OPTIMAL CASUALTY INSURANCE, REPAIR, AND REGULATION IN THE PRESENCE OF A SECURITIES MARKET

AN CHEN[‡] AND PHILIP H. DYBVIG[*]

ABSTRACT. Insurance regulation is often based on keeping probabilities of failure small and not on any explicit analysis of economic trade-offs. We build a simple economic model of optimal casualty insurance based on a story about insuring a house. This is done in the presence of a securities market that is complete over states distinguished by security payoffs. We show that partial insurance might be desirable, i.e. the optimal insurance policy fully insures the casualty loss in less expensive states and does not pay off anything in more expensive states. We also analyze a stylized model of insurance regulation imposing a maximum default probability. Curiously, the two problems have a solution of the same form, and there is a choice for the default probability that implements the optimal insurance contract. The equivalence breaks down outside the simplest benchmark case, but in general there is less than full insurance (even absent moral hazard or adverse selection), and optimal repair is fully insured.

Keywords: optimal casualty insurance, optimal regulation
JEL-Code: G11, G22

1. INTRODUCTION

In the simplest model of optimal insurance, there is actuarially fair insurance, and agents are fully insured in the sense that in equilibrium they are indifferent about whether a loss occurs or not. As discussed by Arrow (1963) and formalized more fully elsewhere in the literature,[1] there is less than full insurance in the presence of various frictions and informational problems (adverse selection and moral hazard). This paper gives an alternative reason why there may be less than full insurance, namely that if there is endogenous repair, there may be less than full insurance if it is not always optimal to undertake the repair. This idea is developed in a stylized example with a possible casualty loss on a house in the absence of aggregate actuarial risk and in the presence of a securities market the insurance company can invest in. In this example, there is less than full insurance

Date. June 15, 2009.

‡ Department of Business Administration III, Faculty of Law and Economics, University of Bonn, Adenauerallee 24-42, 53113 Bonn, Germany, Tel: 0049-228-739227, Fax: 0049-228-735048.

* Olin School of Business, Washington University in Saint Louis, Campus Box 1133 One Brookings Drive Saint Louis, MO 63130-4899, USA, Fax: 001-815-642-4318.

[1]See for example Arrow (1971, 1973), Holmstrom (1979), Shavell (1979)), Rothschild and Stiglitz (1976), Towsend (1979), Rey (2003), and Doherty and Schlesinger (1983).

(even though there is not any friction or informational asymmetry), but optimal repair is fully insured. Specifically, the optimal insurance policy fully insures the casualty loss in less expensive market states when repair is optimal, but does not pay any compensation in more expensive market states. The paper also gives a curious result that in our base model, traditional insurance regulation based on probability of default (which is similar to the more recent concept of Value-at-Risk) can be used to implement the optimal insurance contract. We also provide a number of extensions that suggest the curious result is not very general.

In the present paper, the analysis of the optimal insurance is conducted by introducing a simple economic model of optimal casualty insurance based on a story about insuring a house. This is done in a one–period model in the presence of a securities market that is complete over states distinguished by security payoffs. In other words, in addition to purchasing insurance, the agent may invest his endowment in securities at the initial time. The agent aims to maximize the expected utility of the terminal wealth which is dependent of both the insurance payment and the investment payoff from a securities market (taking account of the casualty loss, state realizations and repair policy). We adopt the usual approach to optimal contracting of computing the optimal payment and afterwards seeking realistic institutions to implement the optimal contract, in this case decomposing the cash flow into an investment portfolio and an insurance policy. We show there is less than full insurance (even absent moral hazard or adverse selection), but optimal repair is fully insured.

In a separate model, we analyze a stylized model of insurance regulation imposing a maximum default probability (under the traditional insurance criterion) in the context of optimal investment. That is, the insurance company maximizes the market value of the payoff to shareholders under the criterion that the probability of default (a net loss, or assets less than liabilities) is no more than some critical probability. In other words, the criterion requires a minimum probability α of solvency. Curiously, in the simplest case the regulatory outcome has the same form as the optimal insurance contract, and the maximum default probability can be chosen to implement the optimal contract.

We examine whether the equivalence still holds in some less stylized settings: 1) we consider several examples of non-additively-separable preferences; 2) the repair policy is allowed to be chosen over a continuum of values (instead of two policies "no repair" or "repair all"); 3) the casualty loss can take any value in $[0, H)$ (lifting the Bernoulli distribution of the loss); 4) the casualty loss is not independent of the states; 5) H is random. In most cases, the equivalence result between the optimal insurance and optimal default regulation breaks down, but it is still optimal to have less than full insurance.

The remainder of the paper is organized as follows. Section 2 investigates optimal casualty wealth and insurance problems in a stylized model. Section 3 presents a stylized model of insurance regulation imposing a maximum default probability. The solutions to both problems are of the same form. Section 4 discusses some extensions to examine the robustness of our equivalence result. Section 5 concludes the paper.

2. Stylized model of optimal casualty insurance

There are two points of time, 0 and 1. A representative agent has endowment only at time 0: cash and a house of size H purchased previously. At time 0, the agent may invest his endowment in securities in a complete market over states distinguished by security payoffs, and he may also purchase insurance through a mutual insurance company. Owning the house carries a risk of a casualty loss (this is what the insurance is about). The agent's casualty loss is given by the random variable C. C is assumed to be Bernoulli with value c with probability p and 0 with probability $1 - p$. The objective probability of the loss is common knowledge for all agents, and equals the true population average that will be realized. Investment in the market and buying insurance is valued by a state-price density ξ, so the agent's budget constraint for terminal financial wealth P is

$$W_0 = E[\xi P].$$

P is the total terminal financial payment, which equals investment plus payment obtained from the insurance policy. Assuming identical additively separable von Neumann-Morgenstern preferences $U_H(H) + U_W(P)$, and identical initial wealth, each agent aims to maximize expected utility:

$$\max_{P,R} E[U_H(H - C + R) + U_W(P - \gamma R)].$$

R is the value of the repair, and the cost of the repair is γR where $\gamma > 0$. We are assuming that the agents never want to sell their houses because the total cost of selling the house and moving elsewhere is large. Therefore, the choice of whether to repair depends on the agent's preferences and need not depend only on whether the increase in house price is bigger than the cost of repair. If $\gamma < 1$, doing the repair increases the market value of the house more than the repair cost, although given that the owner is not selling it still may not be optimal to make the repair. If $\gamma > 1$, the repair increases the value of the house less than the cost of the repair (for example it may be an expensive repair to return the trim to original condition but it is not a structural problem or a big blemish to leave it as it is). Again, given that the owner is not selling the house now, it may be worth it to do the repair. In the special case, $\gamma = 1$, the repair increases the value of the house by exactly the cost of the repair. It is reasonable to assume that R is less than the casualty loss, i.e. $R \in [0, C]$, that is, we are considering a repair, not an addition to the house ($R > C$) or

selling off part of the house ($R < 0$).

