(K_t, X_t) \text{ is strictly concave in } X_t \text{ for all } X_t \leq X_E^{1,F}. \end{aligned}$$

Moreover, the trigger level $X_E^{2,F}$ is given by

$$X_E^{2,F} = \frac{\nu_1 \delta (f_2 - f_1 + r\rho I}{(\nu_1 - 1) r [R_{22}^2 - R_{21}^2]} \tag{20}$$

and that of $X_E^{1,F}$ is given by

$$X_E^{1,F} = \frac{\nu_1 \delta (f_2 - f_1 + rI}{(\nu_1 - 1) r [R_{22}^1 - R_{12}^1]}. \tag{21}$$

Given our assumptions on the revenue function it follows that trigger (20) is greater or equal to (21) for $\rho > 1$.

As mentioned firm 2 does not have an incentive to become the leader if and only if $G(X_t, \rho) \leq 0$ for all $X_t \leq X_E^{2,F}$. Given the properties of $V_L^2(X_t, \rho)$ and $V_F^2(X_t, \rho)$ this holds true if and only if we find (X^*, ρ^*) such that

$$\begin{aligned} G(X^*, \rho^*) &= 0, \\ \frac{\partial G(X^*, \rho^*)}{\partial X} &= 0. \end{aligned} \tag{22}$$

It is straight forward to show that (**??**) and (22) are satisfied if and only if

$$X^* = \frac{\nu_1}{\nu_1 - 1} \frac{\delta \rho^* I}{[R_{12}^2 - R_{21}^2]}. \tag{23}$$

The equations (23) and (**??**) identify ρ^*. At the point (X^*, ρ^*) the value functions of firm 2 acting as the follower is tangent to the value function of firm 2 acting as the leader. Hence, for all $\rho \geq \rho^*$ the follower does not have an incentive to become the leader. Therefore for this set of parameter

[8] Since the value functions depend on the relative cost difference ρ we explicitly use it as an argument.

restrictions the equilibrium outcome is that firm 1 acts as the leader and firm 2 acts as the follower. This is the unique pure strategy Nash equilibrium outcome.

For $\rho < \rho^*$ firm 2 has an incentive to become the leader. This incentive exists for all values of X_t in the interval $[X_E^{2,P}, X_E^{1,F}]$, where $X_E^{2,P}$ is defined by

$$V_F^2(X_E^{2,P}, \rho) = V_L^2(X_E^{2,P}, \rho).$$

We are now in the scenario of a preemptive equilibrium. In this equilibrium firm 1 will either invest at the level $X_E^{2,P}$ or at its leader trigger level X_E^1 if $X_E^1 < X_E^{2,P}$. Further details on the preemptive equilibrium and the non-existence of a pure strategy equilibrium in case of $\rho = 1$ can be found in Boyer et al. (2007).

A.5 Proof of Proposition Five

The value function for the leader ($i = 1$) satisfies the Bellman equation

$$\frac{1}{2}\sigma^2 X^2 V_{XX}^1 + (r - \delta)XV_X^1 - rV^1 + XR_{11}^1 - f_1 = 0$$

together with the boundary conditions

$$\begin{aligned}
V^1(0) &= -\frac{f_0^i}{r}, \\
V^1(X_E^1) &= \frac{X_E^1 R_{21}^1}{\delta} - \frac{f_2}{r} - I + A_3^1 (X_E^1)^{\nu_1}, \\
V_X^1(X_E^1) &= \frac{R_{21}^1}{\delta} + \nu_1 A_3^1 (X_E^1)^{\nu_1 - 1}, \\
V^i(X_E^2) &= \frac{X_E^2 R_{22}^1}{\delta} - \frac{f_2}{r}.
\end{aligned}$$

A general solution is given by

$$V^1(X) = A_0^1 + A_1^1 X + A_2^1 X^{\nu_1}$$

where $A_k^1, k = 0, 1, 2$ are constants that are determined together with the boundary conditions. The constant A_3^1 from above expresses the change of the value function for the leader after the capacity expansion of the follower has taken place. It is determined by the boundary condition

$$V^1(X_E^2) = \frac{X_E^2 R_{22}^1}{\delta} - \frac{f_2}{r}.$$

Solving the Bellman equation together with the boundary conditions results in

$$\begin{aligned}
A_0^1 &= -\frac{f_1}{r} \\
A_1^1 &= \frac{R_{11}^1}{\delta} \\
A_2^1 &= \frac{f_2 - f_1 + rI}{r(\nu_1 - 1)}(X_E^1)^{-\nu_1} + A_3^1 \\
A_3^1 &= \frac{X_E^2[R_{22}^1 - R_{21}^1]}{\delta}(X_E^2)^{-\nu_1}
\end{aligned}$$

which results in the value function for the leader given by (5).

