

Modern Portfolio Management with Conditioning Information

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JEL Classification: C44, G11

Key Words: Portfolio Management; Conditioning Information; Benchmark; Tracking Error; Foreign Exchange

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This paper studies models in which active portfolio managers optimize performance relative to a benchmark and utilize conditioning information unavailable to their clients. We provide explicit solutions for the optimal strategies with multiple risky assets, with or without a risk free asset, and also consider various constraints on portfolio risk. The equilibrium implications of the models are discussed. A currency portfolio example shows that the optimal solutions improve the measured performance by 53% out of sample, compared with portfolios ignoring conditioning information.

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1 Introduction

A common problem in modern portfolio management is to earn higher expected return than a pre-specified unmanaged benchmark portfolio, while minimizing the variance of the difference of the two returns, the “tracking error variance.” Active managers, who are compensated for performance relative to a given benchmark, typically face this kind of problem. While this problem is obviously interesting in practice, it is also important for academic scholars since its distinct portfolio policies and equilibrium implications add new insights to the conventional mean-variance setting.

This paper makes the following contributions. We develop explicit solutions for the optimal active portfolios, with an array of possible constraints, under the assumption that active managers have information about security returns (the “conditioning information,” which this paper interprets to mean any predetermined information active managers can access) that is not available to their clients. We focus in particular, on problems in which the manager uses the information to optimize mean variance performance measures that the client without the conditioning information can observe. The generic form of the optimal portfolios can be characterized by three components: a mean-variance efficient portfolio, a “hedging demand” for the benchmark portfolio, and the portfolio with global minimum conditional second moment. Such a representation generalizes Hansen and Richard (1987), and specializes Fama (1996) and Ferson, Siegel, and Xu (2006). A simulation shows that, abstracting from misspecification and estimation errors, our solutions potentially improve the measured performance by a factor of four when compared to portfolios ignoring conditioning information. When implemented realistically on currency or equity data, the optimal portfolios actually produce robust superior performance.

The problem of tracking error investing is formally posed in Roll (1992). He argues that it takes a long time to reliably measure the value-added from a fund manager, and a benchmark reduces the estimation error in portfolio performance. Starks (1987), Admati and Pfleiderer (1997), and Ou-Yang (2003) study compensation contracts

involving benchmarks and address the agency problems between the fund managers and their clients that arises because the two parties potentially have conflicts of interests. Jorion (2003) argues that active managers may not be willing to disclose their information to the clients, and ex post performance based on realized returns is very noisy. Such issues motivate the use of constraints on portfolio risk profiles. Roll (1992), Jorion (2003), Stutzer (2003), and Alexander and Baptista (2008) consider different constraints and provide analytical solutions. Brennan (1993), Cuoco and Kaniel (2007), Gómez and Zapatero (2003), Stutzer (2003), and Cornell and Roll (2005) derive market equilibrium implications of tracking error variance minimization.

The above studies raise interesting issues related to tracking error variance minimization but ignore the presence of conditioning information. Conditioning information plays a central role in modern portfolio management since a substantial portion of clients delegate their investment decisions to professional money managers in the belief that managers are better informed; see Avramov and Chordia (2006), and Bansal, Dahlquist, and Harvey (2004). This is the first study to explore the optimal portfolios of tracking error investors and their equilibrium implications when conditioning information is explicit.¹

There are alternative ways to exploit conditioning information. An active manager may pursue “conditional tracking efficiency” or CTE, where he uses the conditioning information to optimize *conditional* measures. Even if conditioning information is not explicitly stated, we can interpret the means and variances in previous studies like Roll (1992), Jorion (2003), and Stutzer (2003) as conditional moments to produce CTE solutions. This is what these authors probably have in mind. However, this paper illustrates that when there is conditioning information, an active manager may pursue “unconditional tracking efficiency” or UTE, where he uses conditioning information

¹Zhou (2008) recently introduces conditioning information to the active portfolio management context in a different problem. He revisits the fundamental law of active portfolio management by Grinold (1989) and Grinold and Kahn (2000), and considers how an informed active manager can maximize *unconditional* value-added, approximated by unconditional risk-adjusted portfolio return.

to form portfolios that optimize *unconditional* performance measures.² We argue this is a natural formulation. We call problems which ignore conditioning information altogether “no-information tracking efficiency” or NITE.

The central contribution of this paper is to develop the UTE problem. The information structure of UTE is common in practice. The portfolio manager conducting an optimization uses more information than is available to his clients. If the clients do not have conditioning information, they can only form unconditional performance measures. It is sensible that the active manager uses conditioning information to form portfolios that maximize his performance from the clients’ perspective. Our focus on unconditional measures is consistent with Dybvig and Ross (1985), Hansen and Richard (1987), Ferson and Siegel (2001), Abhyankar, Basu, and Stremme (2005), Ferson, Siegel, and Xu (2006), and Zhou (2008).

The rest of this paper is organized as follows. Section 2 provides solutions to UTE versions of the modern portfolio management problem. It also discusses the properties of the solutions, connecting them to the familiar concepts of mean-variance efficiency and hedging demands. Section 3 gives extensions. Section 4 provides empirical examples. Section 5 presents an international financial market application. Section 6 concludes the paper.

2 The Portfolio Management Problem

Consider an active manager who faces N risky assets. Suppose R is an N -dimensional vector of raw asset returns, R_b is the raw return of the benchmark with unconditional mean μ_b and unconditional variance σ_b^2 . The unconditional moments are the ones we estimate using the usual sample means and variances. At the beginning of the period, the active manager uses conditioning information Z to form the optimal portfolio

²In an Appendix, Corollaries 1 and 4 provide simple analytical illustration of the amount of unconditional inefficiency a CTE strategy produces.

weights, $x(Z)$, and the portfolio return is $R_p = x(Z)'R$.

Under UTE versions of modern portfolio management, the active manager's problem is to minimize the unconditional tracking error variance, $\mathbf{var}(R_p - R_b)$, for a given level of "alpha" $\bar{\alpha}_p$, i.e., $\mathbf{IE}(R_p - R_b) = \bar{\alpha}_p$. The active manager may also face portfolio risk constraints such as $\mathbf{cov}(R_p, R_b) = \bar{\sigma}_{bp}$ or $\mathbf{var}(R_p) = \bar{\sigma}_p^2$ for given values of $\bar{\sigma}_{bp}$ or $\bar{\sigma}_p^2$. In addition, a portfolio weight constraint which restricts the sum of the portfolio weights to be 1 may be imposed. The active manager's problem can be summarized as

$$\begin{aligned}
\min_{x(Z)} \quad & \mathbf{var}(R_p - R_b), \\
\text{s.t.} \quad & \mathbf{IE}(R_p - R_b) = \bar{\alpha}_p, \\
& \mathbf{cov}(R_p, R_b) = \bar{\sigma}_{bp} \quad \text{or} \quad \mathbf{var}(R_p) = \bar{\sigma}_p^2, \\
& x(Z)' \mathbf{1} = 1.
\end{aligned} \tag{1}$$

where $\mathbf{1}$ is an N -dimensional vector of ones. The general solution to the optimization problem (1) is provided in Proposition 1.

Proposition 1 *The general solution for the UTE portfolios has the following generic form,*

$$x(Z) = \lambda_1 x_{mv}(Z) + \lambda_2 x_h(Z) + \lambda_3 x_0(Z) \tag{2}$$

$$\equiv \lambda_1 \Phi(Z) \mu(Z) + \lambda_2 \Phi(Z) \gamma(Z) + \lambda_3 x_0(Z), \tag{3}$$

where

$$\begin{aligned}
\mu(Z) &= \mathbf{IE}(R|Z), \\
\Omega(Z) &= \mathbf{IE}(RR'|Z), \\
\gamma(Z) &= \mathbf{IE}[R(R_b - \mu_b)|Z].
\end{aligned} \tag{4}$$

and

$$\Phi(Z) = \Omega(Z)^{-1} - \frac{\Omega(Z)^{-1} \mathbf{1} \mathbf{1}' \Omega(Z)^{-1}}{\mathbf{1}' \Omega(Z)^{-1} \mathbf{1}}, \tag{5}$$

$$x_0(Z) = \frac{\Omega(Z)^{-1} \mathbf{1}}{\mathbf{1}' \Omega(Z)^{-1} \mathbf{1}}. \tag{6}$$

The parameters λ_1 , λ_2 , and λ_3 are constants, whose values depend on the portfolio constraints, but not on the realizations of the conditioning information Z .

Proof: The solution can be found using calculus of variations, as shown in the Appendix.

2.1 Interpreting the Solution

The optimal portfolio weights in Equation (2) are the sum of three terms, similar to Fama’s (1996) representation of multifactor minimum variance portfolios and Ferson, Siegel, and Xu’s (2006) results for mimicking portfolios with conditioning information. Motivated by Ferson and Siegel (2001) and Ferson, Siegel, and Xu (2006), we interpret the zero net investment portfolios $x_{mv}(Z)$ and $x_h(Z)$ as the “mean-variance component” and the “hedging demand component,” respectively, and the normal portfolio $x_0(Z)$ as the “global minimum conditional second moment portfolio weight vector.”

Hansen and Richard (1987) show that the return on an unconditional minimum variance portfolio, R_{umv} , can be decomposed as

$$R_{umv} = \lambda_e R_e + R_n, \quad (7)$$

where R_e is a zero net investment portfolio, λ_e is a constant, and R_n has portfolio weights that sum to 1. Our generic form of the UTE solution produces³

$$\begin{aligned} R_p &= x(Z)'R = \lambda_1 x_{mv}(Z)'R + \lambda_2 x_h(Z)'R + x_0(Z)'R \\ &\equiv \lambda_1 R_{mv} + \lambda_2 R_h + R_0. \end{aligned} \quad (8)$$

Since $x_0(Z)'1 = 1$, $x_{mv}(Z)'1 = 0$, $x_h(Z)'1 = 0$, and λ_1 , λ_2 are constants, Equation (8) adds an additional term, R_h , to the Hansen and Richard (1987) representation. This is a special case of Ferson, Siegel, and Xu’s (2006) “ $K + 2$ fund separation,”

³Here since $x_{mv}(Z)$ and $x_h(Z)$ are zero net investment portfolios, $x(Z)'1 = 1$ if and only if $\lambda_3 = 1$. Special cases are discussed below.

where K is the number of relevant risk factors to hedge in the mimicking portfolio construction context. In our case, $K = 1$. Thus, a tracking error investor ends up with an induced “hedging demand” relative to the benchmark portfolio.

The component $\lambda_1 x_{mv}(Z)$ captures mean-variance investing behavior. It is interesting to compare this component with Ferson and Siegel’s (2001) unconditionally mean-variance efficient portfolio, the portfolio minimizing unconditional portfolio variance for a given unconditional expected return. For a given expected return restriction, the solutions are not identical.⁴ The difference, in Lagrangian multiplier rather than in conditional mean, is induced by tracking error variance minimization. Intuitively, to maintain the same expected return as the unconditionally efficient portfolio, the UTE portfolio has to adjust its position to offset the expected return generated by the hedging demand component.

The term $\lambda_2 x_h(Z)$ captures the “hedging demand” induced by the benchmark. Intuitively, to minimize tracking error, the manager tends to hedge fluctuations in the benchmark. Appendix A.5 shows that hedging demand for the benchmark portfolio can induce a priced risk factor in equilibrium.

Hedging demand does not always matter in modern portfolio management. Consider two special cases: $\lambda_2 = 0$ or $\gamma(Z) = 0$. In either case, the UTE portfolio will reduce to the unconditionally efficient portfolio of Ferson and Siegel (2001). $\gamma(Z) = 0$ is unlikely, because it implies that all portfolios have zero covariance with benchmark. There is no way to hedge in this case, even the manager has preference for hedging benchmark.⁵

The case $\lambda_2 = 0$ can happen when a portfolio risk constraint is not binding. One possibility is that the active manager has preference unaffected by the portfolio risk

⁴Note imposing an alpha constraint is equivalent to imposing $\mathbf{IE}(R_p) = \bar{\alpha}_p + \mu_b$.

⁵An extreme case is when the benchmark is a risk free rate. In this case, the tracking error variance minimization problem reduces to the classical minimum variance problem, and the optimal portfolio is the same as Ferson and Siegel’s (2001) unconditionally efficient portfolio.

constraint. Another possibility is that the constraint carries a zero shadow price.⁶ In either situation, the shadow price of the portfolio risk constraint is zero, and no hedging demand is induced by the benchmark.

3 Extensions

The generic solution (3) can accommodate different portfolio constraints. This is important because portfolio constraints are common in practice, and can be imposed to improve the out-of-sample performance of classical mean-variance portfolio solutions.⁷ However, the literature on unconditional portfolio efficiency with conditioning information has not developed portfolio constraints. This paper studies portfolio constraints and presents evidence of their effects on the out-of-sample performance of UTE strategies. We consider constraints on systematic (beta) risk and on total portfolio variance, without a risk free asset. In the absence of conditioning information, these problems reduce to the formulations in Roll (1992), Jorion (2003), and Alexandar and Baptista (2008). We allow a risk free asset and a fixed benchmark. A special case reduces to Stutzer's (2003) problem. We then discuss direct upper and lower bound constraints on portfolio weights, e.g. Jagannathan and Ma (2003), and relate our solutions to the norm-constrained portfolios of DeMiguel, Garlappi, Nogales, and Uppal (2009). Finally, we consider a problem in which clients may have

⁶For example, in Proposition 2 (no risk free rate, with a portfolio beta constraint), if we set the constraint

$$\bar{\beta}_p = \frac{1}{\sigma_b^2} \left[\frac{(\bar{\alpha}_p + \mu_b - \mu_0)\psi_2}{\psi_1} + \sigma_{b0} \right], \quad (9)$$

then $\lambda_2 = 0$. Similarly, if we set

$$\bar{\sigma}_p^2 = \frac{(\bar{\alpha}_p + \mu_b - \mu_0)^2}{\psi_1} + \Omega_0 - (\bar{\alpha}_p + \mu_b)^2, \quad (10)$$

in Proposition 3 (no risk free rate, with a total portfolio variance constraint), then $\lambda_2 = 0$ as well.

⁷See, for example, Frost and Savarino (1988), Chan, Karceski, and Lakonishok (1999), Jagannathan and Ma (2003), DeMiguel, Garlappi, and Uppal (2007), Kan and Zhou (2007), and Alexander, Baptista, and Yan (2008), and DeMiguel, Garlappi, Nogales, and Uppal (2009).

a subset of the active manager's information.

3.1 No Risk Free Asset, Constraints on Portfolio Risk

Consider an active manager who faces N risky assets, no risk free asset and a restriction that the portfolio beta on the benchmark equals a target beta $\bar{\beta}_p$, i.e., $\mathbf{cov}(R_p, R_b)/\sigma_b^2 = \bar{\beta}_p$. The beta constraint is equivalent to setting $\bar{\sigma}_{bp} = \bar{\beta}_p\sigma_b^2$ in (1). This constraint is sensible when the clients want to restrict the systematic risk of the portfolio. The active manager's problem is summarized as

$$\begin{aligned} \min_{x(Z)} \quad & \mathbf{var}(R_p - R_b) \\ \text{s.t.} \quad & \mathbf{IE}(R_p - R_b) = \bar{\alpha}_p, \mathbf{cov}(R_p, R_b) = \bar{\beta}_p\sigma_b^2, x(Z)' \mathbf{1} = 1. \end{aligned} \tag{11}$$

Roll (1992) solves this problem without taking into account the conditioning information. The solution to the optimization problem (11) is provided in Proposition 2 in the Appendix. It is a special case of (3) with particular values for the λ 's.

Jorion (2003) considers a total portfolio risk constraint, i.e., the active manager has to maintain $\mathbf{var}(R_p) = \bar{\sigma}_p^2$. The total risk constraint can be translated into a constraint on value-at-risk; see Jorion (2000) and Alexandar and Baptista (2008). The UTE version of this problem is summarized as

$$\begin{aligned} \min_{x(Z)} \quad & \mathbf{var}(R_p - R_b) \\ \text{s.t.} \quad & \mathbf{IE}(R_p - R_b) = \bar{\alpha}_p, \mathbf{var}(R_p, R_b) = \bar{\sigma}_p^2, x(Z)' \mathbf{1} = 1. \end{aligned} \tag{12}$$

The solution to the UTE version of Jorion's (2003) problem with conditioning information is presented in Proposition 3 in the Appendix. Once again the solution is a special case of (3).

3.2 No Risk Free Asset and No Portfolio Risk Constraint

If we relax the constraints on portfolio risk, the active manager's problem reduces to the following problem

$$\begin{aligned} \min_{x(Z)} \quad & \mathbf{var}(R_p - R_b) \\ \text{s.t.} \quad & \mathbf{IE}(R_p - R_b) = \bar{\alpha}_p, x(Z)' \mathbf{1} = 1. \end{aligned} \tag{13}$$

Roll (1992) considers this problem but again no conditioning information is used. The solution is provided in Proposition 4 in the Appendix.