In the following, we take the usual approach to optimal contracting of computing the optimal payment (the direct mechanism) and afterwards interpreting the optimal contract. This decomposes naturally into the sum of two nonnegative payments: one payment looks like investment in a securities market we might see absent any casualty loss, and the other payment looks like insurance subject to default. We first assume that the agent's repair policy R can be chosen to be 0 ("no repair") or c ("repair all") (the case $R \in [0, c]$ is discussed in Section 4). More specifically, it implies

$$C = 0 \Rightarrow R = 0; \quad \text{and} \quad C = c \Rightarrow R = 0 \text{ or } c. \tag{1}$$

Both P and R are functions of ξ and C, i.e. $P : \Re_+ \times \{0, c\} \rightarrow \Re_+$ and $R : \Re_+ \times \{0, c\} \rightarrow \{0, c\}$. The optimal payment P and repair R solve the following problem:

$$\text{Choose} \quad P : \Re_+ \times \{0, c\} \rightarrow \Re_+ \text{ and } R : \Re_+ \times \{0, c\} \rightarrow \{0, c\} \text{ to} \tag{2}$$
$$\text{maximize} \quad E[U_H(H - C + R(\xi, C)) + U_W(P(\xi, C) - \gamma R(\xi, C))]$$
$$\text{subject to} \quad W_0 = E[\xi P(\xi, C)]$$
$$\text{for all} \quad \xi \in \Re_+, \text{ and } C \in \{0, c\}, \ R(\xi, C) \leq C.$$

The resource constraint $W_0 = E[\xi P(\xi, C)]$ is equivalently the individual constraint given actuarially fair pricing of insurance against C conditional on ξ or a mutual insurance company's constraint for the representative customer when the insurance company does not face actuarial risk in aggregate. The analysis of optimal financial wealth and optimal insurance payment is carried out according to the realizations of the states ξ and of the casualty loss C.

To start with, we neglect first the restrictions on the form of repair set, full, partial or some other form. The Lagrangian function is

$$\mathcal{L} = E\left[U_H(H - C + R(\xi, C)) + U_W(P(\xi, C) - \gamma R(\xi, C))\right] + \lambda(W_0 - E\left[\xi P(\xi, C)\right])$$
$$= E\left[U_H(H - C + R(\xi, C)) + U_W(P(\xi, C) - \gamma R(\xi, C)) + \lambda(W_0 - \xi P(\xi, C))\right] \tag{3}$$

with λ being a Lagrangian multiplier. Maximizing the integrand of the Lagrangian function over the final wealth P leads to

$$U'_W(P(\xi, C) - \gamma R(\xi, C)) - \lambda \xi = 0$$

or if the inverse marginal utility I_W exists (for concave utilities, U'_W is strictly decreasing and there exits a unique inverse $(U'_W)^{-1} := I_W$), we obtain

$$P^*(\xi, C) = I_W(\lambda \xi) + \gamma R(\xi, C). \tag{4}$$

When there is some casualty loss, the optimal final wealth P^* depends on R, namely on the choice of the agent whether to repair the house. Interestingly, the monetary shock is

endogenous because repair is endogenous. Note that the optimal wealth can be decomposed into two parts: optimal investment $I_W(\lambda \xi)$ and optimal insurance $\gamma R(\xi, C)$. This form of the solution states that whatever the policy for repair (the R function), insurance will pay for all repairs. The insurance will pay for the repair, if any, but would not pay off if there is a casualty loss but no repair, i.e. the cost of whatever repair is made is insured.

Maximizing the integrand of the Lagrangian (3) over R leads to

$$U'_H(H - C + R(\xi, C)) + U'_W(P(\xi, C) - \gamma R(\xi, C))(-\gamma) = 0$$

At the optimum, substituting $P^*(\xi, C)$ from (4) into the above equation and taking account of the constraints on R, we obtain the optimal repair policy

$$R^*(\xi, C) = \min\{\max\{I_H(\gamma \lambda \xi) - H + C, 0\}, C\}. \tag{5}$$

Obviously, the optimal repair policy is dependent on the state realization ξ. Given a concave utility U_H, the inverse $I_H := (U'_H)^{-1}$ is decreasing in ξ. It implies that given $I_H(\gamma \lambda \xi) - H + C$ is in $[0, C]$, the optimal repair decreases in ξ. Since $C = 0$ implies $R = 0$, the repair policy is only relevant when $C = c$. In particular, if only two repair policies are available: $R = 0$ ("no repair") or $R = c$ ("repair all"), there exists a critical value ξ^*, such that all the values of ξ larger than ξ^* lead to a non-positive value of $I_H(\gamma \lambda \xi) - H + C$ and consequently to $R = 0$. To put it differently, in less expensive states $\xi < \xi^*$, "repair all" is chosen over "no repair" and in more expensive states $\xi \geq \xi^*$, "no repair" is chosen. Let us now take a look at how ξ^* is determined. Given $C = c$, if R is chosen equal to c (corresponding to "repair all" case), it results in the value

$$\mathcal{L}_{R=c} = U_H(H) + U_W(I_W(\lambda \xi)) + \lambda(W_0 - \xi(I_W(\lambda \xi) + \gamma c)),$$

whereas if R is chosen equal to 0 (corresponding to "no repair" case), we have the value

$$\mathcal{L}_{R=0} = U_H(H - c) + U_W(I_W(\lambda \xi)) + \lambda(W_0 - \xi I_W(\lambda \xi)).$$

"Repair all" is at least as good as "no repair" when

$$\mathcal{L}_{R=c} \geq \mathcal{L}_{R=0} \quad \Leftrightarrow \quad \xi \leq \frac{1}{\lambda \gamma c}(U_H(H) - U_H(H - c)) := \xi^*. \tag{6}$$

Condition (6) indicates that in the less expensive states, "repair all" is preferable to "no repair", whereas in the more expensive states, "no repair" will be chosen. In other words, the optimal insurance policy fully insures the casualty loss in less expensive states and does not pay off anything in more expensive states. Furthermore, if λ is given, the higher the γ-value (the cost of the repair), the lower the critical ξ^*, and the better the economy must be in order to make "repair all" a desirable choice for the agent. Hence, the probability of doing a repair is decreasing in γ all the time if λ is given. However, as we will see later, λ is determined endogenously through the budget constraint, the effect of γ becomes more subtle and the argument that the probability of doing a repair is decreasing is γ holds only

State and casualty realization	Optimal investment W^*	Optimal insurance I^*
$\{C = 0\}$	$\frac{1}{\lambda\xi}$	0
$\{C = c \text{ and } \xi > \xi^*\}$	$\frac{1}{\lambda\xi}$	0
$\{C = c \text{ and } \xi \leq \xi^*\}$	$\frac{1}{\lambda\xi}$	γc

TABLE 1. Decomposition of the optimal wealth P^* ($R \in \{0, c\}$).

under certain conditions.

