# The Role of Heterogeneity in Asset Pricing: The Effect of Clustering Approach

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We use novel clustering approach and local Taylor series approximations of the stochastic discount factor to examine the role of heterogeneity in asset pricing. We present evidence that the equity premium is consistent with a stochastic discount factor calculated as the weighted average of the clusters' intertemporal marginal rates of substitution in the 1984-2002 period. This result is driven by the skewness of the cross-sectional distribution of consumption growth, but cannot be explained by the cross-sectional variance and mean alone. We show that the result is sensitive to the construction of clusters as well as the number of the clusters used in the study. We find that nine clusters is sufficient to capture idiosyncratic risk with relative risk aversion coefficient equal to six. The same result cannot be reproduced using individual household data.

*JEL Classification: D12, E21, G10, G12*

*Keywords: incomplete markets, household consumption data, idiosyncratic consumption risk, Euler equations*

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# The Role of Heterogeneity in Asset Pricing: The Effect of Clustering Approach

**Abstract**

We use novel clustering approach and local Taylor series approximations of the stochastic discount factor to examine the role of heterogeneity in asset pricing. We present evidence that the equity premium is consistent with a stochastic discount factor calculated as the weighted average of the clusters' intertemporal marginal rates of substitution in the 1984-2002 period. This result is driven by the skewness of the cross-sectional distribution of consumption growth, but cannot be explained by the cross-sectional variance and mean alone. We show that the result is sensitive to the construction of clusters as well as the number of the clusters used in the study. We find that nine clusters is sufficient to capture idiosyncratic risk with relative risk aversion coefficient equal to six. The same result cannot be reproduced using individual household data.

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# 1 Introduction

In the past few decades, financial economics research has debated whether aggregation problems can stand at the heart of some asset pricing puzzles, namely the equity premium and the risk-free rate puzzles. The assumption of incomplete consumption insurance leads to the impossibility by the heterogeneous consumers to equate the marginal rates of substitution state by state, and therefore, to impossibility of aggregation and use of a "representative" consumer. This assumption has received considerable attention recently. Constantinides and Duffie (1996) show that incomplete consumption insurance where consumers become subject to uninsurable income risk (such as job loss or divorce, for example) is important for explaining key asset pricing facts. Testing this implication is important not only for the purpose of testing a particular model but for a more fundamental reason. If idiosyncratic risk is priced, then the models that assume aggregation have to be reconsidered and financial economists should move from the "representative" agent paradigm to the models where an important role is given to the agents' heterogeneity.

There is an ongoing debate in the literature of whether idiosyncratic risk is priced.[1] Mankiw and Zeldes (1991), Jacobs (1999), Brav, Constantinides, and Geczy (2002), Cogley (2002), Jacobs and Wang (2004), and Balduzzi and Yao (2007) provide tests of asset pricing models with heterogeneous consumers with rather conflicting findings. In particular, Brav, Constantinides, and Geczy (2002) and Cogley (2002) use Taylor series approximations of a stochastic discount factor so that higher moments of cross-sectional consumption growth affect a stochastic discount factor. By testing the implications of their models, they come to different conclusions. In this paper, we show that local Taylor approximations have to be implemented carefully otherwise financial econometricians can come to different conclusions driven by incorrect use of Taylor expansions. The culprit seems to be a convergence radius of Taylor approximation, outside of which series might diverge from the true function's value and yield unreliable results.

[1] In the paper, we focus on the equilibrium intertemporal consumption allocations. So, we refer to idiosyncratic risk as "idiosyncratic consumption risk". Therefore, we use "idiosyncratic risk" and "idiosyncratic consumption risk" interchangeably throughout the paper.

Brav, Constantinides, and Geczy (2002) allow the cross-sectional mean, variance, and skewness to affect the stochastic discount factor (SDF) and find that the cross-sectional skewness is an important component of the SDF. In their set up, they find a low and economically plausible value of a relative risk aversion coefficient, consistent with the historical equity premium. However, using the generalized framework of Constantinides and Duffie (1996), Cogley (2002) does not find that idiosyncratic risk can explain the equity premium. His calibration shows that using individual consumption data one can generate a 2% equity premium only, even for high enough parameters of a relative risk aversion coefficient.[2] In addition, Cogley considers the first three cross-sectional moments of consumption growth and does not find any evidence that these factors are priced in equilibrium. Both studies use Taylor series expansions and obtain conflicting results. However, neither of them investigates the conditions for the correct Taylor approximation. Also, neither Brav, Constantinides, and Geczy (2002) nor Cogley (2002) break the CEX sample into cohorts to deal with the measurement error problem like Jacobs and Wang (2004) do. Cochrane (2006) emphasizes that the conflicting findings in the aforementioned papers deserve an explanation. In particular, he states: "*What are the time-varying cross-sectional moments that drive the result and why did Brav, Constantinides, and Geczy (2002) find them while Cogley (2002) and Lettau (2003) did not?*" The goal of our paper is to examine the factors that drive these differences.

First, we demonstrate that the series used in the Taylor approximation must lie within a certain convergence radius, otherwise Taylor approximation might "approximate" some arbitrary function, which is not necessarily a true stochastic discount factor. This simple observation leads to a possible explanation of the conflicting aforementioned results.

Second, we demonstrate how different aggregation schemes can lead to different empirical results and conclusions. A measurement error problem in individual consumption data is well known. Essentially, there are two ways to mitigate a measurement error. Jacobs and Wang (2004) create synthetic cohorts of individual consumption data. Balduzzi and Yao (2007)

[2]In other studies, not directly related to ours, Jacobs and Wang (2004) show that the cross-sectional mean and variance of consumption growth are priced factors within the framework of multifactor linear asset pricing models by breaking the sample into cohorts using age and education characteristics. In a closely related framework, Balduzzi and Yao (2007) report some asset pricing success by reconciling the magnitude of the U.S. equity premium with the consumption of asset holders given reasonable values of a relative risk aversion coefficient.

explicitly model a measurement error in individual consumption data. We deal with the measurement error in two ways. In our first approach, we aggregate individual consumption data in the spirit of Jacobs and Wang but show that the number of cohorts and the cohorts' construction methodology affects the results. *Ex-ante,* it is not clear what is the optimal number of the cohorts. On one side, cohorts should not be too large in order to preserve enough heterogeneity, but on the other side, small cohorts might suffer the same measurement error problem as individual consumption data. So, we create several sets of cohorts to investigate how aggregation affects our results. In our "representative" cohorts households are classified based on their demographic characteristics such as education and age. In our second approach we use a clustering analysis where we classify households into clusters. We first classify households in three cohorts based on education, and then we cluster households within each education cohort so that households with similar age and income appear in the same cluster. To our knowledge, we are the first to use clustering analysis in this context. The big advantage of this approach is that we let data decide which households are similar and, therefore, get in the same cluster. Of course, the accuracy of this approach depends on the number of clusters we use. Since we do not have any prior guidance on that, we consider several sets of clusters and decide what number of clusters provides the best fit for our model. In general, a stochastic discount factor (SDF) in our model is a function of cohort- or cluster-based cross-sectional moments of consumption growth. The interesting result that we obtain is that cluster-based SDF can explain the equity premium, while cohort-based SDF cannot irrespective of how many demographic cohorts are employed.

