

Interest Rates in Trade Credit Markets

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Abstract

Despite strong evidence that suppliers of inputs are usually informed lenders, the cost of trade credit rarely varies with borrowing firm characteristics. We solve this puzzle by demonstrating that it is optimal for suppliers to keep riskier firms indifferent between trade credit and loans from uninformed banks. Because these bank loans are likely to vary across industries but not with firm characteristics, the same pattern applies to the cost of trade credit. The model predicts that the cost of trade credit is more likely to vary with firm characteristics in industries that are plagued by moral hazard problems or economic distress.

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1 Introduction

In the G7 countries, suppliers of inputs to production processes extend a significant amount of credit to their customers.¹ Smith (1987), Mian and Smith (1992) and Biais and Gollier (1997) argue that the prominence of trade credit is due to an informational advantage: The sales effort of suppliers makes it easier for them to assess their customers' credit risk. Accordingly, Petersen and Rajan (1997) show that, vis-à-vis banks, suppliers extend more credit to firms with current losses and positive growth of sales; a finding that they interpret as a supplier's comparative advantage in identifying firms with growth potential.

Nevertheless, Giannetti, Burkart and Ellingsen (2008) show that credit risk is not an important determinant of interest rates in the U.S. trade credit markets. Apparently, the cost of trade credit varies across industries but not with firm characteristics.² This result is hard to reconcile with the evidence that suppliers are informed lenders. After all, basic economic principles suggest that interest rates should increase with the borrower's risk of credit.

We solve this puzzle by arguing that competition with uninformed banks makes it difficult for suppliers to align the cost of trade credit with their customers' risk. In particular, an attempt to selectively raise the interest rates paid by the riskier firms will induce them to borrow from uninformed banks, whose interest rates overestimate the odds that the debt contract will be honored. We demonstrate that it is optimal for suppliers to keep the riskier firms indifferent between trade credit and loans from uninformed banks. Because the cost of these bank loans is likely to vary across industries but not with firm characteristics, the same pattern applies to the cost of trade credit.

To understand the main ideas of our paper, consider an industry with a continuum of firms, a bank, and a supplier of inputs. The firms seek financing to undertake a profitable project, whose possible outcomes are two: a positive return on the investment (success) or not (failure). While a bank loan is the standard source of financing, trade credit is a more efficient alternative because the supplier has an informational advantage over the bank.

¹See Rajan and Zingales (1995) for the importance of trade credit in the G7 countries (Canada, France, Germany, Italy, Japan, U.K., and the U.S.).

²See Ng, Smith and Smith (1999) and Petersen and Rajan (1994).

To model the supplier's informational advantage, we assume that the probability that the project succeeds depends on firm-specific risk factors – the types – that are distributed in the positive interval $[\underline{t}, 1]$. Without loss of generality, the probability of success of a type- t firm, p^t , increases with t . When firms seek financing to undertake a project, they already know their types and so does the supplier. The bank, on the other hand, does not know the types.

Will the supplier take advantage of its private information to vary the cost of trade credit with the firm-specific risk factors? To answer this question, we build upon a key observation: knowledge of the borrowing firms' types gives market power to the supplier. As suggested by standard monopoly pricing, a sufficiently inelastic demand for inputs (and, by extension, for credit) makes it optimal for the supplier to raise the cost of trade credit until it reaches the maximum level that firms are willing to accept, that is, the interest rate charged by the uninformed bank.³ Since the interest rates of these bank loans cannot vary with information that is privy to the supplier, the cost of trade credit contracts that mimic the bank loans cannot either. Accordingly, we demonstrate that the unique equilibrium of our model implies that the cost of trade credit does not vary with firm characteristics, if the elasticity of the demand for inputs is below a certain threshold $\bar{\epsilon}$.

If the elasticity of demand is larger than $\bar{\epsilon}$, then it is not optimal for the supplier to offer all firms the same terms of trade credit. Instead, the supplier induces a larger volume of loans to its most valuable customers – the safer firms – by offering them lower interest rates. As such, the equilibrium splits the firms into two groups: The riskier ones pay the bank rate while the safer firms pay lower interest rates that decrease with their probability of success.

This implication is interesting, for at least two reasons. First, an equilibrium in which the cost of trade credit is the same for all firms is extreme. As Ng, Smith and Smith (1999) point out, suppliers occasionally waive penalties for late payments. For all practical purposes, waving penalties is equivalent to selectively reducing the cost of trade credit. Second, and more importantly, models of interest rates in trade credit markets should yield some conditions under which the cost of trade credit does vary with firm characteristics. The equilibrium that allows for trade credit contracts to vary with firm characteristics can occur under at least two

³Brennan, Maksimovic and Zechner (1988) also argue that the elasticity of the demand for inputs determines the cost of trade credit. In their model, the supplier is a monopolist in the product market.

phenomena of this sort are present: economic distress and moral hazard problems.

Economic distress reduces the expected debt repayment, inducing the bank to increase interest rates. In turn, the higher bank rate lets the supplier raise the cost of trade credit, thereby increasing the margins of profit of trade credit transactions. Large margins make the volume of loans more important for the supplier, strengthening its incentive to reduce the interest rate paid by the safer firms. As a result, our model predicts that the cost of trade credit is more likely to vary with firm characteristics in economically distressed industries.

A similar argument predicts that the cost of trade credit is more likely to vary with firm characteristics in industries plagued by moral hazard problems. As Burkart and Ellingsen (2004) argue, suppliers can mitigate moral hazard problems more efficiently than banks, because trade credit is extended in kind rather than cash. It then follows that moral hazard problems weaken the bank's ability to compete with the supplier, implying higher margins of profit in trade credit transactions and stronger incentives for the cost of trade credit to vary with the borrower's risk. Moral hazard, therefore, doesn't explain the existing evidence on interest rates in trade credit markets, although it may lead trade credit to be efficient.

This paper builds primarily on Biais and Gollier (1997) and Burkart and Ellingsen (2004), whose main goal is to explain why trade credit is pervasive. In Biais and Gollier, suppliers can identify firms whose credit risk is overestimated by banks. Knowing that these firms' credit lines are unduly low, suppliers are willing to fill their financing needs. Burkart and Ellingsen's paper argues that loans in kind (as opposed to cash) are less vulnerable to moral hazard problems. As such, suppliers may extend credit to firms that have exhausted their ability to borrow from banks. In Biais and Gollier as well as Burkart and Ellingsen's models, the cost of trade credit would vary with the suppliers' private information, were the customers to have different levels of credit risk.

The remainder of the paper is organized as follows. Section 2 describes the basic model, which assumes that the supplier's informational advantage is the reason for trade credit to exist. Section 3 shows how industry characteristics and firm-specific risk factors determine the cost of trade credit. Section 4 introduces moral hazard and shows that it makes the cost of trade credit more sensitive to firm characteristics. Section 5 discusses the robustness of

the results to different information structures and richer trade credit contracts, and Section 6 concludes. Proofs that are not in the text can be found in the appendix.

2 The Model

2.1 Sequence of events and information structure

Consider a risk-neutral economy with a zero risk-free rate, a bank, a supplier of inputs, and a continuum of firms in the interval $[\underline{t}, 1]$, where $\underline{t} > 0$. In this economy, the firms seek financing to undertake a project, whose possible outcomes are only two; they yield a positive return on the investment (success) or not (failure). When we add firm-specific factors to those that are intrinsic to the project, the probability of success increases with the firm's type, that is, $p^t = tp$, with $p \in (0, 1)$ and $t \in [\underline{t}, 1]$.

While a bank loan is the standard source of external financing, trade credit is efficient in our model because the supplier knows the firms' types, but the bank doesn't. Neither the bank nor the supplier can observe the project's return without bearing a verification cost, though. As Townsend (1979) and Gale and Hellwig (1985) demonstrate, verification costs imply that outside equity isn't an optimal financing contract. Hence, firms rely on debt-like instruments to finance the project, whether the lender is the bank or the supplier.

Figure 1: Timing of events



As figure 1 shows, the game begins at date 0, when the firms seek financing to purchase inputs for the project. At this time, they already know their types, and so does the supplier.

In contrast, the bank knows only the cumulative distribution of types, $F_0(t)$, and the project-specific risk factor, p , which is common knowledge.⁴

In addition to allowing a better assessment of the firms' credit risk, the informational advantage gives the supplier a first-mover advantage: We assume that the supplier makes a take-it-or-leave-it offer at date 1, whenever it is in its interest to extend trade credit.⁵ When making trade credit offers, the supplier knows that the bank may also offer credit. The financing decision ends with a bank loan at date 2 if the firm either declines a trade-credit offer or trade credit isn't available. In any event, we assume that the bank does not observe the terms of the trade credit offers, but we let it update the distribution of types, once a firm agrees to take a bank loan.

After securing the funds, the firms buy the inputs and undertake the project at date 2. The payoffs realize at date 3, when the firms repay the debt (if possible) and distribute the residual cash flow to shareholders.

2.2 Technology

In our model, there are only two possible outcomes for the project: success or failure. In case of success, the return of investing I is $Q(I)$. The production function $Q(I)$ is increasing and strictly concave on the investment, satisfying $Q(0) = 0$ and the standard Inada conditions.⁶

In contrast, failure destroys the return on the investment. Frank and Maksimovic (1998) argue that suppliers are more efficient than banks in rescuing the assets of financially distressed firms.⁷ As such, we let the project's salvage value (net of the verification cost) vary with the source of external financing. For simplicity, we assume that the bank cannot rescue the inputs in default, while the supplier rescues a fraction $\delta \in [0, 1]$ of the investment in inputs. If $\delta = 0$, then the supplier does not have an advantage over the bank in default.

⁴See Petersen and Rajan (1997) and McMillan and Woodruff (1999) for empirical evidence that suppliers have an informational advantage over banks in the U.S. and in Vietnam, respectively.

⁵The main results of our paper hold under less extreme assumptions on how the supplier and the firms reach an agreement over the terms of trade credit.

⁶The Inada conditions are $\lim_{I \rightarrow 0} Q'(I) = \infty$ and $\lim_{I \rightarrow \infty} Q'(I) = 0$.

⁷Petersen and Rajan (1997) find evidence that loans to bad lenders are more costly for banks than suppliers, because it is easier for the latter to transform repossessed inputs into liquid assets.

2.3 Loan contracts

We assume that free entry rules out abnormal profits for the uninformed bank; any attempt to extract rents from the firms attracts new (uninformed) lenders. Of course, the break-even payment depends on the firm's type. In particular, riskier firms should pay more, conditional on the project's success.

