Term Premium Dynamics and the Taylor Rule

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Abstract

We explore the bond-pricing implications of an exchange economy where (i) preference shocks result in time-varying term premiums in real yields, and (ii) a monetary policy Taylor rule determines inflation and nominal term premiums. A calibrated version of the model matches the observed term structure of both the mean and volatility of yields. In addition, unlike a comparable model with exogenous inflation, a Taylor rule that matches the properties of observed inflation creates nominal term premiums that remain volatile even at long maturities. Experiments with different parameter values for the Taylor rule demonstrate that the nominal term premiums can be highly sensitive to monetary policy, and that the recent decrease in the level and volatility of the nominal yields could be the result of a more aggressive monetary policy.

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1 Introduction

A challenge for financial economics is understanding how macroeconomic variables affect the term structure of interest rates. Empirical features such as an average upward-sloping term structure, time-varying term premiums, and volatile long-term interest rates are not well captured by standard macroeconomic models. Rather than conclude that the link between the macroeconomy and financial markets is weak, we choose to explore different specifications of macroeconomic models to gain a better understand of these linkages. In particular, we consider the potential for a time-varying price of risk and an endogenous monetary policy rule to help account for some puzzling features of the data, such as an upward sloping average yield curve and the relatively high volatility of long-term rates.

The challenges for a structural macroeconomic model are fairly easy to see. Take as a benchmark an exchange economy in which the representative agent has time-additive expected utility and constant relative risk aversion, and inflation is an exogenous process. (1) Upward sloping average nominal yields: An upward sloping average yield curve requires a nominal pricing kernel that is negatively autocorrelated (see Backus and Zin (1994)). However, the real pricing kernel in this benchmark model will be positively autocorrelated when consumption growth is positively autocorrelated, hence, the term structure of *real* bond yields will be downward sloping on average. In addition, if inflation is positively autocorrelated, the nominal pricing kernel will also be positively autocorrelated when inflation is positively autocorrelated, hence, the nominal term structure will also be downward sloping on average. (2) Volatile yields at long maturities: If investors were risk neutral, then volatility of long yields would require current forecasts of future nominal interest rates to be sensitive to current information. However, at long maturities, these forecasts converge to the mean of the stationary distribution, hence, long yields converge to a constant and volatility converges to zero. Volatility of long yields in the benchmark model must be the consequence of either very persistent risk premiums via very persistent consumption growth, or very persistent inflation. Therefore, the benchmark model is capable of matching these dimensions of the data when either consumption growth or inflation are assumed to have highly counterfactual dynamics (see Piazzesi and Schneider (2007)).

In this paper, we will maintain the structure of an exchange economy with relatively uncomplicated dynamics for exogenous state variable, but with two important extensions of the benchmark model: (1) The representative agent has a preference shock that is sensitive to a latent variable and (external) consumption growth. As in Wachter (2006), we show that this preference shock has important implications for the dynamics of risk premiums. In particular, risk premiums can be highly persistent even when consumption growth is not, which helps account for both the slope and volatility of the nominal yield curve. (2) Monetary policy is determined endogenously through a Taylor rule. We show that the endogenous process for inflation will depend on the same factors as real rates, and that the implied inflation covariance risk will further contribute to a negatively autocorrelated nominal pricing kernel. We accomplish this in a framework that maintains an affine structure for nominal yields, which allows us to give a more structural interpretation to the empirical findings in Ang et al. (2005). Unlike New-Keynesian models of the term structure such as Bekaert et al. (2005), monetary policy in our model plays no role beyond determining the inflation rate. Finally, we also explore the relationship between the preference shock that we infer from properties of the yield curve with external habit formation as in Abel (1990), Campbell and Cochrane (1999).

Our structural model also allows us to conduct policy experiments. That is, we can subject our model to different monetary policies (*i.e.*, different Taylor rule parameters), and ask how these changes would be reflected in the properties of the yield curve. We show that a policy rule that increases the sensitivity of short rates to inflation has the effect of lowering both the average nominal yield curve and the volatility of yields at all maturities.

We conjecture that this might help us understand the changes in the data that we observe after the Volcker disinflation.

2 Affine Term-Structure Models with Stochastic Price of Risk

The structural models we examine below fall within a particular class of arbitrage-free term structure models that are popular in the empirical literature, which we briefly review in this section. The state of the economy is summarized by a k-dimensional vector of variables $\mathbf{s}_t$ that follows a first-order vector autoregression:

$$\mathbf{s}_{t+1} = (\mathbb{I} - \Phi)\theta + \Phi \mathbf{s}_t + \Sigma^{1/2}\varepsilon_{t+1}, \tag{1}$$

where $\{\varepsilon_t\} \sim \text{IID}\mathcal{N}(0, \mathbb{I}_k)$, Φ is a $k \times k$ matrix of autoregressive parameters assumed to be stable, and θ is a $k \times 1$ vector of drift parameters. The conditional covariance matrix, Σ, is constant.

Prices for real and nominal default-free bonds are given by the fundamental equation of asset pricing

$$b_t^{(n)} = \mathbb{E}_t[M_{t+1} b_{t+1}^{(n-1)}], \tag{2}$$

where $b_t^{(n)}$ is the price at date t of a default-free pure-discount bond that pays 1 at date $t + n$ where $b_t^{(0)} = 1$. The asset-pricing kernel, M_{t+1}, will be interpreted as the equilibrium marginal rate of intertemporal substitution for the representative consumer in our structural model below.

We assume that the pricing kernel takes the form

$$-\log M_{t+1} = \Gamma_0 + \Gamma_1^\top \mathbf{s}_t + \frac{1}{2}\lambda(\mathbf{s}_t)^\top \Sigma \lambda(\mathbf{s}_t) + \lambda(\mathbf{s}_t)^\top \Sigma^{1/2}\varepsilon_{t+1}. \tag{3}$$

The $k \times 1$ vector Γ_1 represents the "factor loadings" for the pricing kernel and the $k \times 1$ vector $\lambda(\mathbf{s}_t)$ is the state-dependent price of risk which is also assumed to be affine in the state vector

$$\lambda(\mathbf{s}_t) = \lambda_0 + \lambda_1 \mathbf{s}_t, \tag{4}$$

where λ_0 is a $k \times 1$ vector of constants and λ_1 is a $k \times k$ matrix of constants. The quadratic term $\frac{1}{2}\lambda(\mathbf{s}_t)^\top \Sigma \lambda(\mathbf{s}_t)$ in (3) is a correction term that preserves the linearity of interest rates.

Bond prices of all maturities are linear functions of the state vector

$$-\log b_t^{(n)} = \mathcal{A}^{(n)} + \mathcal{B}^{(n)\top} \mathbf{s}_t,$$

where $\mathcal{A}^{(n)}$ is a scalar, and $\mathcal{B}^{(n)}$ is a $k \times 1$ vector. Equivalently, continuously compounded yields, $i_t^{(n)}$, defined by $b_t^{(n)} \equiv \exp(-n\, i_t^{(n)})$, are also affine functions of the state variables,

$$i_t^{(n)} = \frac{1}{n}\left[\mathcal{A}^{(n)} + \mathcal{B}^{(n)\top} \mathbf{s}_t\right].$$

The parameters defining the bond yields, $\mathcal{A}_n$ and $\mathcal{B}_n$, solve the bond pricing equation (2) resulting in:

$$\begin{aligned} \mathcal{A}_n &= \Gamma_0 + \mathcal{A}_{n-1} + \mathcal{B}_{n-1}^\top \left[(\mathbb{I} - \Phi)\theta - \Sigma\lambda_0\right] - \frac{1}{2}\mathcal{B}_{n-1}^\top \Sigma \mathcal{B}_{n-1}, \\ \text{and} \qquad \mathcal{B}_n^\top &= \Gamma_1^\top + \mathcal{B}_{n-1}^\top \left[\Phi - \Sigma\lambda_1\right]. \end{aligned} \tag{5}$$

Since $b^{(0)} = 1$, the initial conditions for the recursions are $\mathcal{A}_0 = 0$ and $\mathcal{B}_0 = \mathbf{0}$.

Empirical work by Duffee (2002), Dai and Singleton (2002, 2003), Ang and Piazzesi (2003), Brandt and Chapman (2003) and Dai and Philippon (2004), demonstrates that this class of affine models in which the state dependence of the risk premium is driven by state dependence in the price of risk does a good job capturing the salient features of term structure data.

