Inflation Ambiguity and the Term Structure of Arbitrage-Free U.S. Government Bonds †

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Abstract

Inflation plays a very important role in the pricing of nominal bonds. Investors care not only about inflation shocks, but also how the volatility of inflation shocks may change over time. While inflation volatility was low for most of the 1970s and 1990 - 2003, it spiked in the early 1980s. This paper specifies and estimates a three-factor model for the nominal term structure which accounts for two sources of inflation premia. The first premium is determined by the product of risk aversion and the covariance between inflation and consumption. The second premium is determined by the product of model uncertainty aversion and the volatility of inflation. The second premium arises because the investor is uncertain about the true statistical distribution of future inflation. This premium was high in the early 1980's and it subsequently decreased the post-1980 term premium.

Keywords: Affine term structure model, Inflation ambiguity premium, Model misspecification, Robust control, Forecasting

JEL: C13, C32, E43, E44, G12

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1 Introduction

Expected inflation and the variance of inflation move considerably over time. While inflation was rather smooth in the 1970s and in the period 1990 - 2003, it was high in the early 1980s. Investors face two issues in modeling the effect of inflation. First, given the structure of a model, investors face inflation risk and require an inflation risk premium for bearing unanticipated shocks to inflation. In bearing this inflation risk premium, investors take the distribution, especially the conditional volatility of inflation, as known. Second, investors may be uncertain about the true statistical distribution of inflation.[1] If that is the case, investors require an ambiguity premium to protect themselves against a change in the underlying inflation model.

This paper specifies and estimates a three-factor model for the nominal term structure which accounts for the two sources of inflation premia. The first premium is determined by the product of risk aversion and the covariance between inflation and consumption. The second premium is determined by the product of model uncertainty aversion and the volatility of inflation. Investors take into account that the underlying inflation model could be misspecified. The greater the investor's uncertainty aversion, the more the investor is concerned about model misspecification and the higher the ambiguity premium.

In the theoretical part of the paper, I introduce inflation ambiguity into a three-factor structural nominal term structure model. The term structure model contains an inflation risk premium and an inflation ambiguity premium. The inflation ambiguity premium in Treasury bond prices is given by the negative covariance of inflation and the market price of inflation ambiguity. I determine the market price of inflation ambiguity by solving an agent's max-min problem. The max-min approach is useful for the investor if he fears

[1]Uncertainty is understood in the way of Knight (1921). A random variable is uncertain in the Knightian sense, if its statistical distribution is not known for sure.

that future inflation might come from a set of unspecified inflation models which are close to his reference model. The max-min solution can be divided into two separate steps. First, the investor suspects that his inflation model is not the true description of future inflation. The investor therefore seeks an inflation model that works well across a set of models which are close to his reference model and difficult to disentangle. Second, given that the agent has found his robust inflation model, he maximizes his life-time expected utility under this robust inflation model. I determine the term structure of real bonds, inflation expectations and nominal bonds in closed-form. The robust inflation forecast protects the ambiguity-averse agent against unfavorable inflation misspecifications. The difference between the robust inflation forecast and the inflation forecast if the model was perfectly known determines the inflation ambiguity premium.

In the empirical part of the paper, I generate an equivalence between the inflation ambiguity premium and the inflation variance premium by assuming that the upper bound of potential inflation misspecifications moves with the volatility of inflation. Intuitively, when the volatility of inflation increases, it becomes more difficult to estimate the drift of inflation precisely. The agent therefore doubts his underlying inflation model more if he observes more dispersed inflation realizations.

I test the model with U.S. data. I analyze whether there is evidence that investors command an inflation ambiguity premium and whether this premium helps to explain movements in the nominal yield curve. To identify the various components of the model, such as real yields, inflation expectations, inflation risk premium and inflation ambiguity premium, I use a data set comprising nominal Treasury yields and a panel of inflation, consumption growth and money growth, at a monthly frequency from 1970 to 2003.