3.3 Exogenous Benchmark Portfolio Weights

Suppose that the benchmark portfolio weights are exogenous, observable, and fixed at the beginning of the holding period. The weights may vary from period to period, provided that they are in the manager's information set. The observability of fixed exogenous benchmark portfolio weights is assumed in Roll (1992), Jorion (2003), and Stutzer (2003), for example. This implies that the active manager is able to hold or replicate the benchmark. Actual benchmark weights may also evolve during the measurement period. We do not model the effects of intra-period variation in the benchmark weights.⁸

First suppose there is no risk free asset and the benchmark portfolio weight vector is $x_b(Z)$. Define $w(Z) \equiv x(Z) - x_b(Z)$, where $w(Z)$ is a zero net investment portfolio weight vector. We can define the portfolio alpha, beta, variance, and tracking error variance in terms of $w(Z)$ and the conditional moments in (4). The active manager's

⁸In the case that the benchmark is difficult to replicate but an active portfolio manager can engage in a futures contract on the benchmark, an exchange-traded fund indexing the benchmark, or a managed portfolio mimicking the benchmark. Treating a feasible mimicking portfolio as an investable asset, the active manager can apply our solutions as an approximation.

problem is

$$\begin{aligned}
\min_{w(Z)} \quad & \mathbf{var}(R_p - R_b) = \mathbf{IE}[w(Z)'\Omega(Z)w(Z)] - \{\mathbf{IE}[w(Z)'\mu(Z)]\}^2, \\
\text{s.t.} \quad & \mathbf{IE}[w(Z)'\mu(Z)] = \bar{\alpha}_p, \\
& \mathbf{cov}(R_p, R_b) = \bar{\sigma}_{bp} \quad \text{or} \quad \mathbf{var}(R_p) = \bar{\sigma}_p^2, \\
& w(Z)'\mathbf{1} = 0,
\end{aligned} \tag{14}$$

for pre-determined $\bar{\alpha}_p$ and $\bar{\sigma}_{bp}$ or $\bar{\sigma}_p^2$, where the choice variable is $w(Z)$. As in Proposition 1, the solution has the following generic form,

$$\begin{aligned}
x(Z) &= w(Z) + x_b(Z), \\
w(Z) &= \lambda_1 x_{mv}(Z) + \lambda_2 x_h(Z) + \lambda_3 x_0(Z).
\end{aligned} \tag{15}$$

The solutions are provided in Propositions 5–7 in the Appendix, which solve for the relevant values of the λ 's.

3.4 The Presence of a Risk Free Asset

Now we consider examples with a risk free asset with return R_f . The risk free rate can be *conditionally* risk free, i.e. varying from period to period, and observed at the beginning of each period. In this formulation of the problem the active manager faces a benchmark portfolio with return in excess of the risk free rate, i.e., $r_b \equiv R_b - R_f$, and uses conditioning information, Z , to choose an unrestricted weights vector $x(Z)$ of the N risky assets, investing the rest of the investable funds in the risk free asset or borrowing at the rate R_f . Suppose $r \equiv (R - R_f\mathbf{1})$ is an N -vector of risky asset returns in excess of the risk free rate, and r_b is the benchmark return in excess of R_f . The observed returns on the investor's portfolio will be $R_p = x(Z)'r + R_f \equiv r_p + R_f$, where r_p is the excess return of the active portfolio.

The active manager's problem becomes

$$\begin{aligned}
\min_{x(Z)} \quad & \mathbf{var}(r_p - r_b) \\
\text{s.t.} \quad & \mathbf{IE}(r_p - r_b) = \bar{\alpha}_p \\
& \mathbf{cov}(r_p, r_b) / \mathbf{var}(r_b) = \bar{\beta}_p \quad \text{or} \quad \mathbf{var}(r_p) = \bar{\sigma}_p^2.
\end{aligned} \tag{16}$$

The solutions are provided in Propositions 8 and 9 in the Appendix.⁹ Stutzer (2003) solves a special case in which portfolio risk constraints and conditioning information are absent.

3.5 Portfolio Weight Constraints

Direct constraints on portfolio weights are common in practice and well studied in the academic literature. Empirical studies show that portfolio efficiency is gained out of sample when certain portfolio weight constraints are imposed.¹⁰ We can consider a portfolio weight lower bound $\underline{x} = (\underline{x}^1, \dots, \underline{x}^N)'$, where \underline{x}^j corresponds to the minimum weight on asset j . A lower bound becomes no short-sale constraint when $\underline{x} = 0$. Upper bounds can be defined similarly as $\bar{x} = (\bar{x}^1, \dots, \bar{x}^N)'$.¹¹

To impose the above constraints in the UTE problem, we add

$$x_p^j(Z) \geq \underline{x}^j, \tag{17}$$

$$x_p^j(Z) \leq \bar{x}^j, \tag{18}$$

$j = 1, \dots, N$. For simplicity, we ignore portfolio risk constraints for now. The optimal solution to, problem (13) for example, along with constraints (17) and (18), has the

⁹Transforming the N risky asset returns into returns in excess of the benchmark and applying the unconditionally efficient portfolio in Theorem 2 of Ferson and Siegel (2001) yields the UTE solution in Proposition 9(iii). An assumption of this approach is that the active manager can hold or replicate the benchmark portfolio.

¹⁰See Frost and Savarino (1988), Jagannathan and Ma (2003), and DeMiguel, Garlappi, and Uppal (2007), among others.

¹¹Jagannathan and Ma (2003) set $\underline{x} = 0$ and $\bar{x}^1 = \dots = \bar{x}^N$.

first order conditions of

$$\Omega(Z)x_p(Z) = \lambda_1\mu(Z) + \gamma(Z) + \lambda_3(Z)\mathbf{1} + \underline{\Lambda}(Z) - \overline{\Lambda}(Z) \quad \text{almost surely,} \quad (19)$$

$$\mathbf{E}[x_p(Z)'\mu(Z)] = \bar{\alpha}_p + \mu_b, \quad (20)$$

$$\underline{\Lambda}^j(Z) \geq 0, \quad \text{or} \quad \underline{\Lambda}^j(Z) = 0 \quad \text{if} \quad x_p^j(Z) > \underline{x}^j, \quad (21)$$

$$\overline{\Lambda}^j(Z) \geq 0, \quad \text{or} \quad \overline{\Lambda}^j(Z) = 0 \quad \text{if} \quad x_p^j(Z) < \overline{x}^j, \quad (22)$$

where $\underline{\Lambda}(Z)$ and $\overline{\Lambda}(Z)$ are N -vectors of shadow prices of constraints (17) and (18), with typical elements $\underline{\Lambda}^j(Z)$ and $\overline{\Lambda}^j(Z)$, respectively. The solution has the form

$$x_p(Z) = \lambda_1\Phi(Z)\mu(Z) + \Phi(Z)\gamma(Z) + x_0(Z) + \Phi(Z)[\underline{\Lambda}(Z) - \overline{\Lambda}(Z)], \quad (23)$$

subject to (21) and (22). Similar to DeMiguel, Garlappi, and Uppal (2007), we can define an adjusted conditional return vector

$$\mu^\dagger(Z) = \mu(Z) + \frac{\underline{\Lambda}(Z) - \overline{\Lambda}(Z)}{\lambda_1}, \quad (24)$$

and the above solution (23) becomes the familiar generic form (3). As a result, we have a “shrinkage in conditional means” interpretation: assuming $\lambda_1 > 0$, the lower (upper) bound constraint is more likely to be binding when conditional expected return is low (high), and the adjusted conditional means tilt toward the average.

An alternative shrinkage interpretation focuses on shrinkage in the conditional second moment matrix of returns, generalizing Jagannathan and Ma (2003). Define

$$\Omega^\dagger(Z) = \Omega(Z) + [\mathbf{1}\overline{\Lambda}(Z)' + \overline{\Lambda}(Z)\mathbf{1}'] - [\mathbf{1}\underline{\Lambda}(Z)' + \underline{\Lambda}(Z)\mathbf{1}'], \quad (25)$$

and we will have¹²

$$\Omega^\dagger(Z)x_p(Z) = \lambda_1\mu(Z) + \gamma(Z) + [\lambda_3(Z) + \overline{\Lambda}(Z)'\overline{x} - \underline{\Lambda}(Z)'\underline{x}]\mathbf{1} \quad (26)$$

$$\equiv \lambda_1\mu(Z) + \gamma(Z) + \lambda_3^\dagger(Z)\mathbf{1}, \quad (27)$$

which corresponds to (87). In other words, $x_p(Z)$ solves a UTE problem in which conditional second moment matrix is $\Omega^\dagger(Z)$ and no portfolio weight upper or lower

¹²We utilize the relations $\overline{\Lambda}(Z)'[x_p(Z) - \overline{x}] = 0$, $\underline{\Lambda}(Z)'[x_p(Z) - \underline{x}] = 0$, $x_p(Z)'\mathbf{1} = 1$, and (19).

bound constraint is imposed. The optimal UTE solution has the familiar generic form, with $\Phi(Z)$ replaced by

$$\Phi^\dagger(Z) \equiv \Omega^\dagger(Z)^{-1} - \frac{\Omega^\dagger(Z)^{-1} \mathbf{1} \mathbf{1}' \Omega^\dagger(Z)^{-1}}{\mathbf{1}' \Omega^\dagger(Z)^{-1} \mathbf{1}}. \quad (28)$$

Since the upper (lower) bound constraint is more likely to be binding when conditional second moment of an asset is low (high), we have a shrinkage in conditional second moments.

3.6 Relation to Portfolio Norm Constraints

DeMiguel, Garlappi, Nogales, and Uppal (2009) derive a general class nesting a number of portfolio weight constraints. When the UTE problem is considered, their “1-norm” constraint imposes

$$\|x_p(Z)\|_1 \equiv |x_p(Z)|' \mathbf{1} \leq d_1. \quad (29)$$

If we set $d_1 = 1$, the above problem reduces to a shortsale-constrained problem,¹³ and the solution is in (23) and (21) in which $\underline{x} = 0$ and $\bar{\Lambda}(Z)$ is turned off.

DeMiguel, Garlappi, Nogales, and Uppal (2009) also consider an “ \mathcal{A} -norm” constraint, which imposes

$$\|x_p(Z)\|_{\mathcal{A}} \equiv x_p(Z)' \mathcal{A} x_p(Z) \leq d_{\mathcal{A}}. \quad (30)$$

The problem considered in (12) is a special case in which we only consider equality constraint and set \mathcal{A} to unconditional covariance matrix of returns and $d_{\mathcal{A}} = \bar{\sigma}_p^2$.

3.7 Partial Observability of Conditioning Information by the Clients

The UTE portfolios we develop are based on the assumption that the clients do not have access to the conditioning information. In practice it may be possible that some

¹³See Proposition 1 of DeMiguel, Garlappi, Nogales, and Uppal (2009).

sophisticated clients observe part of the active manager's conditioning information and compensate him based on the *conditional* performance measure implied by their information. Suppose the clients have information $z \subset Z$. The active manager's problem becomes a special CTE problem defined over z ,

$$\begin{aligned} \min_{x(Z)} \quad & \mathbf{var}(R_p - R_b|z), \\ \text{s.t.} \quad & \mathbf{IE}(R_p - R_b|z) = \bar{\alpha}_p(z), \quad x(Z)' \mathbf{1} = 1, \end{aligned} \tag{31}$$

and possibly other portfolio risk constraints. The generic form of the optimal portfolio becomes

$$x(Z) = \lambda_1(z)x_{mv}(Z) + \lambda_2(z)x_h(Z) + \lambda_3x_0(Z), \tag{32}$$

where $\lambda_1(z)$ and $\lambda_2(z)$ depend on the clients' information. The values of the conditional $\lambda(z)$'s may be found as straightforward generalizations of our UTE solutions. However, such solutions are more difficult to implement than the UTE portfolios, given that z is heterogeneous across clients and hard to measure.

4 Empirical Examples

We consider two special cases to develop intuition about the relation between the active portfolio weight function and the conditioning information. In the first case we impose no portfolio risk constraint. In the second example we examine the effects of a portfolio risk constraint.

4.1 Behavior of the UTE Portfolio Weights

Suppose the active manager has only two risky assets a and b with raw returns R_a and R_b , respectively, and a risk free asset with return R_f . Assume asset b is the benchmark, i.e., the benchmark portfolio weights are $x_b = (0, 1)'$. The conditioning information is a scalar random variable Z .

Define r_a and r_b as the asset returns in excess of risk free rate and suppose that the conditional expectations of r_a and r_b are linear in Z , and that the covariance matrix of r_a and r_b conditional on Z is constant¹⁴,

$$\mathbf{E} \begin{bmatrix} r_a \\ r_b \end{bmatrix} \Big| Z = \begin{bmatrix} \delta_{0a} + \delta_{1a}Z \\ \delta_{0b} + \delta_{1b}Z \end{bmatrix}, \mathbf{var} \begin{bmatrix} r_a \\ r_b \end{bmatrix} \Big| Z = \begin{bmatrix} \Sigma_{aa} & \Sigma_{ab} \\ \Sigma_{ba} & \Sigma_{bb} \end{bmatrix}. \quad (33)$$

Since there is a risk free asset and the benchmark portfolio weights are known, Proposition 9(iii) gives the active portfolio weight function as $x(Z) = w(Z) + x_b$, where

$$\begin{aligned} w(Z) &= \lambda_1 \Omega(Z)^{-1} \mu(Z) \\ &= \frac{\lambda_1}{|\Omega(Z)|} \begin{bmatrix} \mathbf{E}(r_b^2|Z)\mathbf{E}(r_a|Z) - \mathbf{E}(r_b|Z)\mathbf{E}(r_a r_b|Z) \\ \mathbf{E}(r_a^2|Z)\mathbf{E}(r_b|Z) - \mathbf{E}(r_a|Z)\mathbf{E}(r_a r_b|Z) \end{bmatrix}, \end{aligned} \quad (34)$$

and

$$|\Omega(Z)| = \mathbf{E}(r_a^2|Z)\mathbf{E}(r_b^2|Z) - [\mathbf{E}(r_a r_b|Z)]^2. \quad (35)$$

In Equation (34) $|\Omega(Z)|$ is a function of Z^4 , and both $\mathbf{E}(r_b^2|Z)\mathbf{E}(r_a|Z) - \mathbf{E}(r_b|Z)\mathbf{E}(r_a r_b|Z)$ and $\mathbf{E}(r_a^2|Z)\mathbf{E}(r_b|Z) - \mathbf{E}(r_a|Z)\mathbf{E}(r_a r_b|Z)$ are functions of Z^3 , so we conclude that $w(Z) \rightarrow 0$ and thus $x(Z) \rightarrow x_b$ as $Z \rightarrow \pm\infty$. This result generalizes Ferson and Siegel (2001), who show that the unconditionally efficient portfolio has a conservative response to strong signals. Ferson, Siegel, and Xu (2006) use a similar characterization. The active manager becomes very conservative when the signal is strong – he simply holds the benchmark in the limit. The intuition is that when the conditioning information is extreme, the active manager may easily satisfy the alpha constraint by holding a tiny $w(Z)$, and at the same time minimize the tracking error variance by investing most of the fund in the benchmark.

¹⁴Such a formulation corresponds to a linear predictive model with conditional homoskedasticity, as would be implied by joint normality of (r_a, r_b, Z) .

Figure 1 depicts the relation between the portfolio weights in excess of the benchmark, i.e., $w(Z)$, and the strength of (standardized) signal.¹⁵ Here Z is positively correlated with r_a and negatively correlated with r_b . Consider the case when $Z > 0$. As Z increases from zero, the portfolio weight on asset a (dotted curve) is positive and increases initially. After hitting its peak where the standardized Z is 2.5, the weight on asset a shrinks quickly and approaches zero. The portfolio weight on asset b in excess of the benchmark (solid curve) decreases initially, and then increases and approaches zero after reaching its trough at which the standardized Z is 3.3. When $Z < 0$, the pattern reverses.

4.2 Portfolio Risk Constraints on UTE Weights

We illustrate the effects of portfolio risk constraints in the previous example. The portfolio risk constraint is $\bar{\beta}_p = 1.5$. Proposition 9(i) or 9(ii) gives the optimal portfolio weights as $x(Z) = w(Z) + x_b$, where

$$w(Z) = \frac{\lambda_1 - \lambda_2 \mu_b}{|\Omega(Z)|} \begin{bmatrix} \mathbf{IE}(r_b^2|Z)\mathbf{IE}(r_a|Z) - \mathbf{IE}(r_b|Z)\mathbf{IE}(r_a r_b|Z) \\ \mathbf{IE}(r_a^2|Z)\mathbf{IE}(r_b|Z) - \mathbf{IE}(r_a|Z)\mathbf{IE}(r_a r_b|Z) \end{bmatrix} + \lambda_2 x_b, \quad (36)$$

Under the assumption in (33), the first term of the right hand side of $w(Z)$ approaches $(0, 0)'$ as $Z \rightarrow \pm\infty$, and the second term is a fraction of the benchmark portfolio, proportional to the shadow price of the portfolio risk constraint. Thus $x(Z) \rightarrow (1 + \lambda_2)x_b$ when the active manager receives a very strong signal.

The active manager does not hold the benchmark given an extreme signal, when a portfolio risk constraint is imposed. Figure 2 depicts the influences of the constraint on the portfolio weights in excess of the benchmark, using the same set of parameters as in the first example and adding a $\bar{\beta}_p = 1.5$ constraint. While the weight on asset a

¹⁵We use a currency portfolio return (described below) to draw this picture. The Japanese Yen is asset a , UK Pounds are asset b , and the conditioning information is the lagged return of Japanese Yen. δ 's are estimated by least squares, and the conditional covariance matrix is the covariance matrix of the residuals, scaled by 1%. $\bar{\alpha}_p$ is set to 5% annually.

(dotted curve) displays similar limiting behavior to Figure 1, the portfolio weight on asset b (solid curve) does not approach zero even when Z gets very large. Instead, it approaches $\lambda_2 = 0.702$. The reason is to meet the portfolio risk constraint. Here the target beta is 1.5, while the realized beta is 1.702.