In the paper, we essentially test two hypothesis. First, we test the validity of our SDF by using just cohort- and cluster-based consumption growth. Second, we investigate the importance of higher-order consumption moments using Taylor expansion of the cohort/cluser-based SDF. Our tests use the equity premium of equally- and value-weighted market portfolio returns. Two main findings emerge. First, we do not reject the hypothesis that the equally-weighted sum of a cluster-based intertemporal marginal rates of substitution (IMRS) is a valid stochastic discount factor with a relative risk aversion (RRA) coefficient between five and seven. Such an RRA coefficient, although slightly higher than in the study of Brav, Constantinides, and Geczy (2002), is economically reasonable. However, we do reject the same hypothesis when the SDF is given by a cohort-based average of IMRS.[3]

Second, we find that cross-sectional moments of the household consumption growth do indeed matter for explaining idiosyncratic risk. Consistent with our first finding, we find that the importance of the cross-sectional consumption growth moments emerges only when we use clusters, but not cohorts, to construct the intertemporal marginal rate of substitution. In particular, we find that skewness is important for explaining asset returns. Although this result seems to be in line with Brav, Constantinides, and Geczy (2002) it is not entirely so. Brav, Constantinides, and Geczy (2002) do not partition their households into cohorts. In addition, they average out the individual intertemporal rates of substitution while we perform first the level aggregation to minimize the measurement error, e.g. we first compute consumption levels within a cohort (or cluster), and then compute cohort/cluster-based consumption growth. In this way we sharpen the results of Brav, Constantinides, and Geczy (2002) and provide a new methodology for the estimation of idiosyncratic risk that balances between preserving enough heterogeneity and managing noise inherent in the CEX dataset.[4]

Finally, we use an extended CEX data set from 1984 to 2002, which has never been used before. Most of the empirical studies cited above use the CEX data set that spans a much shorter sample period: from 1982 to 1995. With this approach, using longer and more comprehensive data, we step forward to resolve the debate to which John Cochrane drew the attention of financial economists in his 2006 survey "Financial Markets and Real Economy".

The rest of the paper is organized as follows. In Section 2 we present the model, Taylor approximation, and discuss the necessary conditions for this approximation to converge to a true function. In Section 3 we describe the CEX data, methodology for cohort construction, and asset returns data. In Section 4 we present our empirical setup and discuss the results. We conclude in Section 5.

[3]From now on, we call an SDF constructed as the average of the intertemporal marginal rates of substitution of education-age cohorts as *the cohort-based SDF*, and we call an SDF constructed as the average of the intertemporal marginal rates of substitution of clusters as *the cluster-based SDF*.

[4]We were not able to replicate Brav, Constantinides, and Geczy (2002) results using individual consumption growth data. Our main suspect is the individual consumption growth data might lie outside the convergence radius of Taylor series.

## 2 The Model

This section adopts a standard Lucas (1978) framework that assumes a representative agent economy in which only aggregate consumption risk matters. Complete markets are observationally equivalent to the existence of the representative agent model and have far-going consequences for explaining the equity premium puzzle and the risk-free rate puzzle. Kocherlakota (1996), Campbell, Lo, and MacKinlay (1997), and Cochrane (2001) provide excellent surveys on these issues. Under the assumption of complete markets, individuals can insure themselves against idiosyncratic consumption risk. Therefore, in this economy pricing of the assets is identical to the representative agent framework.

However, a number of studies during the last decades argued that such an assumption is not very realistic (Cochrane (1991), Mace (1991), Brav, Constantinides, and Geczy (2002), Balduzzi and Yao (2007), Constantinides (2002), to name just a few). Therefore, it is interesting to investigate whether market incompleteness can account for the empirical failure of consumption-based asset pricing models. Constantinides (2002) argues that accounting for idiosyncratic consumption risk can potentially explain the statistical properties of asset returns. However, number of authors, for example, Brav, Constantinides, and Geczy (2002), Vissing-Jorgensen (2002), Balduzzi and Yao (2007), Jacobs and Wang (2004) also note that individual consumption data is subject to the measurement error which could drive differences in answer to this question. The striking example of this are the contrasting findings by Brav, Constantinides, and Geczy (2002) and Cogley (2002). In this vein, we present our model that accounts for idiosyncratic risk, uses Taylor series approximation, and different aggregation schemes to test whether idiosyncratic risk can explain observed equity returns data.

### 2.1 No Arbitrage Conditions and Preferences

The economy in the present model is populated by a set of households $i = 1, \ldots, I$ that participate in the financial markets. At time $t$, each individual agent consumes $c_{i,t}$. We assume that households trade in frictionless capital markets, are not taxed, and are not short-sale constrained. They trade a set of securities $j = 1, \dots, J$ with total returns $R_{j,t+1}$ between dates $t$ and $t+1$. The households are assigned to either

- *cohorts* based on demographic characteristics such as education and age (demographic cohorts), or
- *clusters*, where households are classified into different groups based on education, age, and income characteristics. We describe the clustering procedure in detail in Appendix A.

With this in mind, we define the consumption growth $g_{k,t+1}$ of a cohort $k$ as the ratio of cohort per capita consumptions $c_{k,\tau}$ in the two consecutive periods:[5]

$$g_{k,t+1} = \frac{c_{k,t+1}}{c_{k,t}}. \tag{1}$$

Under assumption of the representative consumer the following optimality condition for the $k_{th}$ cohort must hold:

$$\mathbb{E}[M(g_{k,t+1})R_{j,t+1}|\mathcal{F}_t] = 1, \tag{2}$$

where $M_{t+1}$ is a valid pricing kernel that depends on the consumption growth $g_{k,t+1}$ of a cohort $k$. $R_{j,t+1}$ is the return on a risky asset or a risk-free bond $j$ from time $t$ to time $t+1$, and $\mathcal{F}_t$ is the information set at time $t$, common to all households in the sample.[6] We could mitigate the measurement error further by taking the equally-weighted average of cohort-based MRS and test whether this new stochastic discount factor is a valid SDF. The new Euler equation is given by:

$$\mathbb{E}\left[\frac{1}{K}\sum_{k=1}^{K} M\left(g_{k,t+1}\right) R_{j,t+1}|\mathcal{F}_t\right] = 1. \tag{3}$$

We assume time- and state-separable von Neumann-Morgenstern homogeneous constant relative risk aversion (CRRA) preferences for a representative consumer within each cohort

[5]The formal definition of per capita cohort consumption and its construction is given in Appendix A.

[6]Here, in the general exposition of the model, we use cohort to mean both cohorts and clusters. This distinction becomes relevant when we describe our empirical results.

$k$:

$$V_0 = \mathbb{E}\left[\sum_{t=0}^{\infty} \beta \frac{c_{k,t}^{1-\gamma} - 1}{1-\gamma} \middle| \mathcal{F}_0\right], \tag{4}$$

where $0 < \beta < 1$ is the constant subjective time discount factor, $\gamma > 0$ is the RRA coefficient, $c_{k,t}$ is the dollar per capita consumption of the $k^{th}$ cohort at time $t$. Therefore, the following Euler equation lends the base for our first empirical test of the validity of the stochastic discount factor represented as the average of cohort-based marginal rates of substitution:

$$\mathbb{E}\left[\beta \frac{1}{K} \sum_{k=1}^{K} (g_{k,t+1})^{-\gamma} R_{j,t+1} | \mathcal{F}_t\right] = 1. \tag{5}$$

This equation is similar to equation (4) in Brav, Constantinides, and Geczy (2002), except that the authors define equally-weighted average of marginal rates of substitutions using individual household data. To summarize, our procedure deals with the measurement error problem in two ways: first, we compute cohort-based consumption growth, and second, we take an equally-weighted average of cohort-based marginal rates of substitution.

### 2.2 Taylor Approximation

Equation (5) can still be affected by a measurement error when some terms are raised to high enough power of RRA. Therefore, we consider Taylor series approximations up to cubic term. The quadratic approximation of the stochastic discount factor in (5) around the cross-sectional mean $g_t \equiv \frac{1}{K}\sum_{k=1}^{K} g_{k,t}$ is given by:

$$m_t = \beta g_t^{-\gamma}\left[1 + \frac{1}{2}\gamma(\gamma+1)\frac{1}{K}\sum_{k=1}^{K}\left(\frac{g_{k,t}}{g_t} - 1\right)^2\right]. \tag{6}$$

The cubic approximation of the SDF in (5) around $g_t$ is given by:

$$\begin{aligned} m_t &= \beta g_t^{-\gamma}\left[1 + \tfrac{1}{2}\gamma(\gamma+1)\tfrac{1}{K}\textstyle\sum_{k=1}^{K}\left(\tfrac{g_{k,t}}{g_t} - 1\right)^2\right. \\ &- \left.\tfrac{1}{6}\gamma(\gamma+1)(\gamma+2)\tfrac{1}{K}\textstyle\sum_{k=1}^{K}\left(\tfrac{g_{k,t}}{g_t} - 1\right)^3\right]. \end{aligned} \tag{7}$$

Equation (6) is the function of the cross-sectional mean $g_t$ and cross-sectional variance

$\frac{1}{K}\sum_{k=1}^{K}\left(\frac{g_{k,t}}{g_t}-1\right)^2$, while equation (7) is the function of the first two moments and cross-sectional skewness $\frac{1}{K}\sum_{k=1}^{K}\left(\frac{g_{k,t}}{g_t}-1\right)^3$ of the cohort-based consumption growth rate. By testing whether the SDF is given by (6) we test whether the second moment only can explain the cross-sectional variation of the pricing kernel. By testing that the SDF is given by (7) we test whether second and third moments of cross-sectional consumption growth can jointly explain the cross-sectional variation of the stochastic discount factor. In the next section we demonstrate that the Taylor approximation has to be implemented carefully.