The exact optimal terminal wealth and optimal repair policy depend on the law for ξ, the utility function, the probability of a loss, and the parameters c, H, and W_0. In the following, we take log-utility as an example to analyze the optimal insurance and repair policy, i.e. $U_H(z) = \nu \log z$ and $U_W(z) = \log z$.

Given ξ and $C = 0$, the integrand of the Lagrangian function is

$$\mathcal{L} = \nu \log H + \log P(\xi, 0) + \lambda(W_0 - \xi P(\xi, 0)). \tag{7}$$

Maximizing the integrand of the Lagrangian function over the final wealth P, we obtain

$$P^*(\xi, 0) = \frac{1}{\lambda \xi}. \tag{8}$$

Given ξ and $C = c$, the integrand of the Lagrangian function is

$$\mathcal{L} = \nu \log(H - c + R(\xi, c)) + \log(P(\xi, c) - \gamma R(\xi, c)) + \lambda(W_0 - \xi P(\xi, c)).$$

Log-utilities lead to the optimal wealth

$$P^*(\xi, c) = \frac{1}{\lambda \xi} + \gamma R(\xi, c), \tag{9}$$

where $R \in \{0, c\}$, i.e. "no repair" or "repair all". Under the log-utility assumption, "repair all" is at least as good as "no repair" when

$$\xi \leq \frac{\nu}{\lambda \gamma c} \log \frac{H}{H - c} = \xi^*. \tag{10}$$

To sum up, under log-utility the optimal final wealth P^* takes the values:

$$P^*(\xi, C) = \begin{cases} \frac{1}{\lambda\xi} & \text{if } \{C = 0\} \quad \text{or} \quad \{C = c \text{ and } \xi > \xi^*\} \\ \frac{1}{\lambda\xi} + \gamma c & \text{if } \{C = c \text{ and } \xi \leq \xi^*\} \end{cases}. \tag{11}$$

The optimal wealth can be decomposed into two parts: optimal investment from the securities market and optimal insurance. As illustrated in Table 1, the solution to the problem will have no insurance when ξ is high and full insurance otherwise, where the threshold ξ^* for providing the insurance that is a higher level of ξ the smaller the γ (given λ not endogenously determined). Furthermore, the insurance will pay exactly the repair γR and other investments will be made directly. Full insurance is not always the optimal insurance

policy, but the insurance will always pay for the repair.

The parameter still needed to be determined is the optimal Lagrangian multiplier λ, which is chosen to make the budget constraint binding, i.e.

$$W_0 = (1-p)E\left[\frac{1}{\lambda\,\xi}\,\xi\right] + pE\left[\frac{1}{\lambda\,\xi}\,\xi + \gamma\,c\,\xi\,1_{\{\xi\leq\frac{\nu}{\lambda\gamma c}\log\frac{H}{H-c}\}}\right]. \tag{12}$$

Assume, the state price density process is lognormal with

$$\log\xi \sim N\left(-\left(r + \frac{1}{2}\eta^2\right),\eta^2\right), \tag{13}$$

where r is the continuously-compounded yield on a one-period bond and $\eta = \frac{\mu-r}{\sigma}$ is the market price of risk. The assumption is made as it would be in the Black-Scholes model with fixed coefficients and many other stationary models in continuous time with Vasicek term structure (see, for example, Section 3 of Dybvig (1988) or Examples 1 and 2 of Dybvig and Rogers (1997)). The assumption of lognormally distributed state price process ξ implies that (12) can be reformulated as:

$$W_0 = \frac{1}{\lambda} + p\,\gamma\,c\,e^{-r}N\left(\frac{\log(\frac{\nu}{\lambda c\gamma}\log\frac{H}{H-c}) + (r - \frac{1}{2}\eta^2)}{\eta}\right), \tag{14}$$

where $N(x) := \int_0^x \frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}\,dt$. The optimal value for λ can be determined numerically from the above equation.

3. Default probability regulation

With the increasing number of insurer failures since the 1980s, the IASB (International Accounting Standards Board) in Europe and the FASB (Financial Accounting Standards Board) in the United States advocate risk-based insurance regulation in the new accounting standards. The new regulatory standard "Solvency II" which concerns all European insurance companies and is expected by around 2010 emphasizes a "risk-oriented approach" which focuses on downside risk. That is, the insurance regulation is based on keeping probabilities of failure small, namely the probability of a net loss for the whole net position (using the dynamic trading strategy not a fictional buy-and-hold at each point in time) is bigger than the initial investment is no more than some critical amount.[2]

Consider a stylized model of optimal default regulation of an insurance company. To be consistent with Section 2, there are only two points of time: time 0 and 1. At time 0, the insurance company's initial assets value is W_0, which is financed by a representative insured and a representative shareholder. The company's contractual obligation is to repay

---

[2]We do not know whether Solvency II suffers from the same defects of Basle I and II that precipitated the bank failures. This is an important question but is not dealt with in this paper.

the insured a certain amount L at time 1. In our model setup, since we ignore any feature linked to the actuarial risk and focus on the financial risk, L is assumed to be constant, which implies that the solvency requirements are based on financial considerations and aspects only. Due to the limited liability of the shareholder, the insured has no assurance to obtain L at time 1. In order to protect the insured, the regulatory authority has the incentive to ensure the promised amount of L to the insured with at least a probability of α. Taking this insurance regulation criterion into consideration and letting W be the terminal wealth of the insurance company, the insurance company faces the following optimization problem:

$$
\begin{aligned}
\text{choose} \quad & W \geq 0 \quad \text{to} \\
\text{maximize} \quad & E[\xi(W-L)^+] \\
\text{subject to} \quad & E[\xi W] = W_0 \\
& E[1_{\{W \geq L\}}] \geq \alpha
\end{aligned}
\tag{15}
$$

with $(x)^+ := \max\{x, 0\}$ and $1_{\{W \geq L\}}$ is the indicator which is 1 when $W \geq L$ and 0 otherwise. That is, the insurance company aims to maximize the market value of the terminal claim of the shareholder. Hereby we assume there is no agency problem and the insurer acts on behalf of the shareholder. After providing the insurance payment, the insurance company will provide the remaining terminal wealth to the shareholder. Due to the property of limited liability (his terminal payment cannot be negative), the claim of the shareholder on the insurance company can be considered as a European option written on the final wealth with the strike L. Note that some costs might be added upon bankruptcy, for instance by saying that only a fraction of the wealth is left upon liquidation. For simplicity reasons, we assume there is no costs for bankruptcy or other types of costs. This could be implemented by funding this insurance contract with a separate pool of money. Furthermore, we assume there is no recourse to resources from other policies.