The estimation identifies the following cross-sectional properties. First, the term structures of the inflation ambiguity premium and of real yields are upward sloping. The inflation ambiguity premium is negative for short-maturity bonds and positive for long-maturity bonds. This means that model uncertainty with regard to inflation increases the required excess return for long-end Treasury bonds and reduces them for short-term bonds. In times of inflation uncertainty, it is more difficult to predict inflation over the next ten years than it is to forecast future inflation over the next six months. Ambiguity averse investors therefore prefer to hold short-term bonds instead of long-term bonds. This increases the price of short-term bonds and reduces the price of long-term bonds.

Second, the term structure of inflation expectations is flat and the term structure of the inflation risk premium is downward sloping, high for short-maturity bonds and slightly negative for long-maturity bonds. The inflation risk premium is a measure for the conditional covariation of inflation and consumption. This covariation is relatively high for a horizon of up to two years, and quickly mean reverts to zero for longer maturities. Inflation shocks have no persistent effect on consumption.

Third, during the monetary policy experimentation of the $1979-1983$, the term structure of inflation expectations sloped strongly downwards while the term structure of inflation ambiguity sloped strongly upwards. The term structure of the inflation risk premium remained unchanged. During this period, investors expected high inflation to mean revert to lower levels and therefore priced nominal bonds with a lower expected inflation. At the same time, investors charged a steep inflation ambiguity premium, because they were highly concerned that their inflation model might not be the correct one.

Finally, I find that the estimated inflation ambiguity premium plays an important role in explaining the variance of nominal yields. Variations in nominal yields are due to changes in expected inflation and due to changes in the inflation ambiguity premium. Fluctuations in the inflation ambiguity premium explain a big fraction of nominal yield movements at the very short- and the very long-end of the nominal yield curve. For the six-month bond it explains 14.45% and for the ten year bond it explains 54% of the variation. The remaining fluctuations are mostly due to to changes in expected inflation. I also find evidence that inflation ambiguity helps to explain deviations from the expectation hypothesis as measured by the Campbell-Shiller coefficients.

The rest of the paper is organized as follows. Section 2 presents the model, Section 3 presents the data and estimation methodology. Section 4 presents the empirical results, section 5 summarizes related literature and contrasts them to this paper. Section 6 concludes. The proofs to the paper are summarized in a separate technical appendix to this paper, which can be accessed via the author's homepage.

2 The Model

2.1 Real Economy

2.1.1 Representative Agent

The representative agent has time separable and logarithmic preferences in consumption holdings c_t and real monetary holdings m_t[2] The agent holds a capital stock K_t and owns a linear technology A_t which produces an output good Y_t, i.e. $Y_t = A_t K_t$. The exogenous

[2]In particular, $U(x) = \int_0^\infty e^{-\rho t} \log(x_t) dt$, $x_t \equiv c_t m_t^\gamma$, $\gamma \in [0,1]$. Real monetary holdings m_t are equal to the ratio of nominal money supply M_t and the price level p_t in the economy. I follow the assumption that real monetary balances are held in equilibrium because they reduce the total amount of gross resources, $x_t \equiv c_t m_t^\gamma$, needed to obtain a given level of net consumption c_t. Real money balances are said to provide a transaction service to the agent. The size of this transaction service is modeled through the parameter γ. If $\gamma = 0$, money does not provide a transaction service, whereas $\gamma = 1$ means that the agent has to hold one unit of money for every unit of consumption holdings. For $\gamma \equiv 0$ this framework specializes to the standard RBC model.

growth rate of the technology is given by

$$dA_t = (\mu_A + \nu_A z_t)dt + \sigma_A \sqrt{z_t} dW_t^z \tag{2.1}$$

$$dz_t = \kappa_z(\theta_z - z_t)dt + \sigma_z \sqrt{z_t} dW_t^z. \tag{2.2}$$

The stochastic technological innovation z_t follows a stationary square-root process which fulfils the usual stationarity and integrability conditions. The agent can invest ζ_P of his wealth in this production technology and the remaining wealth ζ_N in a nominal risk-free bond. The real return of the investment into the nominal bond is not known ex-ante, because inflation is stochastic. I assume that inflation is affected by three random state variables: the consumption factor z_t, the inflation volatility factor v_t, and the inflation drift factor ω_t. These three state variables make up the state space S_t of the model, i.e. $S_t = \{z_t, v_t, \omega_t\}$. The central bank is assumed to control an additional monetary shock which is denoted $W^{\hat{M}}$ and which is orthogonal to the state shocks.