2.1 Some Properties of the Affine Term-Structure Model

The fundamental pricing equation (2) tells us that long-term bonds can be seen as one-period instruments with the uncertain payoff $b_{t+1}^{(n-1)}$. It implies that, from a one-period holding period perspective, long-term bonds might involve compensations for risk that must be reflected in expected excess returns over the one-period risk-free rate i_t. Define the one-period term premium involved in an n-period bond as

$$\xi_t^{(n)} \equiv i_t^{(n)} - \frac{1}{n}\left[i_t + (n-1)\mathbb{E}_t i_{t+1}^{(n-1)}\right]. \tag{6}$$

Using the recursive equations (5), the term premium of an n-period bond can be written in the affine form

$$\xi_t^{(n)} \;=\; \frac{1}{n}\left[\xi_{\mathcal{A},n} + \xi_{\mathcal{B},n}{}^{\top}\mathbf{s}_t\right], \tag{7}$$

with coefficients given by

$$\xi_{\mathcal{A},n} \;=\; -\mathcal{B}_{n-1}^{\top}\Sigma\left(\lambda_0 + \frac{1}{2}\mathcal{B}_{n-1}\right)$$

and

$$\xi_{\mathcal{B},n}{}^{\top} \;=\; -\mathcal{B}_{n-1}^{\top}\Sigma\lambda_1.$$

From these equations, we can infer that term premiums in the affine framework are time-varying as long as the market price of risk is not constant ($\lambda_1 \neq \mathbf{0}$). This characteristic is essential to capture deviations from the expectations hypothesis. To see this, consider the

Campbell and Shiller (1991) coefficients, $\beta^{(n)}$, associated with the regression

$$i_{t+1}^{(n-1)} - i_t^{(n)} = \alpha^{(n)} + \frac{\beta^{(n)}}{n-1}(i_t^{(n)} - i_t) + \varepsilon_{CS,t}^{(n)}. \tag{8}$$

Under the expectations hypothesis, the $\beta^{(n)}$ coefficients are equal to 1. Using equation (6), these coefficients are

$$\beta^{(n)} = 1 - n\frac{\mathrm{cov}(i_t^{(n)} - i_t, \xi_t)}{\mathrm{var}(i_t^{(n)} - i_t)}.$$

Deviations from the expectations hypothesis are explained by time-varying term premiums whose variation is correlated with the variability of interest-rate spreads. Such a pattern is entirely driven by the existence of time variation in the market price of risk.

The term premium $\xi_t^{(n)}$ multiplied by maturity is equal to the expected one-period holding period return of an n period bond in excess of the one period rate. To see this, denote by $xr_{t,t+1}^{n}$ as the one-period holding period return from time t to $t+1$ of an n period bond in excess of the one period rate. The return is given by

$$xr_{t,t+1}^{(n)} = \log\left(\frac{b_{t+1}^{(n-1)}}{b_t^{(n)}}\right) - i_t = -(n-1)i_{t+1}^{(n-1)} + ni_t^{(n)} - i_t$$

and from equation (6) it follows that $\mathbb{E}_t\left[xr_{t,t+1}^{(n)}\right] = n\xi_t^{(n)}$.

Historically, long-term nominal bond yields are on average higher than short-term nominal yields. From the affine specification, the average spread between an n-period bond yield and a one-period interest rate is

$$\mathbb{E}[i_t^{(n)} - i_t] = \frac{n-1}{n}\mathbb{E}[i_t^{(n-1)} - i_t] + \frac{1}{n}\mathbb{E}[xr_{t,t+1}^{(n)}] = \frac{1}{n}\mathbb{E}\left[\sum_{j=2}^{n} xr_{t,t+1}^{(j)}\right].$$

The recursive representation for the average spread shows that the unconditional spread associated with a specific maturity can be expressed as the weighted average of the unconditional spread linked to a bond with a shorter maturity and a maturity-specific holding-period expected excess return. When interest rates are represented by stationary state variables, expected excess returns must be positive enough to obtain an upward-sloping average yield curve. This imposes restrictions on the parameters of the market price of risk.

To obtain the volatility of long-term interest rates, consider the non-recursive solution for the vector of factor sensitivities in equation (5),

$$\mathcal{B}_n = \left[(\mathbb{I} - \Phi_\lambda)^{-1} (\mathbb{I} - \Phi_\lambda^n) \right]^\top \mathcal{B}_1,$$

where

$$\Phi_\lambda = \left[\Phi - \Sigma \lambda_1 \right].$$

The matrix Φ_λ can be interpreted as the autoregressive matrix for the state variables under the risk-neutral measure. This matrix is different from the autocorrelation matrix under the actual measure as long as the market price of risk is time varying. From this representation, the unconditional variance of interest rates is

$$\mathrm{var}(i_t^{(n)}) = \frac{1}{n^2} \mathcal{B}_1^\top (\mathbb{I} - \Phi_\lambda)^{-1} (\mathbb{I} - \Phi_\lambda^n) \,\mathrm{var}(\mathbf{s}_t) \left[(\mathbb{I} - \Phi_\lambda)^{-1} (\mathbb{I} - \Phi_\lambda^n) \right]^\top \mathcal{B}_1.$$

For the one state variable case ($k = 1$), the volatility of interest rates simplifies to

$$\sigma(i_t^{(n)}) = \frac{1}{n} \frac{1 - \Phi_\lambda^n}{1 - \Phi_\lambda} \sigma(i_t).$$

Figure 1 presents the volatility of long-term interest rates implied by the formula above for different coefficients Φ_λ as a proportion of the volatility of the one-period interest rate. From the figure, volatility dies out quickly unless Φ_λ is very close to one. For models with a

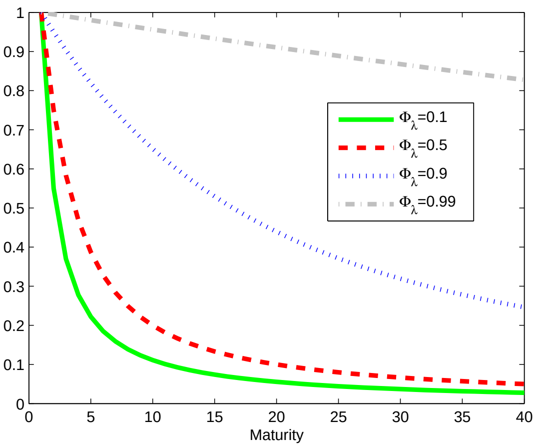

Figure 1: Long-term rate volatility as proportion of short-term rate volatility.

constant market price of risk $\Phi_\lambda = \Phi$, the volatility of interest rates depends entirely on the autocorrelation of the state variables. Thus, in order to capture a slow declining volatility across maturities, stationary state variables need to be very persistent. This is consistent with the result in Backus and Zin (1994) that the volatility of interest rates converges to zero under stationary state variables. The existence of a state-dependent market price of risk, λ_1, such that $\Phi_\lambda - \Phi$ is positive definite, potentially overcomes the lack of persistence in the state variables and helps increase the volatility of longer term interest rates.

3 An Equilibrium Essentially Affine Economy

Empirical estimates of the parameters of the affine model laid out in the previous section are somewhat difficult to interpret. The latent state variables have no direct interpretation, and the parameters of the equilibrium pricing kernel could be determined by preferences, opportunities, government policy, or most likely all of these combined. Therefore, in this section we provide a simple structural model of the macroeconomy that will allow us to better evaluate the content of these empirical models. We will assume a pure-exchange economy with a representative household, and first focus on the role of preferences for determining the term structure of real bonds.

3.1 Consumption and Households

The infinitely-lived representative agent has access to a complete set of date- and state-contingent assets and maximizes lifetime utility subject to a resource constraint. The intertemporal optimization problem is therefore given by

$$\max_{\{C_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} e^{-\delta t} \frac{C_t^{1-\gamma}}{1-\gamma} Q_t \right]$$

subject to the intertemporal budget constraint

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} M_t C_t \right] \leq w_0. \tag{9}$$

Here δ denotes the time preference parameter, γ is the local curvature of the utility function, Q_t is an exogenous preference shock, and w_0 is the household's initial wealth.

Consumption C_t is exogenous in our pure-exchange setting. The process for logarithmic consumption growth, $\Delta c_{t+1} \equiv \log C_{t+1} - \log C_t$, is given by

$$\Delta c_{t+1} = (1-\phi_c)\theta_c + \phi_c \Delta c_t + \sigma_c \varepsilon_{c,t+1} \tag{10}$$

with $\{\varepsilon_{c,t+1}\} \sim$ IID$\mathcal{N}(0,1)$.

The log difference in the exogenous preference shock, $\Delta q_{t+1} \equiv \log Q_{t+1} - \log Q_t$, is linearly related to consumption growth Δc_{t+1}, with a coefficient that varies linearly with the current level of consumption growth and an exogenous variable ν_t interpreted as a pure taste shock:

$$-\Delta q_{t+1} = \frac{1}{2} \left(\eta_c \Delta c_t + \eta_\nu \nu_t\right)^2 \mathrm{var}_t \Delta c_{t+1} + \left(\eta_c \Delta c_t + \eta_\nu \nu_t\right)\left(\Delta c_{t+1} - \mathbb{E}_t \Delta c_{t+1}\right).$$

The preference shock allows for an exogenously varying stochastic risk aversion similar to external habit formation with the addition of a pure taste shock unrelated to consumption growth. The representative household's overall sensitivity to consumption growth is $\gamma + (\eta_c \Delta c_t + \eta_\nu \nu_t)$, where $(\eta_c \Delta c_t + \eta_\nu \nu_t)$ can be interpreted as the stochastic part of the representative household's risk aversion. To complete the specification of the preference shock, ν_t has autoregressive dynamics given by

$$\nu_{t+1} = \phi_\nu \nu_t + \sigma_\nu \varepsilon_{\nu,t+1} \tag{11}$$

with $\{\varepsilon_{\nu,t+1}\} \sim$ IID$\mathcal{N}(0,1)$. The shock $\varepsilon_{\nu,t+1}$ is independent of the consumption growth shock $\varepsilon_{c,t+1}$.

The term $-\frac{1}{2}\left(\eta_c \Delta c_t + \eta_\nu \nu_t\right)^2 \mathrm{var}_t \Delta c_{t+1}$ in the stochastic preference shocks implies that the conditional mean of the growth of the preference shock is

$$E_t\left[\frac{Q_{t+1}}{Q_t}\right] = 1,$$

implying that the process for the preference shock is a martingale. The coefficient η_c measures the sensitivity of the representative household's level of risk-aversion to the current growth rate of aggregate consumption. The coefficient η_ν measures the sensitivity of the representative household's level of risk aversion to the process ν_t which is independent of consumption growth.