### 2.3 Taylor Expansion Approximations

In this section we lay out conditions under which the local approximation for any function should be undertaken. In particular, it is important to identify the convergence interval $(-r, r)$ of the Taylor series expansion, where $r$ is the radius of convergence. For a Taylor series, the radius of convergence is the distance between the approximation point and the closest singularity of the function. For example, for a CRRA utility function the marginal utility is given by $c^{-\gamma}$, where $\gamma$ is the RRA coefficient. Clearly, marginal utility has a singularity at zero, which implies that, if we approximate marginal utility around a point $\bar{c}$, the radius of convergence is given by $\bar{c}$. This observation shows that we cannot use data outside the interval $(0, 2\bar{c})$. To illustrate this point let us consider the marginal utility of a myopic investor $f(c) = \frac{1}{c}$ and approximate it around the point $\bar{c} = 1$. The radius of convergence in this case is one, and we expect the Taylor series to converge absolutely to the true function in the interval $(0, 2)$. In Panel A of Figure 1, we plot five even-order approximations of $f(c)$, up to the $10^{th}$ order, and it can be seen that they do converge to $f(c)$. On the other hand, if we consider larger intervals, not only will the approximation not improve, it will worsen. Panel B of Figure 1 shows the same graph on the wider interval $(0, 2.5)$. Indeed, it is obvious that the approximations are not reliable beyond the supremum of $(0, 2)$.[7]

These arguments show that the radius of convergence for Taylor approximations (6) and

[7] For the clarity of presentation, we plot only even-power approximations, but odd-power approximations would show a similar divergence pattern from the true function outside the convergence interval.

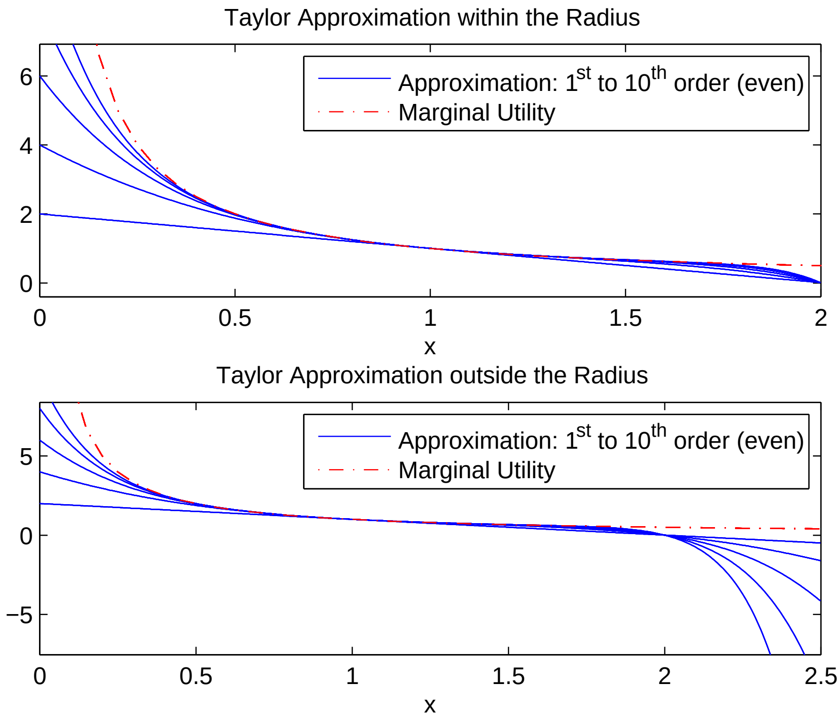


Figure 1: **Taylor Approximation for** $f(c) = \frac{1}{c}$**.**

This figure presents five even-power Taylor approximations of the function $f(c) = \frac{1}{c}$ up to the $10^{th}$ order. Panel A presents approximations in the interval $(-r, r) = (0, 2)$, while Panel B - in the interval $(-r, r) = (0, 2.5)$.

(7) should not exceed (0,2) interval, since we approximate an SDF around 1. Therefore, we argue that cohort- and cluster-based consumption growth observations $g_{k,t}$ should not exceed 2 when we calculate the SDF approximations. With this caveat in mind, we consistently check our cohort- and cluster-based consumption growth data in addition to adapting a few data filters.

# 3 Data

## 3.1 *The Consumer Expenditure Survey (CEX)*

Our data comes from the Consumer Expenditure Survey (CEX), produced by the Bureau of Labor Statistics (BLS), and covers the period from 1984 to 2002. The CEX provides a continuous flow of information on the buying habits of American consumers and furnishes data to support periodic revisions of the Consumer Price Index. Using a stratified sample methodology, the Bureau of Labor Statistics designs the survey to be representative of the US civilian population. This data set is not a proper panel, but rather a series of cross sections with a limited time dimension. The data is organized in quarterly files. Approximately 5,000 households are interviewed 3 months apart for 5 consecutive quarters (a trial interview is conducted in the first quarter). These interviews are staggered evenly throughout the year on a monthly basis and households report their consumption over the 3 months preceding the interview. After the fifth interview, households are replaced by new households. As Vissing-Jorgensen (2002) notes, the attrition is quite substantial, with only about 70 percent of the households having data for all four valid interviews.[8]

For each interview, respondents report consumption over the previous quarter and provide the reference month in which expenditures took place. In theory, this enables us to have up to twelve months of consumption information for each household, making it possible to obtain monthly consumption with monthly frequency. However, inspection of the data reveals that consumption is most often equal to quarterly consumption divided by three and thus it makes sense to aggregate consumption into a quarterly measure.[9] Notice that, although

[8]In the Appendix, we provide more details on the CEX structure and on our dedition of consumption.

[9]This same feature of the CEX data set was observed previously by Brav, Constantinides, and Geczy (2002) and

upon aggregation only quarterly consumption is measurable, the fact that interviews are conducted every month allows us to measure it at monthly frequency.

### 3.2 *Data Partition: Cohorts and Clusters*

To reduce a measurement error and to improve the approximation of the Taylor series expansion of the aggregate stochastic discount factor, we group households into groups based on observable demographic characteristics.[10] Organizing households by similar education, age, and income characteristics, we implicitly assume that a representative agent exists within each group. One issue with this approach is that the choice of grouping method for the construction of synthetic cohorts is not obvious. On one hand, one does not want the groups to be too small, because in that case measurement error and random effects will be very large. On the other hand, if groups are too large, it may not be appropriate to assume a representative agent exists in each cohort. We propose two ways to partition the data. The first way is similar to that proposed by Jacobs and Wang (2004) and uses simple rules to allocate consumers among age and education categories. However, to make sure that our results are not driven by an ad-hoc method of constructing cohorts, we also create cohorts based on clustering analysis. The clusters are obtained with a straight K-means algorithm in which we categorize households according to their attained education, age, and income.[11] In the appendix, we provide detailed definitions of cohorts, clusters, and group-based consumption growth (the groups being either cohorts or clusters).