The agent maximizes his life-time utility with respect to consumption and money demand, while at the same time taking into account that the real return on the nominal bond is risky and uncertain.

$$\max_{c_t, m_t, \zeta_N, \zeta_P} \min_{h(S_t) \in \Theta(S_t)} E^{Q^{h(S_t)}} \left[\int_t^\infty e^{-\rho s} \left(\log\left(c_s\right) + \gamma \log\left(m_s\right) \right) \, ds | \mathcal{F}_t \right] \tag{2.3}$$

subject to the budget- and Entropy constraint

$$\begin{aligned} dW_t = {} & W_t \zeta_P \left((\mu_A + \nu_A z_t)dt + \sigma_A \sqrt{z_t} dW_t^z \right) - c_t dt - m_t dt - \delta \zeta_P W_t dt + \\ & + W_t \zeta_N \left([R(S_t) - \mu_p(S_t)]dt - \sigma_p(S_t) \cdot \begin{pmatrix} dW^z \\ dW^v \\ dW^{\hat{M}, h(S_t)} + h(S_t)dt \end{pmatrix} \right) \end{aligned} \tag{2.4}$$

$$\frac{1}{2} h^2(S_t) \leq \eta(S_t). \tag{2.5}$$

The set $\Theta(S_t)$ is assumed to be a well defined and nonparametric set of potential inflation priors.

Equation (2.3) represents the objective function of the ambiguity averse agent. The agent maximizes his life-time utility function with regard to consumption c_t, money demand m_t and portfolio holdings ζ, while at the same time insuring himself against a worst case inflation distortion, $h(S_t)$, which the central bank chooses. Equation (2.4) is the intertemporal budget constraint. The first line of the budget constraint shows that the agent can invest a fraction ζ_P of his wealth into the production technology, he can consume real consumption and real money holdings and part of the capital stock depreciates at rate δ. The second line shows that the agent can invest a fraction of his wealth ζ_N into the risk-free nominal bond, which is risky and uncertain in real terms. The source of uncertainty is the true monetary policy shock $dW^{\hat{M},h(S_t)}$ that the central bank fully controls. If the agent is not ambiguity averse with regard to this shock, $h(S_t)$ equals zero and the max-min objective function reduces to a standard rational expectations objective function. In the other cases, $h(S_t)$ will be determined endogenously as a solution to the minimization problem. Equation (2.5) of the budget constraint represents the entropy constraint which specifies that the investor wants to protect himself against conditional inflation drift distortions which are smaller or equal to $\eta(S_t)$. Assuming sufficient smoothness conditions, this upper bound $\eta(S_t)$ can be any nonparametric combination of the underlying state variables. This upper bound is therefore allowed to be time-varying and stochastic. The next proposition summarizes the solution to the minimization problem.

Proposition 1 (Equilibrium - Optimal Degree of Inflation Robustness) *The optimal degree of inflation distortion that the ambiguity averse agent takes into account is given by*

$$h^*(S_t) = \sqrt{2\eta(S_t)}. \tag{2.6}$$

Proposition 1 presents the optimal degree of inflation misspecification. The endogenously determined degree of misspecification results from solving the minimization problem within the max-min operation. The agent's model uncertainty with regard to the inflation model causes the agent to adjust his locally expected inflation rate by $h^*(S_t)$. This optimal degree of distortion is a square-root function of the upper bound.