From the household's first-order conditions we obtain a real pricing kernel M_{t+1} given by the intertemporal marginal rate of substitution[1]

$$M_{t+1} = e^{-\delta} \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\frac{Q_{t+1}}{Q_t}\right). \tag{12}$$

Therefore, the logarithmic real pricing kernel $m_{t+1} \equiv \log M_{t+1}$ is

$$\begin{aligned} -m_{t+1} &= \delta + \gamma\, \Delta c_{t+1} - \Delta q_{t+1} \\ &= \delta + \gamma\,(1-\phi_c)\,\theta_c + \gamma\,\phi_c\,\Delta c_t + \frac{1}{2}\,(\eta_c\,\Delta c_t + \eta_\nu\,\nu_t)^2\,\sigma_c^2 \\ &\quad + (\gamma + \eta_c\,\Delta c_t + \eta_\nu\,\nu_t)\,\sigma_c \varepsilon_{c,t+1}. \end{aligned} \tag{13}$$

This real pricing kernel can be seen as a 2-factor stochastic price of risk affine model with state variables $\mathbf{s}_t = (\Delta c_t, \;\; \nu_t)^\top$. Proposition 1 summarizes the link between the equilibrium for this economy and the affine framework presented above.

Proposition 1. *The equilibrium characteristics of the economy and its associated real pricing kernel are represented by equations* (1), (3), *and* (4) *where*

$$\mathbf{s}_t = (\Delta c_t, \;\; \nu_t)^\top$$

and

$$\Phi = diag\{\phi_c, \quad \phi_\nu\}, \quad \theta = (\theta_c, \quad 0)^\top, \quad \Sigma^{1/2} = diag\{\sigma_c, \quad \sigma_\nu\}, \quad \varepsilon = (\varepsilon_c, \quad \varepsilon_\nu)^\top,$$

$$\Gamma_0 = \delta + \gamma\,(1-\phi_c)\,\theta_c - \frac{1}{2}\gamma^2\sigma_c^2, \qquad \Gamma_1 = \left[\gamma\left(\phi_c - \eta_c\sigma_c^2\right), \qquad -\gamma\,\eta_\nu\sigma_c^2\right]^\top,$$

[1] The term $\frac{Q_{t+1}}{Q_t}$ can be seen as a Radon-Nikodym derivative that represents a change of measure from the pricing kernel of a CRRA economy. This representation for the pricing kernel is isomorphic to the Epstein-Zin pricing kernel presented in Gallmeyer et al. (2007) or the model-uncertainty adjusted pricing kernel in Hansen and Sargent (2007). Although the economic underpinnings in these models are different, they share the purpose of shifting the marginal utility of consumption.

$$\lambda_0 = [\gamma, \quad 0]^\top, \qquad and \qquad \lambda_1 = \begin{bmatrix} \eta_c & \eta_\nu \\ 0 & 0 \end{bmatrix}.$$

Proof. Characterize the state-vector process (1) using equations (10) and (11), and express (13) in matrix form. □

This representation allows us to price real discount bonds using equation (5). The equilibrium continuously compounded n-period real interest rate, ${r_t}^{(n)}$, must satisfy the household's first-order condition for n-period real bond holdings

$$e^{-nr_t^{(n)}} = \mathbb{E}_t\left[M_{t+n}\right] = \mathbb{E}_t\left[M_{t+1}e^{-(n-1)r_{t+1}^{(n-1)}}\right]. \tag{14}$$

Therefore, real interest rates can be expressed as linear combinations of consumption growth and the exogenous variable ν_t, with loadings given by functions of deep economic parameters.

Relative to a general essentially-affine model, the model's structural parameters significantly reduce the dimensionality of the parameter space. From the structure of the price of risk $\lambda(\mathbf{s}_t)$, innovations in the pricing kernel are solely driven by shocks to consumption growth $\varepsilon_{c,t+1}$. The preference shock ν_t does however contribute to time variation in the price of risk as long as $\eta_\nu \neq 0$. The preference parameters η_c and η_ν affect the sensitivity of interest rates to the state variables. In particular, a negative value for η_c increases the response of real interest rates to consumption growth and implies a counter-cyclical price of consumption growth risk. Such a feature can lead to an upward-sloping average yield curve. To see this, consider the spread between a 2-period bond and a one-period bond, $r_t^{(2)} - r_t$. Using (14) and Proposition 1, the average spread is

$$\begin{aligned}
\mathbb{E}[r_t^{(2)} - r_t] &= -\frac{1}{2}\mathbb{E}\left[\text{var}_t(r_{t+1})\right] + \mathbb{E}\left[\text{cov}_t(m_{t+1}, r_{t+1})\right] \\
&= -\frac{1}{2}\mathbb{E}\left[\text{var}_t(r_{t+1})\right] - \gamma(\gamma + \eta_c\theta_c)(\phi_c - \eta_c\sigma_c^2)\sigma_c^2.
\end{aligned}$$

Therefore, the model's potential to generate positive spreads depends entirely on its ability to capture a positive covariance between the real price kernel and interest rates. Under power utility, $\eta_c = 0$ and the real yield curve is always downward sloping, unless we assume a counterfactual negative autocorrelation of consumption growth. In contrast, since long-term consumption growth, θ_c, is positive, a positive spread can be obtained when $\eta_c < -\frac{\theta_c}{\gamma}$.

3.2 Nominal Bond Pricing

Define P_t as the money price of goods at date-t. If we assume a frictionless conversion of money for goods, then the nominal pricing kernel is

$$M_{t+1}^{\$} = e^{-\delta} \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(\frac{Q_{t+1}}{Q_t} \right) \left(\frac{P_{t+1}}{P_t} \right)^{-1}. \tag{15}$$

Let ${i_t}^{(n)}$ denote the continuously compounded n-period nominal interest rate. The household's first-order condition for n-period nominal bond is

$$e^{-i_t^{(n)}} \;=\; \mathbb{E}_t \left[M_{t+n}^{\$} \right].$$

The logarithm of the nominal pricing kernel is then $m_{t+1}^{\$} = m_{t+1} - \pi_{t+1}$, where $\pi_{t+1} \equiv \log P_{t+1} - \log P_t$ is the rate of inflation from t to $t+1$.

To close the nominal side of the model, we need to derive a process for the evolution of inflation. For comparisons, we consider two approaches to for modeling inflation dynamics. Our first approach is to directly specify inflation as an exogenous process. Our second approach is to specify a monetary policy rule that links the nominal short rate to the rate of inflation via a Taylor rule, which results in endogenous dynamics for inflation.

3.3 Exogenous Inflation Nominal Pricing Kernel

By expanding the state space to include an exogenous inflation process π_t, the nominal state vector is $\mathbf{s}_t^{\$} = (\Delta c_t, \;\; \nu_t, \;\; \pi_t)^\top$. Further, we assume that the stochastic process for inflation is given by

$$\pi_{t+1} = (1 - \phi_\pi)\theta_\pi + \phi_\pi \pi_t + \sigma_\pi \varepsilon_{\pi,t+1}, \tag{16}$$

where $\{\varepsilon_{\pi,t+1}\} \sim \text{ IID}\mathcal{N}(0,1)$ and is independent of all other shocks in the model. Given the conditional variance of inflation, $\text{var}_t(\pi_{t+1}) = \sigma_\pi^2$, is constant, the nominal state vector still conforms to the essentially affine setting described above.

Based on the equilibrium real and nominal pricing kernels given by equations (13) and (15), the equilibrium nominal term structure from our habit-based pure exchange economy can be expressed as a 3-factor stochastic price of risk affine model characterized in Proposition 2.

Proposition 2. *The equilibrium characteristics of the economy under the exogenous inflation process and its associated nominal pricing kernel are represented by equations* (1)*,* (3)*, and* (4) *where*

$$\mathbf{s}_t^{\$} = (\Delta c_t, \;\; \nu_t, \;\; \pi_t)^\top$$

and

$$\Phi^{\$} = diag\{\phi_c, \, \phi_\nu, \, \phi_\pi\}, \quad \theta^{\$} = (\theta^\top, \, \theta_\pi)^\top, \quad \Sigma^{1/2,\$} = diag\{\sigma_c, \, \sigma_\nu, \, \sigma_\pi\}, \quad \varepsilon^{\$} = (\varepsilon^\top, \, \varepsilon_\pi)^\top,$$

$$\Gamma_0^{\$} = \Gamma_0 + (1-\phi_\pi)\theta_\pi - \frac{1}{2}\sigma_\pi^2, \qquad \Gamma_1^{\$} = [\Gamma_1, \qquad \phi_\pi]^\top,$$

$$\lambda_0^{\$} = \left[\lambda_0^\top, \qquad 1\right]^\top, \qquad and \qquad \lambda_1^{\$} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & 0 \end{bmatrix}.$$

Proof. Characterize the state-vector process (1) using equations (10), (11), and (16), then substitute them into the nominal pricing kernel (15) and express in matrix form. □

From Proposition 2, the market prices of risk related to consumption growth and the exogenous taste shock ν_t are the same as in Proposition 1. Equivalently, the compensations for the risks associated with consumption growth and the exogenous preference variable are the same for assets with real and nominal payoffs. The last term of $\lambda(\mathbf{s}_t)$ contains the price of inflation risk, which in this setting is constant.