We claim that, in our model, the bank cannot screen the firms' types. To see why, consider any two standard debt contracts, $(I^k, A(I)^k)_{k \in \{T, \hat{T}\}}$, that finance I^k in exchange for a promised payment $A(I)^k$.⁸ For the bank to screen some of the firms, there must exist types $t \in T$ and $\hat{t} \in \hat{T}$ such that $(I^T, A(I)^T)$ gives a larger expected profit for t than the contract $(I^{\hat{T}}, A(I)^{\hat{T}})$, with the reverse inequality for the type \hat{t} , that is,

$$p^t[Q(I^T) - A(I)^T] > p^t[Q(I^{\hat{T}}) - A(I)^{\hat{T}}] \quad \text{and} \quad p^{\hat{t}}[Q(I^T) - A(I)^T] \leq p^{\hat{t}}[Q(I^{\hat{T}}) - A(I)^{\hat{T}}].$$

These two conditions cannot hold simultaneously, though, because $p^t[Q(I^T) - A(I)^T] > p^t[Q(I^{\hat{T}}) - A(I)^{\hat{T}}]$ implies that $p^{t'}[Q(I^T) - A(I)^T] > p^{t'}[Q(I^{\hat{T}}) - A(I)^{\hat{T}}]$ for any $t' \neq t$. Hence, if a loan contract is optimal for a type- t firm, then it is also optimal for the other types, thereby preventing the bank from designing a screening device. This feature of the model preserves the supplier's informational advantage, which is crucial to the purpose of our paper, while allowing for risk characteristics to vary across firms.⁹

It then follows that the amount of a bank loan request does not update the bank's prior over the borrower's type; the updating is restricted to the firm's willingness to use the bank as the source of external financing. Here, we take the updated distribution, $F_1(t)$, as given.¹⁰ Because the bank cannot rescue assets in default, the zero profit condition on a standard debt contract $(I, A(I))$ is thus $pE_1[t]A(I) = I$, or equivalently:

$$r^B \equiv \frac{A(I)}{I} - 1 = \frac{1}{pE_1[t]} - 1, \tag{1}$$

with $pE_1[t] = p \int_t^1 t dF_1(t)$.

⁸Townsend (1979) demonstrates that verification costs make a standard debt contract optimal for the bank.

⁹Intuitively, our economy should be interpreted as an industry, whose firms opt for the same production plan but differ with respect to their ability to manage the production process.

¹⁰Proposition 2 characterizes the updated distribution $F_1(t)$.

Equation (1) implies that the promised payment, $A(I)$, varies with the loan amount, I , but the interest rate on the loan doesn't: $r^B = \frac{1}{pE_1[t]} - 1$ for any I . To characterize the optimal loan contract it thus suffices to pin down the loan amount that maximizes the value of a representative firm that borrows from the bank, that is, the investment $I^{E_1[t]}$ that makes the marginal productivity of investment, $pE_1[t]Q'(I^{E_1[t]})$, equal to its marginal cost, 1. The necessary and sufficient condition that characterizes the optimal investment is thus

$$Q'(I^{E_1[t]}) = \frac{1}{pE_1[t]}. \quad (2)$$

The optimal loan contract, therefore, lends $I^{E_1[t]}$, in exchange for the borrower's promise to pay $(1 + r^B)I^{E_1[t]}$ after the project's payoff realizes.

As it turns out, the optimal loan contract can be implemented by a linear debt contract that lets the firms pick the loan amount. To see this, consider the maximization problem that yields the optimal investment of a type- t firm that finances the inputs at an interest rate r :

$$\max_I p^t \left[Q(I) - (1 + r)I \right]. \quad (3)$$

The objective function (3) takes into account that the firm benefits from the project only if it succeeds. With probability $1 - p^t$ the firm gets into operational problems that destroy the project's returns. The first order condition of program (3), which is also sufficient, is

$$Q'(I^*) = 1 + r. \quad (4)$$

Plugging the bank rate $r^B = \frac{1}{pE_1[t]} - 1$ into the first order condition (4) yields $Q'(I^*) = \frac{1}{pE_1[t]}$. But this is exactly the first order condition (2) for the value-maximizing investment of the type $E_1[t]$. Without loss of generality we can thus focus the analysis of bank loans on linear debt contracts at the interest rate $r^B = \frac{1}{pE_1[t]} - 1$.

2.4 Trade credit

We consider a supplier endowed with a constant return-to-scale technology in a market for inputs without barriers to entry. To simplify the notation, we assume that the constant marginal cost of production is one, which is also the equilibrium price of the input. As a result, the supplier does not fetch abnormal profits in the product market.

Nonetheless, the informed supplier is the only lender who can vary the loan contracts with the borrowing firm's type. Due to this informational advantage, it can enjoy abnormal profits in trade credit transactions. The supplier's problem, therefore, is to maximize its expected financial profits, while taking into account that firms can borrow from the bank at the interest rate $r^B = \frac{1}{pE_1[t]} - 1$. When solving this problem, we shall restrict our attention to linear debt contracts: The supplier makes a take-it-or-leave-it offer to finance purchases of inputs at an interest rate r^t that may vary with the firm's type t .

Linear debt contracts are pervasive in trade credit markets. For instance, a common trade credit contract in the U.S. combines a 30 day maturity with a two percent discount for early payment within 10 days of the invoice (2-10 net 30 loans). As Petersen and Rajan (1997) point out, not exercising the discount option is equivalent to accepting a 10-day debt contract at an interest rate of 44 percent a year. Assuming linear contracts from the onset may thus be interpreted as a short-cut to focus our attention on the most relevant determinants of interest rates in standard trade credit transactions.¹¹

3 Equilibrium in the trade credit market

This section is divided in three parts. In the first, we derive the optimal trade credit contracts, taking as given the bank rate r^B . In the second part, we characterize the equilibrium of the game by deriving the posterior belief $F_1(t)$ that determines r^B . The last part of the section then shows how economic shocks and industry characteristics (i.e., the salvage value δ) affect the likelihood that the cost of trade credit varies with firm characteristics.

¹¹Section 5 demonstrates that the main insight of our paper – i.e., competition prevents informed suppliers from aligning the cost of trade credit with the borrower's risk – is robust to optimal nonlinear contracts, if default imposes small economic losses on suppliers.

3.1 The optimal trade credit contracts

The goal of the supplier is to design a linear debt contract that maximizes its expected financial profits. The optimal interest rate r^t of a trade credit transaction with a type- t firm solves

$$\max_{r^t} \left(p^t(1 + r^t) + (1 - p^t)\delta - 1 \right) I^*(r^t) \quad (5)$$

$$\text{subject to } (1 - \delta) \left(\frac{1}{p^t} - 1 \right) \leq r^t \leq r^B. \quad (6)$$

The objective function is the supplier's expected profit from extending trade credit to a t -firm at an interest rate r^t . The interest rate determines the firm's investment in the project, $I^*(r^t)$, from the first order condition (4). With probability p^t , the firm can pay principal plus interest: $(1 + r^t)I^*(r^t)$. But, with probability $1 - p^t \in (0, 1)$, the firm experiences operational problems that destroy the project's return, leaving only a salvage value (net of the verification costs): $\delta I^*(r^t)$. In any event, the supplier bears the cost of the input.

The constraint (6) summarizes the two restrictions faced by the supplier. First, the interest rate r^t must be larger than or equal to the supplier's break-even point: $(1 - \delta) \left(\frac{1}{p^t} - 1 \right)$. The interest rate cannot be too high, though, or else the firm is better off borrowing from the bank. As such, the bank rate r^B is the maximum cost of trade credit.

The only reason for a solution to Program (5) not to exist is the upper bound on the interest rate ($r^t \leq r^B$). It isn't profitable for the supplier to offer trade credit, if the bank rate r^B is lower than the supplier's break-even point, $(1 - \delta) \left(\frac{1}{p^t} - 1 \right)$. If so, the program's opportunity set is empty, meaning that the supplier will not offer trade credit to the type- t firm. Using that $p^t = tp$, a necessary and sufficient condition for trade credit to the type- t firm to be profitable for the supplier is $r^B \geq (1 - \delta) \left(\frac{1}{tp} - 1 \right)$, or equivalently

$$t \geq \bar{t}(\delta) = \frac{1 - \delta}{p(1 - \delta + r^B)}. \quad (7)$$

Having determined the types to whom the supplier has incentive to offer trade credit, our next task is to characterize the interest rate r^t that solves the supplier's maximization problem (5). To do this, note first that the interest rate $(1 - \delta) \left(\frac{1}{p^t} - 1 \right)$ is not optimal for the supplier, regardless of the firm's type. This interest rate yields zero expected profits for the supplier,

while any slightly larger rate implies strictly positive expected profits.¹² The optimal interest rate, therefore, either satisfies $(1 - \delta) \left(\frac{1}{p^t} - 1 \right) < r^t < r^B$ or is the bank rate r^B .

A standard trade-off, between margin of profit and volume of sales, determines the optimal interest rate. On the one hand, raising the interest rate increases financial profits per unit of trade credit. On the other hand, it decreases the demand for inputs. To rule out the uninteresting case that it is always optimal for the supplier to raise the cost of credit as much as possible, we assume:¹³

Assumption 1 Let $\epsilon(r) = -\frac{(1+r)\frac{dI^*(r)}{dr}}{I^*(r)}$ be the interest-elasticity of the demand for inputs. Thus, $\epsilon(r)$ is non-decreasing in r .

Given Assumption 1, Proposition 1 demonstrates that the interest-elasticity of the demand for inputs solves the trade off between margin and volume.

Proposition 1 *It is profitable for the supplier to offer trade credit if and only if the firm's type is $t \geq \bar{t}(\delta)$. In this case, the optimal interest rate is $r^t = r^B$ if and only if $\epsilon(r^B) \leq \bar{\epsilon}(r^B, \delta, t)$ with $\bar{\epsilon}(r^t, \delta, t) = \frac{tp(1+r^t)}{tp(1+r^t-\delta)-(1-\delta)}$. If $\epsilon(r^B) > \bar{\epsilon}(r^B, \delta, t)$, then $(1 - \delta) \left(\frac{1}{p^t} - 1 \right) < r^t < r^B$, with r^t strictly decreasing in δ and t , unless $\delta = 1$, in which case r^t does not vary with t .*

Proposition 1 shows that the supplier raises the interest rate to its upper bound, if and only if the demand for inputs is sufficiently inelastic at the bank rate r^B , that is, $\epsilon(r^B) \leq \bar{\epsilon}(r^B, \delta, t)$. More interestingly, Proposition 1 also shows that the optimal interest rate r^t increases with the firm's risk, with two exceptions: If default does not impose an economic loss ($\delta = 1$) or if r^t reaches the maximum interest rate that firms are willing to pay, that is, the bank rate r^B .

3.2 Equilibrium contracts

From Proposition 1, the cost of trade credit of a type- t firm does not vary with its probability of success, whenever the elasticity at the bank rate, $\epsilon(r^B)$, is smaller than or equal to the

¹²More formally, the unit profit, $p^t(1 + r^t) + (1 - p^t)\delta - 1$, strictly increases with r^t , and, from the Inada condition, $I^*(r^t)$ is always positive for any r^t slightly larger than the break-even rate.

¹³The assumption holds, for example, if $Q(I) = I^\alpha$ with $\alpha \in (0, 1)$.

cutoff $\bar{\epsilon}(r^B, \delta, t)$. The elasticity of the demand for inputs, therefore, is a key determinant of the interest rates in the trade credit markets.

As one can check, the cutoff $\bar{\epsilon}(r^B, \delta, t)$ decreases with the firm's type, t . Hence, a sufficient condition for the optimal trade credit contracts not to vary with firm characteristics is $\epsilon(r^B) \leq \bar{\epsilon}(r^B, \delta, 1)$. If this elasticity condition is not satisfied and default is costly (i.e., $\delta < 1$), then at least some trade credit contracts prescribe interest rates that fall with the borrowing firm's probability of success. Whether the cost of trade credit varies with firm characteristics thus depends on how elastic the demand for inputs is.