The solution to the minimization problem provides the agent with the optimal degree of robustness. Given this endogenous degree of robustness, the agent adjusts his probability measure and solves a standard rational expectations maximization problem. This two step optimization procedure makes the max-min set-up very tractable. The next proposition summarizes the resulting equilibrium policy functions for consumption and money demand.

Proposition 2 (Equilibrium - Optimal Consumption and Money Demand) *The Optimal consumption and money demand is given by*

$$c_t^* = \frac{\rho}{1+\gamma} W_t \tag{2.7}$$

$$m_t^* = \gamma c_t^*. \tag{2.8}$$

These optimal values are financed by a optimal trading strategy. His optimal strategy is to invest his wealth into the production technology:

$$\zeta_P = 1,\ \zeta_N = 0. \tag{2.9}$$

Proposition 2 states that the agent consumes in equilibrium a constant fraction of his wealth. The agent follows an optimal trading strategy in which wealth is totally invested in the production technology. The nominal bond is in zero net supply. The expected return of an investment into the nominal bond is a shadow price for the equilibrium constraint that the representative agent invests all his wealth into the production technology. Inflation ambiguity in my model does not affect the policy function for consumption and money demand. Instead, it affects the expected excess return of the inflation-sensitive nominal bond.

The mirror image of consumption not being affected by inflation ambiguity is that the real interest rate, r_t, and the market price of output risk, $\lambda_{r,Y}(t)$, are not affected by inflation ambiguity. These values coincide with a standard Cox, Ingersoll, and Ross (1985a) economy. The next proposition summarizes this result.

Proposition 3 (Equilibrium - Real Interest Rate and Real Market Price of Risk) *The real interest rate r_t and the market price of output risk $\lambda_{r,Y}(t)$ are given by*

$$r_t = \mu_A + \nu_A\, z_t - \delta - {\sigma_A}^2 z_t \tag{2.10}$$

$$\lambda_{r,Y}(t) = \sigma_A \sqrt{z_t}. \tag{2.11}$$

The first equation presents that the real interest rate accounts for the growth rate of the production process minus capital depreciation and precautionary savings. The second equation summarizes that there is one market price of risk, namely market price of output risk. Having determined the real interest rate and real market price of risk allows the pricing of inflation-indexed bonds. Their equilibrium values are summarized in the next proposition.

2.1.2 Real Term Structure

Given the solution of the equilibrium consumption and investment problem, I solve directly for the term structure of inflation indexed-bonds. Its value is determined by the Euler equation,

$$B_t(\tau) = e^{-\rho\tau} E_t^P \left[\frac{u_c(c^*_{t+\tau}, m^*_{t+\tau})}{u_c(c^*_t, m^*_t)} \right], \tau > 0, \tag{2.12}$$

where u_c denotes the partial derivative of u with respect to c and c^*, m^* denote optimal consumption and money holdings. The following proposition summarizes the analytic solution for the term structure of real bonds:

Proposition 4 (Term Structure of Real Bonds) *The closed-form solution for the equilibrium price of a real zero-coupon bond $B_t(\tau)$ with time to maturity τ is given by a log-linear function of the business cycle state variable z_t:*

$$B_t(\tau) = A(\tau)\, e^{-b_z^r(\tau)\cdot z_t}, \ \forall \tau \in \mathbb{R}^+; \tag{2.13}$$

where $A(\tau)$, $b_z^r(\tau)$ are deterministic functions of the structural parameters of the economy.

The equilibrium term structure for real bonds shows that fluctuations in the real bond values are driven by variations in aggregate output.

To price nominal bonds, I endogeneize inflation. The endogenous process for inflation is affected by the uncertain monetary policy shock. It is optimal for the ambiguity averse agent to distort the expected inflation rate by an amount of $h^*(\theta) = \sqrt{2\eta(\theta)}$. The next section shows that this robust inflation adjustment to the true monetary policy shock $W^{\hat{M}}$ affects the expected growth rate of money supply, inflation and it affects the expected excess return of inflation-sensitive assets.