Assuming ξ is nonatomic[3] and that the problem is feasible, the solution is not usually unique, but the optimal payoff on the insurance $\min\{W, L\}$ is unique. Define $\bar{\xi} \equiv \inf\{x | P(\xi < x) \geq \alpha\}$. Since ξ is nonatomic, $P(\xi < \bar{\xi}) = \alpha$, and if the problem is feasible one solution is

$$
W = W_0 \frac{1_{\{\xi < \bar{\xi}\}}}{E[\xi 1_{\{\xi < \bar{\xi}\}}]}
$$

and the unique insurance payoff is

$$
I = L 1_{\{\xi < \bar{\xi}\}}.
$$

---

[3]If ξ has atoms (mass points) there is a trivial change. In that case, the insurance payoff is unique up to perturbation of states in which $\xi = \bar{\xi}$ (defined in the text). If the state space has atoms, but there are finitely many equally probable states, the result is almost the same but the failure probability of $1 - \alpha$ may not be feasible and must be rounded up to the next feasible value. If the state space has atoms of unequal probability, then we may have something like the knapsack problem and things are more complicated.

In the case $W_0 \geq LE[\xi 1_{\{\xi < \bar{\xi}\}}]$, $I = \min\{W, L\}$ as it should, and otherwise the problem is infeasible. If $W_0 = LE[\xi 1_{\{\xi < \bar{\xi}\}}]$, this is the unique feasible solution and $W = L$, but otherwise the solution is not unique.

The intuition behind these claims are as follows. The insurer wants to give the insured as little value as possible, since the insurer gets to keep what is left over. Therefore, the insurer will pay off in full with probability α (any more wastes value that could be kept if allocated to solvent states), and when not paying off in full pays zero (otherwise value is wasted). And to minimize value given to the insured, the states in which the insurance does not pay off should be the more expensive states (with large $\xi > \bar{\xi}$).

Let us now show these claims formally. First, we want to show that the problem is feasible if and only if $W_0 \geq LE[\xi 1_{\{\xi < \bar{\xi}\}}]$. The "if" part is proven constructively by the claimed solution above. We prove the "only if" part by contrapositive. Suppose that $W_0 < LE[\xi 1_{\{\xi < \bar{\xi}\}}]$ and that W is a feasible solution. For any feasible solution, $P(W \geq L) \geq \alpha$. Therefore,

$$
\begin{aligned}
W_0 =& E[\xi W] \\
\geq& E[\xi\, L\, 1_{\{W \geq L\}}] \\
=& L\, E[\xi\, 1_{\{\xi < \bar{\xi}\}}] + L\, E[\xi\, 1_{\{W \geq L \text{ and } \xi \geq \bar{\xi}\}}] - L\, E[\xi 1_{\{W < L \text{ and } \xi < \bar{\xi}\}}] \\
>& W_0 + L\bar{\xi}\left(P(W \geq L \text{ and } \xi \geq \bar{\xi}) - P(W < L \text{ and } \xi < \bar{\xi})\right) \\
\geq& W_0
\end{aligned}
$$

where the last step follows because $P(W \geq L) \geq \alpha$ and $P(\xi < \bar{\xi}) = \alpha$.

Now, we come to the optimization problem. The objective is $E[\xi(W - L)^+] = E[\xi(W - \min\{W, L\})] = W_0 - E[\xi \min\{W, L\}] = W_0 - E[\xi I]$, so the original problem can be restated in terms of the insurance claim as

$$
\begin{aligned}
\text{Choose} \quad & I \geq 0 \quad \text{to} \\
\text{minimize} \quad & E[\xi I] \\
\text{subject to} \quad & E[1_{\{I \geq L\}}] \geq \alpha.
\end{aligned}
$$

In order to prove that the any feasible strategy with the insurance payment of the claimed optimum is optimal, it suffices to show that $E[\xi I]$ is minimized among feasible strategies. Let $I^* = L1_{\{\xi < \bar{\xi}\}}$ be this claimed optimal insurance payment and let I be any other feasible

9

payment.

$$E[\xi I] \geq LE[\xi 1_{\{I \geq L\}}]$$
$$= LE[\xi 1_{\{\xi < \bar{\xi}\}}] + LE[\xi 1_{\{\xi \geq \bar{\xi} \text{ and } I \geq L\}}] - LE[\xi 1_{\{\xi < \bar{\xi} \text{ and } I < L\}}]$$
$$\geq E[\xi I^*] + L\bar{\xi} \left(P(\xi \geq \bar{\xi} \text{ and } I \geq L) - P(\xi < \bar{\xi} \text{ and } I < L) \right)$$
$$\geq E[\xi I^*]$$

where the last step follows from $P(\xi < \bar{\xi}) = \alpha$ and $P(I \geq L) \geq \alpha$. Note that if $I \neq I^*$, the inequalities are strict, so $I = I^*$ is a necessary condition for an optimum.

Summing up the results, if the problem is feasible ($W_0 \geq L\,E[\xi\,1_{\{\xi < \bar{\xi}\}}]$), the optimal wealth is in the form of

$$W^*(\xi) = \begin{cases} 0 & \text{if } \{\xi \geq \bar{\xi}\} \\ \geq L & \text{if } \{\xi < \bar{\xi}\} \end{cases}. \tag{16}$$

Only when $W_0 = L\,E[\xi\,1_{\{\xi < \bar{\xi}\}}]$, we have a unique feasible solution and $W^* = L$, otherwise the solution is not unique. The optimal payoff on the insurance is unique and given by

$$I^*(\xi) = \begin{cases} 0 & \text{if } \{\xi \geq \bar{\xi}\} \\ L & \text{if } \{\xi < \bar{\xi}\} \end{cases}. \tag{17}$$

Curiously, this problem has a solution of the same form as that in the optimal casualty insurance problem in Section 2. The critical value $\bar{\xi}$ is dependent of the default probability parameter $1-\alpha$ and there is a choice for the default probability that implements the optimal insurance contract. Specifically, the critical value of $\bar{\xi}$ can be expressed as a function of α:

$$\bar{\xi} = \exp\left\{ \eta\,N^{-1}(\alpha) - \left(r + \frac{1}{2}\eta^2 \right) \right\},$$

where N^{-1} is the inverse of the cumulative distribution function of the standard normal distribution.