3.4 A Monetary-Policy Consistent Nominal Pricing Kernel

Alternatively, we can derive the nominal pricing kernel by imposing a monetary policy rule linking inflation to the nominal short rate. Assume that monetary policy follows a nominal interest rate rule of the form

$$i_t = \bar{\imath} + \imath_c \Delta c_t + \imath_\pi \pi_t + u_t, \tag{17}$$

where u_t is a policy shock capturing the non-systematic component of monetary policy. This policy shock follows an autoregressive process with dynamics given by

$$u_{t+1} = \phi_u u_t + \sigma_u \varepsilon_{u,t+1}, \tag{18}$$

where $\{\varepsilon_{u,t+1}\} \sim$ IID$\mathcal{N}(0,1)$ and is independent of all other shocks in the model.

The policy rule (17) is similar to the one proposed in Taylor (1993). The evident difference between the two rules is that, while under the original Taylor (1993) specification the short-term interest rule depends on the output gap level, the rule here reacts to consumption growth. The absence of a production sector with frictions in this endowment economy does not admit an interpretation of an output gap. Therefore, with slight abuse of terminology, we refer to the policy rule as the Taylor rule for the model.

Given that the Taylor rule (17) must be consistent with the nominal pricing kernel (15), we can use the two equations to solve for an internally consistent process for inflation. This process is given by

$$\pi_t = \bar{\pi} + \pi_c \Delta c_t + \pi_\nu \nu_t + \pi_u u_t.$$

The equilibrium constraint imposed by the price of the one-period nominal bond (16) implies loading coefficients for the equilibrium inflation that satisfy

$$\bar{\pi} = \frac{1}{1-\imath_\pi}\left[\bar{\imath} - \delta - (\gamma+\pi_c)(1-\phi_c)\theta_c + \frac{1}{2}(\gamma+\pi_c)^2\sigma_c^2 + \frac{1}{2}\pi_\nu^2\sigma_\nu^2 + \frac{1}{2}\pi_u^2\sigma_u^2\right],$$

$$\pi_c = \frac{\gamma(\phi_c - \sigma_c^2\,\eta_c) - \imath_c}{\imath_\pi - \phi_c + \sigma_c^2\,\eta_c}, \qquad \pi_\nu = -\frac{(\gamma+\pi_c)\sigma_c^2\,\eta_\nu}{\imath_\pi - \phi_\nu}, \qquad \text{and} \qquad \pi_u = -\frac{1}{\imath_\pi - \phi_u}.$$

These coefficients show that the sensitivity of inflation to the state variables are determined by the response of the monetary authority to consumption growth and inflation.

Substituting the monetary-policy consistent inflation process into the nominal pricing kernel (15), we obtain a 3-factor essentially affine nominal term structure model. The nominal state vector is given by $\mathbf{s}_t^{\$} = (\Delta c_t, \quad \nu_t, \quad u_t)^\top$. The dynamics of the nominal state variables and the nominal pricing kernel are characterized in Proposition 3.

Proposition 3. *The equilibrium characteristics of the economy under the endogenous inflation process and its associated nominal pricing kernel are represented by equations* (1), (3), *and* (4) *where*

$$\mathbf{s}_t^{\$} = (\Delta c_t, \quad \nu_t, \quad u_t)^\top$$

and

$$\Phi^{\$} = diag\{\phi_c,\ \phi_\nu,\ \phi_u\}, \quad \theta^{\$} = (\theta^\top, \quad 0)^\top, \quad \Sigma^{1/2,\$} = diag\{\sigma_c,\ \sigma_\nu,\ \sigma_u\}, \quad \varepsilon^{\$} = (\varepsilon^\top,\ \varepsilon_u)^\top,$$

$$\Gamma_0^{\$} = \delta + \bar{\pi} + (\gamma+\pi_c)\,(1-\phi_c)\,\theta_c - \frac{1}{2}(\gamma+\pi_c)^2\sigma_c^2 - \frac{1}{2}\pi_\nu^2\sigma_\nu^2 - \frac{1}{2}\pi_u^2\sigma_u^2,$$

$$\Gamma_1^\$ = \left[(\gamma + \pi_c) \left(\phi_c - \eta_c \sigma_c^2 \right), \qquad \pi_\nu \phi_\nu - (\gamma + \pi_c)\, \eta_\nu \sigma_c^2, \qquad \pi_u \phi_u \right]^\top,$$

$$\lambda_0^\$ = [\gamma + \pi_c, \quad \pi_\nu, \quad \pi_u]^\top, \qquad \textit{and} \qquad \lambda_1^\$ = \begin{bmatrix} \lambda_1 & 0 \\ 0 & 0 \end{bmatrix}.$$

Proof. Characterize the state-vector process (1) using equations (10), (11), and (18), substitute them into the nominal pricing kernel (15) and express in matrix form. □

The vector $\lambda_0^\$$ shows that the constant component of the market prices of risk related to consumption growth and ν_t are affected by the inflation process. Given in equilibrium, the inflation process is determined by consumption growth and ν_t, the nominal compensation for risk depends on the response of inflation to these two processes.

4 Analysis

We compare the term structure implications of the exogenous and endogenous inflation economies presented above. This analysis is useful in understanding what we can learn about interest rate dynamics when a monetary policy rule is imposed. In addition, we conduct a policy experiment by changing the monetary policy rule to analyze its implications for the term structure and macroeconomic variables.

4.1 Data

To understand the main differences in the term-structure dynamics between the exogenous and endogenous inflation models, we calibrate the two models to selected statistics of the U.S. data. We use quarterly U.S. data from 1971:3 to 2005:4 for interest rates, consumption, and consumer prices. The zero-coupon yields for yearly maturities from 1 to 10 years are obtained using the Svensson (1994) methodology applied to off-the-run Treasury coupon securities by the Federal Reserve Board.[2] The short-term nominal interest rate is the 3-month T-bill from the Fama-Bliss risk-free rates database. The consumption growth series is constructed using quarterly data on real per capita consumption of nondurables and services from the Bureau of Economic Analysis. The inflation series is obtained following the methodology in Piazzesi and Schneider (2007). These data capture inflation related only to nondurable consumption and services. Therefore, it is consistent with the consumption data. The construction of the inflation data and a comparison to log-changes in the CPI are presented in the Appendix.

4.2 Calibration: Exogenous inflation vs. Endogenous inflation

For comparison purposes, the two models are calibrated such that they share the same real dynamics. That is, the parameters describing the real side of the economy in the two models are the same. The parameters are chosen by calibrating the endogenous inflation model to match the average level and volatility of nominal interest rates as well as the average, volatility, and first order autocorrelation of consumption growth and inflation. The resulting parameter values for the real side of the economy are used in the exogenous inflation model. The parameters for its exogenous inflation process are chosen to match selected moments of the observed inflation process. Analytical representations of macroeconomic and term structure model-implied statistics for the two models are reported in the Appendix.

Table 1 contains the common parameter values across the two models. The parameters θ_c, ϕ_c, and σ_c were chosen to match the mean, standard deviation, and first-order autocorrelation of consumption growth. The habit parameters η_c and η_ν were calibrated in the endogenous inflation model. They were chosen to match the shape of the average nominal yield curve and its volatility. The negative sensitivity of the habit to consumption growth, $\eta_c < 0$, generates an upward sloping yield curve. It can be interpreted as counter-cyclical risk aversion shifting marginal utility to obtain positive average risk premiums for long-term bonds.

[2] The data series are available at http://www.federalreserve.gov/pubs/feds/2006/200628/feds200628.xls.

Table 1: Common parameter values in the two models.

Parameter	Description	Value
δ	Subjective discount factor	1.7766×10^{-4}
γ	Curvature parameter	0.65
θ_c	Average consumption growth	4.938×10^{-3}
ϕ_c	Autocorrelation of consumption growth	0.4146
σ_c	Conditional volatility of consumption growth	3.962×10^{-3}
η_c	Habit sensitivity to consumption growth	-28805
ϕ_ν	Autocorrelation of latent variable	0.10
σ_ν	Conditional volatility of latent variable	0.055
η_ν	Habit sensitivity to consumption growth	-12500

The autoregressive parameter ϕ_ν is set to capture the volatility of interest rates for intermediate maturities with σ_ν fixed at 0.055. When $\phi_\nu = 0$, we can capture the volatility of short-term and long-term rates, but it implies a quick decline in volatilities for intermediate maturities that is not observed in the data. Allowing for a positive autocorrelation of the latent taste shock variable helps to overcome this limitation. The magnitude of σ_ν is such that η_ν has the same order of magnitude as η_c. The sensitivity of the habit to the latent taste shock variable, η_ν, allows us to capture the volatility of the short-term rate. When $\eta_\nu = 0$ and the model matches the shape of the yield curve, the endogenous inflation model implies a lower volatility for the short-term rate than observed in the data.

Given both the exogenous and endogenous inflation models are calibrated such that they share the same real asset pricing dynamics, Figure 2 presents the common properties of the the real yield curve — its average shape, volatility, and term premium structure. In addition to requiring a latent taste shock variable ν that helps to fit the structure of bond volatilities, our calibrated preference structure is not easily interpreted as habit formation preferences. This is driven by our desire to capture an upward-sloping average real yield curve and still generate an affine-based nominal term structure with endogenous inflation.