As it turns out, Petersen and Rajan (1997) show that, in the U.S., the demand for trade credit seems to be fairly inelastic; a finding that our model relates to interest rates that do not vary with firm characteristics. Ng, Smith and Smith (1999) also show, however, that some firms keep the discount for early payments, despite missing the contractual deadline. For all practical purposes, waiving the deadline for discounts is equivalent to selectively reducing the cost of trade credit. A testable model of interest rates in the trade credit markets, therefore, should yield implications on the likelihood that the cost of trade credit varies with firm characteristics. These implications can be drawn from our model, because Proposition 2 links the equilibrium interest rates not only to the elasticity of demand but also to industry characteristics.

Proposition 2 *There is a unique sequential equilibrium of the game. In this equilibrium, the strategies and beliefs are:*

- Bank: Offers to all firms loan contracts at an interest rate $r^B = \frac{1}{pE_1[t]} - 1$, where $E_1[t] = \int_{\underline{t}}^1 t dF_1(t)$.
- Firms: If the type- t firm has access to trade credit at an interest rate $r^t \leq r^B$, then it borrows $I^*(r^t)$ from the supplier. Otherwise, the t -firm borrows $I^*(r^B)$ from the bank to purchase inputs for the project.
- Supplier: Offers a trade credit contract $(r^t, I^*(r^t))$ to all firms. There exists $\hat{t} \in [\underline{t}, 1]$ such that $t \leq \hat{t}$ implies $r^t = r^B$, and $t > \hat{t}$ implies $r^t = r(\delta, t) \in ((1 - \delta)(\frac{1}{p^t} - 1), r^B)$, with $r(\delta, t)$ decreasing in δ and t . The cutoff type is $\hat{t} = 1$ if $\epsilon(r^B) \leq \bar{\epsilon}(r^B, \delta, 1)$, and

$\hat{t} = \underline{t}$ if $\epsilon(r^B) \geq \bar{\epsilon}(r^B, \delta, \underline{t})$. If $\bar{\epsilon}(r^B, \delta, 1) < \epsilon(r^B) < \bar{\epsilon}(r^B, \delta, \underline{t})$, then \hat{t} is implicitly defined by $\epsilon(r^B) = \bar{\epsilon}(r^B, \delta, \hat{t})$, with $\hat{t} \in (\underline{t}, 1)$.

- Beliefs: $Prob(t = \underline{t} | \text{bank loan}) = 1$, which implies $F_1(t) = 1, \forall t \in [\underline{t}, 1]$.

In our model, there is no economic reason for the uninformed bank to finance the purchase of inputs; the supplier's first-mover advantage lets it displace the bank. Accordingly, Proposition 2 demonstrates that a firm's request for a bank loan is an event off the equilibrium path: All firms use trade credit.

Because a bank loan is off the equilibrium path, Bayes' rule does not pin down the updating of the bank's prior upon a request for bank loan. The proof of Proposition 2 shows, nonetheless, that a distribution concentrated on the \underline{t} -type is the only updated belief that satisfies the consistency requirement of sequential equilibrium (Kreps and Wilson (1982)). The bank, therefore, interprets a request for a loan as a signal that the firm is the riskiest type \underline{t} , setting the interest rate according to this belief, that is, $r^B = \frac{1}{p\underline{t}} - 1$.¹⁴

More importantly, Proposition 2 characterizes the equilibrium of the game through a cutoff type \hat{t} that splits the firms in two groups: The safer firms, $t > \hat{t}$, pay an interest rate $r^t < r^B$ that decreases with their probability of success, while the riskier ones, $t \leq \hat{t}$, pay the bank rate r^B that does not vary with firm characteristics. If $\epsilon(r^B) \leq \bar{\epsilon}(r^B, \delta, 1)$, then $\hat{t} = 1$, implying that the cost of trade credit is r^B for all firms. The safer firms pay an interest rate lower than r^B , though, if $\epsilon(r^B) > \bar{\epsilon}(r^B, \delta, 1)$, in which case $\hat{t} < 1$. In particular, a sufficiently elastic demand ($\epsilon(r^B) > \bar{\epsilon}(r^B, \delta, \underline{t})$) implies that the cost of trade credit always falls with the firm's probability of success, that is, $\hat{t} = \underline{t}$.

The characterization of the equilibrium of the game implies that the cutoff type \hat{t} is a sufficient statistic for the sensitivity of the cost of trade credit with respect to firm-specific risk factors. As the cut-off type \hat{t} decreases, there is an increase in the fraction of trade credit contracts whose interest rates fall with the firm's probability of success. The next section builds on this result to show how industry characteristics shape the likelihood that the cost

¹⁴Intuitively, the riskier types are denied trade credit in any perturbation that makes a bank loan a positive probability event. As the perturbation goes to zero, the beliefs converge to a mass at \underline{t} , which is thus the unique updated belief that satisfies the Kreps-Wilson refinement.

of trade credit varies with firm characteristics.

3.3 The likelihood that the cost of trade credit varies with firm characteristics

Typically, interest rates in the trade credit markets vary across industries but not with firm characteristics. As such, a test with the power to reject our model should be centered around implications that help predict when or where the cost of trade credit varies with firm characteristics. In this spirit, our model yields potentially testable implications that link the cross-sectional variation of interest rates to the salvage value of the project, δ , and the state of the economy.

As the salvage value of the project goes up, so does the supplier's expected return on a trade credit contract. Large margins make the volume of loans more important for the supplier, strengthening its incentive to use the risk of credit to price discriminate its customers. By better aligning the interest rates with the credit risk, the supplier can increase the margins of the riskier trade credit contracts, while inducing larger loans to its most valuable customers, the safer firms. Accordingly, a larger fraction of trade credit contracts should vary with the firm's probability of success. Proposition 3 formalizes this intuition.

Proposition 3 *Define $\underline{\delta} \equiv \frac{1}{1-p} \left\{ p(1+r^B) \left(\frac{1}{r^B} - 1 \right) + 1 \right\}$ and $\bar{\delta} \equiv \frac{1}{1-tp} \left\{ tp(1+r^B) \left(\frac{1}{r^B} - 1 \right) + 1 \right\}$. The cost of trade credit does not vary with firm characteristics if $\delta \leq \underline{\delta}$, while it always falls with the firm's probability of success if $\delta \in [\bar{\delta}, 1)$. If $\underline{\delta} < \delta < \min\{\bar{\delta}, 1\}$, the fraction of firms for whom the cost of trade credit falls with the probability of success increases with δ .*

Petersen and Rajan (1997) show that suppliers extend less credit in industries that keep a high fraction of finished goods in inventory; a finding that they interpret as evidence that it is easier for suppliers to transform repossessed inputs (rather than finished goods) into liquid assets. As such, Proposition 3 predicts that a high fraction of finished goods decreases the likelihood that the cost of trade credit varies with firm characteristics.

Consider now an economic shock that reduces the probability of success of all firms, that is, the support of types shifts from $[\underline{t}, 1]$ to $[t^{min}, t^{max}]$, with $t^{min} < \underline{t}$ and $t^{max} < 1$. After the

shock, the distribution of types changes from $F_0(t)$ to $G_0(t)$, with $\int_{t^{min}}^{t^{max}} tdG_0(t) < \int_t^1 tdF_0(t)$.¹⁵

The impact of the shock on the supplier's expected return is twofold. On the one hand, it lowers the probability that firms honor the trade credit contracts, reducing the supplier's expected return. On the other hand, Proposition 2 shows that the shock carries through to the updating done in response to a bank loan request, implying that the interest rate of a bank loan goes up to $r^B(G_1) = \frac{1}{pt^{min}} - 1 > \frac{1}{pt} - 1 = r^B(F_1)$. The shock, therefore, increases the value of the supplier's private information, allowing it to increase its profit margins.

Of course, further restrictions on the distribution $G_0(t)$ are needed to determine the net effect of the shock on the supplier's expected profit. Nonetheless, our model predicts an unambiguously higher likelihood that the cost of trade credit varies with firm characteristics: The larger margin allowed by the higher bank rate makes it optimal for the supplier to increase the number of firms whose cost of trade credit varies with their probability of success. Proposition 4 formalizes the link between the equilibrium cost of trade credit and a negative shock that harms the economy's perspectives.

Proposition 4 *A shock that lowers the probability of success of all firms decreases $\hat{t}(\delta)$, increasing the fraction of firms whose cost of trade credit falls with the probability of success.*

Proposition 4 predicts that the cost of trade credit is more likely to vary with firm characteristics in industries plagued by economic distress. This implication of our model is consistent with Wilner (2000), who argues that suppliers have incentives to bail-out financially distressed customers in order to preserve long-term business relationships. Anticipating their own incentives, suppliers should embed the expected cost of the potential bail-out in the terms of trade credit. Clearly, this expected cost varies with the customer's risk and is more likely to be relevant in distressed industries.

An example may help illustrate Propositions 3 and 4. The production function is $Q(I) = I^\alpha$, which implies an iso-elastic demand for inputs: $\epsilon(r) = \frac{1}{1-\alpha}$, for any interest rate r .

¹⁵A special case of the shock in the support of t is equivalent to an increase in the project-specific risk factor, that is, a lower p . To see this, let $\Delta p > 0$ be a small change in p that reduces the probability of success of a type- t firm to $(p - \Delta p)t$. This probability of success can be written as $ps = (p - \Delta p)t$, where $s = \frac{1-\Delta p}{p}t$ is the shock in the space of types that is equivalent to the shock in p , if we consider that the new distribution of types is $G_0(s) = F_0\left(\frac{p}{p-\Delta p}s\right)$.

This production function satisfies our technological assumptions if $\alpha \in (0, 1)$. To assure an equilibrium in which the cost of trade credit varies with firm characteristics, we also assume $\delta < 1$ and $\frac{1}{1-\alpha} = \epsilon(r^B) > \bar{\epsilon}(r^B, \delta, 1) = \frac{p(1+r^B)}{p(1+r^B-\delta)-(1-\delta)}$, or equivalently, $\alpha > \frac{1-\delta(1-p)}{p(1+r^B)}$.

From Proposition 2, $\hat{t}(\delta)$ is implicitly defined by $\epsilon(r^B) = \bar{\epsilon}(r^B, \delta, \hat{t}(\delta))$, which yields:

$$\hat{t}(\delta) = \frac{1 - \delta}{p(\alpha(1 + r^B) - \delta)}. \quad (8)$$

Equation (8) characterizes the safer firms that are offered trade credit contracts at a cost that decreases with the probability that the project succeeds, $(\hat{t}(\delta), 1]$, and the riskier firms that pay r^B , regardless of their firm-specific factors, $[\underline{t}, \hat{t}(\delta)]$. A smaller value of $\hat{t}(\delta)$ increases the fraction of trade credit contracts whose interest rate varies with firm characteristics.

As equation (8) shows, $\hat{t}(\delta)$ decreases with r^B , which, in turn, increases with the expected probability that the project fails. Hence, the cost of trade credit is more likely to vary with firm characteristics in economically distressed industries.

To see the impact of the salvage value δ on the cross-sectional variation of interest rates, differentiate $\hat{t}(\delta)$ with respect to δ to obtain $\frac{d\hat{t}(\delta)}{d\delta} = \frac{p(-\alpha(1+r^B)+1)}{(p(\alpha(1+r^B)-\delta))^2}$. One can check that $\frac{d\hat{t}(\delta)}{d\delta} < 0$ if and only if $\alpha > \frac{1}{1+r^B}$. This condition on α is implied by $\alpha > \frac{1-\delta(1-p)}{p(1+r^B)}$, which assures that the equilibrium allows for some trade credit contracts to vary with firm characteristics.