2.2 Incorporating Inflation

2.2.1 Monetary Policy Rule

I model the general structure of the money supply rule under the ambiguity-free measure P similar to Buraschi and Jiltsov (2005). Agents believe that the monetary authority follows a Taylor-type rule for base money. The degree of output and inflation targeting are represented by q_1 and q_2, respectively. Both variables are determined by the central bank. If q_1 and q_2 are different from zero, the output growth rate $\frac{dY_t}{Y_t}$ as well as the equilibrium inflation rate $\frac{dp_t}{p_t}$ influence the drift and volatility of the money supply process.

$$
\begin{aligned}
\frac{dM_t}{M_t} &= \omega_t dt + q_1 \left(\frac{dY_t}{Y_t} - \hat{y} dt \right) + q_2 \left(\frac{dp_t}{p_t} - \hat{\pi} dt \right) + \rho_{Mv} \sigma_M \sqrt{v_t} dW_t^v \\
&\quad + \sqrt{1 - \rho_{Mv}^2} \sigma_M \sqrt{v_t}\, dW_t^{\hat{M}}, \quad \rho_{Mv} \in [0, 1], \qquad (2.14) \\
d\omega_t &= \kappa_\omega \left(\theta_\omega - \omega_t \right) dt + \sigma_\omega \sqrt{\omega_t}\, dW_t^\omega, \qquad (2.15) \\
dv_t &= \kappa_v \left(\theta_v - v_t \right) dt + \sigma_v \sqrt{v_t}\, dW_t^v. \qquad (2.16)
\end{aligned}
$$

First, in equation (2.14) the monetary authority tries to meet its inflation target $\hat{\pi}$ and output growth target $\hat{y}$ with weights q_1 and q_2, respectively. Second, the monetary authority only imperfectly controls the monetary aggregate. Therefore, the money supply rule is affected by two other stochastic processes, ω_t and v_t. The state variables ω_t influences the long-term money growth rate, whereas the latter shock v_t influences the conditional volatility of the money growth rate. All Brownian shocks are orthogonal to each other.

The money supply process is affected by the true monetary policy shock. The endogenously determined drift misspecification of the true monetary policy shock has been determined in (2.6). This misspecification affects the conditional growth rate of the money supply process. The next proposition summarizes how the equilibrium misspecification affects the money growth rate.

Proposition 5 (Ambiguity-Adjusted Dynamic of Nominal Aggregates) *The ambiguity-adjusted dynamic of the nominal aggregates in the economy are given by*

$$\frac{dM_t}{M_t} = \omega_t dt + q_1 \left(\frac{dY_t}{Y_t} - \hat{y}dt \right) + q_2 \left(\frac{dp_t}{p_t} - \hat{\pi}dt \right) + \kappa_M(t)dt + \rho_{Mv}\sigma_M \sqrt{v_t}dW_t^v + \sqrt{1-\rho_{Mv}{}^2}\sigma_M \sqrt{v_t}dW^{\hat{M},h^*(S_t)}, \tag{2.17}$$

$$dW^{\hat{M},h^*(S_t)} = dW^{\hat{M}} - h^*(S_t) \tag{2.18}$$

$$\kappa_M(t) = \frac{\sqrt{1-\rho_{Mv}{}^2}\sigma_M}{1-q_2} \cdot \sqrt{v_t} \cdot h^*(S_t). \tag{2.19}$$

Proposition 5 shows that the misspecification of the true monetary policy shock adds an additional term to the conditional expected value of money growth. This additional term is called $\kappa_M(t)$. It also affects the unconditional variance of the nominal aggregates. The ambiguity adjustment $\kappa_M(t)$ is the product of the market price of inflation ambiguity, $h^*(S_t)$, and the amount of inflation ambiguity in the money supply process, $\frac{\sqrt{1-\rho_{Mv}{}^2}\sigma_M}{1-q_2} \cdot \sqrt{v_t}$.