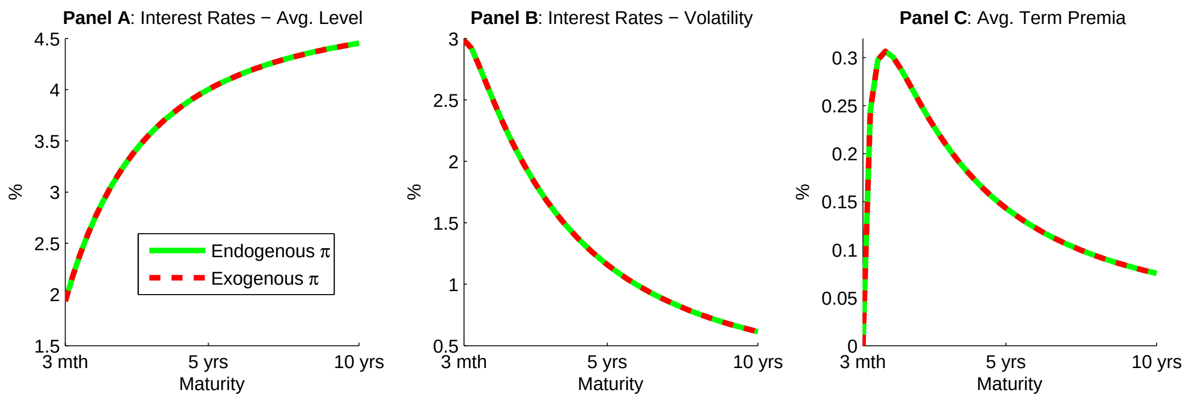

Figure 2: Real Interest Rate Properties - Exogenous and Endogenous Inflation

Habit-based models such as Campbell and Cochrane (1999)[3] and Wachter (2006) generate a countercyclical price of consumption growth risk and real interest rates that are negatively correlated with consumption growth. Our preference specification does not allow us to capture these two properties simultaneously and so cannot be interpreted as a habit-based preference specification. To see this, consider the average spread between a 2-period bond and a one-period bond previously examined in section 3.

$$\begin{aligned} \mathbb{E}[r_t^{(2)} - r_t] &= -\frac{1}{2}\mathbb{E}\left[\text{var}_t(r_{t+1)}\right] + \mathbb{E}\left[\text{cov}_t(m_{t+1}, r_{t+1})\right] \\ &= -\frac{1}{2}\mathbb{E}\left[\text{var}_t(r_{t+1})\right] - \gamma(\gamma + \eta_c\theta_c)(\phi_c - \eta_c\sigma_c^2)\sigma_c^2. \end{aligned}$$

Ignoring the term for the conditional variance of the short-term rate, this specification allows for an upward sloping average curve if either $\phi_c - \eta_c\sigma_c^2 < 0$ or $\gamma + \eta_c\theta_c < 0$.

For $\phi_c - \eta_c\sigma_c^2 < 0$ to hold, η_c has to be positive enough $\left(\eta_c > \frac{\phi_c}{\sigma_c^2}\right)$, since the autocorrelation of consumption growth ϕ_c in the data is positive. Therefore $\gamma + \eta_c\Delta c_t$ is high for high consumption growth and low for low consumption growth, which implies a cyclical price of consumption growth risk rather than a countercyclical one as in a habit formation specifi-

[3] The published version of Campbell and Cochrane (1999) assumed that real interest rates are constant, but the working paper version also studied the case of time-varying interest rates.

cation. In addition, $\phi_c - \eta_c \sigma_c^2 < 0$ implies that the autocorrelation of consumption growth under the risk neutral measure is negative. This leads to instability in the consumption growth process and generates a choppy yield curve where real rates are negatively correlated with consumption growth.

Alternatively, our calibration leads to a positive sloping real yield curve through $\gamma + \eta_c \theta_c < 0$. For this to hold, $\eta_c < 0$ which captures the idea of countercyclical risk aversion. However, in a habit-based interpretation, it also implies a negative average risk aversion precluding interpreting our preference specification as a form of habit formation. Additionally, a negative η_c delivers a positive correlation between real rates and consumption growth, in contrast to what is implied by the usual habit models.

Table 2 contains the model-specific parameters. For the endogenous inflation model, the policy rule parameters imply positive responses of the monetary authority to consumption growth and the level of inflation. To capture the volatility of long term rates, the autoregressive coefficient of the policy shock, ϕ_u, is set close to one. The inflation process in the endogenous inflation model is given by

$$\pi_t = 0.012 - 0.28\Delta c_t + 0.047\nu_t - 1.48u_t,$$

where the negative loading on consumption growth induces the negative correlation between consumption growth and inflation that is observed in the data. For the exogenous inflation model, the parameters θ_π, ϕ_π, and σ_π are chosen to match the mean, standard deviation, and first order autocorrelation of inflation.

Table 3 reports some model-implied statistics for both models. Panel A of the table shows that both models are able to capture important properties of the dynamics of consumption growth and inflation. As mentioned above, the endogenous inflation model has the advantage of capturing the negative correlation between consumption growth and inflation.

Table 2: Model-specific parameter values.

Endogenous-π		Exogenous-π	
$\bar{\imath}$	-0.007	θ_π	1.115×10^{-2}
$\imath_c$	0.79	ϕ_π	0.84
$\imath_\pi$	1.68	σ_π	3.593×10^{-3}
ϕ_u	0.9982		
σ_u	2.5×10^{-4}		

This correlation is zero by construction under the exogenous inflation model.

Panel B of Table 3 and Figure 3 present selected properties of nominal interest rates. The average level of the yield curve implied by the endogenous inflation model match its empirical counterpart. The average nominal short-term rate and the slope of the curve for the calibrated exogenous inflation model are higher than in the data. Panel C in Figure 3 shows that the higher spreads in the exogenous inflation model are explained by differences in risk premiums. The risk premiums in the endogenous inflation model imply expected excess returns that increase monotonically with maturity and vary from 0.22% for the 6-month rate to 2.10% for the 10-year bond yield. In the exogenous inflation model, the implied expected excess returns are 0.48% for the 6-month rate, and reach 2.90% for the 10-year bond yield.

Panel B in Figure 3 demonstrates that the volatilities of the short-term rate and the 10-year rate in the endogenous inflation model match the data. The volatility of the nominal short-term rate in the exogenous inflation model is higher than in the data and the volatility of the 10-year interest rate is significantly lower. While the ratio of the volatility of the 10-year rate to the short-term rate is 78% in the data as well as the endogenous inflation model, it is only 19% in the exogenous inflation model. This failure of the exogenous inflation model is driven by the lack of persistence in the consumption growth and inflation processes. The time-varying prices of risk ($\lambda_1 \neq \mathbf{0}$) given by the habit parameters η_c and η_ν is not strong enough to increase the volatility of long rates when inflation is an exogenous process.

In contrast, the endogenous inflation model is able to capture short-term rate and long-term rate volatility simultaneously since the policy rule allows us to describe inflation, and thus, interest rates, in terms of a very persistent process, the policy shock. That is, the volatility of interest rates does not die out quickly with bond maturity because the non-systematic component of the Taylor rule exhibits significant persistence.

Table 3: Data and model-implied descriptive statistics.

	Data	Endogenous-π	Exogenous-π
Panel A			
$\mathbb{E}\left[\Delta c_t\right] \times 4 \quad (\%)$	1.98	1.98	1.98
$\mathbb{E}\left[\pi_t\right] \times 4 \quad (\%)$	4.46	4.42	4.46
$\sigma\left(\Delta c_t\right) \times 4 \quad (\%)$	1.74	1.74	1.74
$\sigma\left(\pi_t\right) \times 4 \ (\%)$	2.66	2.69	2.67
$\text{corr}\left(\Delta c_t, \Delta c_{t-1}\right)$	0.41	0.41	0.41
$\text{corr}\left(\pi_t, \pi_{t-1}\right)$	0.84	0.85	0.84
$\text{corr}\left(\Delta c_t, \pi_t\right)$	-0.33	-0.18	0
Panel B			
$\mathbb{E}[i_t] \times 4 \quad (\%)$	6.11	6.11	6.39
$\mathbb{E}[i_t^{(20)}] \times 4 \quad (\%)$	7.31	7.36	8.40
$\mathbb{E}[i_t^{(40)}] \times 4 \quad (\%)$	7.68	7.65	8.83
$\sigma\left(i_t\right) \times 4 \quad (\%)$	3.04	3.04	3.73
$\sigma(i_t^{(20)}) \times 4 \quad (\%)$	2.61	2.48	1.35
$\sigma(i_t^{(40)}) \times 4 \quad (\%)$	2.38	2.37	0.71
$\text{corr}(i_t, i_{t-1})$	0.92	0.69	0.39
$\text{corr}(i_t, i_t^{(20)})$	0.86	0.93	0.99
$\text{corr}(i_t, i_t^{(40)})$	0.82	0.88	0.99
Panel C			
$\text{corr}\left(i_t, \Delta c_t\right)$	-0.10	0.19	0.26
$\text{corr}\left(i_t, \pi_t\right)$	0.60	0.91	0.59

One way to increase the volatility of long-term rates relative to the short-term rate in the exogenous inflation model is to increase the autoregressive parameter for the latent preference variable ϕ_ν. However, increasing this parameter leads to counterfactual implications. When the 10-year rate volatility is matched, a hump-shaped pattern for volatility across