4 Trade credit and moral hazard problems

Burkart and Ellingsen (2004) argue that trade credit is a pervasive source of external financing, because loans in kind are less vulnerable to opportunistic behavior. One may thus wonder whether moral hazard problems make it less likely that the cost of trade credit varies with firm-specific risk factors. To answer this question, we introduce a moral hazard problem in the investment decision that, in the spirit of Burkart and Ellingsen, weakens the bank's ability to compete with the supplier.

4.1 The moral hazard problem

In this section, we expand the investment opportunities by letting the firms choose one of two mutually exclusive projects: R and S .

As in the single-project economy, projects R and S are risky. In case of success, the return of investing I in project $i \in \{R, S\}$ is $\rho_i Q(I)$, where ρ_i is a productivity parameter and $Q(I)$ is an increasing and strictly concave production function that satisfies the Inada conditions. In case of failure, both projects leave a salvage value δI for the supplier while destroying the inputs when the bank is the source of external financing. By adding the firm-specific risk factors to those that are intrinsic to the projects, the probability of success of an investment by a firm $t \in [\underline{t}, 1]$ in project $i \in \{R, S\}$ is $p_i^t = tp_i$, with $1 > p_S > p_R > 0$.

To make the investment decision relevant for the firms, we assume that the value of project S is larger than the value of R , but, conditional on success, R delivers the highest return, that is, $\rho_S = 1 < \rho = \rho_R$. To be sure, a necessary condition for S to be the value-maximizing project is $p_S > \rho p_R$. If failure destroys the initial investment ($\delta = 0$), then any gap between p_S and ρp_R suffices for project S to maximize value. If $\delta > 0$, a sufficient condition for S to be the efficient project is $p_S > \rho p_R$ with p_R sufficiently close to zero.

The assumptions on the productivity parameters give rise to a well known moral hazard problem identified by Jensen and Meckling (1976): Leverage induces firms to invest in the riskiest project. Despite the safest project's greater efficiency, a higher return in the successful states of nature creates incentive for the riskiest project R , because leverage keeps the upside gains in the firm while shifting part of the downside losses to the lender. The larger ρ is, the greater are the upside gains of project R and, consequently, the stronger are the incentives for the firms to inefficiently select the riskiest project. Accordingly, ρ parameterizes the strength of the moral hazard problem.

As in Burkart and Ellingsen (2004), the supplier can mitigate moral hazard problems more efficiently than the bank, because trade credit is extended in kind. In particular, we assume that the supplier can tailor the inputs to the safest project. Trade credit contracts, therefore, can be contingent on the type of the firm and the choice of the project. In contrast, the bank may have to distort the loan contract in order to mitigate the moral hazard problem.

4.2 The optimal loan contract

As Jensen and Meckling (1976) point out, leverage may induce firms to inefficiently select the riskiest project. Due to the verification cost, the best that the bank can do to mitigate the moral hazard problem is to offer a standard debt contract that satisfies three constraints. First, competition in the credit market implies that the debt contract cannot fetch a positive expected profit for the bank. Second, the contract must be profitable for the firms (participation constraint). And third, it must induce them to select the targeted project (incentive-compatibility constraint).

To characterize the optimal loan contract, consider a standard debt contract that lends I_S in exchange for the borrower's promise to pay A_S when the project's payoff realizes. Once a firm seeks for a loan contract, the bank updates the prior distribution of types to $F_1(t)$. Given the updated belief, the loan contract leaves no expected profit for the bank if

$$\frac{A_S}{I_S} = \frac{1}{p_S E_1[t]}. \quad (9)$$

Equation (9) implies that the promised return on the loan contract, $r_S^B = \frac{A_S}{I_S} - 1$, doesn't change with the amount of the loan. As such, we can write the loan contract as a pair (I_S, r_S^B) . To characterize the loan amount I_S , we move on to the incentive condition, which ties the loan to the efficient project, given by:

$$p_S^t [Q(I_S) - (1 + r_S^B)I_S] \geq p_R^t [\rho Q(I_S) - (1 + r_S^B)I_S]. \quad (10)$$

Inequality (10) assures that project S is more profitable than R , for any type- t firm that signs for the loan contract (I_S, r_S^B) . After taking into account that $p_S > \rho p_R$ and $p_i^t = t p_i$ for $i \in \{R, S\}$, some simple algebra lets us write the incentive-compatibility condition, (10), as

$$\frac{Q(I_S)}{I_S} \geq (1 + r_S^B) \frac{p_S - p_R}{p_S - \rho p_R}. \quad (11)$$

Condition (11) is a lower bound on the average productivity of the investment on the safest project. Since the production function is concave, average productivity decreases with the investment. Hence, the lower bound on the average productivity implies that the investment on the safe project cannot be larger than a cutoff – call it $\bar{I}_S^B(\rho)$ – that satisfies the restriction

(11) with equality. Proposition 5 shows that $\bar{I}_S^B(\rho)$ decreases with the moral hazard parameter ρ and that severe moral hazard problems implies credit constraint, that is, $\bar{I}_S^B(\rho)$ is smaller than the investment level $I_S^*(r_S^B)$ that maximizes expected profits under the interest rate r_S^B .¹⁶

Proposition 5 *A loan contract (I_S^B, r_S^B) is incentive compatible if and only if $I_S^B \leq \bar{I}_S^B(\rho)$, with $\frac{d\bar{I}_S^B(\rho)}{d\rho} < 0$. Moreover, there is a productivity parameter $\rho^{const} \in (1, \frac{p_S}{p_R})$ such that (I_S^B, r_S^B) implies credit constraint if and only if $\rho > \rho^{const}$.*

The intuition for Proposition 5 is straightforward. Firms lean towards the riskiest project if it delivers a large upside gain (high ρ). If these upside gains increase, the bank responds to the stronger moral hazard problem by reducing the credit line. In particular, the safest project becomes credit constrained once the productivity parameter ρ crosses the cut-off ρ^{const} .

If the incentive-compatible loan contract implies credit constraint, then the value of the safest project decreases relative to that of the riskiest project R . This raises the question of whether it is worthwhile for the bank to distort the loan contract to assure that the firm undertakes the safest project. Proposition 6 shows that the efficiency gains of inducing project S outweigh the cost of credit constraint, if the moral hazard problems are not too severe.¹⁷

Proposition 6 *There is a cutoff $\rho^S \in (\rho^{const}, \frac{p_S}{p_R})$ such that an efficient loan contract induces the firms to undertake the safest project if and only if $\rho \leq \rho^S$.*

In the next section, we shall argue that the supplier, who is immune to the moral hazard problem, always ties trade credit to the safest project. As such, $\rho > \rho^S$ implies that the bank provides funding for the riskiest project, while the supplier finances the safest project. Conceivably, firms do not alter their investment plans simply because they have access to trade credit. We shall thus assume that $\rho \leq \rho^S$, from now on. This assumption, formalized below, implies that the credit line to the safest project is the equilibrium outcome in the market for bank loans.

¹⁶More formally, $\rho > \rho^{const}$ implies that $\bar{I}_S^B(\rho) < I_S^*(r_S^B) = \operatorname{argmax}_I p_S^t [Q(I) - (1 + r_S^B)I]$.

¹⁷The proof of Proposition 6 uses the fact that the bank cannot repossess the assets of bankrupted firms. Without this assumption, Proposition 6 holds if the elasticity of investment is bounded from above.

Assumption 2 *The moral hazard parameter satisfies $\rho \leq \rho^S$. Hence, firms without trade credit undertake the safest project after borrowing from the bank.*

Assumption 2 does not necessarily imply credit constraint. Firms become credit constrained only if $\rho \in (\rho^{const}, \rho^S)$, in which case they borrow the maximum amount allowed by the bank, so that the scale of the project is as close as possible to the optimal investment $I_S^*(r_S^B)$. We can therefore write the expected profit of the type- t firm with a bank loan as

$$\Pi_B^t = p_S^t \Pi_B(\rho) \equiv \begin{cases} p_S^t \left[Q(I_S^*(r_S^B)) - (1 + r_S^B) I_S^*(r_S^B) \right] & \text{if } \rho \leq \rho^{const}, \\ p_S^t \left[Q(\bar{I}_S^B(\rho)) - (1 + r_S^B) \bar{I}_S^B(\rho) \right] & \text{if } \rho \in (\rho^{const}, \rho^S]. \end{cases} \quad (12)$$

Equation (12) characterizes the firms' outside option in their bargaining with the supplier for trade credit. Equipped with it, we can turn our attention to the characterization of the optimal trade-credit contracts.

4.3 The optimal trade credit contracts

The supplier's problem is to design a linear trade credit contract that maximizes its expected financial profit. To achieve this goal, the supplier takes into account its ability to tie the financing of the inputs to the project that maximizes the gains from trade, that is, project S . The best trade credit contract, therefore, is tied to a single project, as in the maximization program (5) that ignores moral hazard problems.

While the supplier does not fear an inefficient selection of projects, the optimal trade credit contract is influenced by moral hazard problems in the market for bank loans. As Proposition 5 shows, severe moral hazard problems force the bank to constrain the supply of loans, weakening its ability to compete in the credit market. In response, the supplier can raise interest rates until the credit-constrained firms are indifferent between a cheaper (but undersized) bank loan and a larger (but costlier) trade credit offer. The maximum cost of trade credit that type- t firms are willing to pay – call it $\bar{r}(\rho)$ – is implicitly defined by

$$p_S^t \left[Q(I_S^*(\bar{r}(\rho))) - (1 + \bar{r}(\rho)) I_S^*(\bar{r}(\rho)) \right] = \Pi_B^t. \quad (13)$$

The left-hand side of equation (13) is the expected profit of a type- t firm that invests in project S , after accepting trade credit at the interest rate $\bar{r}(\rho)$. For $\bar{r}(\rho)$ to be the maximum interest rate that the supplier can charge, it must leave the firm's expected profit at its reservation value: the expected profit, Π_B^t , of borrowing from the bank to invest in project S (see equation (12)). If the type- t firm is not credit-constrained ($\rho \leq \rho^{const}$), then the bank rate is the maximum interest rate that the supplier can charge (i.e., $\bar{r}(\rho) = r_S^B$). Otherwise, the supplier can raise $\bar{r}(\rho)$ above the bank rate. Not surprisingly, $\bar{r}(\rho)$ increases with ρ , because a stronger credit constraint (i.e., a larger ρ) increases firms' willingness to pay for trade credit. We have thus established:

Lemma 1 *The cost of trade credit cannot be higher than the interest rate $\bar{r}(\rho)$ that is implicitly defined by equation (13). If the firm is not credit constrained (i.e., $\rho \leq \rho^{const}$), then $\bar{r}(\rho)$ is the interest rate r_S^B of the bank loan designed to the safest project. Otherwise, $\bar{r}(\rho)$ strictly increases with the productivity parameter ρ , with $\bar{r}(\rho) > r_S^B$, for any $\rho > \rho^{const}$.*

From Lemma 1, we can write the supplier's problem as

$$\max_{r^t} \left(p_S^t(1 + r^t) + (1 - p_S^t)\delta - 1 \right) I_S^*(r^t) \quad (14)$$

$$\text{subject to } (1 - \delta) \left(\frac{1}{p_S^t} - 1 \right) \leq r^t \leq \bar{r}(\rho). \quad (15)$$

The objective function (14) is the expected financial profit of a trade credit contract. The maximization program has two constraints. The interest rate r^t must be larger than the break-even point $(1 - \delta) \left(\frac{1}{p_S^t} - 1 \right)$, but is capped by the maximum interest rate $\bar{r}(\rho)$ that the firm is willing to pay.