We proceed as above when deriving the value function for the follower ($i = 2$) and note that the boundary conditions now become

$$\begin{aligned}
V^2(0) &= -\frac{f_1}{r}, \\
V^2(X_E^2) &= \frac{X_E^2 R_{22}^2}{\delta} - \frac{f_2}{r} - \rho I + A_3^2 (X_E^2)^{\nu_1}, \\
V_X^2(X_E^2) &= \frac{R_{22}^2}{\delta} + \nu_1 A_3^2 (X_E^2)^{\nu_1 - 1}, \\
V^2(X_E^1) &= \frac{X_E^1 R_{21}^2}{\delta} - \frac{f_1}{r}.
\end{aligned}$$

The change of the boundary conditions relative to the proof of the leader is the consequence of the follower's response to the leader's exercise of the option at the trigger level X_E^1. The constant A_3^2 accounts for this change.

At the trigger level X_E^1 when the leader exercises his option the follower's value function needs to satisfy

$$V^2(X_E^1) = \frac{X_E^1 R_{21}^2}{\delta} - \frac{f_1}{r}$$

which implies a follower's value function equal to (7).

A.6 Proof of Proposition 6

Follows from the definition of the value function and beta.

A.7 Proof of Proposition 7

We follow closely the argument used in the proof of Proposition 3.4. . We assume that now $\rho < 1$ so that the salvage value of firm 2 is strictly smaller than that of firm 1. This suggests that firm 2 is the follower in the contraction game, it is the firm that contracts later. The value function of firm 2 when it acts as the follower is given by

$$V_F^2(K_t, X_t) = \begin{cases} \frac{R_{01}^2 X_t}{\delta} - \frac{f_1}{r} + \frac{f_1 - f_0 + r\rho S}{r(1-\nu_2)} \left(\frac{X_t}{X_C^{2,F}}\right)^{\nu_2} & X_t \geq X_C^{2,F} \\ \frac{R_{00}^2 X_t}{\delta} - \frac{f_0}{r} + \rho S & X_t \leq X_C^{2,F}, \end{cases} \tag{24}$$

where $X_C^{2,F}$ is the contraction trigger when firm 2 acts as the follower. Now assume, instead, that firm 2 acts as the leader. The value function of firm 2 at the time when it contracts as the leader becomes

$$V_L^2(K_t, X_t) = \begin{cases} \frac{R_{10}^2 X_t}{\delta} - \frac{f_0}{r} + \rho S + \frac{X_C^{1,F}[R_{00}^2 - R_{10}^2]}{\delta} \left(\frac{X_t}{X_C^{2,F}}\right)^{\nu_2} & X_t \geq X_C^{1,F} \\ \frac{R_{00}^2 X_t}{\delta} - \frac{f_0}{r} + \rho S & X_t \leq X_C^{1,F}, \end{cases} \tag{25}$$

where $X_C^{1,F}$ is the trigger when firm 1 contracts as the follower and firm 2 acts as the leader. The two contraction triggers are given by

$$X_C^{2,F} = \frac{\nu_2 \delta (f_1 - f_0 + r\rho S}{(1-\nu_2) r [R_{00}^2 - R_{01}^2]} \tag{26}$$

and

$$X_C^{1,F} = \frac{\nu_2 \delta (f_1 - f_0 + rS}{(1-\nu_2) r [R_{00}^1 - R_{10}^1]}. \tag{27}$$

Given our assumptions on the revenue function it follows that trigger (27) is strictly greater than trigger (26) for $0 < \rho < 1$. From this property and the value functions (24) and (25) it can be shown that

$$G(X_t, \rho) \equiv V_L^2(X_t, \rho) - V_F^2(X_t, \rho) \leq 0$$

holds for all X_t. Hence, firm 2 never has an incentive to become the leader. Therefore sequential exercise of contraction options is the unique pure strategy Nash equilibrium.