4. Generalizations

In the simplest case, we show that the regulatory outcome has the same form as the optimal contract, and the maximum default probability can be chosen to implement the optimal contract. This is a useful benchmark, although in general the equivalence no longer holds. This section aims to examine the robustness of our results by setting ourselves in more general or different model setups. Besides, some restrictions of the stylized default probability regulation are discussed.

4.1. **Utility assumptions.** Identical additively separable von Neumann-Morgenstern preferences are assumed to analyze the optimal insurance policy in Section 2. Now we will discuss several other preferences characterized by the following utilities:

  (a) $U(aH + W)$, $a > 0$;
  (b) $U(H, W) = -\frac{1}{HW}$;
  (c) $U(H, W) = H^{1/2} W^{1/2}$.

In case (a), housing and wealth are perfect substitutes.[4] In this case, the agent faces the following optimization problem:

$$\text{Choose} \quad P : \Re_+ \times \{0, c\} \to \Re_+ \text{ and } R : \Re_+ \times \{0, c\} \to \{0, c\} \text{ to} \qquad (18)$$

$$\text{maximize} \quad E[U(a(H - C + R(\xi, C)) + P(\xi, C) - \gamma R(\xi, C))]$$

$$\text{subject to} \quad W_0 = E[\xi P(\xi, C)]$$

$$\text{for all} \quad \xi \in \Re_+, \ C \in \{0, c\}), \ R(\xi, C) \le C.$$

The Lagrangian function is

$$\mathcal{L} = E\left[U(a(H - C + R(\xi, C)) + P(\xi, C) - \gamma R(\xi, C)) + \lambda(W_0 - \xi P(\xi, C))\right]$$

with λ being the Lagrangian multiplier. Maximizing the the integrand of the Lagrangian function over the final wealth P leads to

$$U'(a(H - C + R(\xi, C)) + P(\xi, C) - \gamma(R(\xi, C))) - \lambda \xi = 0.$$

If the inverse marginal utility I exists, we can express the optimal terminal wealth as follows:

$$P^*(\xi, C) = I(\lambda \xi) - aH + (\gamma - a)R(\xi, C) + aC.$$

Substituting this back into the utility function, we obtain a utility of $U(I(\lambda \xi))$ which depends on ξ, but not on C. It implies that the optimal insurance policy is full insurance in the sense that the insurance makes the agent indifferent about whether there is a casualty loss or not, given the state ξ of the market. In case (a), the repair policy is also special: optimal repair depends on the parameter a but not on the market state ξ.[5] If $C = 0$, the insurance pay off nothing and the optimal terminal wealth corresponds to $I(\lambda \xi) - aH$, and no repair is needed. If $C = c$, choosing "repair all" leads to

$$\mathcal{L}_{R=c} = U(I(\lambda \xi)) + \lambda(W_0 - \xi(I(\lambda \xi) - aH + \gamma c))$$

and choosing "no repair" leads to

$$\mathcal{L}_{R=0} = U(I(\lambda \xi)) + \lambda(W_0 - \xi(I(\lambda \xi) - aH + ac)).$$

---

[4]Housing and perfect substitutes are in classical sense (the agent is ready to change one goods for another in a constant ratio) which does not depend on substitutability dollar-for-dollar ($a = 1$).

[5]If $a = 1$, the agent is indifferent between repairing and not repairing, and the agent could choose a repair policy that depends on the market state. However, this policy would be no better than choosing a policy that does not depend on the market state.

State and casualty realization	Optimal investment W^*	Optimal insurance I^*
$\{C = 0\}$	$I(\lambda \xi) - a\,H$	0
$\{C = c$ and $a \geq \gamma\}$	$I(\lambda \xi) - a\,H$	$\gamma\,c$
$\{C = c$ and $a < \gamma\}$	$I(\lambda \xi) - a\,H$	$a\,c$

TABLE 2. Decomposition of the optimal wealth P^* ($R \in \{0, c\}$) for the preference $U(aH + W)$.

"Repair all" is at least as good as "no repair" if $a \geq \gamma$ and a reversed consequence results if $a < \gamma$. As a result, the optimal investment from the financial market and insurance is given in Table 2. It is a quite intuitive result: since house and monetary wealth are perfect substitutes, if the increase in the subjective value of the house is larger than the cost of repairing the house ($a\,R \geq \gamma\,R$) the agent always chooses to repair the house, whereas if it is an expensive repair ($\gamma\,R > a\,R$), the agent chooses not to repair. As a result, the optimal insurance policy obtained for $U(a\,H + W)$ leads to an optimal full insurance policy for monetary losses given actuarially fair pricing; optimal repair is fully insured. Besides, if there is a casualty loss, the insurance always pays off, even if there is no repair (it is the case where the optimal insurance corresponds to $a\,c$).

Cases (b) and (c) characterize some general utility functions (not identical additively separable von Neumann-Morgenstern or perfect-substitutes preferences). For $U(H, W) = \frac{-1}{HW}$, the integrand of the Lagrangian function (given ξ and C) is given by

$$\mathcal{L} = \frac{-1}{(H - C + R(\xi, C))(P(\xi, C) - \gamma\,R(\xi, C))} + \lambda(W_0 - \xi\,P(\xi, C))$$

with λ being the Lagrangian multiplier. Maximizing over $P(\xi, C)$, we obtain

$$P^*(\xi, C) = \left(\frac{1}{\lambda\,\xi} \frac{1}{H - C + R(\xi, C)} \right)^{\frac{1}{2}} + \gamma\,R(\xi, C).$$

Substituting this back into the utility function, we obtain a utility as a function of both ξ and C:

$$- (\lambda\,\xi)^{\frac{1}{2}} (H - C + R(\xi, C))^{-\frac{1}{2}}.$$

Particularly, we have different utilities for two cases: $\{C = c, \ R = 0\}$ and $\{C = 0\}$:

$$U_{C=c,R=0} = U_{C=0} \left(\frac{H}{H - c} \right)^{\frac{1}{2}}.$$

State and casualty realization	Optimal investment W^*	Optimal insurance I^*
$\{C = 0\}$	$\left(\frac{1}{\lambda \xi H}\right)^{\frac{1}{2}}$	0
$\{C = c \text{ and } R = 0\}$	$\left(\frac{1}{\lambda \xi H}\right)^{\frac{1}{2}}$	$\left(\frac{1}{\lambda \xi (H-c)}\right)^{\frac{1}{2}} - \left(\frac{1}{\lambda \xi H}\right)^{\frac{1}{2}}$
$\{C = c \text{ and } R = c\}$	$\left(\frac{1}{\lambda \xi H}\right)^{\frac{1}{2}}$	γc

TABLE 3. Decomposition of the optimal wealth P^* ($R \in \{0, c\}$) for the preference $-\frac{1}{HW}$.