Table 1 shows the descriptive statistics for both cohorts (Panel A) and clusters (Panel B). The first column reports the number of groups with which we compute the cross-sectional mean, variance, and skewness that we use to approximate the stochastic discount factor. As can be seen, the cross sectional moments obtained using clustering are relatively more spread out. The cross-sectional average and maximum volatilities of cluster-based consump-

Balduzzi and Yao (2007).

[10]For evidence on the presence of measurement error, see, for example, Jacobs and Wang (2004) and references therein. Given the presence of several outliers that we believe are due to genuine mistakes in the data set, we eliminate observations if the relative consumption growth is below, or above, the $1^{st}$ and $99^{th}$ percentiles respectively. We then group the observations of the resulting filtered data set.

[11]We thank Lukasz Pomorski and George Korniotis for pointing this to us.

tion growth are about .10 and .40 (columns (9) and (10): these estimates vary depending on the number of clusters), while the average and maximum volatilities of cohort-based consumption growth are roughly 0.05 and .10, respectively. It is also obvious that cluster-based consumption growth is more skewed than cohort-based consumption growth (column (12)). Not that the case of three cohorts is identical in both Panels because first we classify all households in three cohorts based on education characteristics.[12] Therefore, there is no clustering done in this case.

### 3.3 *Asset Returns*

The asset returns considered in our study are:

- Market returns: Equally-weighted and value-weighted nominal monthly returns (capital gains plus dividends) on the pool of stocks traded on NYSE, AMEX, and NASDAQ. The market returns sample corresponds to the sample of available consumption data. The returns are from March 1984 to November 2002. We deflate market returns using the seasonally unadjusted inflation rate obtained from the Consumer Price Index for All Urban Consumers. We obtain both market returns and CPI data from the Center for Research in Security Prices (CRSP) via Wharton Research Data Base.

- Risk-free Rate: Our measure of risk-free rate is the monthly returns on 30-day Treasury Bill Portfolio. The returns are from March 1984 to November 2002. market returns using the seasonally unadjusted inflation rate obtained from the Consumer Price Index for All Urban Consumers. The returns on the risk-free portfolio are from the Treasury and Inflation files from CRSP in the Wharton Research Data Base.

We get quarterly returns by compounding the monthly returns. Table 2 presents summary statistics for quarterly real asset returns. The returns are overlapping and available at a monthly frequency to match consumption growth.

[12]The education characteristics are: high-school degree only, some college years (not a completed college degree), and a college degree.

# 4 Empirical Analysis

## 4.1 *Unexplained Equity Premium*

First, we test the hypothesis (5) outlined in Section 2.1 that the equally-weighted average of the cohort-based marginal rates of substitution is a valid SDF. Specifically, we test whether the following statistic $w$ is equal to zero:

$$w_t = \frac{1}{TK} \sum_{t=1}^{T} \sum_{k=1}^{K} (g_{k,t})^{-\gamma} (R_{M,t} - R_{F,t}). \tag{8}$$

Table 3 presents the estimates of the unexplained equity premium (8) for various sets of cohorts and clusters. We set $K = 3$, 9, 18, 24, and 30.[13] Panels A and B report cohort-based $w_t$ estimates along with their $p-$values for equally- and value-weighted equity premia, respectively. The $p-$value of $w = 0$ against an unspecified alternative is computed based on the $t-$statistic. Panels C and D report cluster-based $w_t$ estimates along with their $p-$values for equally-weighted and value-weighted equity premia, respectively. We vary the coefficient of the relative risk aversion $\gamma$ from 0 and 8. When $K = 3$ the stochastic discount factor represents the average of the intertemporal marginal rates of substitution of three synthetic cohorts constructed on education information only. When $K = 6$, the cohort-based SDF is computed as the average of the intertemporal marginal rates of substitution of six synthetic cohorts constructed on education and age information: every education cohort is split into two age cohorts: households with the age lower and higher than the median, respectively. $\gamma = 0$ (first row in all panels) corresponds to the quarterly sample average of the equity premium in our sample, equal to 1.42% per quarter. For any $\gamma > 0$ and any number of cohorts, we reject zero unexplained equity premium. In fact, all $w_t$ estimates are higher than the sample average. Intermediate cases for $K$ provide the same result.[14] Note that for $K = 3$ the results are identical for both cohorts and clusters (second and third columns are identical in Panels A and C, and B and D, respectively) because we classify households into

[13]Following Brav, Constantinides, and Geczy (2002), we set $\beta = 1$ in our tests. This does not seem to affect our results. Results for omitted cohort sets are similar to the ones presented and not reported for space consideration, but available upon request.

[14]Not reported but available upon request.

three cohorts based on education characteristics before constructing clusters out of these three education cohorts.

The situation is different when we test $w_t = 0$ for the cluster-based SDF (Panels C and D). We cannot reject that the equity premium is zero anymore for certain sets of clusters. The unexplained equally-weighted equity premium estimates cross zero when $\gamma$ is between 6 and 7 and $K = 9$ or 18 (Panel C). For higher $K$, the equity premium can be explained with slightly higher $\gamma$, which is between 7 and 8. Panel D (the value-weighted equity premium) reconfirms the same result. This result does not hold if one uses too few clusters for an SDF construction. These simple tests show that idiosyncratic consumption risk indeed helps to explain the equity premium with reasonable values of the risk aversion coefficient when sufficiently many clusters are employed to generate enough cross-sectional distribution variation. Although we do not provide a formal test of what is the optimal number of clusters to be used, our analysis indicates that using 9 to 18 cohorts provides a good balance between the amount of heterogeneity and some data noise-reduction.

Next, we use Taylor series approximations to study which properties of the cross-sectional consumption growth drive our results. The set up of Tables 4 and 5 is identical to the one in Table 3. There are two main findings in these tables. First, as Panels A and B (both Table 4 and 5) show, neither quadratic nor cubic Taylor approximations of the cohort-based SDF have the ability to explain the equity premium irrespective of the $\gamma$ coefficient. We reject $w_t = 0$ hypothesis everywhere in this case. This is indeed consistent with the results in Table 3 (Panels A and B). This result indicates that by constructing cohorts using demographic characteristic only, we do not capture necessary cross-sectional variation present in the data (see summary statistics in Table 1).[15] Second, cubic Taylor approximation (7) of the cluster-based SDF helps to explain the equity premium (both equally- and value-weighted, Panels C and D, Table 5). The estimates of the $w_t$ cross zero when $K = 18$ and $\gamma$ is between 6 or 7. For higher $K$ the $w_t$ estimates do not cross zero, however, they are

[15]This result is consistent with our earlier version of the paper, where we used GMM to estimate the RRA coefficient of the household consumption data. But this finding remains at odds with Brav, Constantinides, and Geczy (2002) paper. Although we have tried to replicate this paper, we were unable to do so. Our primary suspect and explanation would be that individual consumption growth data lie outside of the Taylor series convergence radius, even if authors impose some filters on the data.

not significantly different from zero. Panels C and D of Table 4 manifest that the quadratic Taylor approximation (6) cannot hold responsible for a good fit of the equity premium with a cluster-based SDF. Results of these three tables, therefore, can be summarized by three robust findings. First, cluster-based SDF can capture the equity premium, while cohort-based SDF cannot. Second, cross-sectional skewness of cluster-based representative agents is a key property in explaining the equity premium with a reasonable risk aversion coefficient. Third, to obtain this effect, we need to consider sufficiently many clusters of similar households, so enough cross-sectional variation is generated.

## 5 Conclusion

In this study, we examine the role of heterogeneity in asset pricing. By using CEX data set of individual households consumption growth data, we show that the cluster-based stochastic discount factor is a valid pricing kernel with the relative risk aversion coefficient between 6 and 7. Interestingly, we cannot obtain the same result when we use education-age demographic cohorts for the construction of the stochastic discount factor. Demographic cohorts seem to wash out heterogeneity in the data and do not produce enough cross-sectional variation in order to explain the equity premium.