The price level in the economy is determined via the money market clearing condition, $p_t^* := \frac{M_t}{m_t^*}$. Its endogenously determined degree of misspecification is summarized in the following proposition.

Proposition 6 (Equilibrium Price Level) *The equilibrium inflation dynamic is given by*

$$\frac{dp_t^*}{p_t^*} = \mu_p(S_t)\, dt + \sigma_p(S_t) \cdot \begin{pmatrix} dW^z \\ dW^v \\ dW^{\hat{M},h^*(S_t)} + h^*(S_t)dt \end{pmatrix} \tag{2.20}$$

and the endogenous degree of inflation misspecification is given by $\sigma_p(S_t)h^*(S_t)$.

The endogenously determined degree of model misspecification $h^*(S_t)$ affects the conditional expected value of inflation. The optimal amount of inflation drift distortion is given by the product of the market price of inflation ambiguity $h^*(S_t)$ and inflation volatility.

The functions $\mu_p(S_t), \sigma_p(S_t)$ are specified in the technical appendix of this paper. Proposition 6 shows that if the agent does not know the true inflation model, he adds to his conditional expected inflation rate $\mu_p(S_t)$ an inflation ambiguity premium, $\sigma_p(S_t) \cdot (0,\, 0,\, h^*(S_t))$. The ambiguity premium in inflation is the product of the market price of ambiguity times inflation volatility. The results so far do not depend on any particular parametric structure of the upper bound of potential inflation drift distortions.

After having endogeneized the inflation process, we can immediately determine the nominal interest rate $R(t)$ and the nominal market prices of risk λ_R that nominal assets have to pay. The next proposition summarizes its equilibrium values.

Proposition 7 (Nominal Interest Rate and Nominal Market Price of Risk) *The nominal interest rate R_t and the nominal market price for output risk $\lambda_{R,Y}(t)$ as well as the nominal market price of monetary risk $\lambda_{R,\hat{M}}(t)$ and $\lambda_{R,v}(t)$ are given by*

$$R(t) = \delta_0 + \delta_1' \begin{pmatrix} z_t \\ v_t \\ \omega_t \end{pmatrix} + \frac{\sqrt{1-\rho_{Mv}^2}\sigma_M}{1-q_2}\sqrt{v_t}h^*(S_t) \tag{2.21}$$

$$\lambda_{R,Y} = \frac{\sigma_A \sqrt{z(t)}\,(q_1 - q_2)}{1-q_2} \tag{2.22}$$

$$\lambda_{R,v}(t) = \frac{\rho_{Mv}\,\sigma_M\,\sqrt{v(t)}}{1-q_2} \tag{2.23}$$

$$\lambda_{R,\hat{M}}(t) = \frac{\sqrt{1-\rho_{Mv}^2}\,\sigma_M\,\sqrt{v(t)}}{1-q_2} \tag{2.24}$$

where δ_0 and δ_1 are deterministic functions of the underlying economy. There parametric form is specified in the technical appendix of this paper.

The endogenous inflation ambiguity adjustment $\frac{\sqrt{1-\rho_{Mv}^2}\sigma_M}{1-q_2}\sqrt{v_t}h^*(S_t)$ affects the otherwise standard equilibrium nominal interest rate in my economy. The nominal market prices of risk are not affected by the agent's ambiguity. The intuitive reason for this is that market prices of risk coincide with the volatilities of the intertemporal marginal rate of substitution (IMRS), whereas market prices of uncertainty coincide with its drift misspecification. The endogenous inflation ambiguity adjustment affects the nominal interest rate. It will therefore, also affect the term structure of nominal bonds.