5 Empirical Analysis

We assess the performance of the tracking efficient portfolios, with or without conditioning information, in this section. We provide an application to international financial data. We choose a currency portfolio management example for several reasons. First, conditioning information is likely to be important in currency trading given the large swings in conditional risk premiums (e.g. Hansen and Hodrick (1980) and Fama (1984)). Second, the currency market is very large and liquid. Microstructure and default risk issues are relatively minor.¹⁶ Third, it is relatively easy to short currencies. The models without constraints may therefore be more realistic in currency markets where short selling is relatively inexpensive. Fourth, there is room for active management since many currency market participants, such as central banks, equity managers, tourists, and international businesses, do not concentrate on exchange rate exposure. Note that, our approach can be applied to other asset classes as well; an application to equity portfolio management is presented in Section 5.6.3.

Suppose that a US-based active manager speculates in currencies and forms currency portfolios using four major currencies: US Dollars, Japanese Yen, UK Pounds, and Euro. The manager faces an unmanaged currency benchmark portfolio of the

¹⁶For example, the bid-ask spread of Eurocurrencies could be as low as 15 basis points (Grabbe, 1996).

above four currencies¹⁷, defined by $R_{b,t} = x'_{b,t-1}R_t$, where $x_{b,t-1}$ is a vector of exogenous benchmark weights known at $t - 1$, and the return on currency j is

$$R_t^j = \frac{s_t^j(1 + y_{t-1}^j)}{s_{t-1}^j} - 1, \quad (39)$$

where s_t^j is the spot exchange rate for currency j , and y_{t-1}^j is the one-period Eurocurrency deposit rate or Treasury rate for currency j , observed at time $t - 1$.¹⁸

We use monthly data for spot and forward exchange rates, and deposit rates for the above four currencies for the period from August 1978 to January 2005. We deflate all returns by US inflation rates¹⁹ so that all returns are risky real returns. We annualize monthly real returns, and report annual rates by multiplying them by 12.

¹⁷In practice, currency benchmarks are often defined in the following fashion,

$$R_{b,t} = x'_{b,t-1}[(1 - \omega_h)R_t^{uc} + \omega_h R_t^c], \quad (37)$$

where $x_{b,t-1}$ depends on relative equity market value of each country, R_t^{uc} is a vector of uncovered returns, defined in (39), R_t^c is a vector of covered returns, defined as

$$R_t^{c,j} = \frac{f_{t-1}^j(1 + y_{t-1}^j)}{s_{t-1}^j} - 1, \quad (38)$$

where f^j is the forward exchange rate for currency j . Conventionally the pre-determined scalar ω_h is called ‘‘hedge ratio,’’ usually set to 0, 0.5, or 1 in practice. The case we consider in our empirical study is a special case where ω_h is set to zero. While not tabulated here, the UTE portfolios remain advantageous regardless the choice of ω_h . This is because the predictable component in R_b is increasing with ω_h .

¹⁸We acquire exchange rate data for Japanese Yen from the Federal Reserve Bank of St. Louis website, and the UK Pounds and synthetic Euro exchange rates from Datastream. The Eurodollar, Euroyen, and Europound deposit rates are also available in Datastream. We construct the monthly interest rate for synthetic Euro using the same method Datastream uses to calculate synthetic Euro interest rate, which is a GDP-weighted average of Euro region interest rates. Should some interest rates be unavailable, we rescale the other weights so that they sum to one.

¹⁹We define the inflation rate as the growth rate of the not-seasonally-adjusted Consumer Price Index for all urban consumers (all items less food and energy), available from Federal Reserve Bank of St. Louis website.

Motivated by Hansen and Hodrick (1980), Bilson (1981), and Fama (1984), we consider the following conditioning information: past *forecast errors*, defined as the difference between the spot exchange rate and the lagged forward exchange rate²⁰; the *forward premium*, defined as the difference between the logarithms of the forward exchange rate and the contemporaneous spot exchange rate; past *depreciation rates*, defined as the difference between the logarithms of the spot exchange rate and the lagged spot exchange rate; and *lagged real dollar returns* on the individual currencies.

5.1 Estimation

We describe the estimation of the conditional moments in this section. As in the previous section, we assume a very simple and conservative formulation for the return generating process: Suppose returns are linear in predictor variables and the conditional covariance matrix is constant.²¹

Let R_t^j denote the returns on currency j at time t . The conditioning information available to the active manager at time t is Z_{t-1} . The conditional expected return for currency j , $\mu^j(Z_{t-1}) = \mathbf{E}(R^j|Z_{t-1})$, is the fitted value of the following time series regression,

$$R_t^j = Z_{t-1}'\delta^j + \varepsilon_t^j. \quad (40)$$

The conditional second moment matrix $\Omega(Z)$ in the presence of conditioning information is

$$\Omega(Z_{t-1}) = \Sigma^\varepsilon + \mu(Z_{t-1})\mu(Z_{t-1})', \quad (41)$$

²⁰All forward exchange rate data, except for synthetic Euro, are from Datastream. We construct the implied forward exchange rates by assuming covered interest rate parity holds. We use similar methods to construct the forward exchange rate for other currencies whenever explicit forward rate data are missing.

²¹Such a formulation is conservative in the sense that we model only a first order effect for conditional means and assume conditional homoskedasticity for the covariance matrix estimator. As described in a later subsection, a factor model capturing the first two moments of currency returns provides substantial improvement to the UTE portfolios.

where Σ^ε , assumed to be constant over time, is the covariance matrix of the ε_t^j 's in (40). $\Omega(Z_{t-1})$ is time-varying due to the time-varying nature of $\mu(Z_{t-1})$. Estimators δ and Σ^ε are the maximum likelihood estimators under joint normality of R and Z .

We assume $x_{b,t-1}$ is known throughout our empirical study. With the knowledge of $\mu(Z)$ and $\Omega(Z)$, we infer $\gamma(Z)$ by $\gamma(Z) = \Omega(Z)x_b - \mu_b\mu(Z)$. With $\mu(Z)$, $\gamma(Z)$, and $\Omega(Z)$, we estimate the portfolio parameters μ_0 , σ_{b0} , Ω_0 , ψ 's, and η 's by taking the sample means.

5.2 In-Sample Predictability and Model Selection

For in-sample study, we use the sample from October 1978 to December 1998, a total of 243 observations. The Euro was formally introduced in January 1999, and we leave the post-1998 data, 73 observations, for an out-of-sample study.

Time series regressions like (40) of currency returns on all the instruments jointly suggest that the instruments have explanatory power.²² To mitigate overfitting in the out-of-sample experiments, we consider a parsimonious specification in which every predictor variable has joint explanatory power for all currencies.²³ The final instruments are a constant, the depreciation rates of the Japanese Yen and UK Pounds, the lagged returns of the Japanese Yen, UK Pounds, and US Dollars. These instruments produce p -values below 0.2% in joint tests of significance with White's (1980) correction, but the adjusted R^2 is small; typically only half of one percent. Presumably in practice some active managers could find conditioning information that works better. If we find that the conditioning information in the examples is important, it is likely to be more important in practice.

²²The estimation results are available upon request.

²³We use the following model selection procedure. We use GMM to estimate (40) for all assets simultaneously. For each predictor variable Z_i , we test the joint hypothesis of $\delta_i^j = 0$ for all i , where Z_i and δ_i^j are the i -th element of Z and δ_j , respectively. We eliminate Z_i if the δ_i^j 's are not jointly significant. Each equation is forced to have the same set of instruments.

5.3 Potential Benefits of the UTE Portfolios

This section explores the potential advantage of our unconditionally tracking efficient (UTE) solutions over the no-information tracking efficient (NITE) and the conditionally tracking efficient (CTE) portfolios.²⁴ The comparison abstracts from misspecification and estimation errors of the statistical moments. The reason we want to abstract these issues is to allow the active manager to use the “correct” models for the conditional moments in the simulations (i.e. the same ones that generate the simulated data).

We abstract from these issues because in the simulation we are able to observe a large simulated data set and know its data generating process, we can use the correctly specified models for the conditional moments and estimate the parameters precisely.

We simulate 1,000 paths, each of them containing an artificial data set of 12,000 observations. This is equivalent to the number of monthly observations in 1,000 years.²⁵ Since the parameters of the data generating process are fixed and the manager knows the structure of the data generating process, the maximum likelihood estimates

²⁴We consider a special case of CTE solutions which can be obtained by reinterpreting all the moments in NITE solutions as conditional moments.

²⁵We first use a block bootstrap to resample the instruments and the growth rate of equity market values in pairs, creating an artificial data set of 12,000 observations. The benchmark weights can be calculated by simulated equity market values: We first randomly draw an observation from the equity market value data and treat it as the initial value for the simulated series, then apply the simulated growth rates to that initial value and then construct the whole artificial series for equity values. Then we form the returns for individual currencies and the benchmark, using the return generating process in the parsimonious regression model (using the model selection procedure in footnote 23) for first moment and a constant covariance matrix for second moment. When estimating the conditional moments, we use exactly the same specification used to calibrate the simulation. The unconditional moments and other parameters can be obtained by taking sample averages. Then we form the NITE, the CTE and the UTE portfolios and compute the performance measures. The exercise is repeated 1,000 times.

that he uses will essentially be at the probability limits.

The performance of the active portfolios is summarized in Table I. We first consider the case where there is no portfolio risk constraint and no riskless rate. The target alphas are set to 1%, 3%, and 5% per year. For each portfolio, we compute the difference between the active portfolio return and the benchmark return. We report the sample average of the difference between the two returns, “alpha,” and the standard error of the difference, “tracking error volatility.” We also report the statistic summarizing the ratio of alpha to tracking error volatility, the “information ratio.”²⁶

The advantage of using conditioning information to form dynamic portfolios is obvious, and the dominance of the UTE portfolio is substantial. While Table I shows that all of the active portfolios on average achieve target alphas precisely, the tracking error volatilities of the NITE portfolios are larger than that of the UTE and the CTE portfolios. With an information ratio of 0.358, the UTE portfolios almost double that of the CTE portfolios (0.193) and are four times better than that of the NITE portfolios (0.072).²⁷ None of the 1,000 simulation paths presents NITE portfolios or CTE portfolios with smaller tracking error volatilities or greater information ratios than the UTE portfolios.

The potential benefit of the UTE portfolios can be visualized by the tracking efficiency frontiers, defined as the graphical relation between the alphas of the portfolios and their tracking error volatilities. Figure 3 shows the tracking efficiency frontiers, averaged across the 1,000 simulation trials. Clearly the CTE portfolios dominate the NITE portfolios, and UTE portfolios dominate the NITE and the CTE portfolios, in the sense that for any given alpha, the superior portfolios have much lower tracking

²⁶Note this is different from Treynor and Black’s (1973) “appraisal ratio,” i.e. Jensen’s (1969) alpha divided by residual volatility, which is also sometimes referred as information ratio (e.g. Grinold and Kahn, 1999).

²⁷Note in this case, since the Lagrangian multipliers are proportional to $\bar{\alpha}_p$, the information ratio is invariant to $\bar{\alpha}_p$ for non-zero alphas. See Corollary 2.

error volatility than do the inferior portfolios.

5.4 In-Sample Performance

We next turn to the data to evaluate the performance of the tracking efficient portfolios in a realistic finite sample. First we use the data for the period October 1978 to December 1998 to estimate unconditional and conditional moments, assuming linear conditional first moment and constant conditional covariance structure, and apply the optimal portfolio solutions to the data.

Besides the information ratio, empirically we also consider the following measures: the first one is an “incentive,” expressed as the additional performance required by the NITE portfolio such that it is on par with a dynamic portfolio. Suppose uninformed clients are indifferent between the performance of an enhanced NITE portfolio and the performance of an actively managed portfolio when we equate their unconditional information ratios:

$$\frac{\alpha_p}{\sqrt{\text{var}(R_p - R_b)}} = \frac{\alpha_{NITE} + \mathcal{I}_p}{\sqrt{\text{var}(R_{NITE} - R_b)}}. \quad (42)$$

Solving,

$$\mathcal{I}_p = y_p \alpha_p - \alpha_{NITE}, \quad (43)$$

$$y_p \equiv \frac{\sqrt{\text{var}(R_{NITE} - R_b)}}{\sqrt{\text{var}(R_p - R_b)}}, \quad (44)$$

which has an equivalent representation

$$\mathcal{I}_p = [y_p R_p + (1 - y_p) R_b] - R_{NITE} \quad (45)$$

$$\equiv R_H - R_{NITE}. \quad (46)$$

Therefore \mathcal{I}_p is a (vertical) distance measure extending M^2 (see Graham and Harvey (1997) and Modigliani and Modigliani (1997)) to the tracking-error investment context: we use a management portfolio and the benchmark portfolio to form a hypothetical portfolio H whose tracking error volatility is the same as that of the NITE

portfolio, then we find the return differential of the hypothetical portfolio and the NITE portfolio. Our incentive measure is consistent with the information measure. In addition, it also illustrates an economic magnitude, i.e. the amount by which a managed portfolio outperforms a NITE portfolio, if the managed portfolio's tracking error volatility is normalized to the level of the NITE portfolio's tracking error volatility.

Although when we impose $\alpha_p = \alpha_{NITE} = \bar{\alpha}$, we have

$$\mathcal{I}_p = \bar{\alpha} \left(\frac{\sqrt{\text{var}(R_{NITE} - R_b)}}{\sqrt{\text{var}(R_p - R_b)}} - 1 \right) \quad (47)$$

ex ante, empirically we use ex post alpha estimates instead the ex ante ones to include the possibility that estimation errors may drive resulting alphas away from the target values.

Motivated by Fleming, Kirby, and Ostdiek (2001) and Ferson and Siegel (2001), we also consider a management fee measure, which represents the percentage of assets uninformed investors are willing to pay to switch from the NITE portfolio to an active portfolio. Assuming uninformed investors have quadratic utility defined over tracking errors, a management fee f_p solves

$$\sum_{t=1}^T [(R_{p,t} - R_{b,t} - f_p) - \Delta(R_{p,t} - R_{b,t} - f_p)^2] = \sum_{t=1}^T [(R_{NITE,t} - R_{b,t}) - \Delta(R_{NITE,t} - R_{b,t})^2], \quad (48)$$

where $\Delta = 0.5RRA/(1 + RRA)$, and RRA is the relative risk aversion. Following Fleming, Kirby, and Ostdiek (2001), we set $RRA = 1$ or 10 to represent risk tolerant or highly risk averse behaviors. Note while the management fee measure is an intuitive utility-based measure, it has several shortcomings. First, this approach may mismatch alpha and its implied risk aversion. Second, an aggressive strategy may lead to better performance as the risk is not normalized.²⁸

²⁸As an analogy, when two portfolios are compared, the portfolio with lower Treynor's (1966) measure (reward-to-beta ratio) may have higher Jensen's alpha if it takes excessive systematic risk.

Finally, we also include the familiar unconditional Sharpe (1966) ratio as a performance measure for the sake of completeness. Jorion (2003) argues that putting a total variance constraint on a tracking portfolio enhances its mean-variance performance. Note that, however, our optimal tracking efficient portfolio does not necessarily have the highest unconditional Sharpe ratio since its main objective is maximum unconditional information ratio.

Panel A of Table II does not impose portfolio risk constraints. The NITE and the UTE portfolios generate alphas close to the target alphas, while the alphas of the CTE portfolios are a bit above the target alphas. The UTE portfolios feature the lowest tracking error volatilities, while the NITE portfolios have the highest tracking error volatilities. The resulting information ratios of the UTE portfolios (0.415) are four times better than that of the NITE portfolios (0.079) and 51% better than that of the CTE portfolios (0.275). The incentives of UTE are uniformly higher than those of CTE. Note that it is not surprising to have a higher incentive than a portfolio alpha since we have to use leverage to construct hypothetical portfolios in this case. In addition, the management fees for the dynamic portfolios are all positive. CTE portfolios have the highest Sharpe ratios.

5.5 Out-of-Sample Performance

We use a “fixed window” evaluation scheme to explore the out-of-sample performance of the active portfolios using the data after the introduction of the Euro.²⁹ For the UTE and the CTE portfolios, we estimate the models for conditional moments using

²⁹The fixed window evaluation scheme highlights the time-invariant feature of NITE portfolio weights (Grinblatt and Titman (1993) address similar issues). Other evaluation schemes, e.g., recursive or rolling window, allow for re-estimation of unconditional as well as conditional moments. They allow the NITE portfolios to use new return data, a type of conditioning information. Of course, in practice one observes that NITE solutions are often paired with rolling windows. Thus, this combination is worth evaluating as well on its own merits (rolling NITE). See Section 5.6.4 for an evaluation of various methods.

pre-1999 data, and apply the parameter estimates directly to the new conditioning information to form the new conditional moments out of sample. For the NITE portfolios, we estimate the unconditional moments in sample only and do not update them out of sample.

Panel B of Table II summarizes the out-of-sample performance of the active portfolios. It shows that the NITE portfolios always produce alphas higher than the target alphas. Neither the UTE nor the CTE portfolios achieve the target alphas, however. The tracking error volatilities for the UTE portfolios are always smaller than those of the NITE and the CTE portfolios. In terms of the size of information ratios, the UTE portfolios (0.177) still outperform the CTE (0.124) and the NITE (0.116) portfolios, but the advantage is not as substantial as we have seen in sample. The information ratios of the UTE portfolios are 43% better than that of the CTE portfolios and 53% better than that of the NITE portfolios. Consistent with information ratios, the incentives of UTE portfolios are all positive and higher than those of CTE portfolios. The management fee of UTE is higher than that of CTE only when clients are highly risk averse and ask for 5% alpha. CTE portfolios also have the highest Sharpe ratios out of sample.

Figures 4–7 depict out-of-sample portfolio weights in excess of the benchmark weights. Since we use fixed-window evaluation scheme, NITE portfolio excess weights are constant over time. Both CTE and UTE strategies display time-varying excess weights, but the paths of CTE excess weights are much more volatile. This result casts serious concern on the CTE strategy as it will induce high transaction cost. On the other hand, the transaction cost incurred by the UTE portfolios should be moderate.³⁰

³⁰Note that even the NITE strategy has transaction cost since it rebalances periodically.