A straightforward comparison of Programs (5) and (14) shows that the upper bound on the cost of trade credit, $\bar{r}(\rho) \geq r_S^B$, summarizes the impact of the moral hazard problem on the optimal trade credit contract: it may give more leeway for the supplier to raise interest rates. Moral hazard, therefore, does not change the essence of the supplier's problem. In particular, the optimal cost of trade credit is characterized by a first order condition that solves a trade off between margin of profit and volume of sales.

As in Proposition 2, the first order condition of Program (14) implies that the supplier raises the cost of trade credit to the maximum rate $\bar{r}(\rho)$ if and only if the elasticity of demand at

$\bar{r}(\rho)$ is smaller than or equal to a cutoff, $\bar{\epsilon}(\bar{r}(\rho), \delta, t) = \frac{tp_S(1+\bar{r}(\rho))}{tp_S(1+\bar{r}(\rho)-\delta)-(1-\delta)}$, which decreases with the firm's type, t , and the salvage value δ . A sufficient condition for the cost of trade credit not to vary with firm characteristics is thus $\epsilon(\bar{r}(\rho)) \leq \bar{\epsilon}(\bar{r}(\rho), \delta, 1)$. If $\epsilon(\bar{r}(\rho)) > \bar{\epsilon}(\bar{r}(\rho), \delta, 1)$ and $\delta < 1$, then there is a cutoff type, $\hat{t}(\rho, \delta) \in [\underline{t}, 1)$ such that $r^t = \bar{r}(\rho)$ for any $t \in [\underline{t}, \hat{t}(\rho, \delta)]$. If $t > \hat{t}(\rho, \delta)$, then $(1 - \delta) \left(\frac{1}{p_S^t} - 1 \right) < r^t < \bar{r}(\rho)$ and r^t decreases with δ and t .

In other words, the characterization of the equilibrium in Proposition 2 remains valid, provided that we substitute the interest rate $\bar{r}(\rho)$ for the bank rate r_S^B . As such, moral hazard problems change the equilibrium of the game in two ways only: it increases the cost of trade credit for the constrained firms, $\bar{r}(\rho) > r_S^B$, and, as Proposition 7 below shows, decreases the cutoff type $\hat{t}(\rho, \delta)$ that splits the firms that pay $\bar{r}(\rho)$ from the firms whose cost of trade credit increases with firm-specific risk factors.

Proposition 7 *Let $\hat{t}(\rho, \delta) \in [\underline{t}, 1]$ be the safest type that pays the interest rate $\bar{r}(\rho)$. Thus $\hat{t}(\rho, \delta)$ decreases with ρ and there is a cutoff value, $\bar{\rho}$, such that $\rho \leq \bar{\rho}$ implies that the cost of trade credit never varies with firm characteristics (i.e., $\hat{t}(\rho, \delta) = 1$), in the unique sequential equilibrium of the game. If $\rho > \bar{\rho}$ and $\delta < 1$, then the cost of trade credit decreases with the probability of success for any $t \geq \hat{t}(\rho, \delta) > \underline{t}$.*

Intuitively, moral hazard problems weaken the bank's ability to provide financing, allowing the supplier to increase margins by raising interest rates. As we have already argued, large margins make the volume of loans more important for the supplier, strengthening its incentive to vary the interest rates with the borrowers' risk of credit. Accordingly, Proposition 7 shows that stronger moral hazard problems (i.e., larger ρ) increase the the set of firms, $(\hat{t}(\rho, \delta), 1]$, whose cost of trade credit falls with the probability that the firm succeeds.

As Jensen and Meckling (1976) point out, distressed firms have greater incentives to gamble with risky projects. Hence, Proposition 7 suggests that financially-distressed industries are likely places for empiricists to detect trade credit contracts with interest rates that vary with the borrower's creditworthiness.¹⁸

¹⁸In principle, financial distress could create moral hazard problems not only for the banks but also for the supplier. Nonetheless, Cuñat (2002) argues that suppliers are less vulnerable to moral hazard problems, because they can threaten to stop supplying vital intermediate goods. In this case, the suppliers are likely to be spared of moral hazard problems, making the banks the main targets of opportunistic behavior.

5 Discussion

5.1 A richer information structure for the bank

In the basic model, the bank treats all firms equally. Everything works as if the bank couldn't distinguish, for instance, blue chip corporations from highly leveraged start-up companies. In this section, we endow the bank with a richer information structure and explore its impact on the equilibrium interest rates in the trade credit market.

Consider a partition of the interval of types $[\underline{t}, 1]$, that is, a set of intervals $\{\mathcal{A}_k\}_{k=1}^K$ such that $\cup_{k=1}^K \mathcal{A}_k = [\underline{t}, 1]$ and $\mathcal{A}_k \cap \mathcal{A}_j = \emptyset$, for any $k \neq j$. The partition is the outcome of public signals that cluster the firms in classes of risk: $t < t'$ for any $t \in \mathcal{A}_k$, $t' \in \mathcal{A}_l$ and $l > k$. With this information structure, the bank knows the prior distribution of each class of risk, but cannot identify the types of firms that belong to the same class.

In this setting, Proposition 2 holds for each class of risk. In particular, the equilibrium bank rate in the class k is $r_k^B = \frac{1}{p\underline{t}_k} - 1$, where \underline{t}_k is the riskiest type in \mathcal{A}_k , that is, $\underline{t}_k = \min\{t; t \in \mathcal{A}_k\}$. And the cost of trade credit for firms in the k^{th} -class is the bank rate r_k^B , if the elasticity of demand at r_k^B is below a certain threshold.

Of course, the same argument applies to firms in the class of risk \mathcal{A}_j with $j \neq k$. If the demand for inputs is sufficiently inelastic, then the cost of trade credit for firms in \mathcal{A}_j is $\frac{1}{p\underline{t}_j} - 1$. But then $\frac{1}{p\underline{t}_j} - 1 > \frac{1}{p\underline{t}_k} - 1$ if and only if $\underline{t}_j < \underline{t}_k$, that is, the cost of trade credit is equal for firms in the same class of risk, but varies across classes. This pattern is consistent with the evidence that the cost of trade credit varies across industries but not with firm characteristics, if we interpret the partition of types as the set of industries in the economy.

One may argue, nonetheless, that public signals convey information that help banks screen the credit risk of firms in a same industry. For instance, the risk of a short term loan to a cash-cow firm with no significant debt is negligible, possibly inducing the banks to offer lower interest rates that, ultimately, either rule out trade credit or force suppliers to lower interest rates. If these public signals are pervasive, our model predicts that the cost of trade credit is likely to vary not only across industries but also with firm characteristics.

In contrast, our model predicts that suppliers are unlikely to vary interest rates with

firm characteristics, if moral hazard problems are not severe and informative public signals are concentrated in a small group of cash-cow firms or distressed companies. In this case, uninformed banks probably infer the credit standing of a typical borrower from the average risk of the industry. And informed suppliers can fetch abnormal profits in most trade credit transactions by asking for interest rates that keep firms indifferent between trade credit and loans from uninformed banks.

5.2 Competition among suppliers

Suppose that inputs can be purchased from two equally informed suppliers. In this event, competition between the suppliers should reduce interest rates until they reach the break even point, which depends on the probability that the borrower will pay its debt obligations. Competition between informed suppliers, therefore, implies that the cost of trade credit varies with firm characteristics.

Of course, assuming two equally informed suppliers is equivalent to ruling out the existence of a supplier with private information about the credit risk of a firm. Nonetheless, it is unlikely that multiple suppliers have an informational advantage over banks, if knowledge about the firms' businesses grows mostly from repeated purchases that give rise to special relationships. In this line of reasoning, private information on the credit risk of a given firm is a privilege of its main supplier.

Accordingly, consider an extension of our model that allows for $N > 1$ suppliers. In this extension, each firm in $[t, 1]$ has a most-favored supplier, who knows the firm's type. For instance, m could be the informed supplier of types in a set T^m , while the other suppliers join the bank as uninformed sources of financing, whose beliefs on the types in T^m are represented by the prior $F_0(t)$. Likewise, n could be the informed supplier of types in T^n , with the other suppliers joining the bank as uninformed lenders. From Proposition 2, a sufficiently inelastic demand implies that the cost of trade credit for all firms is $\frac{1}{pE_1[t]} - 1$, whether the informed supplier is m or n ; all that matters is the prior F_0 of the uninformed lenders and the first-mover advantage of the informed suppliers.

It then follows that the cost of trade credit should not vary with firm characteristics,

if private information is due to repeated purchases that give rise to most-favored business relationships between firms and their main suppliers. For the cost of trade credit to vary with firm-specific risk factors, proprietary information must be costlessly conveyed to suppliers. If so, one would wonder why the signal is not conveyed for the banks as well, ruling out the notion that suppliers have an informational advantage over banks.

5.3 Trade credit with alternative outside options

In the basic model, trade credit does not vary with firm characteristics because the supplier competes with uninformed banks in the credit market. It is plausible, though, that the relevant outside option of a firm is a sale of assets, rather than a bank loan.

For example, consider the impact on the airline industry of an increase in the price of fuel. In response to the price increase, an airline may want to buy planes that consume less fuel. In the bargaining for trade credit with its supplier, it is possible that the company's outside option is to reduce its fleet. While the return on the older planes certainly depends on the company's idiosyncratic ability to schedule flights, purchase inputs, and negotiate salaries, these firm-specific characteristics are not so important for the resale value of the airplanes, which are mainly determined by the economic conditions of the airline industry. When bargaining with the riskier airlines, the supplier may want to raise the interest rate until they are indifferent between the two reorganization plans: buying new planes or reducing the fleet. If so, the cost of trade credit will not vary significantly with the firms' characteristics.¹⁹

5.4 Incentive for supplier to disclose private information

Banks often seek information from suppliers about the payment history of their customers. If suppliers answer these inquires truthfully, then their informational advantage disappears and so does the reason we give for the cost of trade credit not to vary with firm characteristics: competition with uninformed banks.

To investigate the suppliers' incentives to disclose information, consider the firms that

¹⁹Felli and Harris (1996) explore the role of outside options in a model of investment decisions in human capital.

pay the interest rate that keeps them indifferent between trade credit and bank loans. In our model, the supplier does not raise the cost of trade credit for these firms because the uninformed bank provides them with an outside option. If the supplier lets the bank know that the market mistakenly perceives these firms as overly risky, then the bank rates go down, forcing the supplier to reduce the cost of trade credit. Hence, the supplier does not have the proper incentive to disclose private information that enhances firms' credit standings.

To be sure, banks may offer some revelation mechanism to suppliers. For instance, profit-sharing mechanisms between a bank and a supplier should provide incentive for the latter to reveal private information. Still, we are not aware of a study that documents revelation mechanisms between banks and suppliers in standard trade-credit transactions. It is conceivable, though, that some sort of revelation mechanism is in place in project loans, which are typically structured around complex contracts. If so, competition with an uninformed bank is no longer relevant for the supplier, implying that, in project loans, the cost of trade credit should vary with the risk of credit of the borrower.