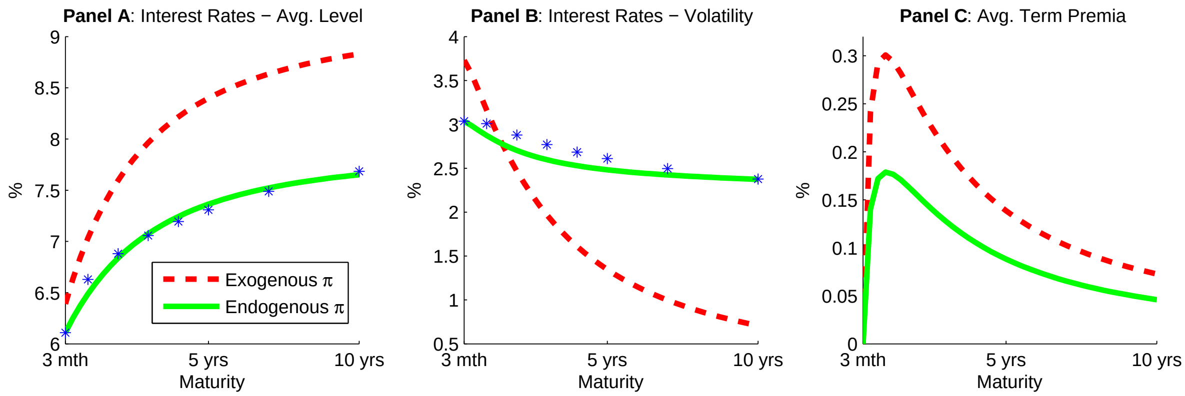

Figure 3: Nominal Interest Rate Properties - Exogenous and Endogenous Inflation. The (∗) denotes data.

maturities is obtained: the volatility of interest rates for some intermediate maturities is significantly higher than the volatility of short and long term rates. Therefore, the exogenous inflation model is unable to jointly capture macroeconomic behavior and the average level and volatility of nominal interest rates.

Table 3 also shows other properties of the endogenous inflation model implied by the calibration. The first-order autocorrelation of the short-term interest rate is higher in the endogenous inflation model, but it is still too low relative to the autocorrelation implied by the data. The correlation between short-term and long-term rates is also too high relative to the data. The endogenous inflation model is also unable to fully capture the correlation structure between the short rate, consumption growth, and inflation. The correlation between the short rate and consumption growth is positive in the endogenous inflation model, while in the data it is negative. The correlation between the short rate and inflation is also higher in the endogenous inflation model than in the data.

To understand the differences across the two models, we can compare the dynamics of real and nominal interest rates. Since the two models share the same parameters for the real side of the economy, the dynamics for real interest rates in the two models are the same. By comparing the real yield curve given in Figure 2 with the nominal yield curve in the exogenous inflation model given in Figure 3, note that the shape of the two average curves, their volatilities, and their risk premiums are very similar. This is not the case if we compare the real yield curve and the nominal yield curve in the endogenous inflation model.

These differences can be understood by comparing the prices of risk in Propositions 1 through 3. The prices of risk and the loading coefficients associated to consumption growth and the exogenous preference variable for assets with real payoffs are the same as those for nominal payoffs in the exogenous inflation model. Here, inflation is modeled as a process that is uncorrelated with these two factors so that the prices of risk in the nominal term structure are the same as in the real term structure. This is no longer true in the endogenous inflation model. Here, inflation depends on consumption growth and the exogenous preference variable. Since $\pi_c < 0$, it implies that the price of consumption growth risk for real payoffs is higher than the price for nominal payoffs. It translates into lower nominal risk premiums than real risk premiums in the calibration. A monetary policy rule generates a negative correlation between consumption growth and inflation that reduces the price of consumption risk in the nominal pricing kernel. This reduction provides hedging properties for nominal bonds and investors require lower expected excess returns to hold them.

The effects of the policy rule are also reflected in differences in the volatilities of nominal and real rates. While the volatility of the short-term nominal rate is similar to that of the short-term real rate, the volatilities of long-term real rates are significantly lower than the volatilities of long-term nominal rates. This difference is explained by the persistence of the policy shock that do not influence real rates and drives the higher volatility of nominal rates.

We can also compare the sensitivity of interest rates and term premiums to the two common state factors in the models: consumption growth and the exogenous preference variable ν_t. Figures 5 and 6 in the Appendix show that nominal loadings in the endogenous inflation model for interest rates and risk premiums are significantly lower than in the exogenous inflation model.

5 A Policy Experiment

The endogenous inflation model is useful to analyze the effects of monetary-policy changes on the dynamics of interest rates. Such policy experiments can be captured by changes in the functional form of the policy rule or changes in the reaction coefficients of the policy rule presented in Section 3.4. Here we follow the latter. We analyze the effects on the dynamics of interest rates to changes in the reaction coefficients on inflation and consumption growth in the policy rule. The motivation for this exercise is provided by empirical evidence presented in Clarida et al. (2000). They estimate reaction functions for monetary policy in the U.S. for different periods and find that the policy rule that describes the most recent period in the U.S. economy has a higher reaction coefficient to the level of inflation than in previous periods. Our objective is to analyze the implications of changes in the reaction to macroeconomic variables on the dynamics of interest rates and try to determine whether these changes are consistent with the evolution of interest rates in recent years.

Table 4 presents regression results for the policy rule (17) for the periods analyzed in Clarida et al. (2000). The table shows that the coefficient of inflation in the rule is significantly higher during the Volcker-Greenspan period (1979-2005) than in the pre-Volcker era (1960-1979).

Table 4: Regressions $i_t = \bar{\imath} + \imath_c \Delta c_t + \imath_\pi \pi_t + u_t$ for different samples.

Sample	$\bar{\imath}$	$\imath_c$	$\imath_\pi$	R^2	$\text{corr}(u_t, u_{t-1})$
$1960:1 - 2005:4$	0.01	0.07	0.74	0.42	0.80
	(0.00)	(0.10)	(0.07)		
$1960:1 - 1979:3$	0.00	0.13	0.60	0.73	0.58
	(0.00)	(0.06)	(0.04)		
$1979:4 - 2005:4$	0.00	0.21	1.12	0.49	0.67
	(0.00)	(0.16)	(0.12)		

Standard errors are reported in parenthesis.

We use the calibration for the endogenous inflation model in Section 4.2 as the baseline calibration. We conduct two policy experiments. In each experiment we modify one reaction coefficient, $\imath_\pi$ or $\imath_c$, to match the average level of the short-term rate for the Greenspan (1987-2005) period, keeping all the other parameters as in the baseline calibration. We refer to these two experiments, as $\Delta\imath_\pi$ and $\Delta\imath_c$, respectively. These experiments allow us to see the term-structure implications of changes in the reaction coefficient to consumption growth and inflation.

Table 5 shows some descriptive statistics associated with the two experiments. Experiment $\Delta\imath_\pi$ requires an increase in i_π to 2.14 from 1.67 to match the average short term interest rate for the Greenspan era. Experiment $\Delta\imath_c$ requires an increase in i_c to 1.07 from 0.79 to do the same. However, the implications for the dynamics of interest rates are significantly different.

The $\Delta\imath_\pi$ experiment is successful in reducing the level of inflation, its volatility, and autocorrelation, as well as the less negative correlation between inflation and consumption growth. The $\Delta\imath_c$ experiment does not capture these features of the inflation process seen in the data.

With respect to the term structure properties, despite the fact that $\imath_\pi$ is the only parameter that is changed in the $\Delta\imath_\pi$ experiment, Figure 4 shows that the implied average yield curve resembles the one observed in the Greenspan era. In particular, an increase in the reaction coefficient to inflation increases the slope of the curve.

The $\Delta\imath_c$ experiment delivers a flat yield curve. The difference between the two experiments can be explained by observing Panel C of Figure 4. The term premiums associated with the $\Delta\imath_\pi$ experiment are positive and those of the $\Delta\imath_c$ experiment are close to zero. While an increase in the $\imath_\pi$ coefficient decreases the negative sensitivity of inflation to consumption growth to -0.25 from -0.43, the increase in the $\imath_c$ coefficient increases the negative correlation to -0.81. A stronger reaction of short-term interest rates in monetary policy to

Table 5: Data and model-implied descriptive statistics for the policy experiments.

	Data		Policy Experiment		
	(1971-2005)	(1987-2005)	Baseline	$\Delta\imath_\pi$	$\Delta\imath_c$
Panel A					
$\mathbb{E}\left[\Delta c_t\right] \times 4$ (%)	1.98	1.83	1.98	1.98	1.98
$\mathbb{E}\left[\pi_t\right] \times 4$ (%)	4.46	2.95	4.42	2.71	3.11
$\sigma\left(\Delta c_t\right) \times 4$ (%)	1.74	1.35	1.74	1.74	1.74
$\sigma\left(\pi_t\right) \times 4$ (%)	2.66	1.26	2.69	1.80	2.67
$\text{corr}\left(\Delta c_t, \Delta c_{t-1}\right)$	0.41	0.28	0.41	0.41	0.41
$\text{corr}\left(\pi_t, \pi_{t-1}\right)$	0.84	0.54	0.85	0.70	0.90
$\text{corr}\left(\Delta c_t, \pi_t\right)$	-0.33	-0.17	-0.28	-0.23	-0.41
Panel B					
$\mathbb{E}[i_t] \times 4$ (%)	6.11	4.49	6.11	4.49	4.49
$\mathbb{E}[i_t^{(20)}] \times 4$ (%)	7.31	5.83	7.36	6.04	4.56
$\mathbb{E}[i_t^{(40)}] \times 4$ (%)	7.68	6.40	7.65	6.39	4.56
$\sigma\left(i_t\right) \times 4$ (%)	3.04	2.05	3.04	2.70	2.43
$\sigma(i_t^{(20)}) \times 4$ (%)	2.61	1.73	2.65	1.66	2.39
$\sigma(i_t^{(40)}) \times 4$ (%)	2.38	1.50	2.39	1.47	2.35
$\text{corr}(i_t, i_{t-1})$	0.92	0.97	0.69	0.38	0.99
$\text{corr}(i_t, i_t^{(20)})$	0.86	0.89	0.93	0.89	0.99
$\text{corr}(i_t, i_t^{(40)})$	0.82	0.79	0.88	0.77	0.99
Panel C					
$\text{corr}\left(i_t, \Delta c_t\right)$	-0.10	0.08	0.19	0.26	0.01
$\text{corr}\left(i_t, \pi_t\right)$	0.60	0.44	0.91	0.84	0.91

inflation therefore increases the riskiness of longer bonds. In contrast, a stronger reaction of short-term interest rates to consumption growth increases the hedging benefits of longer bonds.