It implies that partial insurance might be desirable in this case. Concerning the optimal repair policy, for $C = c$, we have

$$\mathcal{L}_{R=c} = -(\lambda \xi)^{\frac{1}{2}} H^{-\frac{1}{2}} + \lambda \left(W_0 - \xi \left(\frac{1}{\lambda \xi} \frac{1}{H} \right)^{\frac{1}{2}} - \xi \gamma c \right)$$

$$\mathcal{L}_{R=0} = -(\lambda \xi)^{\frac{1}{2}} (H - c)^{-\frac{1}{2}} + \lambda \left(W_0 - \xi \left(\frac{1}{\lambda \xi} \frac{1}{H - c} \right)^{\frac{1}{2}} \right).$$

"Repair all" is chosen over "no repair" when the following condition is satisfied:

$$\left((\lambda \xi)^{\frac{1}{2}} - 1 \right) \left((H - c)^{-\frac{1}{2}} - H^{-\frac{1}{2}} \right) > \lambda \xi \gamma c.$$

Table 3 summarizes the results for the optimal investment in the financial market and optimal insurance. If there is no casualty loss, the optimal wealth is $(1/(\lambda \xi H))^{1/2}$; if there is casualty loss and "no repair" is chosen, the resulting optimal wealth is $(1/(\lambda \xi (H - c)))^{1/2}$; if there is casualty loss and "repair all" is chosen, the optimal wealth is $(1/(\lambda \xi H))^{1/2} + \gamma c$. As observed in Table 3, partial insurance might be desirable, depending on the optimal repair policy which is further dependent on the realization of ξ and C. It still holds the optimal repair is fully insured.

Let us take a look at the last case where we have a Cobb-Douglas utility: $U = H^{1/2} W^{1/2}$. The Lagrangian function can be expressed as

$$\mathcal{L} = E\left[(H - C + R(\xi, C))^{\frac{1}{2}} (P(\xi, C) - \gamma R(\xi, C))^{\frac{1}{2}} + \lambda(W_0 - \xi P(\xi, C)) \right].$$

Maximization over P results in

$$P^*(\xi, C) = \frac{1}{4 (\lambda \xi)^2} (H - C + R(\xi, C)) + \gamma R(\xi, C).$$

Plugging this back in the utility function, we have a utility of

$$\frac{1}{2 \lambda \xi} (H - C + R(\xi, C)),$$

which depends on ξ and casualty loss C. It means that full insurance is not an optimal insurance policy. The agent's utility is not equalized in all circumstances, particularly for

State and casualty realization	Optimal investment W^*	Optimal insurance I^*
$\{C = 0\}$	$\frac{H}{4(\lambda \xi)^2}$	0
$\{C = c \text{ and } R = 0\}$	$\frac{H}{4(\lambda \xi)^2}$	$\frac{H-c}{4(\lambda \xi)^2} - \frac{H}{4(\lambda \xi)^2} < 0$
$\{C = c \text{ and } R = c\}$	$\frac{H}{4(\lambda \xi)^2}$	γc

TABLE 4. Decomposition of the optimal wealth P^* ($R \in \{0, c\}$) for the preference $H^{1/2} W^{1/2}$.

the two scenarios: if there is no casualty loss; and if there is casualty loss and "no repair" is chosen. Concerning the optimal repair policy, for $C = c$, we have

$$\mathcal{L}_{R=c} = \frac{H}{2\lambda \xi} + \lambda \left(W_0 - \xi \frac{H}{(2\lambda \xi)^2} - \xi \gamma c \right)$$

$$\mathcal{L}_{R=0} = \frac{H - c}{2\lambda \xi} + \lambda \left(W_0 - \xi \frac{H - c}{(2\lambda \xi)^2} \right).$$

"Repair all" is chosen over "no repair" when the following condition is satisfied:

$$\xi < \sqrt{\frac{3}{4\gamma \lambda^2}}.$$

The optimal investment from the financial market and the optimal insurance are illustrated in Table 4. Very interestingly, we obtain a quite counterintuitive result. If the agent chooses not to repair, the optimal insurance is negative, i.e. the insurance is not going to pay off anything for the casualty and furthermore ask the agent to provide it a certain amount. It is an anti-insurance scenario and results from the fact that the utility here leads to an extremely low relative relation aversion coefficient (RRA < 1). It implies the possibility of insurance might be eliminated under certain circumstances.

To sum up, under different preferences, there is generally less than full insurance or even anti-insurance (except the preference $U(a H + W)$), but optimal repair is fully insured. For some preferences (for example $U(H, W) = -\frac{1}{H W}$), our equivalence result between optimal insurance and optimal regulation still holds.

4.2. $C \in [0, H)$. Lifting the Bernouli assumption of the casualty loss (only two possible realizations of the casualty loss), we allow the loss to take any value in $[0, H)$ and examine the validity of the equivalence result. The preferences remain identical additively separable von Neumann-Morgenstern. In this case, given ξ and C, the equations containing the first-order derivatives of U_W and U_H are given as follows:

$$U_W'(P(\xi, C) - \gamma R(\xi, C)) - \lambda \xi = 0$$

$$U_H'(H - C + R(\xi, C)) + U_W'(P(\xi, C) - \gamma R(\xi, C))(-\gamma) = 0.$$

Due to the assumption of concave utilities, we have

$$P^*(\xi, C) = I_W(\lambda \xi) + \gamma R(\xi, C)$$

$$R^*(\xi, C) = \min\{\max\{I_H(\gamma \lambda \xi) - H + C, 0\}, C\}.$$

Note for a given realization of casualty loss C, the optimal repair policy decreases (more precisely, non-increasing) in the state realization ξ. In other words, for a *given* C, we need a critical ξ to implement an optimal insurance policy. Since the realization of the casualty loss might be any value in $[0, H)$, as a result, there might exist more than one default probability constraints to implement the optimal insurance policy. Hence, in this more general case, our equivalence result between optimal regulation and optimal insurance breaks down.