5.5 Nonlinear trade credit contracts

Linear trade credit contracts – pervasive in the U.S. – should probably be understood as mechanisms to facilitate repeated interactions among sellers and buyers. To avoid a costly bargaining for credit, suppliers make one-for-all offers that do not necessarily extract all of the firms' rents. Nonetheless, large transactions – like a sale of a commercial jet – should raise incentives for suppliers to design tailor-made contracts that enhance their ability to extract rents. We argue below that a richer space of trade credit contracts does not unravel the main insights of our paper.

Clearly, the supplier's ability to extract rents is maximized, if firms seek financing for a value-maximizing investment plan. Our first task, therefore, is to solve the maximization program that yields the efficient level of investment of a type- t firm that accepts trade credit:

$$\max_I p^t (Q(I) - I) + (1 - p^t) (\delta I - I). \quad (16)$$

The first term in the objective function, (16), is the economic reason for a type- t firm to seek financing for the project. With probability p^t , the project succeeds and the value added

of the investment is $Q(I) - I$. But, with probability $1 - p^t$, the firm runs into problems that destroy the output $Q(I)$, leaving a salvage value δI , where $\delta \in [0, 1]$. The necessary and sufficient first order condition of problem (16) is

$$Q'(I^{eff}) = 1 + (1 - \delta) \left(\frac{1}{p^t} - 1 \right). \quad (17)$$

The first-order condition shows the impact of the firm-specific risk factors on the efficient level of investment. In the absence of risk, the efficient investment makes the marginal productivity of investment, $Q'(I^{eff})$, equal to the marginal social cost, 1. Firm specific factors matter because riskier firms are more likely to get into problems that imply an economic loss, thereby depressing the efficient investment level. Accordingly, we can write the efficient investment as a function, $I^{eff}(p^t, \delta)$, of the probability that the project succeeds and the salvage value of the investment in the event that the project fails.

We claim that an optimal trade credit contract with a type- t is $(I^{eff}(p^t, \delta), A_t, \delta I^{eff}(p^t))$, where $I^{eff}(p^t, \delta)$ is the input that the supplier sells and A_t is the payment to the supplier in the state of nature that the project succeeds. In the default state, the contract stipulates that the supplier captures the residual value δI . Since $I^{eff}(p^t, \delta)$ is the efficient investment level, this contract is optimal if the promised payments assure a positive expected financial profit for the supplier and keeps the firm on its reservation value, that is,

$$p^t A_t + (1 - p^t) \delta I^{eff}(p^t, \delta) \geq 0, \quad (18)$$

$$p^t \left(Q(I^{eff}(p^t, \delta)) - A_t \right) = p^t \Pi_B. \quad (19)$$

If the contract does not satisfy inequality (18), then it is unprofitable for the supplier to extend trade credit. In addition to being profitable for the supplier, the optimal contract must leave the firm with its reservation value. For this to happen, the firm's expected profit under the contract (the left-hand side of equation (19)) must be equal to its expected profit under a bank loan, $p^t \Pi_B$.

To determine whether the optimal trade credit contract varies with firm-specific risk factors, assume that inequality (18) holds and solve for A_t in equation (19) to obtain

$$A_t = Q(I^{eff}(p^t, \delta)) - \Pi_B. \quad (20)$$

Equation (20) shows that the efficient investment, $I^{eff}(p^t, \delta)$, links the cost of trade credit, A_t , to the firm's type: economic efficiency dictates that firms with a higher probability of success (i.e., larger p^t) invest more, enhancing the supplier's ability to extract rents. As a result, safer firms pay a higher cost of trade credit.

But, the impact of firm-specific risk factors on the optimal trade credit contract may be small. One can check in equation (17) that the probability p^t vanishes from the first order condition of the efficient investment, if the supplier can rescue all inputs in default, that is, if $\delta I = I$. In this case, the cost of trade credit depends only on the industry's technology, $Q(\cdot)$, and the interest rates set by the uninformed bank – summarized in Π_B – which, by construction, cannot vary with the information that is privy to suppliers.

Of course, it is unlikely that suppliers can repossess sold inputs at no cost. However, some simple algebra shows that the sensitivity of the efficient investment with respect to p^t decreases monotonically with the salvage factor, converging to zero as δ approaches one. The model thus predicts that it is harder for empiricists to detect a link between firm characteristics and the cost of trade credit, if default imposes small economic losses on suppliers.

In contrast, Proposition 3 finds that the cost of trade credit becomes more sensitive to firm-specific characteristics, as the salvage value increases. The space of contracts explains the different predictions. With linear contracts, an increase in the salvage value strengthens the supplier's incentive to sustain the volume of sales by varying interest rates with the borrowers' risk. With a richer space of contracts, there is no need to distort the trade credit contract, because nonlinear contracts let the supplier fine tune the cost of trade credit to an efficient investment plan.

Petersen and Rajan (1997) argue that it is easier for suppliers to transform repossessed inputs (rather than finished goods) into liquid assets. As such, industries with a high fraction of intermediate goods in inventory should be associated with a large salvage value. Assuming that nonlinear trade credit contracts are pervasive in concentrated markets for inputs, our model predicts that, in these markets, a high fraction of intermediate goods decreases the likelihood that the cost of trade credit varies with firm characteristics. In contrast, the likelihood of cross-sectional variation increases with the fraction of intermediate goods in inventory,

in markets with dispersed suppliers built around a small volume of frequent transactions; in these markets, linear trade credit contracts should be pervasive.

6 Conclusion

Several studies have documented that the cost of trade credit in the U.S. varies across industries but not with firm characteristics. At first glance, this finding is hard to reconcile with the evidence that, vis-à-vis banks, suppliers are better informed about the economic health of their customers. After all, basic economic principles suggest that riskier firms should pay higher interest rates.

We argue that competition with uninformed banks explains why interest rates in the trade credit markets do not seem to vary with firm characteristics. The uninformed banks do not let the supplier raise the cost of trade credit above the cost of their loans. Because the cost of these bank loans is likely to vary across industries but not with firm characteristics, the same pattern applies to the cost of trade credit.

Of course, we are not the first ones to propose an explanation for the perceived lack of cross-sectional variation of the cost of trade credit. Ng, Smith and Smith (1999) argue, for instance, that suppliers may account for firm characteristics by granting discounts for payments after the due date or by selectively reducing the price of the inputs. Alternatively, suppliers may take into account conformity with local practice when setting the cost of trade credit.²⁰ To be sure, these alternative theories are plausible. However, they do not offer implications that could test their predictive power. In contrast, our model predicts that interest rates in trade credit markets vary more strongly with firm characteristics in industries that are plagued by economic distress and moral hazard problems. These implications could be the subject of an interesting empirical test.

²⁰Young and Burke (2001) argue that customs explain the high degree of uniformity of cropsharing contracts.

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Appendix: Proofs of Propositions

Proof of Proposition 1: If $t \geq \bar{t}(\delta) = \frac{1-\delta}{p(1-\delta+r^B)}$, there exists a solution to the supplier's problem because the objective function (5) is continuous and the opportunity set is compact. Using $\epsilon(r) = -\frac{(1+r)\frac{dI^*(r)}{dr}}{I^*(r)}$, write the derivative of the objective function with respect to r^t as

$$\Psi(r^t, p^t, \delta) = I^*(r^t) \frac{1}{1+r^t} \left[-\epsilon(r^t) \left(p^t(1+r^t) + (1-p^t)\delta - 1 \right) + p^t(1+r^t) \right]. \quad (21)$$

For any $t \geq \bar{t}(\delta)$, some simple algebra shows that

$$\Psi(r^t, p^t, \delta) > 0 \iff \epsilon(r^t) < \frac{tp(1+r^t)}{tp(1+r^t-\delta) - (1-\delta)} \equiv \bar{\epsilon}(r^t, \delta, t). \quad (22)$$

Case 1: $\epsilon(r^B) \leq \bar{\epsilon}(r^B, \delta, t)$. From (22), $\epsilon(r^B) \leq \bar{\epsilon}(r^B, \delta, t)$ implies that r^B satisfies the first order condition: $\Psi(r^B, p^t, \delta) \geq 0$. For r^B to be the unique solution to the problem, it suffices to show that, for any $r^t \in \left[(1-\delta) \left(\frac{1}{p^t} - 1 \right), r^B \right)$, the objective function increases with r^t , that is, $\Psi(r^t, p^t, \delta) > 0$, or equivalently, $\epsilon(r^t) < \bar{\epsilon}(r^t, \delta, t)$. This latter inequality follows from $\epsilon(r^B) \leq \bar{\epsilon}(r^B, \delta, t)$, because $\epsilon(r^t)$ is non-decreasing in r^t (see Assumption 1) and $\frac{\partial \bar{\epsilon}(r^t, \delta, t)}{\partial r^t} < 0$.

Case 2: $\epsilon(r^B) > \bar{\epsilon}(r^B, \delta, t)$. From (22), $\epsilon(r^B) > \bar{\epsilon}(r^B, \delta, t)$ implies $\Psi(r^B, p^t, \delta) < 0$, proving that r^B does not satisfy the first order condition. Moreover, $(1-\delta) \left(\frac{1}{p^t} - 1 \right)$ cannot be optimal either, because it implies zero expected profits while any slightly larger interest rate yields strictly positive expected profits. We conclude that any solution to the supplier's problem must lie in the interval $\left((1-\delta) \left(\frac{1}{p^t} - 1 \right), r^B \right)$, satisfying the first order condition $\Psi(r^t, p^t, \delta) = 0$. To see that the optimal interest rate is unique, suppose that there exist \hat{r}^t and r^t with $\hat{r}^t < r^t$ and $\Psi(r^t, p^t, \delta) = \Psi(\hat{r}^t, p^t, \delta) = 0$, which, from (22), implies $\epsilon(r^t) = \bar{\epsilon}(r^t, \delta, t)$ and $\epsilon(\hat{r}^t) = \bar{\epsilon}(\hat{r}^t, \delta, t)$. A contradiction follows because $\epsilon(r^t)$ non-decreasing in r^t , $\bar{\epsilon}(r^t, \delta, t)$ decreasing in r^t , $\epsilon(r^t) = \bar{\epsilon}(r^t, \delta, t)$, and $r^t < \hat{r}^t$ jointly imply that $\epsilon(\hat{r}^t) > \bar{\epsilon}(\hat{r}^t, \delta, t)$.

For the comparative statics of r^t with respect to p^t , note first that $\delta = 1$ reduces the objective function (5) to $p^t r^t I^*(r^t)$, implying that the optimal r^t does not depend on p^t . Assume now that $\delta \in [0, 1)$. Applying the Implicit Function Theorem to $\Psi(r^t, p^t, \delta) = 0$ yields $\frac{dp^t}{dr^t} = \frac{-\frac{\partial \Psi(r^t, p^t, \delta)}{\partial r^t}}{\frac{\partial \Psi(r^t, p^t, \delta)}{\partial p^t}}$. To sign this derivative, note that $\frac{\partial \Psi(r^t, p^t, \delta)}{\partial p^t} = \frac{I^*(r^t)}{1+r^t} \left(-\epsilon(r^t)(1+r^t-\delta) + (1+r^t) \right) < 0$ if and only if $\epsilon(r^t) > \frac{1+r^t}{1+r^t-\delta}$. This latter inequality holds because the optimal

r^t satisfies $\epsilon(r^t) = \bar{\epsilon}(r^t, \delta, t) = \frac{tp(1+r^t)}{tp(1+r^t-\delta)-(1-\delta)} > \frac{1+r^t}{1+r^t-\delta}$, for any $\delta \in [0, 1)$. It then follows that $\frac{dp^t}{dr^t} \leq 0$ because r^t must satisfy the second order necessary condition, $\frac{\partial \Psi(r^t, p^t, \delta)}{\partial r^t} \leq 0$. Hence, $p^t(r^t)$ is non-increasing, and so is the inverse function $r^t(p^t)$.