5.6 Robustness

5.6.1 Portfolio Risk Constraints

Now we examine whether the advantage of the UTE portfolios persists when a portfolio risk constraint is imposed. Panels A and B of Table III consider a portfolio beta constraint of $\bar{\beta}_p = 1$, and Panels C and D of Table III consider a total risk constraint of $\bar{\sigma}_p^2 = \sigma_b^2$.³¹ These are also interesting in view of Frost and Savarino (1988), who find that constraints can actually improve the out-of-sample performance of mean variance optimized portfolios.

Table III shows that the UTE portfolios still dominate the NITE and the CTE portfolios in and out of sample.³² Compared with the results in Table II, the performance of the UTE portfolios is virtually unaffected when a portfolio risk constraint is imposed. Interestingly, the extra penalty from the portfolio risk constraint is relatively more pronounced for the CTE portfolios. The intuition is that, when benchmark weights are known, the Lagrangian multiplier for the hedging component of the portfolio in excess of benchmark portfolio is zero when no portfolio risk constraint is imposed, but it is nonzero when a portfolio risk constraint is imposed. Since the Lagrangian multiplier is time-varying for the CTE portfolios, their portfolio weights become too volatile, compared with the NITE and the UTE portfolios, and therefore their performance is severely impacted as their tracking error volatilities inflate dramatically.

The incentives for UTE portfolios are higher than corresponding ones in Table II since the adverse effects of portfolio constraints on UTE portfolios are weaker than those on other portfolios. Management fees are usually positive in sample, but only UTE portfolio is able to deliver positive fee out of sample, when clients are highly

³¹Similar constraints are studied by Roll (1993) and Jorion (2003), respectively.

³²Note that we do not report the results for target alphas of 3% and 5% when a total risk constraint is imposed since such targets produce optimal NITE portfolio with complex numbers and thus are infeasible. Jorion (2003) does not find alphas higher than 2% in a different data set.

risk averse and targeting higher alphas. NITE portfolios tend to have higher Sharpe ratios out of sample.

5.6.2 Conditional Heteroskedasticity

Motivated by Chan, Karceski, and Lakonishok (1999), we consider a conditional version of a “common factor representation,” a factor model for the conditional moments whose time variation is driven by factors as well as predictor instruments.³³ We use a constant and the growth rate of a trade weighted US dollar exchange rate index³⁴ as the common factors and call them F . The statistical model follows,

$$\begin{aligned} F_t &= Z'_{t-1}\xi + u_t, \\ R_t^j &= F_t' B^j(Z_{t-1}) + v_t^j, \\ B^j(Z_{t-1}) &= Z'_{t-1}\pi^j, \end{aligned} \tag{49}$$

where $B^j(Z_{t-1})$ is the conditional regression beta of R_t^j on F_t , conditional on Z_{t-1} . Thus, $\mathbf{E}(v_t^j|Z) = \mathbf{E}(v_t^j F_t|Z) = \mathbf{E}(v_t^j F_t Z_{t-1}|Z) = 0$. We model

$$\Omega(Z_{t-1}) = [B(Z_{t-1})'\Sigma^u B(Z_{t-1}) + \Sigma^v] + \mu(Z_{t-1})\mu(Z_{t-1})', \tag{50}$$

where Σ^u and Σ^v are covariance matrices for u_t and v_t , respectively, and $\mu^j(Z_{t-1}) = \xi' Z_{t-1} Z'_{t-1} \pi^j$.

Modeling conditional heteroskedasticity improves the performance of the UTE portfolios substantially. Table IV shows that, when no portfolio risk constraint is imposed, the UTE portfolios have information ratios of 0.859 in sample, almost twice as good as the CTE portfolios and ten times better than the NITE portfolios. Out of sample, UTE portfolios produce information ratios of 0.240, which is 36% better than the CTE portfolios and 107% better than the NITE portfolios.

³³We also implement multivariate ARCH-type specifications but they are numerically unstable.

³⁴We use the index “for major currencies,” available from Federal Reserve Bank of St. Louis website.

Similarly, UTE tends to have higher incentives and only generate positive management fees when clients have higher risk aversion and aim at higher alphas. NITE has higher Sharpe ratios out of sample.

5.6.3 An Equity Market Example

It is interesting to examine how our optimal UTE portfolios fare in other financial markets. We present a small exercise using US equity data. We consider an active manager using four portfolios, including a large cap portfolio, a mid- and small-cap portfolio, a value stock portfolio, and a growth stock portfolio³⁵, to beat the Standard and Poor 500 index. We use the predictive variables in Ferson and Harvey (1999) to proxy the conditioning information available to the active manager.

Table V uses data for the sample period February 1979 to December 1998 for the in-sample analysis and leaves the post-1998 data for out-of-sample evaluation³⁶. Without imposing portfolio risk constraints and without modeling conditional heteroskedasticity, Panels A and B of Table V shows that the UTE portfolios are still the preferred portfolios. The UTE portfolios have the highest information ratios, in and out of sample. The advantage of the UTE portfolios over the NITE portfolios is not very large, however. Modeling conditional heteroskedasticity may improve the performance of the UTE portfolios. For example, using CRSP index return as a factor in the single factor model setting, the UTE portfolios have information ratios 5% better than those of the NITE portfolios. The results are summarized in Panels C and D of Table V.

In all cases, UTE portfolios have the highest incentives and positive management fees, and CTE portfolios have the highest Sharpe ratios.

³⁵They are proxied by the Standard and Poor 500, Russell 2500, Russell 1000 (Value), and Russell 1000 (Growth), respectively. The data are from Datastream.

³⁶The sample period is subject to data availability, while we make the division of in and out of sample consistent with our currency portfolio example.

5.6.4 Alternative Estimation and Evaluation Schemes

So far we focus on the fixed window evaluation scheme for out-of-sample study. We have used the pre-Euro sample period for initial estimation and do not allow for re-estimation of statistical moments.

This section considers various sizes of initial estimation periods and also investigates alternative out-of-sample evaluation schemes. The active manager may use 60, 120, 180, or 240 months of data for initial estimation³⁷. As new information arrives, he may maintain all historical data (recursive scheme) or drop the most distant observation (rolling window scheme), and then re-estimate the statistical moments.

Table VI reports the information ratios of active currency portfolios when conditional homoskedasticity is assumed and no portfolio risk constraint is imposed³⁸. We find the following patterns consistent across different sizes of initial samples. The UTE portfolios clearly dominate the CTE and the NITE portfolios in sample. Out of sample, UTE portfolios beat the NITE portfolios in 10 of the 12 occasions. UTE portfolios are preferred to the CTE portfolios under the fixed window and recursive schemes, but the CTE portfolios perform better than the UTE portfolios under the rolling window scheme.

5.6.5 Naive Equal-Weighted Portfolio (“1/N”)

We compare our portfolio performance with that of a naive equal-weighted portfolio. DeMiguel, Garlappi, and Uppal (2007) compare this naive strategy with 14 sample-based mean-variance portfolios, and find that none of the 14 portfolios can consistently beat the naive equal-weighted strategy in terms of out-of-sample mean-variance performance measures.

We assess the performance of this naive strategy using the currency dataset and report the results in Table VII. We consider several ways to split the sample such that

³⁷The numbers of observations out of sample are then 256, 196, 136, and 76, respectively.

³⁸In this case, information ratio is neutral to the target alphas as long as they are non-negative.

the results are directly comparable to Tables II and VI. Interestingly, we find that UTE portfolio is able to outperform the naive strategy most of the time. Indeed $1/N$ strategy might occasionally outperform the other tracking portfolios when we vary in-sample estimation window, but generally speaking, the UTE portfolio has robust out-of-sample performance.³⁹ UTE portfolio remains the better one when we switch to equity dataset.⁴⁰

5.6.6 Robustness Check: A Simulation Analysis

Our empirical evidence has shown that the UTE portfolios have outstanding ability to optimize the trade-off between alpha and tracking error volatility. A few concerns should be addressed here, however. First, the UTE portfolios tend to have poor alphas out of sample. Second, the UTE portfolios are inferior to the CTE portfolios when rolling window evaluation scheme is used. Third, although the advantage of the UTE portfolios is economically significant, its statistical significance remains an open question. It is possible that the above patterns are sample specific, given the possible model misspecification and estimation errors.

We examine the robustness of our results by a simulation of 1,000 paths, allowing

³⁹However, the advantage of dynamic portfolios may be eroded as the number of assets under management increases and misspecification and estimation errors are amplified. DeMiguel, Garlappi, and Uppal (2007) consider as many as 24 assets when studying actual data.

⁴⁰For example, the $1/N$ strategy delivers an out-of-sample information ratio of 0.088, while the UTE strategy delivers 0.386 when a constant covariance structure is imposed (see Table V, Panel B).

for misspecification and estimation errors.^{41,42} Panel A of Table VIII shows that, in sample, the NITE and the UTE portfolios on average produce precise alphas for every given target alpha, while the CTE portfolios produce alphas slightly higher than the target alphas.⁴³ The average information ratios of the UTE portfolios (0.531) are 1.6 times as good as CTE portfolios (0.339), and 3.8 times as good as NITE portfolios (0.141). Out of the 1,000 trials, the NITE portfolios never have higher information ratios than do the UTE portfolios, while the CTE portfolios beat the UTE portfolios for only six times. Overall the in-sample simulation evidence is consistent with the evidence from actual data.

Our simulation results illustrate the statistical significance of the out-of-sample dominance of the UTE portfolios. Panels B to D of Table VIII shows that the UTE portfolios have substantial advantage out of sample, whatever the evaluation scheme is. While none of the average alphas achieves the target alphas, the UTE portfolios have alphas closest to the targets. Out of the 1,000 simulation paths, the NITE portfolios produce alphas closer to the targets than do the UTE portfolios in fewer than 24.3% of the times, and the CTE portfolios have alphas closer to the targets than do the UTE portfolios in fewer than 49.8% of the times. In addition, the average

⁴¹We block bootstrap the individual currency returns, all of the instruments, the factor, and the growth rate of equity market values, and get the same number of observations as our empirical data. We divide the data into “in-sample” and “out-of-sample,” both matching the number of observations in Table II. We implement a new model selection, re-estimate the first and second moments assuming conditional homoskedasticity, and form the active portfolios without imposing portfolio constraints. The out-of-sample performance is evaluated by fixed window, recursive, and rolling window schemes. The procedure is repeated 1,000 times. Since neither a known return-generating process is assumed nor a large data set is available for estimation purpose, this simulation allows for misspecification and estimation errors.

⁴²Note a parametric bootstrap is not appropriate here due to the dimension of the instruments.

⁴³Although in sample, the NITE portfolios have alphas closer to the target alphas than the UTE portfolios in every path, the difference is very tiny and indistinguishable (around 2% of the target alpha). This result is a numerical issue: The alphas of the UTE portfolios are calculated by averaging products of conditional moments and rounding errors may occur in floating point computation.

alphas for UTE portfolios are at least threefold as the numbers in Table II. These results reverse the evidence of poor alphas in the actual data.

On average the UTE portfolios produce clearly better out-of-sample information ratios than those of the NITE and the CTE portfolios. Out of the 1,0000 simulation paths, the NITE portfolios produce higher out-of-sample information ratios than do the UTE portfolios for no more than 45 instances. Indeed, under the rolling window scheme, the CTE portfolios may have better chance to beat the UTE portfolios, but the probability is only 19.9%.

6 Concluding Remarks

We study models in which an active portfolio manager may use conditioning information, i.e., the information about security returns that is unavailable to his clients. Uninformed clients delegate their investment decisions to active managers, inducing them try to beat a benchmark and minimize tracking error variance. The active manager uses conditioning information to optimize the unconditional performance measures, which are observable by the uninformed client. The resulting optimal strategy is “unconditionally tracking efficient” (UTE).

We provide solutions for the UTE portfolios and study their properties and performance. From practitioners’ standpoint, the UTE solutions show how to use conditioning information efficiently. From academic perspective, the UTE problem suggests a specific interpretation of hedging demand in an economy with delegated portfolio management.

We find the portfolio risk constraints are crucial to the active manager’s response to conditioning information. Without portfolio risk constraints, the manager is conservative in the face of strong signals and his limiting behavior is to hold the benchmark. When there are portfolio risk constraints the manager holds more or less in the benchmark portfolios, depending on the shadow price of the portfolio risk constraint.

We also briefly discuss the equilibrium implications of the presence of UTE investors in an economy with delegated portfolio management. The implied asset pricing model features testability in the stochastic discount factor representation and time-varying factor loadings and premiums in the multibeta representation.

The economic significance of the advantages of the UTE portfolios is illustrated by a realistic international financial market example. When implementing the strategies with data for the pre-Euro period, we find the UTE portfolios outperform the NITE and the CTE portfolios dramatically. The advantage of the UTE portfolios remains robust to alternative model specifications, financial market data, estimation periods and out-of-sample evaluation schemes.

A Appendix

A.1 Specific Solutions

To develop the UTE version of the solutions, define the following parameters for convenience,

$$\begin{aligned}\psi_1 &= \mathbf{IE}[\psi_1(Z)], & \psi_1(Z) &\equiv \mu(Z)' \Phi(Z) \mu(Z), \\ \psi_2 &= \mathbf{IE}[\psi_2(Z)], & \psi_2(Z) &\equiv \mu(Z)' \Phi(Z) \gamma(Z), \\ \psi_3 &= \mathbf{IE}[\psi_3(Z)], & \psi_3(Z) &\equiv \gamma(Z)' \Phi(Z) \gamma(Z).\end{aligned}\tag{51}$$

Let $R_0 = x_0(Z)'R$ denote the global minimum conditional second moment portfolio return. It has expected value of,

$$\mu_0 = \mathbf{IE}[\mu_0(Z)], \quad \mu_0(Z) = \mu(Z)'x_0(Z),\tag{52}$$

and covariance with the benchmark return of,

$$\sigma_{b0} = \mathbf{IE} \left[\frac{\mathbf{1}'\Omega(Z)^{-1}\gamma(Z)}{\mathbf{1}'\Omega(Z)^{-1}\mathbf{1}} \right].\tag{53}$$

R_0 has unconditional second moment of Ω_0 with any portfolio $y(Z)$ with weights that sum to 1,⁴⁴

$$\mathbf{IE}[y(Z)'\Omega(Z)x_0(Z)] = \mathbf{IE} \left[\frac{y(Z)'\Omega(Z)\Omega(Z)^{-1}\mathbf{1}}{\mathbf{1}'\Omega(Z)^{-1}\mathbf{1}} \right] = \mathbf{IE} \left[\frac{1}{\mathbf{1}'\Omega(Z)^{-1}\mathbf{1}} \right] \equiv \Omega_0.\tag{54}$$

Or to be more general $\mathbf{IE}[y(Z)'\Omega(Z)x_0(Z)] = c\Omega_0$ if $\mathbf{1}'y(Z) = c$.⁴⁵

Let $\mathbb{I}\{\cdot\}$ denote an indicator function, taking value of 1 when the statement inside $\{\cdot\}$ is true and 0 otherwise.

⁴⁴Note this corresponds to the result $\mathbf{IE}(R_i R_n) = \mathbf{IE}(R_n^2)$ of Hansen and Richard (1987).

⁴⁵Thus for a zero net investment portfolio, its conditional second moment (and hence unconditional second moment) with the global minimum conditional second moment portfolio is 0.

A.1.1 No Risk Free Asset, Constraints on Portfolio Risk

Proposition 2 *The unique solution to the problem in (11) is determined by the weight function (2) where*

$$\begin{aligned}\lambda_1 &= \frac{[(\bar{\alpha}_p + \mu_b) - \mu_0]\psi_3 - (\bar{\beta}_p\sigma_b^2 - \sigma_{b0})\psi_2}{\psi_1\psi_3 - \psi_2^2}, \\ \lambda_2 &= \frac{(\bar{\beta}_p\sigma_b^2 - \sigma_{b0})\psi_1 - [(\bar{\alpha}_p + \mu_b) - \mu_0]\psi_2}{\psi_1\psi_3 - \psi_2^2}, \\ \lambda_3 &= 1.\end{aligned}\tag{55}$$

Proposition 3 *The unique solution to the problem in (12) is determined by the weight function (2) where*

$$\begin{aligned}\lambda_1 &= \frac{\bar{\alpha}_p + \mu_b - \mu_0}{\psi_1} - \frac{\psi_2}{\psi_1}\lambda_2, \\ \lambda_2 &= (-1)^{\mathbb{I}\{\psi_1(\psi_2^2 - \psi_1\psi_3) > 0\}}\sqrt{\kappa}, \\ \kappa &= \frac{[\bar{\sigma}_p^2 + (\bar{\alpha}_p + \mu_b)^2 - \Omega_0]\psi_1 - (\bar{\alpha}_p + \mu_b - \mu_0)^2}{\psi_1\psi_3 - \psi_2^2} \geq 0, \\ \lambda_3 &= 1.\end{aligned}\tag{56}$$

For λ_2 a real number, we require κ to be nonnegative. The non-negativity restriction constrains the feasible set of target alphas.

A.1.2 No Risk Free Asset and No Portfolio Risk Constraint

Proposition 4 *The unique solution to the problem in (13) is determined by the weight function (2) where*

$$\begin{aligned}\lambda_1 &= \frac{[(\bar{\alpha}_p + \mu_b) - \mu_0] - \psi_2}{\psi_1}, \\ \lambda_2 &= \lambda_3 = 1.\end{aligned}\tag{57}$$

Lemma 1 *For random variables X and Y , and X/Y also stochastic,*

$$\mathbf{E}\left(\frac{X^2}{Y^2}\right) > \left[\mathbf{E}\left(\frac{X}{Y}\right)\right]^2 \geq \mathbf{E}(X^2)\mathbf{E}\left(\frac{1}{Y^2}\right) > \frac{[\mathbf{E}(X)]^2}{[\mathbf{E}(Y)]^2} > \frac{[\mathbf{E}(X)]^2}{\mathbf{E}(Y^2)},\tag{58}$$

by Cauchy-Schwartz and Jensen's inequalities.