In our opinion, clustering analysis presents a superior way to deal with the measurement error problem on one side, and leave enough heterogeneity in the data, on the other side. In this way we sharpen the results of Brav, Constantinides, and Geczy (2002), but show that the cross-sectional variation is best captured by construction of clusters, where similar households are organized in the groups according to a $L1$ metric. We also find that our result is driven by the skewness of the cross-sectional distribution of consumption growth, but cannot be explained by the cross-sectional variance and mean alone. We show that the result is sensitive to the construction of synthetic groups (cohorts vs. clusters) as well as the number of household groups used in the study. We find that nine to 18 cohorts seems to be sufficient to capture idiosyncratic risk with relative risk aversion coefficient equal to six. The same result cannot be reproduced using individual household data.

We also show that local Taylor approximations have to be executed carefully, in a sense that the variables have to lie within a function's convergence radius, otherwise Taylor approximation might not converge to the true function. In this case, a stochastic discount factor can suffer from the same measurement problem, and, therefore, researchers might come to different conclusions based on incorrect use of local Taylor approximations.

## A Appendix: Data Section Details

In this appendix we provide a detailed explanation on the definitions and procedures introduced in Section 3.

### A.1 Definition of Consumption

The CEX data are stored in four major files: FMLY, MTAB, MEMBER, and ITAB. We extract all the variables we need from the CEX family files (FMLY), which give information about household characteristics and income, and the detailed expenditures files (MTAB), which we use to construct our aggregate consumption measure of services and non-durables. We calculate per capita monthly nondurable and services (NDS) consumption by aggregating data in the following CEX categories: food, alcoholic beverages, tobacco, gas, utilities, apparel, public transportation, household operations, and personal care. This definition of NDS consumption is consistent with the definition adopted by Jacobs and Wang (2004). Gift items are excluded from aggregation. We obtain per capita consumption by dividing each household consumption by the number of household members. We use consumption data obtained in interviews conducted from January 1984 to December 2002. Nominal consumption is corrected for inflation using the seasonally unadjusted CPI level obtained from CRSP (variable CPIIND from crsp.mcti file).

### A.2 Cohorts and Clusters

The cohorts proposed in Section 3.2 are base on the age and education of the head of a household. We construct three educational categories by grouping households with: at most a high-school diploma; some college experience; and at least bachelor degree. We also consider up to 10 age groups depending on which age quantile the household falls into. Such a partitioning leaves us with up to 30 education-age cohorts. For instance, if we partition households by creating two age groups, i.e. below and above median age, then we end up with 6 cohorts (2 age groups times 3 education groups).

The partition of the consumers' sample into clusters is based on education, age, and total household income. For each of the three education groups described above, we implement a straight bivariate K-means algorithm along the age and income dimension and construct up to 10 clusters. As in the construction of cohorts, such partitioning leaves us with up to 30 education-age-income groups.

K-means clustering is a way to partition a data set in K non overlapping groups. Each group is characterized by a centroid, which can be seen as a representative, central, number for the group. Intuitively, the objective of a K-means algorithm is to create groups such that the distance between group member and their centroid is minimal, while the distance between observations belonging to different clusters is sufficiently high. There are several metrics to define the distance between two points. We use the L1 distance (the L2 being the standard Euclidean distance) because it is less sensitive to the existence of outliers within a cluster. In particular, we use the following SAS lines of codes to generate the clusters:

```
%macro clust;
%do n = 1 %to 10; /*least refers to metric*/
proc fastclus data=consumption out=clust maxclusters=&n least=1
noprint; var age_ref inc; /*cluster on age and income*/ by education
date; /*for each education group and month*/ run; proc sql; create
table cluster&n as select date , sum(c)/sum(clag) as cg , count(c)
as n from clust group by date , cluster; quit;
%put &n; /*print current step*/
%end;
%mend;
```

LEAST=p causes PROC FASTCLUS to optimize an Lp criterion. More information on this procedure can be found on the SAS help file. For a detailed description of the K-means algorithm see also chapter 3 of the book by Mirkin (2005).

The definition of cohort-based (or cluster-based) consumption growth follows Jacobs and Wang (2004) and deserves some explanation. We do not aggregate consumption growth in a given group, but rather aggregate the levels of consumption within a cohort. We then compute consumption growth for the group as the ratio of the aggregate levels. This approach is consistent with the existence of certain types of measurement error in consumption (such as additive measurement error) that can be removed by aggregating on consumption levels but not necessarily by aggregating on consumption growth (see Balduzzi and Yao (2007)). Formally, the average consumption of a representative cohort $k$ at time $t$ is given by

$$c_{k,t} = \frac{1}{N_{k,t}} \sum_{i=1}^{N_{k,t}} c_{k,i,t}, \quad \forall\, t = 1, \ldots, T, \tag{9}$$

where $N_{k,t}$ is the number of observations in the cohort at time t. Furthermore, let $lag(c_{k,t})$ be the lagged average consumption of the set of consumers that belongs to cohort $k$ at time $t$. We define cohort consumption growth as

$$g_{k,t} = \frac{c_{k,t}}{lag(c_{k,t})}. \tag{10}$$

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Table 1: Summary Statistics for Cohort-Based Consumption Growth

This table presents summary statistics for the household consumption growth organized in synthetic groups. Panel A presents descriptive statistics for cohort-based consumption growth, where cohorts are formed based on demographic characteristics such as age and income. Panel B presents descriptive statistics for cluster-based consumption growth, where clusters are formed based on clustering procedure described in Appendix A. Column 1 represents the number of cohorts into which households are sampled. Columns 2, 3, and 4 report the time-series minimum, median and maximum number of households falling in each cohort. Columns 5, 6, and 7 present time-series minimum, mean and maximum of the cross-sectional mean of the cohort-based consumption growth. Columns 8, 9, and 10 present time-series minimum, mean and maximum of the cross-sectional cohort-based consumption growth standard deviation. Columns 11, 12, and 13 present time-series minimum, mean and maximum of the cross-sectional cohort-based consumption growth skewness. The sample period for the is from March 1984 to November 2002. The data is quarterly sampled at monthly frequency.

| Cohort | N Observaions | | | Sample Statistics | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) |
| | | | | | Mean | | | Standard Deviation | | | Skewness | |
| | Min | Med | Max | Min | Mean | Max | Min | Mean | Max | Min | Mean | Max |
| Panel A: Cohorts | | | | | | | | | | | | |
| 3 | 85 | 796 | 284 | 0.9324 | 0.9958 | 1.0670 | 0.0014 | 0.0183 | 0.0545 | -0.7071 | -0.0072 | 0.7071 |
| 6 | 38 | 407 | 155 | 0.9338 | 0.9959 | 1.0646 | 0.0085 | 0.0301 | 0.0669 | -1.7354 | 0.0410 | 1.5536 |
| 9 | 22 | 290 | 101 | 0.9328 | 0.9962 | 1.0666 | 0.0175 | 0.0385 | 0.0837 | -1.8792 | 0.0237 | 1.9912 |
| 12 | 16 | 228 | 77 | 0.9333 | 0.9966 | 1.0669 | 0.0215 | 0.0450 | 0.0815 | -1.6275 | 0.1125 | 2.1600 |
| 15 | 11 | 196 | 61 | 0.9328 | 0.9969 | 1.0690 | 0.0250 | 0.0492 | 0.0816 | -1.6487 | 0.1211 | 1.7253 |
| 18 | 9 | 162 | 53 | 0.9315 | 0.9971 | 1.0658 | 0.0239 | 0.0549 | 0.0896 | -2.0647 | 0.1387 | 1.6857 |
| 21 | 7 | 146 | 45 | 0.9316 | 0.9974 | 1.0679 | 0.0365 | 0.0596 | 0.1037 | -1.6419 | 0.1972 | 2.3295 |
| 24 | 6 | 135 | 39 | 0.9341 | 0.9979 | 1.0676 | 0.0358 | 0.0643 | 0.1104 | -1.3747 | 0.1757 | 1.7728 |
| 27 | 6 | 123 | 34 | 0.9346 | 0.9982 | 1.0700 | 0.0399 | 0.0678 | 0.1110 | -1.0690 | 0.1533 | 2.3323 |
| 30 | 4 | 111 | 31 | 0.9367 | 0.9985 | 1.0780 | 0.0410 | 0.0716 | 0.1152 | -1.3499 | 0.1956 | 2.4651 |
| Panel B: Clusters | | | | | | | | | | | | |
| 3 | 85 | 796 | 284 | 0.9324 | 0.9958 | 1.0670 | 0.0014 | 0.0183 | 0.0545 | -0.7071 | -0.0072 | 0.7071 |
| 6 | 1 | 784 | 148 | 0.8875 | 0.9965 | 1.1419 | 0.0091 | 0.0448 | 0.3764 | -1.7637 | -0.0445 | 1.7735 |
| 9 | 1 | 530 | 104 | 0.8884 | 0.9961 | 1.1194 | 0.0189 | 0.0725 | 0.3195 | -2.3725 | -0.0184 | 2.4262 |
| 12 | 1 | 417 | 78 | 0.8944 | 0.9999 | 1.1497 | 0.0224 | 0.0925 | 0.4026 | -2.8572 | 0.1910 | 2.9629 |
| 15 | 1 | 316 | 62 | 0.8920 | 1.0045 | 1.1411 | 0.0268 | 0.1127 | 0.3705 | -2.8053 | 0.4303 | 3.3375 |
| 18 | 1 | 311 | 51 | 0.9055 | 1.0057 | 1.1200 | 0.0337 | 0.1244 | 0.3409 | -2.9152 | 0.4823 | 3.7090 |
| 21 | 1 | 288 | 45 | 0.8939 | 1.0073 | 1.1349 | 0.0353 | 0.1379 | 0.3766 | -2.5377 | 0.5678 | 3.6326 |
| 24 | 1 | 294 | 37 | 0.8963 | 1.0079 | 1.1231 | 0.0492 | 0.1431 | 0.4419 | -3.0933 | 0.7795 | 3.7958 |
| 27 | 1 | 261 | 33 | 0.9033 | 1.0074 | 1.1389 | 0.0504 | 0.1461 | 0.4188 | -2.8729 | 0.7921 | 3.7894 |
| 30 | 1 | 259 | 29 | 0.9056 | 1.0076 | 1.1308 | 0.0524 | 0.1506 | 0.3993 | -2.3879 | 0.8623 | 4.0817 |