To show that $r^t(p^t)$ is strictly decreasing in p^t , suppose, by absurd, that there exist $p^{\bar{t}}$ and $p^{t'}$ with $r^{\bar{t}}(p^{\bar{t}}) = r^{t'}(p^{t'}) = \bar{r}$ and $p^{\bar{t}} < p^{t'}$. Since the previous paragraph demonstrated that $r^t(p^t)$ is non-increasing, it follows from $r^{\bar{t}}(p^{\bar{t}}) = r^{t'}(p^{t'})$ that $r^t(p^t) = \bar{r}$ for any $p^t \in (p^{\bar{t}}, p^{t'})$. Taking the total derivative of $\Psi(\bar{r}, p^t, \delta) = 0$ with respect to p^t yields $\frac{\partial \Psi(\bar{r}, p^t, \delta)}{\partial p^t} = 0$, for all $p^t \in (p^{\bar{t}}, p^{t'})$, contradicting $\frac{\partial \Psi(r^t, p^t, \delta)}{\partial p^t} < 0$, for any $t \in [\bar{t}(\delta), 1]$. We thus conclude that $\frac{dr^t}{dp^t} < 0$, if $\delta \in [0, 1)$. The argument to sign $\frac{dr^t}{d\delta}$ is analogous to the one we used to sign $\frac{dr^t}{dp^t}$. ■

Proof of Proposition 2: Our first task is to show that the proposed strategies and beliefs form a sequential equilibrium. After that, we show that the sequential equilibrium is unique. Beliefs and the Bank's strategy: Given the firms'- and the supplier's strategies, a request of a bank loan is off-the-equilibrium path, implying that Bayes' rule does not impose restrictions on the updating of the firms' types. We can thus take $Prob(s = \underline{t} | \text{bank loan}) = 1$ as the updated belief. With this belief, the analysis of section 2.3 implies that it is optimal for the bank to offer credit to all firms at the interest rate $r^B = \frac{1}{p\underline{t}} - 1$. To prove that the selected updating satisfies the consistency requirement of Kreps and Wilson (1982), consider a sequence of shocks that induce some types to request a bank loan with positive probability. Each shock defines a bank rate $r^B(F_1^n) = \frac{1}{p \int_{\underline{t}}^1 t dF_1^n(t)} - 1$, where $F_1^n(t)$ is the updated belief under the n^{th} -shock. Given $r^B(F_1^n)$, it is strictly profitable for the supplier to undercut the bank so that any type $t > t^n \equiv \bar{t}(\delta, F_1^n) = \frac{1-\delta}{p(1-\delta+r^B(F_1^n))}$ opts for trade credit. Hence, t^n is the safest type that may ask for a bank loan, and $\lim_{n \rightarrow \infty} t^n = \underline{t}$. Bayesian updating upon a request of a bank loan assigns probability zero for any $t \geq t^n$, with $Prob(s \leq t | \text{bank loan}) = \frac{F_0(t)}{F_0(t^n)}$ for $t < t^n$. For any type t , the updated distribution $F_1^n(t)$ is given by

$$F_1^n(t) = \frac{F_0(t)}{F_0(t^n)} \mathbf{1}_{[t < t^n]} + \mathbf{1}_{[t \geq t^n]}, \quad (23)$$

where $\mathbf{1}_{[t < z]}$ is the indicator function that takes value one if $t < z$ and zero otherwise. Taking limits in equation (23) yields $\lim_{n \rightarrow \infty} F_1^n(t) = 1$ for any t , proving that $Prob(t = \underline{t} | \text{bank loan}) = 1$ is consistent.

The supplier's strategy: From Proposition 1, trade credit is profitable for the supplier if $t \geq \bar{t}(\delta) = \frac{1-\delta}{p(1-\delta+r^B)}$. Plugging $1+r^B = \frac{1}{p\underline{t}}$ into $\frac{1-\delta}{p(1-\delta+r^B)}$ yields $\bar{t}(\delta) = \frac{(1-\delta)\underline{t}}{1-p\delta\underline{t}} \leq \underline{t}$, proving that it is optimal for the supplier to offer trade credit to all firms. Assume first that $\epsilon(r^B) \leq \bar{\epsilon}(r^B, \delta, 1)$, which implies that $\epsilon(r^B) < \bar{\epsilon}(r^B, \delta, t)$ for any $t < 1$, because $\bar{\epsilon}(r^B, \delta, t)$ decreases in t . In this case, Proposition 1 implies that $r^t = r^B$ for any t , and the supplier's pricing strategy is optimal if we set $\hat{t} = 1$. Assume now that $\epsilon(r^B) \geq \bar{\epsilon}(r^B, \delta, \underline{t})$, which implies that $\epsilon(r^B) > \bar{\epsilon}(r^B, \delta, t)$ for any $t > \underline{t}$. Here, Proposition 1 implies that $r^{\underline{t}} = r^B$ and $r^t = r(\delta, t)$ for any $t \in (\underline{t}, 1]$, with $r(\delta, t) \in \left((1-\delta) \left(\frac{1}{p^t} - 1 \right), r^B \right)$ implicitly defined by $\epsilon(r(\delta, t)) = \bar{\epsilon}(r(\delta, t), \delta, t)$. From Proposition 1, $r(\delta, t)$ decreases in δ and t , and we conclude that the supplier's pricing strategy is optimal if we set $\hat{t} = \underline{t}$. Finally, $\bar{\epsilon}(r^B, \delta, \underline{t}) < \epsilon(r^B) < \bar{\epsilon}(r^B, \delta, 1)$ and $\frac{\partial \bar{\epsilon}(r^B, \delta, t)}{\partial t} < 0$ imply that there is \hat{t} such that $\epsilon(r^B) = \bar{\epsilon}(r^B, \delta, \hat{t})$, $\epsilon(r^B) > \bar{\epsilon}(r^B, \delta, t)$ for any $t > \hat{t}$, and $\epsilon(r^B) < \bar{\epsilon}(r^B, \delta, t)$ for any $t < \hat{t}$. Given this \hat{t} , Proposition 1 implies $r^t = r^B$ for $t \leq \hat{t}$, and $r^t = r(\delta, t)$ for $t > \hat{t}$.

Firm strategy: Expected profit maximization implies that it is optimal for any type- t to accept trade credit if and only if $r^t \leq r^B$. If this condition is not satisfied or trade credit is not available, it is optimal for the firm to borrow from the bank $I^*(r^B)$, characterized by the first order condition (4) of the firm's investment problem.

Uniqueness: The arguments that show that $Prob(t = \underline{t} | \text{bank loan}) = 1$ is a consistent belief also prevent any other belief from being consistent, if bank loans are off-the-equilibrium path. Hence, there is a single sequential equilibrium in which all firms accept a trade credit offer. Now, suppose that there is an equilibrium in which some firms borrow from the bank with positive probability. In this proposed equilibrium, let $\alpha(t)$ be the probability that type- t borrows from the bank, $F_1^\alpha(t)$ be the updated distribution upon a request of a bank loan, and $r^B(F_1^\alpha)$ be the bank rate. Since it is in the supplier's interest to undercut the bank whenever trade credit is profitable, there is a safest type, $\bar{t}(\delta, F_1^\alpha) = \frac{(1-\delta)E_1^\alpha[t]}{1-p\delta E_1^\alpha[\underline{t}]}$, that borrows from the bank with positive probability. Given $\bar{t}(\delta, F_1^\alpha)$, Bayes' rule implies that the updated distribution is

$$F_1^\alpha(t) = \frac{G^\alpha(t)}{G^\alpha(\bar{t}(\delta, F_1^\alpha))} \mathbf{1}_{[t < \bar{t}(\delta, F_1^\alpha)]} + \mathbf{1}_{[t \geq \bar{t}(\delta, F_1^\alpha)]},$$

where $dG^\alpha(t) = \alpha(t)dF_0(t)$ is the probability that the borrower's type is no more than t , given the probability $\alpha(t)$ that type t borrows from the bank.

Define $T(\bar{t})$ as a function that, for each mixed-strategy equilibrium α , maps the safest type that borrows from bank, \bar{t} , into the cutoff type induced by the Bayesian updating, that is,

$$T(\bar{t}) := \frac{(1 - \delta)E_1^\alpha[t]}{1 - p\delta E_1^\alpha[t]}, \quad (24)$$

where $E_1^\alpha[t] = \frac{1}{G^\alpha(\bar{t})} \int_{\underline{t}}^{\bar{t}} tdG^\alpha(t)$ is the expected type given the Bayesian belief $G^\alpha(t)$.

For a sequential equilibrium with bank loans to exist, the function $T(\bar{t})$ must have a fixed point, i.e., $T(\bar{t}) = \bar{t}$ with $\bar{t} \neq \underline{t}$. To show that $T(\cdot)$ does not have a fixed point, plug $E_1^\alpha[t] = \frac{1}{G^\alpha(\bar{t})} \int_{\underline{t}}^{\bar{t}} tdG^\alpha(t)$ into equation (24) to obtain $T(\bar{t}) = \frac{(1-\delta) \int_{\underline{t}}^{\bar{t}} tdG^\alpha(t)}{G^\alpha(\bar{t}) - \delta p \int_{\underline{t}}^{\bar{t}} tdG^\alpha(t)}$. We claim that, $T(\bar{t}) < \bar{t}$, for any $\bar{t} \in [\underline{t}, 1]$. To see this note that $T(\bar{t}) < \bar{t}$ is equivalent to $(1-\delta+\delta p\bar{t}) \int_{\underline{t}}^{\bar{t}} tdG^\alpha(t) < \bar{t}G^\alpha(\bar{t})$, which holds because $G^\alpha(t)$ is a distribution and $1 - \delta + \delta p\bar{t} < 1$.

■

Proof of Proposition 3: Solving for δ in $\epsilon(r^B) \leq \bar{\epsilon}(r^B, \delta, 1)$ yields $\delta \leq \underline{\delta}$. Hence, $\delta \leq \underline{\delta}$ implies, from Proposition 2, that $r^t = r^B$ for any t . Likewise, solve for δ in $\epsilon(r^B) > \bar{\epsilon}(r^B, \delta, \underline{t})$ to obtain $\delta > \bar{\delta}$, implying that $\delta \in (\bar{\delta}, 1)$ is necessary and sufficient for $r^t = r(\delta, t)$ for any t . Now, by construction of $\underline{\delta}$ and $\bar{\delta}$, $\delta \in (\underline{\delta}, \min\{\bar{\delta}, 1\})$ is equivalent to $\bar{\epsilon}(r^B, \delta, 1) < \epsilon(r^B) < \bar{\epsilon}(r^B, \delta, \underline{t})$, in which case Proposition 2, implies that $r^t = r(\delta, t)$ for any $t > \hat{t}$, where \hat{t} is implicitly defined by $\epsilon(r^B) = \bar{\epsilon}(r^B, \delta, \hat{t}(\delta))$, or equivalently, $r(\delta, \hat{t}(\delta)) = r^B$. Since $r(\delta, t)$ strictly decreases with respect to t and δ , $r(\delta, \hat{t}(\delta)) = r^B$ implies that $\hat{t}(\delta)$ strictly decreases with δ .