Corollary 1 *The solution in Proposition 4 yields ex ante unconditional tracking error variance⁴⁶*

$$\mathbf{var}(R_{UTE} - R_b) = \frac{(\bar{\alpha}_p + \mu_b - \mu_0 - \psi_2)^2}{\psi_1} - \psi_3 + \Omega_0 - 2\sigma_{b0}. \quad (59)$$

Given the same $\bar{\alpha}_p$, the CTE counterpart has ex ante unconditional tracking error variance of

$$\mathbf{var}(R_{CTE} - R_b) = \mathbf{E} \left(\frac{[\bar{\alpha}_p + \mathbf{E}(R_b|Z) - \mathbf{E}(R_0|Z) - \psi_2(Z)]^2}{\psi_1(Z)} \right) - \psi_3 + \Omega_0 - 2\sigma_{b0}. \quad (60)$$

Recognizing that $\psi_1(Z)$ is a quadratic form, $\mathbf{var}(R_{CTE} - R_b) > \mathbf{var}(R_{UTE} - R_b)$ by Lemma 1.

A.1.3 Solutions with Fixed Exogenous Benchmark Portfolio Weights

Propositions 5–7 provide the solutions to the optimization problem (11), (12), and (13), respectively, with fixed exogenous benchmark portfolio weights. All moments in Propositions 5–7 are defined in (4), (5), and (51).

Proposition 5 *The unique solution to the problem in (11), given fixed exogenous benchmark portfolio weights x_b , is determined by the portfolio weight function (15), where*

$$\lambda_1 = \frac{\bar{\alpha}_p \psi_3 - (\bar{\beta}_p - 1) \sigma_b^2 \psi_2}{\psi_1 \psi_3 - \psi_2^2}, \lambda_2 = \frac{(\bar{\beta}_p - 1) \sigma_b^2 \psi_1 - \bar{\alpha}_p \psi_2}{\psi_1 \psi_3 - \psi_2^2}, \lambda_3 = 0. \quad (61)$$

Proposition 6 *The unique solution to the problem in (12), given fixed exogenous benchmark portfolio weights x_b , is determined by the portfolio weight function (15),*

⁴⁶It follows from $\Phi(Z)' \Omega(Z) \Phi(Z) = 0$ and $\Phi(Z) \Omega(Z) x_0(Z) = 0$.

where

$$\begin{aligned}\lambda_1 &= \frac{\bar{\alpha}_p}{\psi_1} - \frac{\psi_2}{\psi_1} \lambda_2, \\ \lambda_2 &= (-1)^{\mathbb{I}\{\psi_1(\psi_2 - \psi_1\psi_3) > 0\}} \left[1 - \frac{\bar{\alpha}_p^2 - 2\bar{\alpha}_p\psi_2 - (\bar{\sigma}_p^2 + \bar{\alpha}_p^2 - \sigma_b^2)\psi_1}{\psi_1\psi_3 - \psi_2^2} \right]^{0.5} - 1 \in \mathbf{R}, \\ \lambda_3 &= 0.\end{aligned}\quad (62)$$

Proposition 7 *The unique solution to the problem in (13), given fixed exogenous benchmark portfolio weights x_b , is determined by the portfolio weight function (15), where*

$$\lambda_1 = \frac{\bar{\alpha}_p}{\psi_1}, \lambda_2 = \lambda_3 = 0. \quad (63)$$

Corollary 2 *The solution in Proposition 7 yields ex ante unconditional tracking error variance quadratic in $\bar{\alpha}_p$,*

$$\mathbf{var}(R_{UTE} - R_b) = \bar{\alpha}_p^2 \left(\frac{1}{\psi_1} - 1 \right). \quad (64)$$

As a result, the ex ante information ratio is,

$$\left(\frac{1}{\psi_1} - 1 \right)^{-\frac{1}{2}} (-1)^{\mathbb{I}\{\bar{\alpha}_p < 0\}}, \quad (65)$$

i.e., given the sign of $\bar{\alpha}_p$, ex ante information ratio is invariant to $\bar{\alpha}_p$.

Corollary 3 *Given the same $\bar{\alpha}_p$,*

$$\mathbf{var}(R_{CTE} - R_b) = \bar{\alpha}_p^2 \mathbf{E} \left(\frac{1}{\mu(Z)' \Phi(Z) \mu(Z)} - 1 \right) \geq \mathbf{var}(R_{UTE} - R_b), \quad (66)$$

by Jensen's inequality.

A.1.4 Solutions with a Risk Free Asset

To save notation, re-define

$$\begin{aligned}
\mu_b &= \mathbf{E}(r_b), & \mu(Z) &= \mathbf{E}(r|Z), \\
\sigma_b^2 &= \mathbf{var}(r_b), & \Omega(Z) &= \mathbf{E}(rr'|Z), \\
\Phi(Z) &= [\mathbf{E}(rr'|Z)]^{-1}, & \gamma(Z) &= \mathbf{E}[r(r_b - \mu_b)|Z],
\end{aligned} \tag{67}$$

where $\Omega(Z)$ is non-singular and positive definite. Also define

$$\begin{aligned}
\eta_1 &= \mathbf{E}[\mu(Z)'\Omega(Z)^{-1}\mu(Z)], \\
\eta_2 &= \mathbf{E}[\mu(Z)'\Omega(Z)^{-1}\gamma(Z)], \\
\eta_3 &= \mathbf{E}[\gamma(Z)'\Omega(Z)^{-1}\gamma(Z)],
\end{aligned} \tag{68}$$

where the conditional moments are defined in (67). The solutions to the UTE problem in (16) when there are N risky assets and a risk free asset, with or without portfolio risk constraints, are provided in Proposition 8. When there are fixed exogenous benchmark portfolio weights, the solutions to the problem in (16) are summarized in Proposition 9, in which all moments are defined in (67) and (68).

Proposition 8 (i) *The unique solution to the problem in (16), given constraint on portfolio beta $\bar{\beta}_p$, is determined by the weight function (2) where*

$$\lambda_1 = \frac{(\bar{\alpha}_p + \mu_b)\eta_3 - \bar{\beta}_p\sigma_b^2\eta_2}{\eta_1\eta_3 - \eta_2^2}, \lambda_2 = \frac{\bar{\beta}_p\sigma_b^2\eta_1 - (\bar{\alpha}_p + \mu_b)\eta_2}{\eta_1\eta_3 - \eta_2^2}, \lambda_3 = 0. \tag{69}$$

(ii) *The unique solution to the problem in (16), given constraint on portfolio variance $\bar{\sigma}_p^2$, is determined by the weight function (2) where*

$$\begin{aligned}
\lambda_1 &= \frac{\bar{\alpha}_p + \mu_b}{\eta_1} - \frac{\eta_2}{\eta_1}\lambda_2, \\
\lambda_2 &= (-1)^{\mathbb{I}\{\eta_2^2 > \eta_1\eta_3\}} \left[\frac{[\bar{\sigma}_p^2 + (\bar{\alpha}_p + \mu_b)^2]\eta_1 - (\bar{\alpha}_p + \mu_b)^2}{\eta_1\eta_3 - \eta_2^2} \right]^{0.5} \in \mathbf{R}, \\
\lambda_3 &= 0.
\end{aligned} \tag{70}$$

(iii) The unique solution to the problem in (16), without constraint on portfolio risk, is determined by the weight function (2) where

$$\lambda_1 = \frac{(\bar{\alpha}_p + \mu_b) - \eta_2}{\eta_1}, \lambda_2 = 1, \lambda_3 = 0. \quad (71)$$

Proposition 9 (i) The unique solution to the problem in (16), given fixed exogenous benchmark portfolio weights x_b and constraint on portfolio beta $\bar{\beta}_p$, is determined by the weight function (15) where

$$\lambda_1 = \frac{\bar{\alpha}_p \eta_3 - (\bar{\beta}_p - 1) \sigma_b^2 \eta_2}{\eta_1 \eta_3 - \eta_2^2}, \lambda_2 = \frac{(\bar{\beta}_p - 1) \sigma_b^2 \eta_1 - \bar{\alpha}_p \eta_2}{\eta_1 \eta_3 - \eta_2^2}, \lambda_3 = 0. \quad (72)$$

(ii) The unique solution to the problem in (16), given fixed exogenous benchmark portfolio weights x_b and constraint on portfolio variance $\bar{\sigma}_p^2$, is determined by the weight function (15) where

$$\begin{aligned} \lambda_1 &= \frac{\bar{\alpha}_p}{\eta_1} - \frac{\eta_2}{\eta_1} \lambda_2, \\ \lambda_2 &= (-1)^{\mathbb{I}_{\{\eta_2^2 > \eta_1 \eta_3\}}} \left[1 - \frac{\bar{\alpha}_p^2 - 2\bar{\alpha}_p \eta_2 - (\bar{\sigma}_p^2 + \bar{\alpha}_p^2 - \sigma_b^2) \eta_1}{\eta_1 \eta_3 - \eta_2^2} \right]^{0.5} - 1 \in \mathbf{R}, \\ \lambda_3 &= 0. \end{aligned} \quad (73)$$

(iii) The unique solution to the problem in (16), given fixed exogenous benchmark portfolio weights x_b and without constraint on portfolio risk, is determined by the weight function (15) where

$$\lambda_1 = \frac{\bar{\alpha}_p}{\eta_1}, \lambda_2 = \lambda_3 = 0. \quad (74)$$

Corollary 4 The solution in Proposition 9(iii) yields unconditional tracking error variance of

$$\mathbf{var}(R_{UTE} - R_b) = \mathbf{var}(r_{UTE} - r_b) = \bar{\alpha}_p^2 \left(\frac{1}{\mathbf{E}[\mu(Z)' \Omega(Z)^{-1} \mu(Z)]} - 1 \right), \quad (75)$$

while a CTE counterpart, subject to the same $\bar{\alpha}_p$ but minimizing the conditional tracking error variance, yields unconditional tracking error variance of

$$\mathbf{var}(R_{CTE} - R_b) = \mathbf{var}(r_{CTE} - r_b) = \bar{\alpha}_p^2 \left(\mathbf{E} \left[\frac{1}{\mu(Z)' \Omega(Z)^{-1} \mu(Z)} \right] - 1 \right), \quad (76)$$

which is greater than $\mathbf{var}(R_{UTE} - R_b)$ by Jensen's inequality.

A.2 Proof of Propositions 1–4

A.2.1 Generic Form of the Solutions

The agent's optimization problem is (1). With the conditional moments defined in (4), we can rewrite the portfolio alpha and tracking error variance as

$$\mathbf{E}(R_p - R_b) = \mathbf{E}[x(Z)' \mu(Z)] - \mu_b, \quad (77)$$

$$\mathbf{var}(R_p - R_b) = \mathbf{E}[x(Z)' \Omega(Z) x(Z)] - \{\mathbf{E}[x(Z)' \mu(Z)]\}^2 + \sigma_b^2 - 2\mathbf{E}[x(Z)' \gamma(Z)]. \quad (78)$$

First we consider the constraint $\mathbf{cov}(R_p, R_b) = \bar{\sigma}_{bp}$. Given the restrictions on portfolio alpha and covariance with the benchmark, the agent's optimization problem is equivalent to

$$\begin{aligned} \min_{x(Z)} \quad & \mathbf{E}[x(Z)' \Omega(Z) x(Z)], \\ \text{s.t.} \quad & \mathbf{E}[x(Z)' \mu(Z)] = \bar{\alpha}_p + \mu_b, \mathbf{E}[x(Z)' \gamma(Z)] = \bar{\sigma}_{bp}, x(Z)' \mathbf{1} = 1. \end{aligned} \quad (79)$$

Set the Lagrangian as

$$\begin{aligned} \mathcal{L}[x(Z)] = \mathbf{E}[x(Z)' \Omega(Z) x(Z)] & - 2\lambda_1 \{\mathbf{E}[x(Z)' \mu(Z)] - (\bar{\alpha}_p + \mu_b)\} \\ & - 2\lambda_2 \{\mathbf{E}[x(Z)' \gamma(Z)] - \bar{\sigma}_{bp}\} - 2\mathbf{E}\{\lambda_3(Z)[x(Z)' \mathbf{1} - 1]\}. \end{aligned} \quad (80)$$

Consider a perturbation $\hat{x}(Z) = x(Z) + \varepsilon y(Z)$, where $x(Z)$ is the optimal solution, $y(Z)$ is any other portfolio weight function, and ε is a constant. If the weight $x(Z)$ is optimal, the derivative of the Lagrangian for $\hat{x}(Z)$ with respect to ε must be zero when evaluated at $\varepsilon = 0$, i.e.,

$$\frac{\partial \mathcal{L}[\hat{x}(Z)]}{\partial \varepsilon} = 2\mathbf{E}[y(Z)' \{\Omega(Z)x(Z) - \lambda_1 \mu(Z) - \lambda_2 \gamma(Z) - \lambda_3(Z) \mathbf{1}\}] = 0, \quad (81)$$

when evaluated at $\varepsilon = 0$. Since it must hold for all $y(Z)$, it implies that the term in $\{\cdot\}$ must be zero, almost surely in Z . Then we have,

$$x(Z) = \Omega(Z)^{-1}[\lambda_1\mu(Z) + \lambda_2\gamma(Z) + \lambda_3(Z)\mathbf{1}]. \quad (82)$$

Imposing the restriction $\mathbf{1}'x(Z) = 1$, we get the following expression for $\lambda_3(Z)$,

$$\lambda_3(Z) = -\lambda_1 \frac{\mathbf{1}'\Omega(Z)^{-1}}{\mathbf{1}'\Omega(Z)^{-1}\mathbf{1}}\mu(Z) - \lambda_2 \frac{\mathbf{1}'\Omega(Z)^{-1}}{\mathbf{1}'\Omega(Z)^{-1}\mathbf{1}}\gamma(Z) + \frac{1}{\mathbf{1}'\Omega(Z)^{-1}\mathbf{1}}. \quad (83)$$

Plug (83) in (82) and the optimal portfolio is

$$\begin{aligned} x(Z) &= \Omega(Z)^{-1} \left[\lambda_1\mu(Z) + \lambda_2\gamma(Z) - \lambda_1 \frac{\mathbf{1}\mathbf{1}'\Omega(Z)^{-1}}{\mathbf{1}'\Omega(Z)^{-1}\mathbf{1}}\mu(Z) - \lambda_2 \frac{\mathbf{1}\mathbf{1}'\Omega(Z)^{-1}}{\mathbf{1}'\Omega(Z)^{-1}\mathbf{1}}\gamma(Z) + \frac{\mathbf{1}}{\mathbf{1}'\Omega(Z)^{-1}\mathbf{1}} \right] \\ &= \lambda_1\Phi(Z)\mu(Z) + \lambda_2\Phi(Z)\gamma(Z) + x_0(Z), \end{aligned} \quad (84)$$

where $\Phi(Z)$ and $x_0(Z)$ are defined in (5) and (6), respectively. It has the form of (2) if we reinterpret $\lambda_3 = 1$.

Now we consider another portfolio risk constraint $\mathbf{var}(R_p) = \bar{\sigma}_p^2$. Using the perturbation argument, we have the optimal solution

$$\begin{aligned} x(Z) &= -\hat{\lambda}_2^{-1}\Omega(Z)^{-1}[\hat{\lambda}_1\mu(Z) + \gamma(Z) + \hat{\lambda}_3(Z)\mathbf{1}] \\ &\equiv \Omega(Z)^{-1}[\lambda_1\mu(Z) + \lambda_2\gamma(Z) + \lambda_3(Z)\mathbf{1}]. \end{aligned} \quad (85)$$

where $\lambda_1 = -\hat{\lambda}_1/\hat{\lambda}_2$, $\lambda_2 = -1/\hat{\lambda}_2$, and $\lambda_3(Z) = -\hat{\lambda}_3(Z)/\hat{\lambda}_2$. Imposing $\mathbf{1}'x(Z) = 1$ allows us to express $\lambda_3(Z)$ as in (83). The optimal portfolio weights function becomes

$$x(Z) = \lambda_1\Phi(Z)\mu(Z) + \lambda_2\Phi(Z)\gamma(Z) + x_0(Z). \quad (86)$$

It has the form of (2) if we reinterpret $\lambda_3 = 1$.

Now we consider the case where no portfolio risk constraint is imposed. Using the perturbation argument, we have the first order condition

$$\Omega(Z)x(Z) = \lambda_1\mu(Z) + \gamma(Z) + \lambda_3(Z)\mathbf{1}, \quad (87)$$

almost surely, and the optimal solution

$$x(Z) = \Omega(Z)^{-1}[\lambda_1\mu(Z) + \gamma(Z) + \lambda_3(Z)\mathbf{1}]. \quad (88)$$

Impose the restriction $\mathbf{1}'x(Z) = 1$ and express $\lambda_3(Z)$ as in (83). The optimal portfolio is then

$$x(Z) = \lambda_1\Phi(Z)\mu(Z) + \Phi(Z)\gamma(Z) + x_0(Z), \quad (89)$$

where $\Phi(Z)$ and $x_0(Z)$ are defined in (51). It has the form of (2) if we reinterpret $\lambda_2 = \lambda_3 = 1$.

From (84), (86) and (89), we conclude that the UTE solution has generic form of (2).