4.3. $R \in [0, c]$. Letting the casualty loss remain Bernoulli, we instead allow the repair policy to be any value in the interval $[0, c]$, i.e. instead of "no repair" or "repair all", we allow "partial repair" too. Again, identical additively separable von Neumann-Morgenstern preferences are adopted for the following analysis. The Lagrangian function associated with the maximization problem is given by

$$\mathcal{L} = E\Big[U_H(H - C + R(\xi, C)) + U_W(P(\xi, C) - \gamma R(\xi, C)) + \lambda(W_0 - \xi P(\xi, C))$$

$$+ \mu_1 R(\xi, C) + \mu_2(C - R(\xi, C))\Big] \tag{19}$$

with λ, μ_1 and μ_2 denoting nonnegative Lagrangian multiplier. The complementary slackness conditions say the following two conditions should hold:

a) $\mu_1 \geq 0$, $R(\xi, C) \geq 0$ and $\mu_1 R(\xi, C) = 0$;
b) $\mu_2 \geq 0$, $C - R(\xi, C) \geq 0$ and $\mu_2 (C - R(\xi, C)) = 0$.

Since $R(\xi, C)$ and $C - R(\xi, C)$ are not binding for different realizations of the casualty loss, we must have $\mu_1 = 0$ and $\mu_2 = 0$ at optimum.

Carrying out a similar analysis as in Section 2, we obtain the optimal terminal wealth (given ξ and C) and the optimal repair policy:

$$P^*(\xi, c) = I_W(\lambda \xi) + \gamma R(\xi, C)$$

$$R^*(\xi, C) = \min\{\max\{I_H(\gamma \lambda \xi) - H + C\}, C\}.$$

Note that in addition to "repair all" and "no repair", "partial repair" with proportional benefit might be a desirable optimal repair policy if there is casualty loss $(C = c)$, hinging on the realization of ξ. More specifically,

$$R^*(\xi, C) = \begin{cases} 0 & \text{if } \{C = 0\} \text{ or } \{C = c \text{ and } I_H(\gamma \lambda \xi) < H - c\} \\ I_H(\gamma \lambda \xi) - H + c & \text{if } \{C = c \text{ and } H - c \leq I_H(\gamma \lambda \xi) < H\} \\ c & \text{if } \{C = c \text{ and } I_H(\gamma \lambda \xi) \geq H\}. \end{cases}$$

State and casualty realization	Optimal investment	Optimal insurance
$\{C = 0\}$	$\frac{1}{\lambda\xi}$	0
$\left\{C = c \text{ and } \xi > \frac{\nu}{\lambda(H-c)}\right\}$	$\frac{1}{\lambda\xi}$	0
$\left\{C = c \text{ and } \frac{\nu}{\lambda H} \le \xi \le \frac{\nu}{\lambda(H-c)}\right\}$	$\frac{1}{\lambda\xi}$	$\frac{\nu}{\lambda\xi} + \gamma(c-H)$
$\left\{C = c \text{ and } \xi < \frac{\nu}{\lambda H}\right\}$	$\frac{1}{\lambda\xi}$	γc

TABLE 5. Decomposition of the optimal wealth P^* for $R \in [0, c]$.

If we take log-utilities as examples again: $U_H(z) = \nu \log z$ and $U_W(z) = \log z$, we can write down the optimal terminal wealth in the following form (depending on ξ and C):

$$P^*(\xi, C) = \begin{cases} \frac{1}{\lambda\xi} & \text{if } \{C = 0\} \quad \text{or} \quad \{C = c \text{ and } \xi > \frac{\nu}{\gamma\lambda(H-c)}\} \\ \frac{1}{\lambda\xi} + \frac{\nu}{\lambda\xi} + \gamma(c-H) & \text{if } \{C = c \text{ and } \frac{\nu}{\gamma\lambda H} \le \xi < \frac{\nu}{\gamma\lambda(H-c)}\} \\ \frac{1}{\lambda\xi} + \gamma c & \text{if } \{C = c \text{ and } \xi \le \frac{\nu}{\gamma\lambda H}\}. \end{cases} \tag{20}$$

Again the optimal final wealth can be decomposed into the optimal investment and optimal insurance. As shown in Table 5, the optimal insurance policy fully insures the casualty loss in very cheap states (and ξ is small), does not pay off anything to rebuild the house when the economy is down (and ξ is large). Now we have some additional states, i.e. moderately expensive states, full insurance is not optimal and instead a partial insurance is recommended. Note that the optimal insurance looks like insurance with a state-dependent deductible and has a different form than the optimal default regulation, hence, the equivalence result breaks down when the agent can choose a partial repair policy. However, as in all the other scenarios, the optimal repair is fully insured.

4.4. C **not independent of** ξ. So far we have assumed that the casualty loss C is independent of ξ, this section aims to examine the robustness of the equivalence result when C is not independent of ξ. In this case, the analysis for the optimal contract looks identical to our base case. Only now the Lagrangian multiplier will depend on the joint distribution of C and ξ and is going to be different. For the stylized insurance regulation case, if we still assume that there is not any actuarial risk at the insurance company level, the liability is only the conditional expectation $E[C|\xi]$ (not a constant L). Letting W be the terminal wealth of the insurance company, the insurance company now faces the following optimization problem:

$$\begin{aligned} \text{choose} \quad & W \ge 0 \quad \text{to} \\ \text{maximize} \quad & E[\xi(W - E[C|\xi])^+] \\ \text{subject to} \quad & E[\xi W] = W_0 \\ & E[1_{\{W \ge E[C|\xi]\}}] \ge \alpha. \end{aligned} \tag{21}$$

When the liability is a constant, the company wants to default when ξL (equivalently ξ) is high. Whereas when the liability depends on the state realization ξ, there is default when $\xi E[C|\xi]$ is high. Intuitively speaking, if $\xi E[C|\xi]$ increases in ξ (as ξL does), then we will obtain the same form of policy. The following equivalence property of conditional covariance (see e.g. Wallenius (1971))

$$\text{Cov}\,[\xi, E[C|\xi]] = \text{Cov}\,[\xi, C]$$

states that if ξ and C are positively correlated, a higher ξ will lead to a higher $\xi E[C|\xi]$ and if ξ and C are negatively correlated, a higher ξ will lead to a lower $\xi E[C|\xi]$. It implies that if there are more losses when the economy is bad, this model setup will still give the same form of policy. While when the bad economy does not generate more losses, we will not obtain the same form of policy. Therefore, in general, the equivalence might fail.