■

Proof of Proposition 4: If $\epsilon(r^B) \leq \bar{\epsilon}(r^B, \delta, 1)$, then no trade credit contract varies with firm characteristics (i.e., $\hat{t} = 1$). As shown in the paragraph that precedes the statement of Proposition 4, a negative shock increases the bank rate r^B . From Assumption 1, the elasticity of demand is non-decreasing on the interest rate, while $\bar{\epsilon}(r^B, \delta, 1)$ decreases with r^B . Hence, an increase in r^B makes it easier that $\epsilon(r^B) > \bar{\epsilon}(r^B, \delta, 1)$, implying that \hat{t} either stays unaltered or falls below one. If $\epsilon(r^B) > \bar{\epsilon}(r^B, \delta, 1)$, then the safest type that pays r^B , $\hat{t}(\delta)$, is implicitly defined by $\epsilon(r^B) = \bar{\epsilon}(r^B, \delta, \hat{t}(\delta))$, or equivalently, $r(\delta, \hat{t}(\delta)) = r^B$. Fixed δ , $r(\delta, \hat{t}(\delta)) = r^B$ implies that $\hat{t}(\delta)$ strictly decreases with r^B , because $r(\delta, t)$ strictly decreases with t .

■

Proof of Proposition 5: The largest incentive-compatible loan to the safest project, $\bar{I}_S^B(\rho)$, satisfies the incentive-compatibility condition (11) with equality. If we define $\Psi(I_S^B, \rho) = \frac{Q(I_S^B)}{I_S^B} - (1 + r_S^B) \frac{p_S - p_R}{p_S - \rho p_R}$, then $\bar{I}_S^B(\rho)$ is implicitly defined by $\Psi(\bar{I}_S^B(\rho), \rho) = 0$. Applying the Implicit Function Theorem to $\Psi(\bar{I}_S^B(\rho), \rho) = 0$ yields $\frac{d\bar{I}_S^B(\rho)}{d\rho} < 0$.

To characterize the cut-off ρ^{const} , note first that

$$\lim_{\rho \rightarrow \frac{p_S}{p_R}} \Psi(\bar{I}_S^B(\rho), \rho) = 0 \Leftrightarrow \lim_{\rho \rightarrow \frac{p_S}{p_R}} \frac{Q(\bar{I}_S^B(\rho))}{\bar{I}_S^B(\rho)} = \infty \Leftrightarrow \lim_{\rho \rightarrow \frac{p_S}{p_R}} \bar{I}_S^B(\rho) = 0, \quad (25)$$

with the last equivalence following from the Inada conditions. Analogously, $\lim_{\rho \rightarrow 1} \frac{Q(\bar{I}_S^B(\rho))}{\bar{I}_S^B(\rho)} = 1 + r_S^B = Q'(I_S^*(r_S^B))$, with the last equality following from the first order condition (4). Concavity of the production function and the Inada conditions imply that $\lim_{\rho \rightarrow 1} \frac{Q(\bar{I}_S^B(\rho))}{\bar{I}_S^B(\rho)} = Q'(I_S^*(r_S^B))$ only if $\lim_{\rho \rightarrow 1} \bar{I}_S^B(\rho) = \infty$. Hence, $\bar{I}_S^B(\rho)$ is arbitrarily large when ρ is close to 1 and decreases to 0 when $\rho \rightarrow \frac{p_S}{p_R}$. Continuity thus implies that there is $\rho^{const} \in \left(1, \frac{p_S}{p_R}\right)$ such that $\bar{I}_S^B(\rho^{const}) = I_S^*(r_S^B)$, where $r_S^B = \frac{1}{p_S E_1[t]} - 1$ is the equilibrium interest rate.

■

Proof of Proposition 6: Conditioned on providing incentive for the borrower to invest in project S , an efficient loan contract yields zero expected profit for the bank and keeps the scale of the project as close as possible to the first best. With such a contract, credit constraint (i.e., $\rho > \rho^{const}$) implies that the expected profit of a type- t firm is

$$\Pi_S^t(r_S^B, \rho) = p_S^t Q(\bar{I}_S^B(\rho)) - \frac{t}{E_1[t]} \bar{I}_S^B(\rho), \quad (26)$$

where $r_S^B = \frac{1}{p_S E_1[t]} - 1$ and $\bar{I}_S^B(\rho)$ is implicitly defined by the binding incentive condition (11).

From Proposition 5, $\bar{I}_S^B(\rho)$ decreases with ρ , and so does the expected profit $\Pi_S^t(r_S^B, \rho)$, because the profit function is concave on the investment and $\bar{I}_S^B(\rho) < \bar{I}_S^B(\rho^{const}) = I_S^*(r_S^B) \equiv \arg \max_I p_S^t [Q(I) - (1 + r_S^B)I]$. Taking limits in both sides of the incentive condition (11) yields $\lim_{\rho \rightarrow \frac{p_S}{p_R}} \frac{Q(\bar{I}_S^B(\rho))}{\bar{I}_S^B(\rho)} \geq \lim_{\rho \rightarrow \frac{p_S}{p_R}} (1 + r_S^B) \frac{p_S - p_R}{p_S - \rho p_R} = \infty$. From the Inada conditions, we must have $\lim_{\rho \rightarrow \frac{p_S}{p_R}} \bar{I}_S^B(\rho) = 0$, which, in turn, implies that $\lim_{\rho \rightarrow \frac{p_S}{p_R}} \Pi_S^t(r_S^B, \rho) = 0$. As a result, the expected profit of a type- t that undertakes project S converges to 0 as $\rho \rightarrow \frac{p_S}{p_R}$ and grows

monotonically as ρ decreases, reaching its peak, $\Pi_S^t(r_S^B) = p_S^t[Q(I_S^*(r_S^B)) - (1 + r_S^B)I_S^*(r_S^B)] = \Pi_S^t(r_S^B, \rho^{const})$, at any $\rho \leq \rho^{const}$.

Now, conditioned on the borrower's investing in project R , an efficient loan contract is $(I_R^*(r_R^B), r_R^B)$, where $I_R^*(r_R^B) \equiv \arg \max_I \left[\rho Q(I) - (1 + r_R^B)I \right]$ and $r_R^B = \frac{1}{p_R E_1[t]} - 1$. With this contract, the expected profit of a type- t firm is

$$\Pi_R^t(r_R^B, \rho) = \rho p_R^t Q(I_R^*(r_R^B)) - \frac{t}{E_1[t]} I_R^*(r_R^B). \quad (27)$$

From the Envelope Theorem, $\Pi_R^t(r_R^B, \rho)$ increases with ρ . Hence, $\lim_{\rho \rightarrow \frac{p_S}{p_R}} \Pi_R^t(r_R^B, \rho) > \Pi_R^t(r_R^B, \rho^{const}) > 0 = \lim_{\rho \rightarrow \frac{p_S}{p_R}} \Pi_S^t(r_S^B, \rho)$, implying that the type- t firm strictly prefers the loan contract designed to the riskiest project, if ρ is close to $\frac{p_S}{p_R}$. In contrast, we argue below that type- t firms prefer the loan contract designed to the safest project, if the moral hazard problem does not imply credit constraint, i.e., $\rho \leq \rho^{const}$.

To see this note that $p_S^t > \rho p_R^t$ implies $\Pi_R^t(r_R^B, \rho) < p_S^t Q(I_R^*(r_R^B)) - \frac{t}{E_1[t]} I_R^*(r_R^B)$. A sufficient condition for $\rho \leq \rho^{const}$ to imply $\Pi_S^t(r_S^B) > \Pi_R^t(r_R^B, \rho)$ is thus $\Pi_S^t(r_S^B) > p_S^t Q(I_R^*(r_R^B)) - \frac{t}{E_1[t]} I_R^*(r_R^B)$. This is true because $I_R^*(r_R^B) < I_S^*(r_S^B)$ and the profit function is strictly concave with $I_R^*(r_R^B)$ feasible for the program that picked $I_S^*(r_S^B)$ as the optimal unconstrained investment in project S . Continuity and monotonicity of $\Pi_R^t(r_R^B, \rho)$ and $\Pi_S^t(r_S^B, \rho)$ thus imply that there is $\rho^S \in \left(\rho^{const}, \frac{p_S}{p_R} \right)$ such that $\Pi_R^t(r_R^B, \rho^S) = \Pi_S^t(r_S^B, \rho^S)$, with $\Pi_R^t(r_R^B, \rho) < \Pi_S^t(r_S^B, \rho)$ for any $\rho \in [1, \rho^S)$ and $\Pi_R^t(r_R^B, \rho) > \Pi_S^t(r_S^B, \rho)$ for any $\rho \in \left(\rho^S, \frac{p_S}{p_R} \right)$. ■

Proof of Proposition 7: Following the same steps of the analysis of the optimal trade credit contract without moral hazard problems, a sufficient condition for trade credit not to vary with firm characteristics is $\epsilon(\bar{r}(\rho)) \leq \bar{\epsilon}(\bar{r}(\rho), \delta, 1)$. If $\delta < 1$, then $\epsilon(\bar{r}(\rho)) > \bar{\epsilon}(\bar{r}(\rho), \delta, 1)$ for any $\rho \in [1, \rho^S]$ implies that there is always a trade credit contract whose interest rate increases with the firm's risk, in which case Proposition 7 holds for any $\bar{\rho} < 1$. Likewise, no trade credit contract varies with firm characteristics, if $\epsilon(\bar{r}(\rho)) < \bar{\epsilon}(\bar{r}(\rho), \delta, 1)$ for any $\rho \in [1, \rho^S]$. In this case, Proposition 7 holds for $\bar{\rho} = \rho^S$. Finally, assume that $\epsilon(\bar{r}(\hat{\rho})) = \bar{\epsilon}(\bar{r}(\hat{\rho}), \delta, 1)$ for some $\hat{\rho} \in [1, \rho^S]$. From Assumption 1, the elasticity $\epsilon(r)$ is non-decreasing in the interest rate r and, by construction, the cutoff $\bar{\epsilon}(\bar{r}(\rho), \delta, 1)$ decreases with $\bar{r}(\rho)$, which is an increasing function

of ρ . As a result, $\epsilon(\bar{r}(\hat{\rho})) = \bar{\epsilon}(\bar{r}(\hat{\rho}), \delta, 1)$ at $\hat{\rho}$ only, with $\epsilon(\bar{r}(\rho)) < \bar{\epsilon}(\bar{r}(\rho), \delta, 1)$ for any $\rho < \hat{\rho}$. Proposition 7 thus holds for $\bar{\rho} = \hat{\rho}$.

To complete the proof, we argue that moral hazard does not allow for new equilibria to arise. To see why, suppose first that the moral hazard problem does not imply credit constraint. In this case, the firms'- and the supplier's optimal responses are unchanged, and uniqueness of the equilibrium follows from the same arguments in the proof of Proposition 2. Suppose now that moral hazard leads to credit constraint. In this case, the supplier can raise interest rates, increasing its expected profits accordingly. Since trade credit is more profitable for the supplier, the set of types that receive and accept a trade credit offer cannot decrease. But then the result follows because we have already argued that all firms get and accept a trade credit offer in any sequential equilibrium without credit constraint.

■