A.2.2 Proof of Proposition 2

The agent's optimization problem is equivalent to (79) if $\bar{\sigma}_{bp} = \bar{\beta}_p\sigma_b^2$. The optimal solution has the form of (84). To find the values for λ_1 and λ_2 , impose that $\mathbf{E}[\mu(Z)'x(Z)] = \bar{\alpha}_p + \mu_b$,

$$\mathbf{E}[\mu(Z)'x(Z)] = \lambda_1\psi_1 + \lambda_2\psi_2 + \mu_0 = (\bar{\alpha}_p + \mu_b), \quad (90)$$

where ψ 's are also defined in (51). Rearrange,

$$\psi_1\lambda_1 + \psi_2\lambda_2 = (\bar{\alpha}_p + \mu_b) - \mu_0. \quad (91)$$

Similarly, imposing $\mathbf{E}[\gamma(Z)'x(Z)] = \bar{\beta}_p\sigma_b^2$ yields

$$\psi_2\lambda_1 + \psi_3\lambda_2 = \bar{\beta}_p\sigma_b^2 - \sigma_{b0}. \quad (92)$$

Now we have the two-equation system (91) and (92), and we solve for λ_1 and λ_2 by Cramer's Rule,

$$\begin{aligned} \lambda_1 &= \frac{[(\bar{\alpha}_p + \mu_b) - \mu_0]\psi_3 - (\bar{\beta}_p\sigma_b^2 - \sigma_{b0})\psi_2}{\psi_1\psi_3 - \psi_2^2}, \\ \lambda_2 &= \frac{(\bar{\beta}_p\sigma_b^2 - \sigma_{b0})\psi_1 - [(\bar{\alpha}_p + \mu_b) - \mu_0]\psi_2}{\psi_1\psi_3 - \psi_2^2}. \end{aligned} \quad (93)$$

A.2.3 Proof of Proposition 3

The agent's optimization problem is equivalent to (12), whose optimal solution is (86). Imposing $\mathbf{E}[x(Z)'\mu(Z)] = \bar{\alpha}_p + \mu_b$ produces,

$$\psi_1\lambda_1 + \psi_2\lambda_2 = \bar{\alpha}_p + \mu_b - \mu_0. \quad (94)$$

Similarly, imposing $\mathbf{E}[x(Z)'\Omega(Z)x(Z)] = \bar{\sigma}_p^2 + (\bar{\alpha}_p + \mu_b)^2$ yields

$$\lambda_1(\bar{\alpha}_p + \mu_b - \mu_0) + \lambda_1\lambda_2\psi_2 + \lambda_2^2\psi_3 + \Omega_0 = \bar{\sigma}_p^2 + (\bar{\alpha}_p + \mu_b)^2. \quad (95)$$

Rearrange (94) to express λ_1 in terms of λ_2

$$\lambda_1 = \frac{\bar{\alpha}_p + \mu_b - \mu_0}{\psi_1} - \frac{\psi_2}{\psi_1}\lambda_2. \quad (96)$$

Plug in (95) and solve for λ_2 ,

$$\lambda_2 = \pm \left[\frac{[\bar{\sigma}_p^2 + (\bar{\alpha}_p + \mu_b)^2 - \Omega_0]\psi_1 - (\bar{\alpha}_p + \mu_b - \mu_0)^2}{\psi_1\psi_3 - \psi_2^2} \right]^{1/2}. \quad (97)$$

The resulting tracking error variance is

$$\begin{aligned} \mathbf{var}(R_p - R_b) &= \bar{\sigma}_p^2 + \sigma_b^2 - 2\mathbf{E}[\gamma(Z)'x(Z)] \\ &= \bar{\sigma}_p^2 + \sigma_b^2 - 2[\lambda_1\psi_2 + \lambda_2\psi_3] - 2\sigma_{b0} \\ &= \bar{\sigma}_p^2 + \sigma_b^2 - 2\sigma_{b0} - 2 \left[\frac{(\bar{\alpha}_p + \mu_b - \mu_0)\psi_2}{\psi_1} + \lambda_2 \frac{\psi_1\psi_3 - \psi_2^2}{\psi_1} \right]. \end{aligned} \quad (98)$$

To minimize tracking error variance, we pick positive root for λ_2 when $(\psi_1\psi_3 - \psi_2^2)/\psi_1 > 0$ and negative root for λ_2 when $(\psi_1\psi_3 - \psi_2^2)/\psi_1 < 0$. Imposing such a selection rule yields the optimal solution in Proposition 3.

A.2.4 Proof of Proposition 4

The agent's optimization problem is equivalent to (13) and the optimal solution is (89). Impose that $\mathbf{E}[\mu(Z)'x(Z)] = \bar{\alpha}_p + \mu_b$,

$$\mathbf{E}[\mu(Z)'x(Z)] = \lambda_1\psi_1 + \psi_2 + \mu_0 = (\bar{\alpha}_p + \mu_b). \quad (99)$$

Solving for λ_1 ,

$$\lambda_1 = \frac{(\bar{\alpha}_p + \mu_b) - \mu_0 - \psi_2}{\psi_1}. \quad (100)$$

A.3 Relation to Portable Alpha

Consider an active manager with information set Z is assigned a benchmark portfolio b and uses other N assets with return vector R to form a zero net investment portfolio with portfolio weight vector $w(Z)$ in order to beat the benchmark while minimizing tracking error variance. His optimization problem can be reinterpreted as the problem (1) with an $(N + 1)$ weight vector $x(Z)$ with the last element being the weight on the benchmark portfolio, which is restricted to be one. We can rewrite $x(Z) = \tilde{w}(Z) + \tilde{x}_b$, $\tilde{w}(Z) = (w(Z)', 0)'$, and $\tilde{x}_b = (0'_N, 1)'$, and treat \tilde{x}_b as a “benchmark weight vector.” Since we can rewrite

$$\mathbf{E}[\tilde{x}(Z)' \mu_{N+1}(Z)] = \mathbf{E}[w(Z)' \mu_N(Z)] + \mu_b, \quad (101)$$

$$\mathbf{E}[\tilde{x}(Z)' \Omega_{N+1}(Z) \tilde{x}(Z)] = \mathbf{E}[w(Z)' \Omega_N(Z) w(Z)] + \mathbf{E}(R_b^2), \quad (102)$$

$$\mathbf{E}[\tilde{x}(Z)' \gamma_{N+1}(Z)] = \mathbf{E}[w(Z)' \gamma_N(Z)] + \sigma_b^2, \quad (103)$$

the portable alpha problem reduces to (14).

A.4 Relation to Utility Maximization

Fama (1996) shows how agents in Merton’s (1973) economy choose multifactor minimum variance portfolios under normality. Ferson, Siegel, and Xu (2006) extend the results to incorporate conditioning information. If we treat the benchmark return as a state variable, and use Ferson, Siegel, and Xu’s (2006) results, we know the intertemporal optimization problem with wealth \mathcal{W} , consumption \mathcal{C} , and indirect utility function $J(., .)$, has the form

$$\begin{aligned} \max_{x(Z_{t-1}), \mathcal{C}_{t-1}} \quad & u(\mathcal{C}_{t-1}) + \mathbf{E}[J(\mathcal{W}_t, R_{b,t}) | Z_{t-1}] \\ \text{s.t.} \quad & \mathcal{W}_t = (\mathcal{W}_{t-1} - \mathcal{C}_{t-1})x(Z_{t-1})' R_t, x(Z_{t-1})' \mathbf{1} = 1, \end{aligned} \quad (104)$$

which has the same solution $x(Z)$ as the minimization problem

$$\begin{aligned} \min_{x(Z)} \quad & \mathbf{var}(R_p | Z), \\ \text{s.t.} \quad & \mathbf{E}(R_p | Z) = m(Z), \mathbf{E}(R_p R_b | Z) = n(Z), x(Z)' \mathbf{1} = 1. \end{aligned} \quad (105)$$

Notice that the problem (105) is equivalent to

$$\begin{aligned} \min_{x(Z)} \quad & \mathbf{var}(R_p - R_b|Z), \\ \text{s.t.} \quad & \bar{\alpha}_p(Z) = a_p(Z), \bar{\beta}_p(Z) = b_p(Z), x(Z)' \mathbf{1} = 1, \end{aligned} \tag{106}$$

if the conditional mean and variance of R_b are given. This problem is the CTE version of the modern portfolio management problem with a beta constraint. Since unconditional efficiency is nested in conditional efficiency as a special case, an active portfolio manager may solve the unconditional version optimization problem of (106). We then try to answer what kind of preference will induce the manager to solve the UTE problems.

Ferson and Siegel (2001) have shown that unconditional mean-variance portfolios are optimal for agents with quadratic utility functions in a single period model. Motivated by their result, we consider the following active manager's utility function,

$$u(R_p - R_b) = (R_p - R_b) - \zeta(R_p - R_b)^2, \tag{107}$$

where $\zeta > 0$ is a constant characterizing the concavity of the utility function. The absolute risk aversion (ARA) is

$$ARA = -\frac{u(R_p - R_b)''}{u(R_p - R_b)'} = \frac{2\zeta}{1 - 2\zeta(R_p - R_b)}. \tag{108}$$

When $(R_p - R_b)$ is higher, the agent becomes more risk averse. We can use the results of Ferson, Siegel, and Xu (2006) directly and show the utility function of the active managers for whom our UTE solution is optimal, when a beta constraint is imposed. In the intertemporal setting, managers with the indirect utility function

$$J(\mathcal{W}, R_b) = (1 - \nu R_b)\mathcal{W} - \theta\mathcal{W}^2. \tag{109}$$

where ν and θ are constants, and who observe conditioning information, choose portfolio weights minimizing unconditional tracking error variance for a given pair of portfolio alpha and beta. This model is a good description for managers under possible portfolio risk constraints.

A.5 Equilibrium Implications of UTE Portfolios

With the rapid growth of delegated portfolio management it is natural to conjecture that tracking error investing behavior may affect asset prices in equilibrium. Brennan (1993), Stutzer (2003), Gómez and Zapatero (2003), and Cornell and Roll (2005) derive equilibrium implications of tracking error investing. They show that equilibrium expected asset returns display a multibeta representation⁴⁷ in the presence of tracking error investors. However, these models do not explicitly incorporate conditioning information.

This section presents an asset pricing model with tracking error investors who are assumed to optimally hold UTE portfolios. The model features explicit consideration of asymmetric information. We explore two representations. In the stochastic discount factor representation the stochastic discount factor does not depend on the conditioning information. This feature largely mitigates the testability problem raised by Hansen and Richard (1987)⁴⁸. In the multibeta representation, the factor loadings and premiums are both functions of conditioning information.

Consider a market with N risky assets and one risk free asset. The economy has two agents: an uninformed representative client and an informed active manager with conditioning information Z . The client possesses all of the wealth in the economy. She optimally delegates w_a portion of her wealth to the manager, invests w_b in an unmanaged benchmark portfolio with weight x_b , and puts the rest of her wealth in the risk free asset. The active portfolio manager's task is to outperform the bench-

⁴⁷Cornell and Roll (2005) show that expected asset returns are linear in market portfolio return and an additional adjustment term that is a function of betas. With some more algebra, one can show an equivalent multibeta representation.

⁴⁸Hansen and Richard (1987) show that, an econometrician can make inferences about an asset pricing model without using the full information set as long as the stochastic discount factor is a measurable function of the coarser information set or observable variables. If the stochastic discount factor depends on unobserved information it is not generally possible to test the model on a subset of the information.

mark, while minimizing tracking error variance. The manager is assumed to take the benchmark portfolio as exogenously specified. His optimization problem leads to the UTE strategy:

$$x(Z) = \lambda_1 \Omega(Z)^{-1} \mu(Z) + \lambda_2 \Omega(Z)^{-1} \gamma(Z) + x_b, \quad (110)$$

where the conditional moments are defined in (67) and the parameters are defined in Proposition 9 in the Appendix.

In equilibrium, the market portfolio weight is:

$$x_m(Z) = w_a [\lambda_1 \Omega(Z)^{-1} \mu(Z) + \lambda_2 \Omega(Z)^{-1} \gamma(Z) + x_b] + w_b x_b. \quad (111)$$

Let $r_m \equiv x_m(Z)'r$ denote the excess return on market portfolio. Premultiplying $x_m(Z)$ by $\Omega(Z)$ and rearranging terms yields the equilibrium conditional expected return vector,

$$\mu(Z) = \mathbf{IE}(r|Z) = \lambda_m \mathbf{IE}(r_m r|Z) + \lambda_b \mathbf{IE}(r_b r|Z), \quad (112)$$

where $\lambda_m \equiv 1/w_a(\lambda_1 - \mu_b \lambda_2)$ and $\lambda_b \equiv -(w_a \lambda_2 + w_a + w_b)/w_a(\lambda_1 - \mu_b \lambda_2)$ are scalar constants. Equation (112) is the asset pricing model with UTE investors. It says the conditional expected excess return of any asset r_j is a linear function of the conditional comoment of r_j with the market excess return, r_m , and the conditional comoment of r_j with the benchmark excess return, r_b . It is characterized by only two parameters, λ_m and λ_b , which are constants across different assets. It is important to recognize that the model implies λ_m and λ_b are constants that depend on unconditional moments, but not on the conditioning information, Z . It is this feature that allows the model to escape the ‘‘Hansen and Richard (1987) critique.’’ For example, if the conditioning information held by the active portfolio managers is finer than the information available to the econometrician, Equation (112) can still be tested, after taking iterated expectations using the coarser information.

A stochastic discount factor representation of the asset pricing model can be ob-

tained by subtracting the right-hand-side of Equation (112) from its left-hand-side,

$$\mathbf{IE}(\mathcal{M}r|Z) \equiv \mathbf{IE}[(1 - \lambda_m r_m - \lambda_b r_b)r|Z] = \mathbf{IE}(e|Z) = 0, \quad (113)$$

where e is an N -vector of the “pricing errors,” orthogonal to the conditioning information Z held by the active manager. Intuitively, the active manager exploits Z in his trading until (113) holds. If his information can predict pricing errors then his portfolio is not yet optimal. The scalar random variable $\mathcal{M} \equiv (1 - \lambda_m r_m - \lambda_b r_b)$ is the “stochastic discount factor.” Again, because λ_m and λ_b do not depend on unobservable conditioning information, the model is empirically testable on subsets of the information.

Equation (113) is an empirically appealing representation of the model, as argued above. Of course, the model can also be expressed in the more familiar multibeta presentation⁴⁹,

$$\mathbf{IE}(r_j|Z) \equiv \mathcal{B}(m, j, b; Z)\mathbf{IE}(r_m|Z) + \mathcal{B}(b, j, m; Z)\mathbf{IE}(r_b|Z), \quad (114)$$

where

$$\mathcal{B}(i, j, k; Z) = \frac{\beta_{i,k}(Z)\beta_{k,j}(Z) - \beta_{i,j}(Z)}{\beta_{i,k}(Z)\beta_{k,i}(Z) - 1}, \quad (115)$$

$$\beta_{i,j}(Z) = \frac{\mathbf{IE}(r_i r_j|Z)}{\mathbf{IE}(r_i^2|Z)}. \quad (116)$$

The multibeta representation shows that conditional expected asset returns are linear in their conditional betas. The conditional betas are time-varying and nonlinear functions of the conditioning information. This version of the model generalizes the models of Brennan (1993), Stutzer (2003), Gómez and Zapatero (2003), and Cornell and Roll (2005), if we interpret the moments in their models as conditional moments given Z . Equations (112) and (113), however, are empirically more appealing.

⁴⁹The proof is available by request.

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Table I: Potential Benefit of the Unconditionally Tracking Efficient Portfolios

This table reports the potential benefit of using the unconditionally tracking efficient (UTE) portfolios, compared with the no-information tracking efficient (NITE) portfolios and the conditionally tracking efficient (CTE) portfolios. For each portfolio, we compute the difference between the portfolio return and the benchmark return, and then calculate the following metrics: (1) the average difference between the two returns, “alpha,” (2) the standard error of the difference, “tracking error volatility,” and (3) the ratio of alpha to tracking error volatility, “information ratio.” The alphas and tracking error volatilities are annualized and in percentage points. The alphas, tracking error volatilities, and information ratios are the average numbers across 1,000 simulation paths. Each of the paths contains 12,000 simulated observations. The data generating process features the parsimonious regression specification and constant covariance structure. It is correctly specified in estimation.

Target Alpha	Alpha			Tracking Error Volatility			Information Ratio		
	NITE	CTE	UTE	NITE	CTE	UTE	NITE	CTE	UTE
1.000	1.000	0.999	1.000	14.209	5.241	2.793	0.072	0.192	0.358
3.000	3.000	2.996	3.001	42.627	15.723	8.379	0.072	0.192	0.358
5.000	5.000	4.994	5.001	71.045	26.205	13.965	0.072	0.192	0.358

Table II: In-Sample and Out-of-Sample Performance

This table reports the in-sample (October 1978 to December 1998) and out-of-sample (January 1999 to January 2005) performance of the no-information tracking efficient (NITE) portfolios, the conditionally tracking efficient (CTE) portfolios, and the unconditionally tracking efficient (UTE) portfolios. For each portfolio, we compute the difference between the active portfolio return and the benchmark return. We report the sample average of the difference between the two returns, “alpha,” the standard error of the difference, “tracking error volatility,” and the ratio of alpha to tracking error volatility, “information ratio.” “Incentive” represents the maximum fee a NITE investor is willing to pay to achieve UTE performance. “Management Fee (RRA= x)” represents a fee charged on a dynamic portfolio so that an investor with relative risk aversion of x is indifferent between the dynamic portfolio and the NITE portfolio. The alphas, tracking error volatilities, incentives, and management fees are annualized and in percentage points.