4.5. H **random.** The last generalization considered is that H is random and assumed to be a function of ξ. This is not the most realistic assumption we can make if we want to make the house price random, since house prices depend on local as well as aggregate economic conditions, and are not really functions of economy-wide variables. However, a more general formulation would probably run in to the informational issues discussed in the following subsection. As before, the casualty loss is assumed to be Bernoulli, independent of ξ, taking on the value c with probability p and the value 0 with probability $1 - p$. We assume essinf $H(\xi) > c$ to ensure that the house price is still positive even after a casualty loss. Additively separable von Neumann-Morgenstern preferences are assumed as before. The agent faces the following optimization problem:

$$
\begin{aligned}
\text{Choose} \quad & P : \Re_+ \times \{0, c\} \to \Re_+, \text{ and } R : \Re_+ \times \{0, c\} \to \{0, c\} \text{ to} \qquad (22)\\
\text{maximize} \quad & E[U_H(H(\xi) - C + R(\xi, C)) + U_W(P(\xi, C) - \gamma R(\xi, C))]\\
\text{subject to} \quad & W_0 = E[\xi P(\xi, C)]\\
& \text{for all } \xi \in \Re^+ \text{ and } C \in \{0, c\}, \ R(\xi, C) \le C.
\end{aligned}
$$

The Lagrangian function is

$$\mathcal{L} = E[U_H(H(\xi) - C + R(\xi, C)) + U_W(P(\xi, C) - \gamma R(\xi, C)) + \lambda(W_0 - \xi\, P(\xi, C))].$$

Therefore, the optimal terminal wealth is

$$P^*(\xi, C) = I_W(\lambda\,\xi) + \gamma\, R(\xi, C).$$

Let us now take a look at the optimal repair policy given there is casualty loss ($C = c$). If we compare the repair policy $R = c$ ("repair all") and $R = 0$ ("no repair") by substituting the value of R and the optimal terminal wealth back into the integrand of the Lagrangian function, we get "repair all" is at least as good as "no repair" if

$$U_H(H(\xi)) - U_H(H(\xi) - c) \ge \lambda\,\gamma\,c\,\xi. \qquad (23)$$

It is noted that the validity of our equivalence result relies on whether the condition given in (23) holds. The inequality can hold under certain circumstances, depending on U_H, $H(\xi)$ and other parameters. Both the left-hand and right-hand side of (23) are increasing in ξ given that $H' < 0$ and $U_H'' < 0$. If the derivative of the left-hand side is everywhere less than $\lambda \gamma c$ then it is optimal to repair for low ξ and not for high ξ and the stylized regulation supports the first-best. However, in general, this is a strong requirement not satisfied by many reasonable functional forms, and we do not have much to say about the shape of the region of ξ for which repair is optimal and the insurance will optimally pay off. Since the equivalence depends on repair being optimal precisely for all sufficiently low ξ, the equivalence does not hold in general.

4.6. **Informational issues.** We have not emphasized this previously, but the results in the paper depend critically on the informational assumptions. The main result in the base case says that a stated contract that depends only on whether there is a casualty loss can implement the optimal contract that depends both on whether there is a casualty loss and on the state of the economy. If insurance companies write credible contracts that depend on all the required state variables, in the base case the casualty loss C and the state of the economy ξ, then the first-best can be implemented trivially with the regulator requiring a probability zero of default.

We have avoided analyzing the traditional problems of adverse selection and moral hazard, but they would also have an impact on our results, because they would affect whether first-best contracts can credibly be written. For example, if we expanded the model of the previous section to admit (reasonably) a house price that has a conditional distribution that depends on ξ but is not conditionally constant, implementing the first-best contract would require that the insurance company has some way of learning about H. It would not be good enough for an insured agent who knows H to report it, because there would an incentive to report a value of H that implies repair is optimal. This is a form of classic adverse selection model. Moral hazard would also affect the form of the optimal contracting and typically would make it impossible to implement the first-best.

5. Conclusion

In this paper, we show there might be less than full insurance when there is endogenous repair. In a separate model, we analyze a stylized model of insurance regulation imposing a maximum default probability. We show the regulatory outcome has the same form as the optimal contract, and the maximum default probability can be chosen to implement the optimal contract. This is a useful benchmark, although in general the equivalence no longer holds.

The paper's model is very simple and it would be interesting to see how the results hold up in richer environments. For regulation, it would be interesting to consider uncertainty

about the distribution of asset payments, both relating to asymmetric information between the regulator and the insurance company, and so-called "model risk" under which reasonable people would disagree about the distribution of payoffs. For example, an insurance company have relatively reliable liability cash flows, suggesting it would be able to change the premiums if it is invested in illiquid assets. However, this also implies a level of risk that maybe hard to quantify. How does a regulator produce the right incentives for this situation?

REFERENCES

Arrow, K.J. (1963): Uncertainty and the Welfare Economics of Medical Care, *American Economic Review*, 53, 941–973.

Arrow, K.J. (1971): On Optimal Insurance Policies, *Appendix to Essay 8 in "Essays in the Theory of Risk Bearing"*, Chicago, Markham.

Arrow, K.J. (1973): Optimal Insurance and Generalized Deductibles, *Rand Report.*

Doherty, N.A. and H. Schlesinger (1983): Optimal Insurance in Incomplete Markets. *Journal of Political Economy* 91, 1045-1054.

Dybvig, P.H. (1988): Inefficient Dynamic Portfolio Strategies, or, How to Throw Away a Million Dollars in the Stock Market, *Reviews of Financial Studies*, 1, 67–88.

Dybvig, P.H. and L.C.G. Rogers (1997): Recovery of Preferences from Observed Wealth in a Single Realization, *Reviews of Financial Studies*, 10, 151–174.

Holmstrom, B. (1979): Moral Hazard and Observability. *The Bell Journal of Economics* 10(2), 74–91.

Rey, B. (2003): A Note on Optimal Insurance in the Presence of a Nonpecuniary Background Risk, *Risk Theory and Decision*, 54, 73–83.

Rotschild, M. and J.E. Stiglitz (1976): Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information. *Quarterly Journal of Economics* 90, 639–649.

Shavell, S. (1979): On Moral Hazard and Insurance, *The Quarterly Journal of Economics*, 93(4) (Nov., 1979), 541–562.

Townsend, R. (1979): Optimal Contracts and Competitive Markets with Costly State Verification. *Journal of Economic Theory* 22, 265–293.

Wallenius, K.T. (1971): A Conditional Covariance Formula with Applications, *The American Statistician*, 25(3), 32–33.