2.3 Role of Inflation Risk in the Term Structure of Nominal Bonds

The price of a nominal bond in an economy without inflation ambiguity is well studied in the literature (Bakshi and Chen (1996), Wachter (2006), Buraschi and Jiltsov (2005)). Such a framework is a special case of an economy that is subject to model uncertainty. In my model, I define the nominal bond price in an economy that is NOT subject to inflation ambiguity as $\hat{N}_t(\tau)$. Due to the lack of inflation ambiguity, the set of potential priors collapses to a single prior. The ambiguity-free bond price is then determined via an Euler equation, where the expectation is solved under this single prior. As usual, u_c denotes the partial derivative of u with respect to c and the $*$ variables denote optimal equilibrium values.

$$\begin{aligned}\hat{N}_t(\tau) &= e^{-\rho\tau} E_t^P \left[\frac{u_c(c^*_{t+\tau}, m^*_{t+\tau})}{u_c(c^*_t, m^*_t)} \frac{p^*_t}{p^*_{t+\tau}} \right] \\ &= \underbrace{B_t(\tau)}_{RealBondPrice} \quad E_t^P \left[\frac{p^*_t}{p^*_{t+\tau}} \right] + \underbrace{cov_t^P \left(\frac{u_c(c^*_{t+\tau}, m^*_{t+\tau})}{u_c(c^*_t, m^*_t)}, \frac{p^*_t}{p^*_{t+\tau}} \right)}_{InflationRiskPremium}, \ \forall \tau \in \mathbb{R}^+. \qquad (2.25)\end{aligned}$$

This Euler equation coincides with the one in a rational expectations model. The difference between the nominal bond price and the product of real bond price and price deflator is entirely attributed to the inflation risk premium. The inflation risk premium can therefore be derived in analytical form.

Proposition 8 (Identification of the Inflation Risk Premium) *The term structure of inflation risk premia is given by the nominal ambiguity-free term structure minus the product of the term structure of real bonds and the term structure of price deflators, i.e.*

$$cov_t^P\left(\frac{u_c(c^*_{t+\tau}, m^*_{t+\tau})}{u_c(c^*_t, m^*_t)}, \frac{p^*_t}{p^*_{t+\tau}}\right) = \hat{N}_t(\tau) - B_t(\tau)E_t^P\left[\frac{p^*_t}{p^*_{t+\tau}}\right]. \tag{2.26}$$

The closed-form solution for the equilibrium price of an ambiguity-free nominal zero-coupon bond $\hat{N}_t(\tau)$ with time to maturity τ is given by a log-linear function of the state variable vector S_t :

$$\hat{N}_t(\tau) = \hat{Z}(\tau)\, e^{-\hat{b}_z(\tau)\cdot z_t - \hat{b}_\omega(\tau)\cdot \omega_t - \hat{b}_v(\tau)\cdot v_t}, \ \forall \tau \in \mathbb{R}^+; \tag{2.27}$$

where $\hat{Z}(\tau)$, $\hat{b}_z(\tau)$, $\hat{b}_\omega(\tau)$, $\hat{b}_v(\tau)$ are deterministic functions of the structural parameters of the economy.

The corresponding nominal yield curve is affine in the underlying state variables,

$$\hat{y}_t(\tau) = -\frac{1}{\tau}\left(\ln \hat{Z}(\tau) - \hat{b}_z(\tau)z_t - \hat{b}_\omega(\tau)\omega_t - \hat{b}_v(\tau)v_t\right). \tag{2.28}$$

The closed-form solution for the equilibrium term structure of the price deflator is a log-linear function of the nominal and real state variables, i.e.

$$E_t^P\left[\frac{p^*_t}{p^*_{t+\tau}}\right] = A^p(\tau)e^{-b^p_z(\tau)z(t) - b^p_\omega \omega(t) - b^p_v(\tau)v(t)}, \ \forall \tau \in \mathbb{R}^+; \tag{2.29}$$

where $A^p(\tau)$, $b^p_z(\tau)$, $b^p_\omega(\tau)$, $b^p_v(\tau)$ are deterministic functions of the structural parameters of the economy.

The characterization of the deterministic functions and the proof of the proposition can be found in the technical appendix of this paper.