Table 2: Summary Statistics for Asset Returns

This table presents summary statistics for quarterly real asset returns. The returns are overlapping and available at a monthly frequency to match consumption growth. We report means, standard deviations, skewness and kurtosis for the equally weighted (EW) and value weighted (VW) market returns and for the risk-free rate. The market return is the return on value-weighted NYSE-AMEX-NASDAQ index (CRSP variable VWINDD).The risk-free rate is the monthly return on 30-day Treasury Bill Portfolio (CRSP variable T30RET). In addition to the first four moments, we also report the first four autocorrelations for market returns and the risk-free rate. The sample period is from March 1984 to November 2002.

| Assets | Central Moments | | | | Autocorrelations | | | |
|---|---|---|---|---|---|---|---|---|
| | Mean | St Dev | Skew | Kurt | Lag 1 | Lag 2 | Lag 3 | Lag 4 |
| Risk-free Rate | 0.006 | 0.005 | 0.337 | 4.213 | 0.811 | 0.516 | 0.295 | 0.268 |
| VW market return | 0.023 | 0.081 | -0.672 | 4.434 | 0.653 | 0.241 | -0.119 | -0.119 |
| EW market return | 0.022 | 0.104 | -0.375 | 3.996 | 0.691 | 0.213 | -0.186 | -0.256 |

## Table 3: Unexplained Equity Premium

This table presents unexplained equity premium and the corresponding $p$-values for different sets of cohorts of consumers sorted on education and age. Unexplained equity premium is given by:

$$w_t = \frac{1}{TK} \sum_{t=1}^{T} \sum_{k=1}^{K} (g_{k,t})^{-\gamma} (R_{M,t} - R_{F,t}). \tag{11}$$

$K$ represents the number of household cohorts, $g_{k,t}$ is the cohort-based consumption growth, $R_{M,t} - R_{F,t}$ is the excess market return, either equally- or value-weighted. The sample period for the consumption growth data is from March 1984 to November 2002 and includes quarterly observations at a monthly frequency.

| Cohorts | 3 | | 9 | | 18 | | 24 | | 30 | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\gamma$ | UEP | p-val | UEP | p-val | UEP | p-val | UEP | p-val | UEP | p-val |
| Panel A: Equally-Weighted Equity Premium, Cohorts | | | | | | | | | | |
| 0 | 1.42 | 0.02 | 1.42 | 0.02 | 1.42 | 0.02 | 1.42 | 0.02 | 1.42 | 0.02 |
| 1 | 1.44 | 0.02 | 1.44 | 0.02 | 1.44 | 0.02 | 1.44 | 0.02 | 1.44 | 0.02 |
| 2 | 1.46 | 0.02 | 1.47 | 0.02 | 1.47 | 0.02 | 1.47 | 0.02 | 1.48 | 0.02 |
| 3 | 1.48 | 0.02 | 1.50 | 0.02 | 1.51 | 0.02 | 1.52 | 0.02 | 1.53 | 0.02 |
| 4 | 1.51 | 0.02 | 1.53 | 0.02 | 1.55 | 0.02 | 1.57 | 0.02 | 1.59 | 0.02 |
| 5 | 1.54 | 0.02 | 1.57 | 0.02 | 1.61 | 0.02 | 1.63 | 0.02 | 1.66 | 0.02 |
| 6 | 1.57 | 0.02 | 1.62 | 0.02 | 1.67 | 0.02 | 1.71 | 0.01 | 1.75 | 0.01 |
| 7 | 1.61 | 0.02 | 1.67 | 0.01 | 1.75 | 0.01 | 1.81 | 0.01 | 1.87 | 0.01 |
| 8 | 1.65 | 0.02 | 1.73 | 0.01 | 1.83 | 0.01 | 1.92 | 0.01 | 2.02 | 0.01 |
| Panel B: Value-Weighted Equity Premium, Cohorts | | | | | | | | | | |
| 0 | 1.52 | 0.00 | 1.52 | 0.00 | 1.52 | 0.00 | 1.52 | 0.00 | 1.52 | 0.00 |
| 1 | 1.53 | 0.00 | 1.54 | 0.00 | 1.54 | 0.00 | 1.54 | 0.00 | 1.54 | 0.00 |
| 2 | 1.55 | 0.00 | 1.56 | 0.00 | 1.56 | 0.00 | 1.57 | 0.00 | 1.57 | 0.00 |
| 3 | 1.57 | 0.00 | 1.59 | 0.00 | 1.60 | 0.00 | 1.61 | 0.00 | 1.61 | 0.00 |
| 4 | 1.59 | 0.00 | 1.62 | 0.00 | 1.64 | 0.00 | 1.66 | 0.00 | 1.67 | 0.00 |
| 5 | 1.62 | 0.00 | 1.65 | 0.00 | 1.69 | 0.00 | 1.72 | 0.00 | 1.74 | 0.00 |
| 6 | 1.65 | 0.00 | 1.70 | 0.00 | 1.76 | 0.00 | 1.81 | 0.00 | 1.83 | 0.00 |
| 7 | 1.68 | 0.00 | 1.75 | 0.00 | 1.83 | 0.00 | 1.91 | 0.00 | 1.95 | 0.00 |
| 8 | 1.71 | 0.00 | 1.81 | 0.00 | 1.92 | 0.00 | 2.03 | 0.00 | 2.09 | 0.00 |
| Panel C: Equally-Weighted Equity Premium, Clusters | | | | | | | | | | |
| 0 | 1.42 | 0.02 | 1.42 | 0.02 | 1.42 | 0.02 | 1.42 | 0.02 | 1.42 | 0.02 |
| 1 | 1.44 | 0.02 | 1.43 | 0.02 | 1.40 | 0.03 | 1.43 | 0.02 | 1.44 | 0.02 |
| 2 | 1.46 | 0.02 | 1.44 | 0.03 | 1.38 | 0.03 | 1.47 | 0.02 | 1.50 | 0.02 |
| 3 | 1.48 | 0.02 | 1.42 | 0.04 | 1.33 | 0.05 | 1.54 | 0.03 | 1.60 | 0.02 |
| 4 | 1.51 | 0.02 | 1.35 | 0.08 | 1.19 | 0.10 | 1.64 | 0.04 | 1.78 | 0.03 |
| 5 | 1.54 | 0.02 | 1.14 | 0.19 | 0.79 | 0.26 | 1.74 | 0.09 | 2.06 | 0.05 |
| 6 | 1.57 | 0.02 | 0.61 | 0.39 | -0.26 | 0.55 | 1.72 | 0.20 | 2.44 | 0.11 |
| 7 | 1.61 | 0.02 | -0.74 | 0.57 | -2.96 | 0.79 | 1.20 | 0.38 | 2.87 | 0.23 |
| 8 | 1.65 | 0.02 | -4.12 | 0.69 | -9.82 | 0.88 | -0.98 | 0.54 | 2.92 | 0.37 |
| Panel D: Value-Weighted Equity Premium, Clusters | | | | | | | | | | |
| 0 | 1.52 | 0.00 | 1.52 | 0.00 | 1.52 | 0.00 | 1.52 | 0.00 | 1.52 | 0.00 |
| 1 | 1.53 | 0.00 | 1.51 | 0.00 | 1.50 | 0.00 | 1.52 | 0.00 | 1.53 | 0.00 |
| 2 | 1.55 | 0.00 | 1.49 | 0.01 | 1.48 | 0.01 | 1.54 | 0.00 | 1.56 | 0.00 |
| 3 | 1.57 | 0.00 | 1.44 | 0.01 | 1.43 | 0.01 | 1.58 | 0.01 | 1.64 | 0.00 |
| 4 | 1.59 | 0.00 | 1.32 | 0.04 | 1.28 | 0.04 | 1.60 | 0.01 | 1.72 | 0.01 |
| 5 | 1.62 | 0.00 | 1.04 | 0.15 | 0.85 | 0.20 | 1.50 | 0.06 | 1.79 | 0.03 |
| 6 | 1.65 | 0.00 | 0.39 | 0.40 | -0.33 | 0.58 | 1.03 | 0.25 | 1.66 | 0.15 |
| 7 | 1.68 | 0.00 | -1.10 | 0.65 | -3.42 | 0.83 | -0.52 | 0.57 | 0.88 | 0.39 |
| 8 | 1.71 | 0.00 | -4.56 | 0.78 | -10.00 | 0.91 | -5.10 | 0.76 | -1.96 | 0.60 |