Panel B of Figure 4 shows the implications of the experiments on the volatility of interest rates. The $\Delta\imath_\pi$ experiment implies a higher volatility for short-term rates than implied in the Greenspan period and a quick decline in volatility with maturity. The ratio of 10-year rate volatility to short-rate volatility decreases to 55% from 78%. This ratio is low in comparison to the 73% ratio observed on average during the Greenspan era. Therefore, policy shocks lose

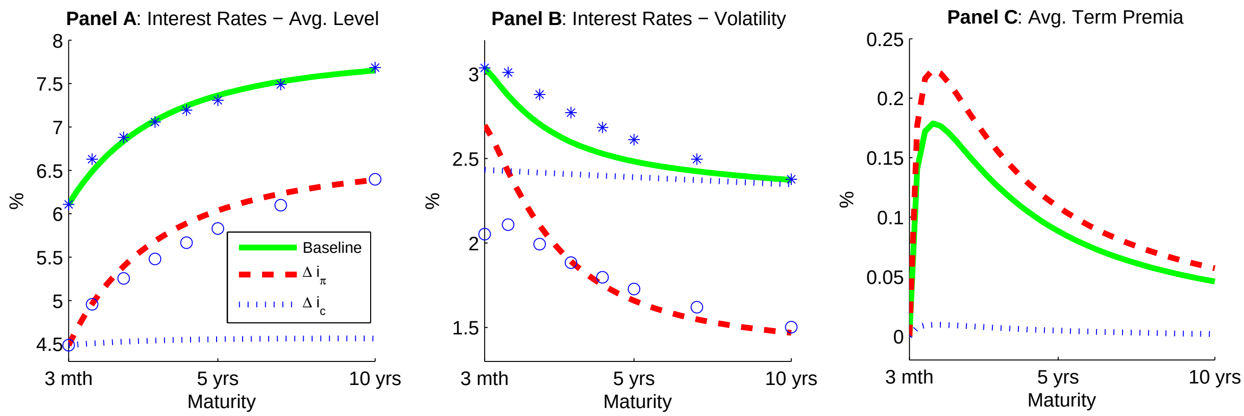

Figure 4: Nominal Interest Rate Properties - Policy Experiment. The (∗) denotes 1971-2005 data and the (∘) denotes 1987-2005 data.

some of their ability to generate long-term rate volatility. The reason is a reduced response in inflation to policy shocks that is also reflected in the reduced persistence in inflation observed during the period. The $\Delta\imath_c$ experiment reduces the volatility of short-term rates, but long-term rate volatility is unaffected.

Other implications of the $\Delta\imath_\pi$ that are consistent with interest rate developments during the Greenspan era are the increase in the correlation between consumption growth and the short-term interest rate, and a decrease in the correlation between inflation and the interest rate. The autocorrelation of the short-term rate decreases in the policy experiment while it increased during the Greenspan era.

6 Conclusion

We show that a consumption-based affine term-structure model is able to capture an important property of long-term interest rates — that they are almost as volatile as short-term rates. We do this by incorporating preferences that lead to a stochastic price of risk and an interest-rate rule for monetary policy. Affine term structure models require, in general, highly persistent state factors to avoid a quick decline in volatility across maturities. This requirement apparently disqualifies macroeconomic variables such as consumption growth or inflation as explanatory variables in these models. However, when a monetary policy rule endogenously makes inflation correlated to real economic activity and a highly autocorrelated monetary policy shock, it is possible to simultaneously obtain a high volatility of long-term rates and reproduce the observed persistence in inflation dynamics.

Our model also allows for the analysis of bond-pricing implications of policy changes. This feature provides term structure restrictions that could be potentially used to identify changes in policy regimes. We show that a policy rule with a higher reaction to inflation appears to captures recent macroeconomic and term structure developments.

References

Abel, A. B., 1990, "Asset Prices Under Habit Formation and Catching Up with the Joneses," *American Economic Review*, 80, 38–42.

Ang, A., S. Dong, and M. Piazzesi, 2005, "No-Arbitrage Taylor Rules," working paper, Columbia University, working paper.

Ang, A., and M. Piazzesi, 2003, "A No-Arbitrage Vector Autoregression of the Term Structure Dynamics with Macroeconomic and Latent Variables," *Journal of Monetary Economics*, 50(4), 745–787.

Backus, D., and S. Zin, 1994, "Reverse Engineering the Yield Curve," working paper, National Bureau of Economic Research.

Bekaert, G., S. Cho, and A. Moreno, 2005, "New-Keynesian Macroeconomics and the Term Structure," working paper, Working paper, Columbia University, April.

Brandt, M., and D. Chapman, 2003, "Comparing Multifactor Models of the Term Structure," working paper, Duke University.

Campbell, J. Y., and J. H. Cochrane, 1999, "By Force of Habit: A Consumption Based Explanation of Aggregate Stock Market Behavior," *Journal of Political Economy*, 107(2), 205–251.

Campbell, J. Y., and R. J. Shiller, 1991, "Yield Spreads and Interest Rates: A Bird's Eye View," *Review of Economic Studies*, 58, 495–514.

Clarida, R., J. Galí, and M. Gertler, 2000, "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *The Quarterly Journal of Economics*, February, 147–180.

Dai, Q., and T. Philippon, 2004, "Government Deficits and Interest Rates: A No-Arbitrage Structural VAR Approach," working paper, New York University.

Dai, Q., and K. J. Singleton, 2002, "Expectations Puzzles, Time-Varying Risk Premia, and Affine Models of the Term Structure," *The Journal of Financial Economics*, 63, 405–443.

Dai, Q., and K. J. Singleton, 2003, "Term Structure Dynamics in Theory and Reality," *Review of Financial Studies*, 16, 631–678.

Duffee, G. R., 2002, "Term Premia and Interest Rate Forecasts in Affine Models," *Journal of Finance*, 57(1), 02, 405–443.

Gallmeyer, M., B. Hollifield, F. Palomino, and S. Zin, 2007, "Arbitrage-Free Bond Pricing with Dynamic Macroeconomic Models," *The Federal Reserve Bank of St. Louis Review*, July/August, 305–326.

Hansen, L., and T. J. Sargent, 2007, *Robustness*, Princeton University Press.

Piazzesi, M., and M. Schneider, 2007, "Equilbrium Yield Curves," in D. Acemoglu, K. Rogoff, and M. Woodford (eds.), *NBER Macroeconomics Annual 2006*, MIT Press, July.

Svensson, L. E., 1994, "Estimating and Interpreting Forward Rates: Sweden 1992-4," working paper, National Bureau of Economic Research.

Taylor, J., 1993, "Discretion versus Policy Rules in Practice," *Carnegie-Rochester Conferences Series on Public Policy*, 39, 195–204.

Wachter, J. A., 2006, "A Consumption-Based Model of the Term Structure of Interest Rates," *Journal of Financial Economics*, 79(2), February, 365–399.

Appendix

A Macroeconomic Data

We present a comparison of statistical properties of two different data sets for aggregate consumption and inflation. We use quarterly U.S. data from 1971:3 to 2005:4. In the first set, inflation is constructed using quarterly data on the consumer price index from the Center for Research in Security Prices (CRSP) and the consumption growth series was constructed using quarterly data on real per capita consumption of nondurables and services from the Bureau of Economic Analysis. This data set considers inflation related to aggregate output, and therefore includes durable goods. In the second set, inflation is obtained following the methodology in Piazzesi and Schneider (2007). This data set captures inflation related only to non-durables and services consumption. Therefore, it represents the adequate measure of inflation for the representative agent economy considered here. Inflation is computed as the log-difference in the price index, PI,

$$PI_t = PI_{t-1}\sqrt{\frac{P_tQ_{t-1}}{P_{t-1}Q_{t-1}}\frac{P_tQ_t}{P_{t-1}Q_t}}.$$

The details of the construction of P and Q can be found in http://faculty.chicagogsb.edu/monika.piazzesi/research/macroannual/.

The second series for consumption growth was constructed following the Piazzesi and Schneider methodology, but adjusting it to extract the effect of population growth. Consumption growth is the log-difference in the quantity index, QI, given by

$$QI_t = QI_{t-1}\frac{N_{t-1}}{N_t}\sqrt{\frac{P_{t-1}Q_t}{P_{t-1}Q_{t-1}}\frac{P_tQ_t}{P_tQ_{t-1}}}.$$

where N denotes population. The population series is obtained from the Bureau of Economic Analysis.