Panel A: In-Sample Performance

Target Alpha	Alpha			Tracking Error Volatility			Information Ratio			Incentive		Management Fee (RRA=1)		Management Fee (RRA=10)		Sharpe Ratio		
	NITE	CTE	UTE	NITE	CTE	UTE	NITE	CTE	UTE	CTE	UTE	CTE	UTE	CTE	UTE	NITE	CTE	UTE
1.000	1.000	1.342	1.023	12.583	4.878	2.463	0.079	0.275	0.415	2.462	4.227	0.678	0.404	1.010	0.778	0.047	0.070	0.057
3.000	3.000	4.026	3.069	37.748	14.633	7.388	0.079	0.275	0.415	7.386	12.682	4.063	3.503	6.996	6.801	0.071	0.160	0.141
5.000	5.000	6.711	5.115	62.913	24.389	12.313	0.079	0.275	0.415	12.311	21.137	10.117	9.604	17.806	18.162	0.076	0.209	0.208

Panel B: Out-of-Sample Performance

Target Alpha	Alpha			Tracking Error Volatility			Information Ratio			Incentive		Management Fee (RRA=1)		Management Fee (RRA=10)		Sharpe Ratio		
	NITE	CTE	UTE	NITE	CTE	UTE	NITE	CTE	UTE	CTE	UTE	CTE	UTE	CTE	UTE	NITE	CTE	UTE
1.000	1.070	0.393	0.199	9.244	3.179	1.127	0.116	0.124	0.177	0.073	0.564	-0.490	-0.662	-0.306	-0.456	0.145	0.106	0.094
3.000	3.210	1.180	0.598	27.731	9.537	3.382	0.116	0.124	0.177	0.220	1.692	-0.338	-0.723	1.331	1.137	0.154	0.137	0.120
5.000	5.350	1.966	0.997	46.219	15.895	5.637	0.116	0.124	0.177	0.367	2.820	1.331	0.907	5.879	5.942	0.143	0.148	0.141

Table III: Portfolio Risk Constraints

This table reports the in-sample (October 1978 to December 1998) and out-of-sample (January 1999 to January 2005) performance of the no-information tracking efficient (NITE) portfolios, the conditionally tracking efficient (CTE) portfolios, and the unconditionally tracking efficient (UTE) portfolios, with a portfolio risk constraint. Panels A and B considers a beta constraint $\bar{\beta}_p = 1$. Panels C and D considers a total risk constraint $\bar{\sigma}_p^2 = \sigma_b^2$. For each portfolio, we compute the difference between the active portfolio return and the benchmark return. We report the sample average of the difference between the two returns, “alpha,” the standard error of the difference, “tracking error volatility,” and the ratio of alpha to tracking error volatility, “information ratio.” “Incentive” represents the maximum fee a NITE investor is willing to pay to achieve UTE performance. “Management Fee (RRA= x)” represents a fee charged on a dynamic portfolio so that an investor with relative risk aversion of x is indifferent between the dynamic portfolio and the NITE portfolio. The alphas, tracking error volatilities, incentives, and management fees are annualized and in percentage points.

Panel A: In-Sample Performance: With Beta Constraint $\bar{\beta}_p = 1$

Target Alpha	Alpha			Tracking Error Volatility			Information Ratio			Incentive		Management Fee (RRA=1)		Management Fee (RRA=10)		Sharpe Ratio		
	NITE	CTE	UTE	NITE	CTE	UTE	NITE	CTE	UTE	CTE	UTE	CTE	UTE	CTE	UTE	NITE	CTE	UTE
1.000	1.000	1.482	1.025	12.652	9.927	2.468	0.079	0.149	0.415	0.888	4.255	0.636	0.410	0.788	0.789	0.049	0.074	0.058
3.000	3.000	4.445	3.075	37.957	29.782	7.404	0.079	0.149	0.415	2.665	12.765	2.840	3.547	4.220	6.880	0.074	0.133	0.144
5.000	5.000	7.409	5.125	63.261	49.637	12.339	0.079	0.149	0.415	4.442	21.274	6.298	9.719	10.081	18.361	0.078	0.146	0.215

Panel B: Out-of-Sample Performance: With Beta Constraint $\bar{\beta}_p = 1$

Target Alpha	Alpha			Tracking Error Volatility			Information Ratio			Incentive		Management Fee (RRA=1)		Management Fee (RRA=10)		Sharpe Ratio		
	NITE	CTE	UTE	NITE	CTE	UTE	NITE	CTE	UTE	CTE	UTE	CTE	UTE	CTE	UTE	NITE	CTE	UTE
1.000	1.015	-1.296	0.191	9.665	13.363	1.144	0.105	-0.097	0.167	-1.952	0.599	-2.522	-0.595	-2.731	-0.370	0.148	-0.006	0.095
3.000	3.044	-3.888	0.573	28.996	40.090	3.431	0.105	-0.097	0.167	-5.856	1.798	-8.860	-0.406	-10.869	1.617	0.149	-0.063	0.121
5.000	5.073	-6.480	0.955	48.327	66.817	5.719	0.105	-0.097	0.167	-9.760	2.996	-17.016	1.623	-23.205	7.068	0.136	-0.077	0.143

Panel C: In-Sample Performance: With Total Risk Constraint $\bar{\sigma}_p^2 = \sigma_b^2$

Target Alpha	Alpha			Tracking Error Volatility			Information Ratio			Incentive		Management Fee (RRA=1)		Management Fee (RRA=10)		Sharpe Ratio		
	NITE	CTE	UTE	NITE	CTE	UTE	NITE	CTE	UTE	CTE	UTE	CTE	UTE	CTE	UTE	NITE	CTE	UTE
1.000	1.000	3.452	1.023	14.983	19.281	2.463	0.067	0.179	0.415	1.683	5.222	2.083	0.569	1.716	1.105	0.056	0.095	0.057

Panel D: Out-of-Sample Performance: With Total Risk Constraint $\bar{\sigma}_p^2 = \sigma_b^2$

Target Alpha	Alpha			Tracking Error Volatility			Information Ratio			Incentive		Management Fee (RRA=1)		Management Fee (RRA=10)		Sharpe Ratio		
	NITE	CTE	UTE	NITE	CTE	UTE	NITE	CTE	UTE	CTE	UTE	CTE	UTE	CTE	UTE	NITE	CTE	UTE
1.000	0.790	2.023	0.201	10.858	19.514	1.126	0.073	0.104	0.178	0.335	1.145	0.581	-0.301	-0.070	-0.017	0.143	0.096	0.094

Table IV: Conditional Heteroskedasticity

This table reports the in-sample (October 1978 to December 1998) and out-of-sample (January 1999 to January 2005) performance of the no-information tracking efficient (NITE) portfolios, the conditionally tracking efficient (CTE) portfolios, and the unconditionally tracking efficient (UTE) portfolios, when the conditional covariance matrix of returns is time-varying. For each portfolio, we compute the difference between the active portfolio return and the benchmark return. We report the sample average of the difference between the two returns, “alpha,” the standard error of the difference, “tracking error volatility,” and the ratio of alpha to tracking error volatility, “information ratio.” “Incentive” represents the maximum fee a NITE investor is willing to pay to achieve UTE performance. “Management Fee (RRA= x)” represents a fee charged on a dynamic portfolio so that an investor with relative risk aversion of x is indifferent between the dynamic portfolio and the NITE portfolio. The alphas, tracking error volatilities, incentives, and management fees are annualized and in percentage points.

Panel A: In-Sample Performance

Target Alpha	Alpha			Tracking Error Volatility			Information Ratio			Incentive		Management Fee (RRA=1)		Management Fee (RRA=10)		Sharpe Ratio		
	NITE	CTE	UTE	NITE	CTE	UTE	NITE	CTE	UTE	CTE	UTE	CTE	UTE	CTE	UTE	NITE	CTE	UTE
1.000	1.000	0.949	1.043	12.583	1.970	1.214	0.079	0.482	0.859	5.063	9.807	0.335	0.435	0.715	0.821	0.047	0.054	0.059
3.000	3.000	2.848	3.128	37.748	5.909	3.642	0.079	0.482	0.859	15.190	29.420	3.330	3.664	6.672	7.055	0.071	0.139	0.154
5.000	5.000	4.746	5.213	62.913	9.849	6.069	0.079	0.482	0.859	25.317	49.034	9.368	9.981	18.031	18.760	0.076	0.213	0.245

Panel B: Out-of-Sample Performance

Target Alpha	Alpha			Tracking Error Volatility			Information Ratio			Incentive		Management Fee (RRA=1)		Management Fee (RRA=10)		Sharpe Ratio		
	NITE	CTE	UTE	NITE	CTE	UTE	NITE	CTE	UTE	CTE	UTE	CTE	UTE	CTE	UTE	NITE	CTE	UTE
1.000	1.070	0.635	0.126	9.244	3.597	0.526	0.116	0.176	0.240	0.562	1.146	-0.255	-0.733	-0.078	-0.525	0.145	0.121	0.089
3.000	3.210	1.905	0.378	27.731	10.792	1.577	0.116	0.176	0.240	1.685	3.437	0.323	-0.920	1.932	0.960	0.154	0.171	0.106
5.000	5.350	3.175	0.630	46.219	17.987	2.628	0.116	0.176	0.240	2.808	5.728	2.364	0.602	6.754	5.691	0.143	0.189	0.123

Table V: An Equity Market Example

This table reports the in-sample (February 1979 to December 1998) and out-of-sample (January 1999 to October 2004) performance of the no-information tracking efficient (NITE) portfolios, the conditionally tracking efficient (CTE) portfolios, and the unconditionally tracking efficient (UTE) portfolios without a portfolio risk constraint. Panels A and B assume conditional homoskedasticity, and Panels C and D assume conditional heteroskedasticity. The portfolios are formed by Standard and Poor 500, Russell 2500, Russell 1000 (Value), and Russell 1000 (Growth) portfolios. We compute the difference between the active portfolio return and the benchmark return. We report the sample average of the difference between the two returns, “alpha,” the standard error of the difference, “tracking error volatility,” and the ratio of alpha to tracking error volatility, “information ratio.” “Incentive” represents the maximum fee a NITE investor is willing to pay to achieve UTE performance. “Management Fee (RRA= x)” represents a fee charged on a dynamic portfolio so that an investor with relative risk aversion of x is indifferent between the dynamic portfolio and the NITE portfolio. The alphas, tracking error volatilities, incentives, and management fees are annualized and in percentage points.

Panel A: In-Sample Performance: Constant Covariance Structure

Target Alpha	Alpha			Tracking Error Volatility			Information Ratio			Incentive		Management Fee (RRA=1)		Management Fee (RRA=10)		Sharpe Ratio		
	NITE	CTE	UTE	NITE	CTE	UTE	NITE	CTE	UTE	CTE	UTE	CTE	UTE	CTE	UTE	NITE	CTE	UTE
1.000	1.000	1.031	1.022	0.885	0.909	0.888	1.130	1.134	1.151	0.004	0.019	0.031	0.022	0.031	0.022	0.216	0.217	0.217
3.000	3.000	3.093	3.066	2.656	2.728	2.664	1.130	1.134	1.151	0.011	0.056	0.092	0.066	0.091	0.066	0.253	0.255	0.254
5.000	5.000	5.154	5.110	4.426	4.547	4.440	1.130	1.134	1.151	0.018	0.094	0.152	0.109	0.149	0.109	0.289	0.292	0.291

Panel B: Out-of-Sample Performance: Constant Covariance Structure

Target Alpha	Alpha			Tracking Error Volatility			Information Ratio			Incentive		Management Fee (RRA=1)		Management Fee (RRA=10)		Sharpe Ratio		
	NITE	CTE	UTE	NITE	CTE	UTE	NITE	CTE	UTE	CTE	UTE	CTE	UTE	CTE	UTE	NITE	CTE	UTE
1.000	0.588	0.825	0.703	1.570	2.579	1.818	0.374	0.320	0.386	-0.086	0.019	0.227	0.113	0.217	0.111	0.001	0.005	0.003
3.000	1.763	2.475	2.108	4.709	7.736	5.454	0.374	0.320	0.386	-0.257	0.057	0.618	0.326	0.524	0.307	0.022	0.035	0.028
5.000	2.938	4.125	3.513	7.849	12.893	9.091	0.374	0.320	0.386	-0.428	0.095	0.924	0.522	0.659	0.468	0.043	0.064	0.053

Panel C: Out-of-Sample Performance: Common Factor Structure

Target Alpha	Alpha			Tracking Error Volatility			Information Ratio			Incentive		Management Fee (RRA=1)		Management Fee (RRA=10)		Sharpe Ratio		
	NITE	CTE	UTE	NITE	CTE	UTE	NITE	CTE	UTE	CTE	UTE	CTE	UTE	CTE	UTE	NITE	CTE	UTE
1.000	1.000	1.063	1.046	0.885	0.895	0.842	1.130	1.188	1.242	0.051	0.100	0.063	0.046	0.063	0.046	0.216	0.218	0.217
3.000	3.000	3.190	3.137	2.656	2.686	2.526	1.130	1.188	1.242	0.154	0.299	0.189	0.139	0.189	0.141	0.253	0.258	0.257
5.000	5.000	5.316	5.229	4.426	4.476	4.209	1.130	1.188	1.242	0.257	0.498	0.315	0.233	0.314	0.238	0.289	0.297	0.296

Panel D: Out-of-Sample Performance: Common Factor Structure

Target Alpha	Alpha			Tracking Error Volatility			Information Ratio			Incentive		Management Fee (RRA=1)		Management Fee (RRA=10)		Sharpe Ratio		
	NITE	CTE	UTE	NITE	CTE	UTE	NITE	CTE	UTE	CTE	UTE	CTE	UTE	CTE	UTE	NITE	CTE	UTE
1.000	0.588	1.309	0.596	1.570	5.003	1.414	0.374	0.262	0.422	-0.177	0.074	0.666	0.010	0.610	0.011	0.001	0.014	0.001
3.000	1.763	3.927	1.789	4.709	15.010	4.241	0.374	0.262	0.422	-0.531	0.223	1.658	0.036	1.150	0.046	0.022	0.060	0.022
5.000	2.938	6.545	2.981	7.849	25.016	7.069	0.374	0.262	0.422	-0.885	0.372	2.190	0.072	0.728	0.101	0.043	0.103	0.044

Table VI: Estimation Periods and Evaluation Schemes

This table reports the in-sample and out-of-sample information ratios of the no-information tracking efficient (NITE) portfolios, the conditionally tracking efficient (CTE) portfolios, and the unconditionally tracking efficient (UTE) portfolios. The out-of-sample evaluation scheme is fixed-window (Fixed), recursive (Recursive), or rolling-window (Rolling). The number of observations in the initial window is 60, 120, 180, or 240. For each portfolio, we report the information ratio.

	NITE	CTE	UTE
Number of Initial Observations: 60			
In-Sample	0.312	0.547	0.828
Fixed	-0.059	0.166	0.232
Recursive	0.014	0.111	0.145
Rolling	-0.016	0.137	0.097
Number of Initial Observations: 120			
In-Sample	0.100	0.528	0.586
Fixed	-0.013	0.056	0.155
Recursive	0.040	0.085	0.117
Rolling	0.007	0.125	0.103
Number of Initial Observations: 180			
In-Sample	0.075	0.413	0.510
Fixed	0.007	0.054	0.057
Recursive	0.070	0.056	0.093
Rolling	-0.126	0.107	0.080
Number of Initial Observations: 240			
In-Sample	0.092	0.322	0.456
Fixed	0.055	0.055	0.014
Recursive	0.043	0.022	0.054
Rolling	0.096	0.050	0.038

Table VII: Naive Strategy

This table reports the in-sample and out-of-sample performance of the naive ($1/N$) portfolios. We split the sample of currency data (February 1979 to October 2004) by using the initial 243, 60, 120, 180, or 240 observations as “in-sample” and the rest as “out-of-sample,” such that the results are directly comparable to Tables II and VI. We report the sample average of the portfolio return in excess of benchmark return, “alpha,” the standard error, “tracking error volatility,” the ratio of alpha to tracking error volatility, “information ratio,” and the Sharpe ratio.

	Obs.	Alpha	Tracking Error Volatility	Information Ratio	Sharpe Ratio
In-Sample	243.000	0.277	8.058	0.034	0.019
Out-of-Sample	73.000	0.554	6.443	0.086	0.086
In-Sample	60.000	-1.195	8.933	-0.134	-0.234
Out-of-Sample	256.000	0.701	7.363	0.095	0.097
In-Sample	120.000	-0.195	9.187	-0.021	-0.013
Out-of-Sample	196.000	0.669	6.643	0.101	0.066
In-Sample	180.000	0.227	8.623	0.026	0.031
Out-of-Sample	136.000	0.491	6.319	0.078	0.033
In-Sample	240.000	0.211	7.878	0.027	0.012
Out-of-Sample	76.000	0.750	7.171	0.105	0.107

Table VIII: Robustness Check: Simulation Evidence

This table reports the simulation results for the in-sample (Panel A) and out-of-sample (Panels B to D) performance of the no-information tracking efficient portfolios (NITE), the conditionally tracking efficient (CTE) portfolios, and the unconditionally tracking efficient (UTE) portfolios. For each portfolio, we compute the difference between the active portfolio return and the benchmark return and the following metrics: (1) the average difference between the two returns, “alpha,” (2) the standard error of the difference, “tracking error volatility,” and (3) the ratio of alpha to tracking error volatility, “information ratio.” The alphas and tracking error volatilities are annualized and in percentage points. We take averages of the above three metrics from 1,000 simulation paths, each of the paths contains the same number of observations as the actual data. The out-of-sample evaluation scheme is fixed-window (Panel B), recursive (Panel C), or rolling-window (Panel D). The numbers in the parentheses are the probabilities of the NITE or the CTE portfolios with (1) alphas closer to the target alphas, or (2) smaller tracking error volatilities, or (3) larger information ratios, than those of the UTE portfolios.