Table 4: Unexplained Equity Premium: Quadratic Approximation

This table presents unexplained equity premium and the corresponding $p$-values for different sets of cohorts of households. Unexplained equity premium is based on the quadratic approximation of the stochastic discount factor given by equation (6) in Section 2.2. The sample period March 1984 to November 2002 and includes quarterly observations at a monthly frequency.

| Cohorts | 3 | | 9 | | 18 | | 24 | | 30 | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\gamma$ | UEP | p-val | UEP | p-val | UEP | p-val | UEP | p-val | UEP | p-val |
| Panel A: Equally-Weighted Equity Premium, Cohorts | | | | | | | | | | |
| 0 | 1.42 | 0.02 | 1.42 | 0.02 | 1.42 | 0.02 | 1.42 | 0.02 | 1.42 | 0.02 |
| 1 | 1.44 | 0.02 | 1.44 | 0.02 | 1.44 | 0.02 | 1.44 | 0.02 | 1.44 | 0.02 |
| 2 | 1.46 | 0.02 | 1.47 | 0.02 | 1.47 | 0.02 | 1.47 | 0.02 | 1.48 | 0.02 |
| 3 | 1.48 | 0.02 | 1.50 | 0.02 | 1.51 | 0.02 | 1.51 | 0.02 | 1.52 | 0.02 |
| 4 | 1.51 | 0.02 | 1.53 | 0.02 | 1.55 | 0.02 | 1.56 | 0.02 | 1.57 | 0.02 |
| 5 | 1.54 | 0.02 | 1.57 | 0.02 | 1.60 | 0.02 | 1.62 | 0.02 | 1.64 | 0.02 |
| 6 | 1.57 | 0.02 | 1.62 | 0.02 | 1.65 | 0.02 | 1.68 | 0.02 | 1.71 | 0.02 |
| 7 | 1.61 | 0.02 | 1.67 | 0.02 | 1.72 | 0.02 | 1.76 | 0.01 | 1.80 | 0.02 |
| 8 | 1.65 | 0.02 | 1.72 | 0.01 | 1.79 | 0.02 | 1.84 | 0.01 | 1.90 | 0.02 |
| Panel B: Value-Weighted Equity Premium, Cohorts | | | | | | | | | | |
| 0 | 1.52 | 0.00 | 1.52 | 0.00 | 1.52 | 0.00 | 1.52 | 0.00 | 1.52 | 0.00 |
| 1 | 1.53 | 0.00 | 1.54 | 0.00 | 1.54 | 0.00 | 1.54 | 0.00 | 1.54 | 0.00 |
| 2 | 1.55 | 0.00 | 1.56 | 0.00 | 1.56 | 0.00 | 1.57 | 0.00 | 1.57 | 0.00 |
| 3 | 1.57 | 0.00 | 1.58 | 0.00 | 1.59 | 0.00 | 1.60 | 0.00 | 1.60 | 0.00 |
| 4 | 1.59 | 0.00 | 1.62 | 0.00 | 1.63 | 0.00 | 1.65 | 0.00 | 1.65 | 0.00 |
| 5 | 1.62 | 0.00 | 1.65 | 0.00 | 1.68 | 0.00 | 1.71 | 0.00 | 1.71 | 0.00 |
| 6 | 1.65 | 0.00 | 1.69 | 0.00 | 1.73 | 0.00 | 1.77 | 0.00 | 1.79 | 0.00 |
| 7 | 1.68 | 0.00 | 1.74 | 0.00 | 1.79 | 0.00 | 1.85 | 0.00 | 1.87 | 0.00 |
| 8 | 1.71 | 0.00 | 1.79 | 0.00 | 1.86 | 0.00 | 1.94 | 0.00 | 1.96 | 0.00 |
| Panel C: Equally-Weighted Equity Premium, Clusters | | | | | | | | | | |
| 0 | 1.42 | 0.02 | 1.42 | 0.02 | 1.42 | 0.02 | 1.42 | 0.02 | 1.42 | 0.02 |
| 1 | 1.44 | 0.02 | 1.43 | 0.02 | 1.41 | 0.02 | 1.43 | 0.02 | 1.44 | 0.02 |
| 2 | 1.46 | 0.02 | 1.45 | 0.03 | 1.44 | 0.03 | 1.49 | 0.02 | 1.50 | 0.02 |
| 3 | 1.48 | 0.02 | 1.45 | 0.03 | 1.50 | 0.03 | 1.58 | 0.02 | 1.60 | 0.02 |
| 4 | 1.51 | 0.02 | 1.46 | 0.04 | 1.59 | 0.03 | 1.72 | 0.02 | 1.74 | 0.02 |
| 5 | 1.54 | 0.02 | 1.45 | 0.06 | 1.70 | 0.03 | 1.89 | 0.02 | 1.93 | 0.02 |
| 6 | 1.57 | 0.02 | 1.44 | 0.08 | 1.83 | 0.04 | 2.11 | 0.02 | 2.15 | 0.02 |
| 7 | 1.61 | 0.02 | 1.41 | 0.12 | 1.97 | 0.04 | 2.37 | 0.02 | 2.42 | 0.02 |
| 8 | 1.65 | 0.02 | 1.36 | 0.17 | 2.12 | 0.05 | 2.67 | 0.02 | 2.74 | 0.02 |
| Panel D: Value-Weighted Equity Premium, Clusters | | | | | | | | | | |
| 0 | 1.52 | 0.00 | 1.52 | 0.00 | 1.52 | 0.00 | 1.52 | 0.00 | 1.52 | 0.00 |
| 1 | 1.53 | 0.00 | 1.51 | 0.00 | 1.51 | 0.00 | 1.52 | 0.00 | 1.53 | 0.00 |
| 2 | 1.55 | 0.00 | 1.51 | 0.00 | 1.53 | 0.00 | 1.57 | 0.00 | 1.59 | 0.00 |
| 3 | 1.57 | 0.00 | 1.50 | 0.01 | 1.58 | 0.00 | 1.65 | 0.00 | 1.68 | 0.00 |
| 4 | 1.59 | 0.00 | 1.48 | 0.01 | 1.64 | 0.01 | 1.77 | 0.00 | 1.82 | 0.00 |
| 5 | 1.62 | 0.00 | 1.46 | 0.02 | 1.72 | 0.01 | 1.92 | 0.00 | 2.00 | 0.00 |
| 6 | 1.65 | 0.00 | 1.42 | 0.04 | 1.82 | 0.01 | 2.11 | 0.00 | 2.21 | 0.00 |
| 7 | 1.68 | 0.00 | 1.36 | 0.08 | 1.92 | 0.01 | 2.32 | 0.00 | 2.46 | 0.00 |
| 8 | 1.71 | 0.00 | 1.28 | 0.13 | 2.03 | 0.02 | 2.57 | 0.01 | 2.75 | 0.00 |