The comparison of statistics for the two sets of data is presented in Table 6. While the properties of consumption growth are very similar across the two sets, the properties of inflation are significantly different. The series that captures inflation related only to non-durables and services is much less volatile and much more persistent than the series for changes in the consumer price index.

Table 6: Consumption Growth and Inflation Statistics. 1971 - 2005

	Set I	P & S (adj.)
$\mathbb{E}[\Delta c_t] \times 4$	2.03%	1.98%
$\mathbb{E}[\pi_t] \times 4$	4.58%	4.46%
$\sigma(\Delta c_t) \times 4$	1.70%	1.74%
$\sigma(\pi_t) \times 4$	3.66%	2.66%
$\text{corr}(\Delta c_t, \Delta c_{t-1})$	0.41	0.41
$\text{corr}(\pi_t, \pi_{t-1})$	0.53	0.84
$\text{corr}(\Delta c_t, \pi_t)$	-0.30	-0.34

B Moment Conditions

Inflation independent processes:

$$\mathbb{E}[\Delta c_t] = \theta_c, \quad \sigma^2(\Delta c_t) = \frac{\sigma_c^2}{1-\phi_c^2}, \quad \text{corr}(\Delta c_{t+1}, c_t) = \phi_c.$$

$$\sigma^2(\nu_t) = \frac{\sigma_\nu^2}{1-\phi_\nu^2}, \quad \text{corr}(\nu_{t+1}, \nu_t) = \phi_\nu.$$

Exogenous inflation:

$$\mathbb{E}[\pi_t] = \theta_\pi, \quad \sigma^2(\pi_t) = \frac{\sigma_\pi^2}{1-\phi_\pi^2}, \quad \text{corr}(\pi_{t+1}, \pi_t) = \phi_\pi$$

$$\text{corr}\left(\Delta c_t, \pi_t\right) = 0.$$

$$\mathbb{E}[i_t] = \delta + \gamma\theta_c(1 - \eta_c\sigma_c^2) + \theta_\pi - \frac{1}{2}\gamma^2\sigma_c^2 - \frac{1}{2}\sigma_\pi^2.$$

Endogenous inflation:

$$\mathbb{E}\left[\pi_t\right] = \bar{\pi} + \pi_c\theta_c, \quad \sigma\left(\Delta\pi_t\right) = \left(\pi_c^2\sigma^2\left(\Delta c_t\right) + \pi_\nu^2\sigma^2\left(\nu_t\right) + \pi_u^2\sigma^2\left(u_t\right)\right)^{1/2},$$

$$\text{corr}(\pi_{t+1}, \pi_t) = 1 - (1-\phi_c)\pi_c^2\frac{\sigma^2(\Delta c_t)}{\sigma^2(\pi_t)} - (1-\phi_\nu)\pi_\nu^2\frac{\sigma^2(\nu_t)}{\sigma^2(\pi_t)} - (1-\phi_u)\pi_u^2\frac{\sigma^2(u_t)}{\sigma^2(\pi_t)},$$

$$\text{corr}(\Delta c_t, \pi_t) = \pi_c\frac{\sigma(\Delta c_t)}{\sigma(\pi_t)},$$

$$\sigma^2\left(u_t\right) = \frac{\sigma_u^2}{1-\phi_u^2}.$$

$$\mathbb{E}[i_t] = \delta + \bar{\pi} + (\gamma + \pi_c)\theta_c(1 - \eta_c\sigma_c^2) - \frac{1}{2}(\gamma+\pi_c)^2\sigma_c^2 - \frac{1}{2}\pi_\nu^2\sigma_\nu^2 - \frac{1}{2}\pi_u^2\sigma_u^2.$$

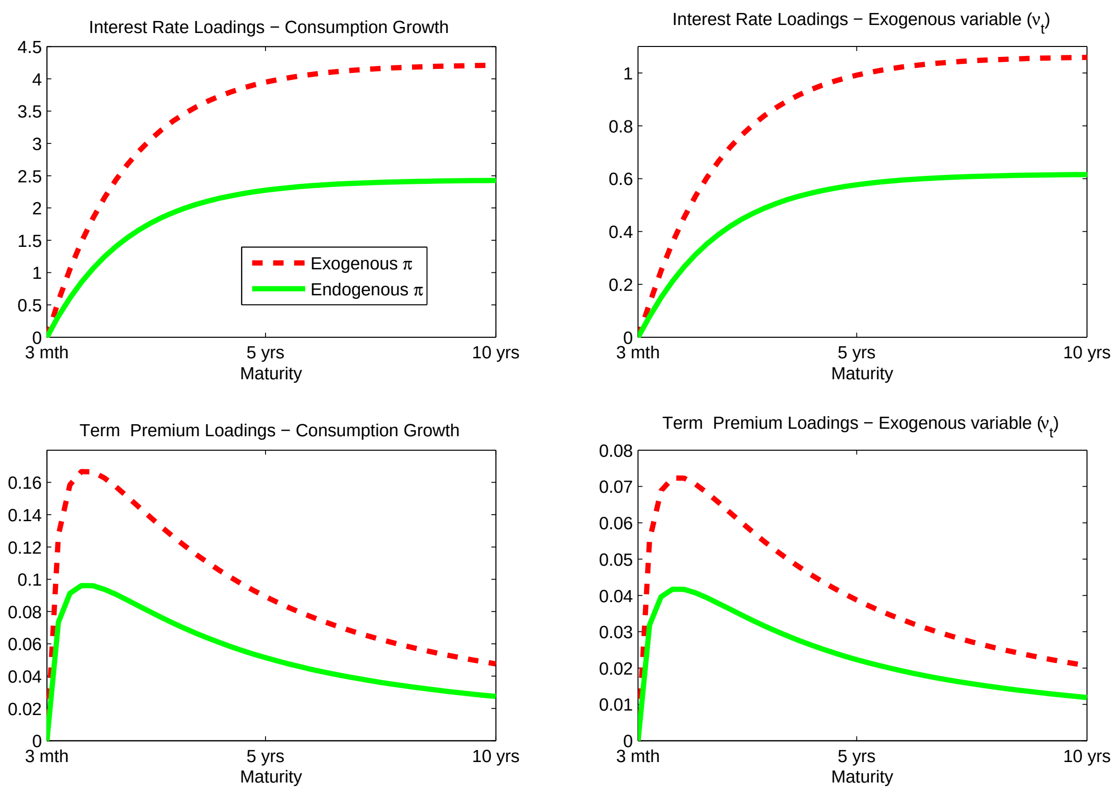

Figure 5: Nominal Interest Rates and Term premiums Loadings

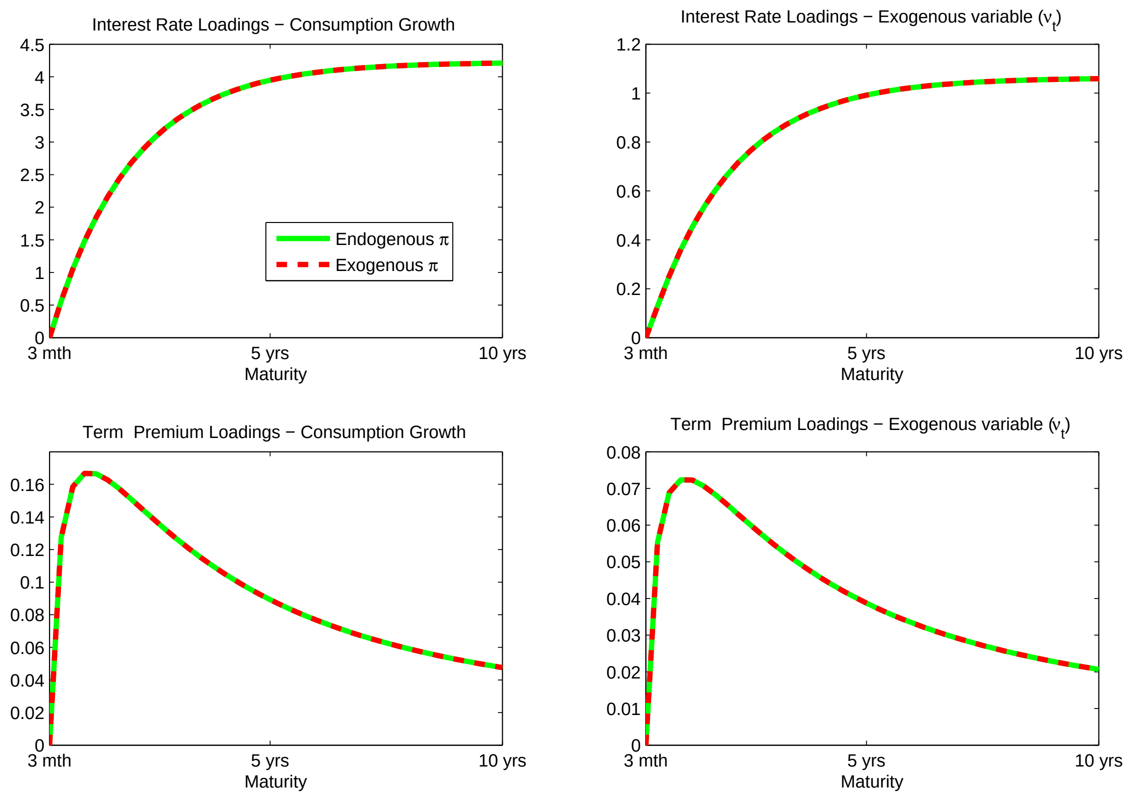

Figure 6: Real Interest Rates and Term premiums Loadings