A.8 Proof of Propositions 8 and 9

The logic is the same as in Proposition 5 with the only change that because of the contraction the call option has to be changed to a put option with the corresponding terminal conditions.

A.9 Proof of Proposition 10

Follows from the definition of beta and the value functions.

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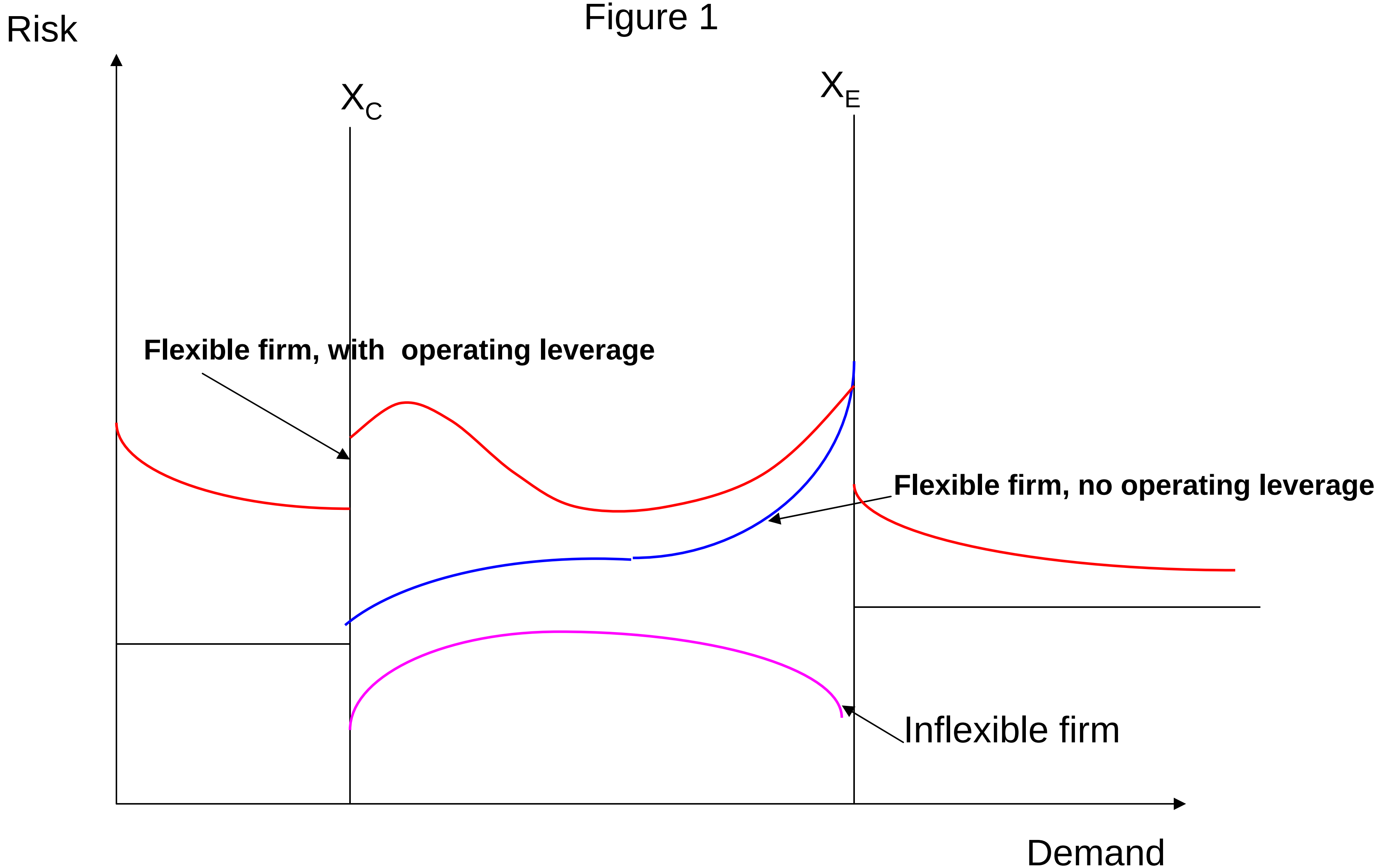
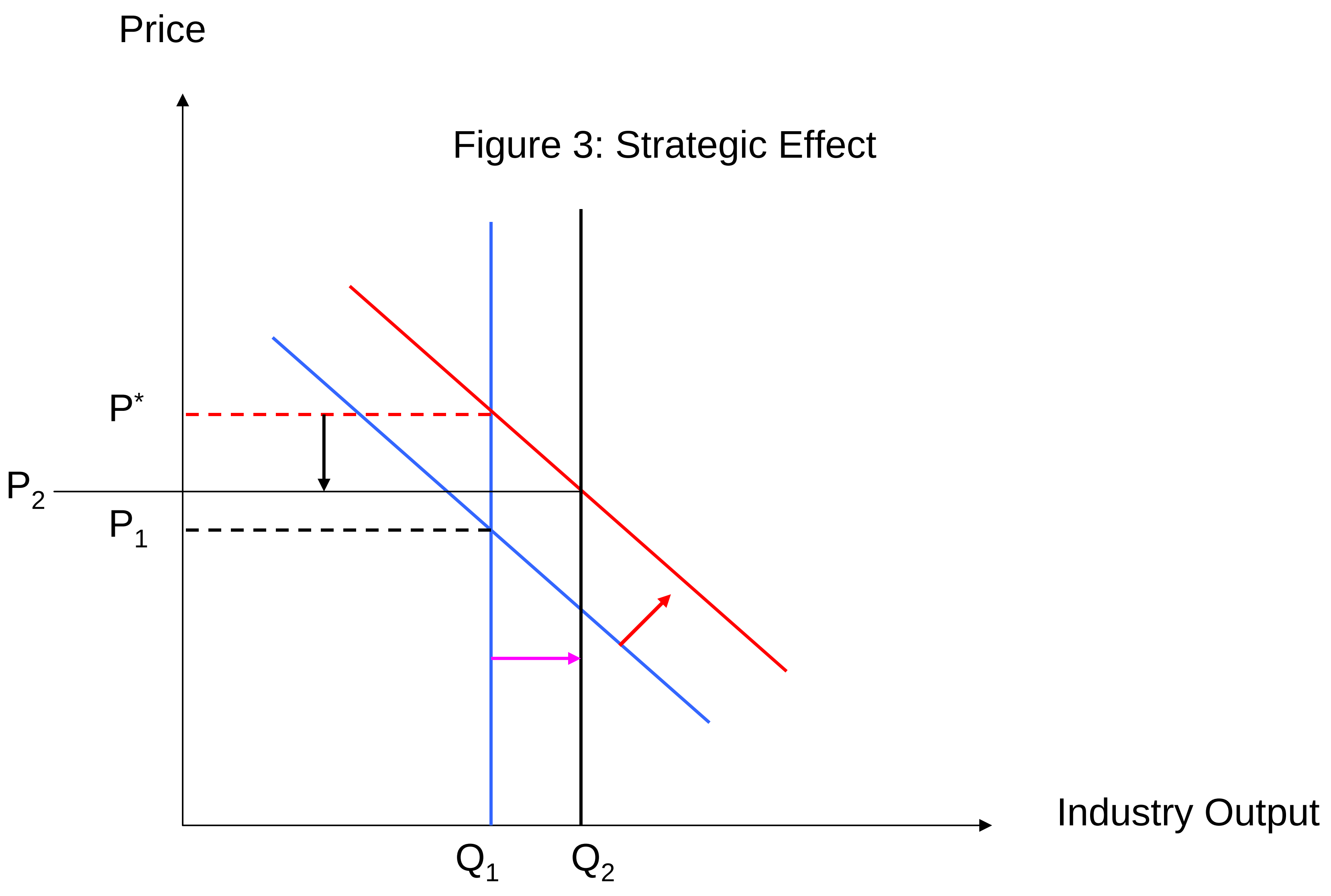
Figure 1


Risk
X_C
X_E
Flexible firm, with operating leverage
Flexible firm, no operating leverage
Inflexible firm
Demand


Price
Figure 3: Strategic Effect
P*
P2
P1
Q1
Q2
Industry Output

Duopoly Expansion