Table 5: Unexplained Equity Premium: Cubic Approximation

This table presents unexplained equity premium and the corresponding $p$-values for different sets of cohorts of households. Unexplained equity premium is based on the cubic approximation of the stochastic discount factor given by equation (7) in Section 2.2. The sample period March 1984 to November 2002 and includes quarterly observations at a monthly frequency.

| Cohorts | 3 | | 9 | | 18 | | 24 | | 30 | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\gamma$ | UEP | p-val | UEP | p-val | UEP | p-val | UEP | p-val | UEP | p-val |
| Panel A: Equally-Weighted Equity Premium, Cohorts | | | | | | | | | | |
| 0 | 1.42 | 0.02 | 1.42 | 0.02 | 1.42 | 0.02 | 1.42 | 0.02 | 1.42 | 0.02 |
| 1 | 1.44 | 0.02 | 1.44 | 0.02 | 1.44 | 0.02 | 1.44 | 0.02 | 1.44 | 0.02 |
| 2 | 1.46 | 0.02 | 1.47 | 0.02 | 1.47 | 0.02 | 1.47 | 0.02 | 1.48 | 0.02 |
| 3 | 1.48 | 0.02 | 1.50 | 0.02 | 1.51 | 0.02 | 1.51 | 0.02 | 1.52 | 0.02 |
| 4 | 1.51 | 0.02 | 1.53 | 0.02 | 1.55 | 0.02 | 1.56 | 0.02 | 1.58 | 0.02 |
| 5 | 1.54 | 0.02 | 1.57 | 0.02 | 1.60 | 0.02 | 1.62 | 0.02 | 1.65 | 0.02 |
| 6 | 1.57 | 0.02 | 1.62 | 0.02 | 1.66 | 0.02 | 1.69 | 0.01 | 1.73 | 0.02 |
| 7 | 1.61 | 0.02 | 1.67 | 0.02 | 1.73 | 0.01 | 1.78 | 0.01 | 1.82 | 0.01 |
| 8 | 1.65 | 0.02 | 1.73 | 0.01 | 1.81 | 0.01 | 1.87 | 0.01 | 1.93 | 0.01 |
| Panel B: Value-Weighted Equity Premium, Cohorts | | | | | | | | | | |
| 0 | 1.52 | 0.00 | 1.52 | 0.00 | 1.52 | 0.00 | 1.52 | 0.00 | 1.52 | 0.00 |
| 1 | 1.53 | 0.00 | 1.54 | 0.00 | 1.54 | 0.00 | 1.54 | 0.00 | 1.54 | 0.00 |
| 2 | 1.55 | 0.00 | 1.56 | 0.00 | 1.56 | 0.00 | 1.57 | 0.00 | 1.57 | 0.00 |
| 3 | 1.57 | 0.00 | 1.58 | 0.00 | 1.60 | 0.00 | 1.60 | 0.00 | 1.61 | 0.00 |
| 4 | 1.59 | 0.00 | 1.62 | 0.00 | 1.64 | 0.00 | 1.65 | 0.00 | 1.66 | 0.00 |
| 5 | 1.62 | 0.00 | 1.65 | 0.00 | 1.69 | 0.00 | 1.71 | 0.00 | 1.72 | 0.00 |
| 6 | 1.65 | 0.00 | 1.70 | 0.00 | 1.74 | 0.00 | 1.78 | 0.00 | 1.79 | 0.00 |
| 7 | 1.68 | 0.00 | 1.74 | 0.00 | 1.81 | 0.00 | 1.87 | 0.00 | 1.88 | 0.00 |
| 8 | 1.71 | 0.00 | 1.80 | 0.00 | 1.89 | 0.00 | 1.96 | 0.00 | 1.98 | 0.00 |
| Panel C: Equally-Weighted Equity Premium, Clusters | | | | | | | | | | |
| 0 | 1.42 | 0.02 | 1.42 | 0.02 | 1.42 | 0.02 | 1.42 | 0.02 | 1.42 | 0.02 |
| 1 | 1.44 | 0.02 | 1.44 | 0.02 | 1.39 | 0.03 | 1.42 | 0.02 | 1.43 | 0.02 |
| 2 | 1.46 | 0.02 | 1.45 | 0.03 | 1.34 | 0.03 | 1.44 | 0.02 | 1.47 | 0.02 |
| 3 | 1.48 | 0.02 | 1.47 | 0.03 | 1.24 | 0.05 | 1.47 | 0.03 | 1.52 | 0.02 |
| 4 | 1.51 | 0.02 | 1.48 | 0.05 | 1.06 | 0.09 | 1.47 | 0.03 | 1.56 | 0.02 |
| 5 | 1.54 | 0.02 | 1.48 | 0.07 | 0.77 | 0.19 | 1.45 | 0.05 | 1.58 | 0.03 |
| 6 | 1.57 | 0.02 | 1.47 | 0.11 | 0.35 | 0.36 | 1.38 | 0.08 | 1.57 | 0.05 |
| 7 | 1.61 | 0.02 | 1.44 | 0.16 | -0.24 | 0.58 | 1.25 | 0.14 | 1.50 | 0.09 |
| 8 | 1.65 | 0.02 | 1.37 | 0.23 | -1.02 | 0.78 | 1.04 | 0.23 | 1.35 | 0.16 |
| Panel D: Value-Weighted Equity Premium, Clusters | | | | | | | | | | |
| 0 | 1.52 | 0.00 | 1.52 | 0.00 | 1.52 | 0.00 | 1.52 | 0.00 | 1.52 | 0.00 |
| 1 | 1.53 | 0.00 | 1.51 | 0.00 | 1.49 | 0.00 | 1.50 | 0.00 | 1.52 | 0.00 |
| 2 | 1.55 | 0.00 | 1.50 | 0.00 | 1.45 | 0.00 | 1.49 | 0.00 | 1.53 | 0.00 |
| 3 | 1.57 | 0.00 | 1.47 | 0.01 | 1.38 | 0.01 | 1.48 | 0.01 | 1.54 | 0.00 |
| 4 | 1.59 | 0.00 | 1.42 | 0.02 | 1.26 | 0.02 | 1.43 | 0.01 | 1.53 | 0.01 |
| 5 | 1.62 | 0.00 | 1.35 | 0.04 | 1.06 | 0.06 | 1.35 | 0.03 | 1.48 | 0.01 |
| 6 | 1.65 | 0.00 | 1.23 | 0.09 | 0.76 | 0.16 | 1.19 | 0.07 | 1.37 | 0.03 |
| 7 | 1.68 | 0.00 | 1.06 | 0.18 | 0.33 | 0.36 | 0.95 | 0.17 | 1.18 | 0.08 |
| 8 | 1.71 | 0.00 | 0.81 | 0.29 | -0.24 | 0.59 | 0.61 | 0.31 | 0.89 | 0.19 |