Panel A: In-Sample Performance

Target Alpha	Alpha			Tracking Error Volatility			Information Ratio		
	NITE	CTE	UTE	NITE	CTE	UTE	NITE	CTE	UTE
1.000	1.000 (1.000)	1.062 (0.119)	1.021	8.972 (0.000)	3.390 (0.000)	1.978	0.141 (0.000)	0.339 (0.006)	0.531
3.000	3.000 (1.000)	3.186 (0.119)	3.062	26.917 (0.000)	10.171 (0.000)	5.934	0.141 (0.000)	0.339 (0.006)	0.531
5.000	5.000 (1.000)	5.310 (0.119)	5.103	44.862 (0.000)	16.952 (0.000)	9.890	0.141 (0.000)	0.339 (0.006)	0.531

Panel B: Out-of-Sample Performance: Fixed Window

Target Alpha	Alpha			Tracking Error Volatility			Information Ratio		
	NITE	CTE	UTE	NITE	CTE	UTE	NITE	CTE	UTE
1.000	0.393 (0.217)	0.703 (0.467)	0.637	9.024 (0.000)	3.520 (0.026)	1.997	0.057 (0.038)	0.227 (0.187)	0.329
3.000	1.179 (0.217)	2.109 (0.467)	1.910	27.072 (0.000)	10.561 (0.026)	5.992	0.057 (0.038)	0.227 (0.187)	0.329
5.000	1.965 (0.217)	3.515 (0.467)	3.183	45.120 (0.000)	17.602 (0.026)	9.987	0.057 (0.038)	0.227 (0.187)	0.329

Panel C: Out-of-Sample Performance: Recursive

Target Alpha	Alpha			Tracking Error Volatility			Information Ratio		
	NITE	CTE	UTE	NITE	CTE	UTE	NITE	CTE	UTE
1.000	0.557 (0.243)	0.741 (0.452)	0.665	9.268 (0.000)	3.565 (0.017)	2.060	0.083 (0.045)	0.235 (0.189)	0.336
3.000	1.670 (0.243)	2.223 (0.452)	1.995	27.805 (0.000)	10.695 (0.017)	6.180	0.083 (0.045)	0.235 (0.189)	0.336
5.000	2.783 (0.243)	3.706 (0.452)	3.324	46.341 (0.000)	17.824 (0.017)	10.299	0.083 (0.045)	0.235 (0.189)	0.336

Panel D: Out-of-Sample Performance: Rolling Window

Target Alpha	Alpha			Tracking Error Volatility			Information Ratio		
	NITE	CTE	UTE	NITE	CTE	UTE	NITE	CTE	UTE
1.000	0.347 (0.238)	0.716 (0.498)	0.623	9.758 (0.000)	3.463 (0.016)	2.034	0.063 (0.045)	0.229 (0.199)	0.320
3.000	1.042 (0.238)	2.147 (0.498)	1.870	29.274 (0.000)	10.390 (0.016)	6.102	0.063 (0.045)	0.229 (0.199)	0.320
5.000	1.737 (0.238)	3.579 (0.498)	3.117	48.790 (0.000)	17.317 (0.016)	10.170	0.063 (0.045)	0.229 (0.199)	0.320

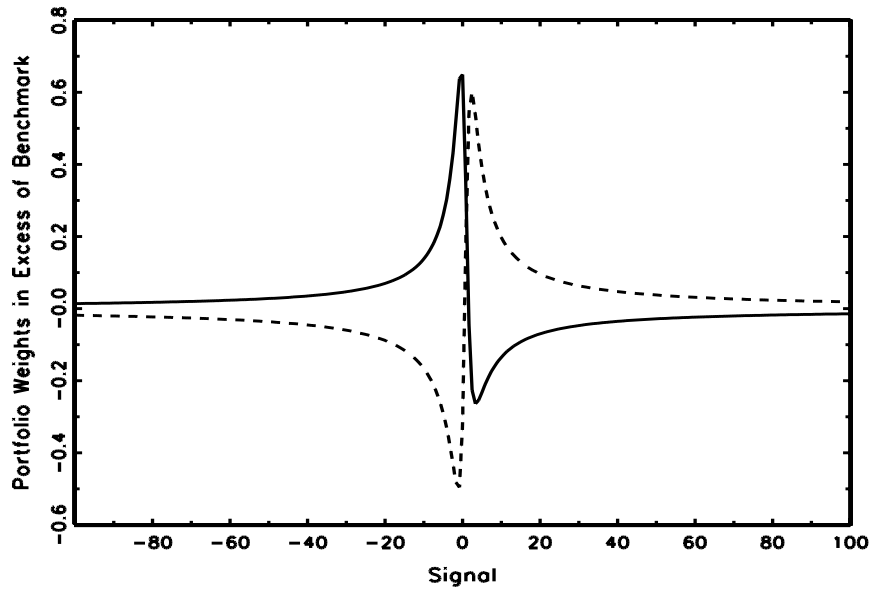


Figure 1: Unconditionally Tracking Efficient Portfolio Weights in Response to Conditioning Information

The dashed curve depicts the unconditionally tracking efficient (UTE) portfolio weights in asset a , and the solid curve depicts the UTE portfolio weights in asset b , both in excess of the benchmark portfolio weights, for various outcomes of conditioning information (signal). Signal is pre-standardized. It is positively correlated with the return of asset a and negatively correlated with the return of asset b . The target alpha is 5%.

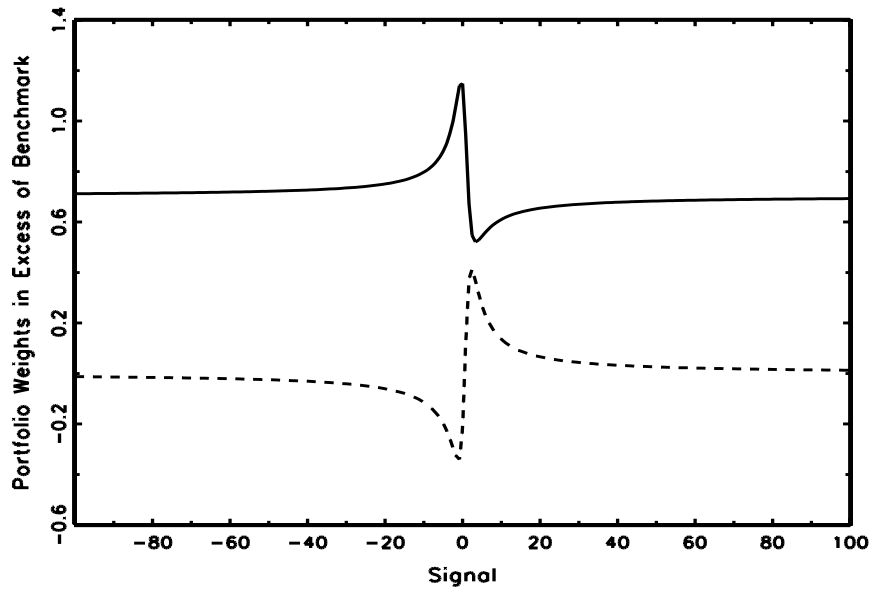


Figure 2: Unconditionally Tracking Efficient Portfolio Weights in Response to Conditioning Information: With Beta Constraint

The dashed curve depicts the unconditionally tracking efficient (UTE) portfolio weights in asset a , and the solid curve depicts the UTE portfolio weights in asset b , both in excess of the benchmark portfolio weights, for various outcomes of conditioning information (signal). Signal is pre-standardized. It is positively correlated with the return of asset a and negatively correlated with the return of asset b . The target alpha is 5%, and the target beta is 1.5.

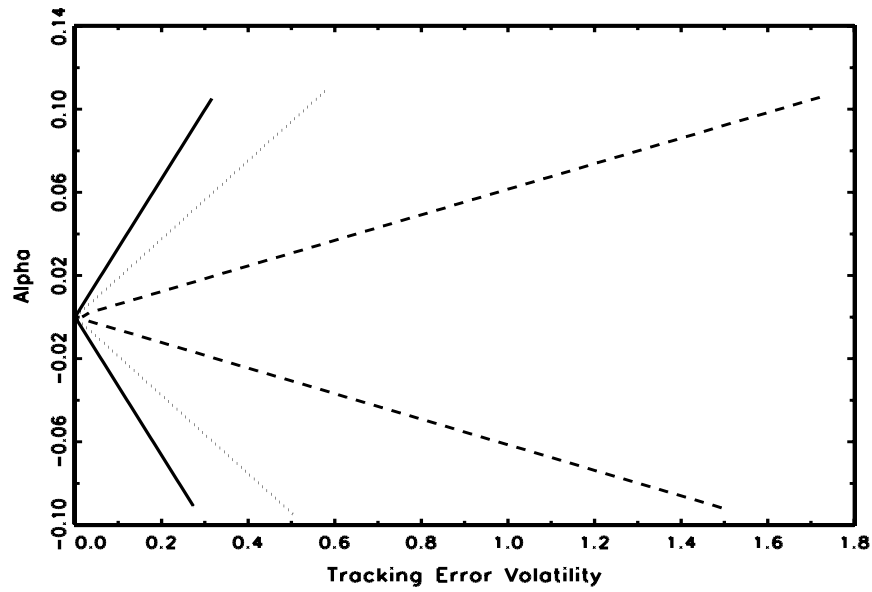


Figure 3: Tracking Efficiency Frontiers

The dashed curve is the tracking efficiency frontier for the no-information tracking efficient (NITE) portfolios, the dotted curve is the tracking efficiency frontier for the conditionally tracking efficient (CTE) portfolios, and the solid curve is the tracking efficiency frontier for the unconditionally tracking efficient (UTE) portfolios. The portfolios are constructed by conditional or unconditional moments based on 12,000 simulated observations. All of the conditional moments are constructed by the true data generating process used to calibrate the simulation.

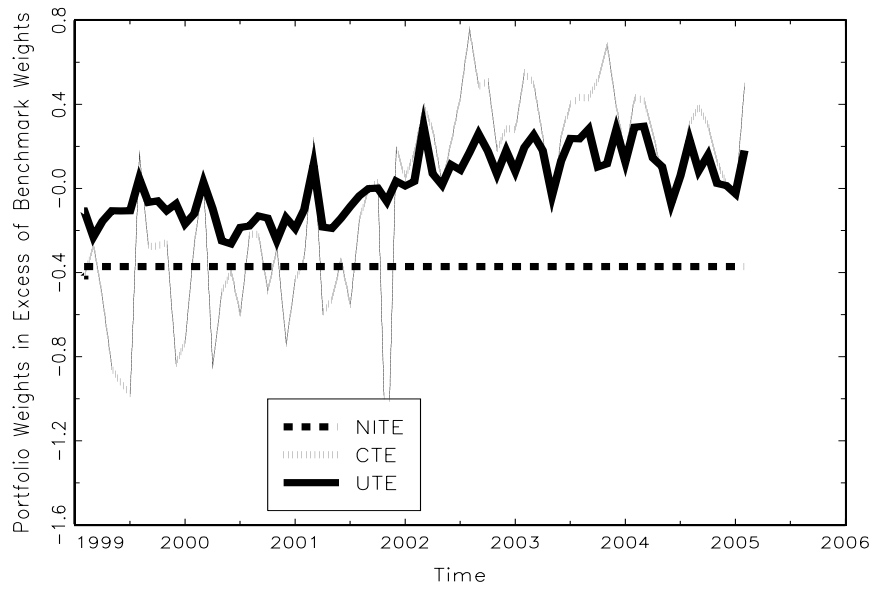


Figure 4: Portfolio Weights in Excess of Benchmark Weights: Japanese Yen

Dashed, dotted, and solid curves are out-of-sample NITE, CTE, and UTE portfolio weights on Japanese Yen, respectively, in excess of the weight in the benchmark portfolio. The target alpha is 5%.

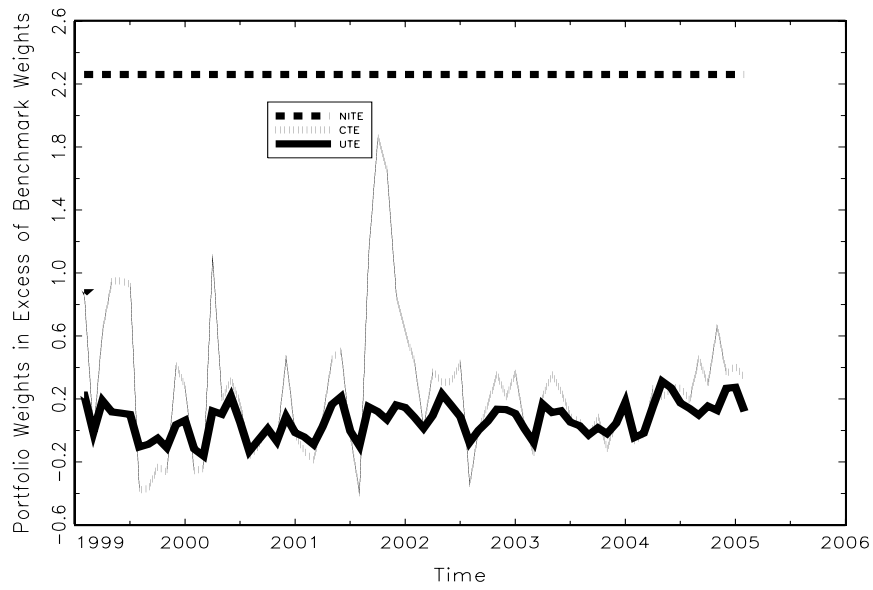


Figure 5: Portfolio Weights in Excess of Benchmark Weights: UK Pounds

Dashed, dotted, and solid curves are out-of-sample NITE, CTE, and UTE portfolio weights on UK Pounds, respectively, in excess of the weight in the benchmark portfolio. The target alpha is 5%.

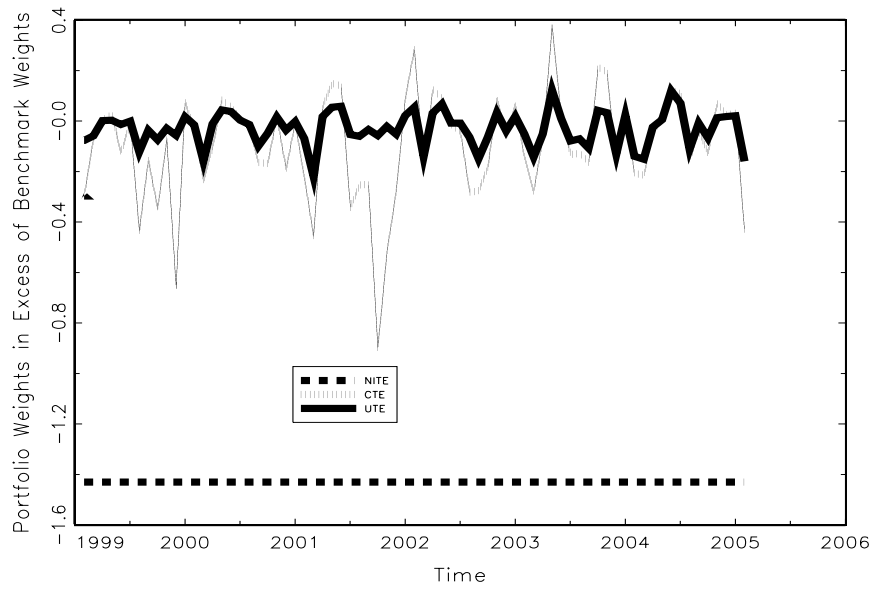


Figure 6: Portfolio Weights in Excess of Benchmark Weights: Euro

Dashed, dotted, and solid curves are out-of-sample NITE, CTE, and UTE portfolio weights on Euro, respectively, in excess of the weight in the benchmark portfolio. The target alpha is 5%.

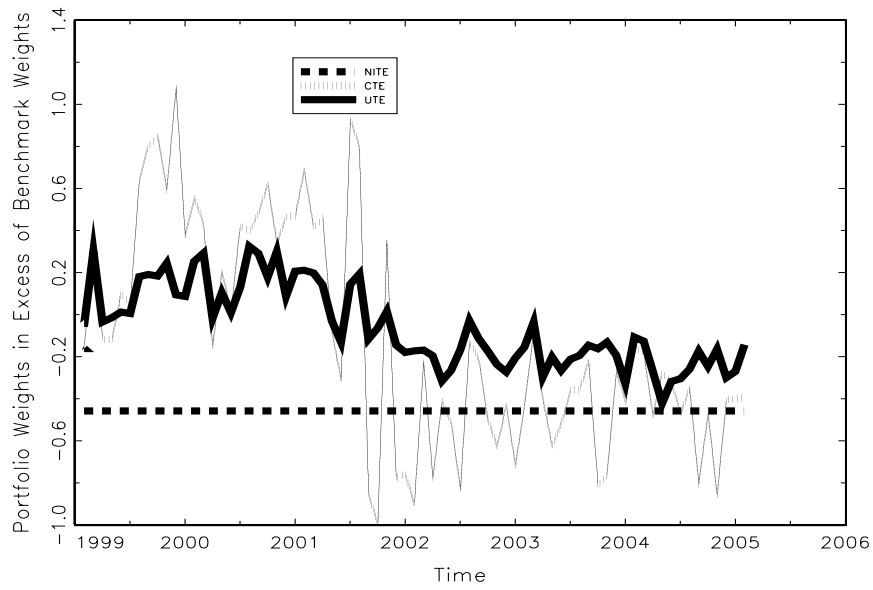


Figure 7: Portfolio Weights in Excess of Benchmark Weights: US Dollars

Dashed, dotted, and solid curves are out-of-sample NITE, CTE, and UTE portfolio weights on US Dollars, respectively, in excess of the weight in the benchmark portfolio. The target alpha is 5%.