2.4 Role Of Inflation Ambiguity in the Term Structure of Nominal Bonds

The role of inflation ambiguity in the term structure of nominal bonds can be derived in a straight forward way. The ambiguity averse agent holds the bond in zero net supply iff its price is determined according to the "robust" Euler equation.

$$N_t(\tau) = e^{-\rho\,\tau} E_t^{Q^{h^*(S_t)}} \left[\frac{u_c(c^*_{t+\tau}, m^*_{t+\tau})}{u_c(c^*_t, m^*_t)} \frac{p^*_t}{p^*_{t+\tau}} \right], \ \forall \tau \in \mathbb{R}^+. \tag{2.30}$$

$$= e^{-\rho\,\tau} E_t^{P} \left[\frac{dQ^{h^*(S_t)}}{dP} \frac{u_c(c^*_{t+\tau}, m^*_{t+\tau})}{u_c(c^*_t, m^*_t)} \frac{p^*_t}{p^*_{t+\tau}} \right], \ \forall \tau \in \mathbb{R}^+. \tag{2.31}$$

The difference between this Euler equation and the Euler equation in a rational expectations model is that the expectation is taken under the endogenously determined inflation ambiguity adjusted probability measure. The difference between both bond prices coincides with the inflation ambiguity premium. To shed light on the role that inflation ambiguity might have in the term structure of nominal bonds, one can re-write the above expectation to get

$$N_t(\tau) = \underbrace{E_t^P \left[e^{-\rho\tau} \underbrace{\frac{u_c(c^*_{t+\tau}, m^*_{t+\tau})}{u_c(c^*_t, m^*_t)}}_{IMRS} \right]}_{RealBondPrice} \cdot \underbrace{E_t^P \left[\frac{p^*_t}{p^*_{t+\tau}} \right]}_{PriceDeflator} + \underbrace{cov_t^P \left(\frac{u_c(c^*_{t+\tau}, m^*_{t+\tau})}{u_c(c^*_t, m^*_t)}, \frac{p^*_t}{p^*_{t+\tau}} \right)}_{InflationRiskPremium} +$$

$$\underbrace{+\, cov_t^P \left(\underbrace{\frac{u_c(c^*_{t+\tau}, m^*_{t+\tau})}{u_c(c^*_t, m^*_t)} \frac{p^*_t}{p^*_{t+\tau}}}_{NominalRiskKernel}, \underbrace{\frac{dQ^{h^*(S_t)}_{t,t+\tau}}{dP_{t,t+\tau}}}_{AmbiguityKernel} \right)}_{AmbiguityPremium} \tag{2.32}$$

$$\frac{dQ^{h^*(S_t)}_{t,t+\tau}}{dP_{t,t+\tau}} := e^{-\frac{1}{2}\int_t^{t+\tau} ||h^*(S(u))||^2 du + \int_t^{t+\tau} h^*(S(u)) dW^{\hat{M}}(u)}. \tag{2.33}$$

The first line on the rhs coincides with the nominal bond price in an economy without ambiguity, i.e. $\hat{N}_t(\tau)$. It therefore holds that

$$N_t(\tau) = \hat{N}_t(\tau) + \underbrace{cov_t^P\left(\underbrace{\frac{u_c(c^*_{t+\tau}, m^*_{t+\tau})}{u_c(c^*_t, m^*_t)}\frac{p^*_t}{p^*_{t+\tau}}}_{NominalRiskKernel}, \underbrace{\frac{dQ^{h^*(S_t)}_{t,t+\tau}}{dP_{t,t+\tau}}}_{AmbiguityKernel}\right)}_{AmbiguityPremium}. \tag{2.34}$$

The last equation shows that investor's ambiguity with regard to the true inflation model enters the Treasury yield curve through a covariance term, the covariance of the ambiguity kernel with the nominal risk kernel. An investor in an inflation ambiguous economy requests an additional premium, which is characterized through the last covariance term. Splitting up this term shows that in general, inflation ambiguity has two channels to enter the term structure. One is through covariation of the ambiguity kernel with the IMRS and the second is through the covariation of the ambiguity kernel with the price deflator. i.e.

$$\underbrace{cov_t^P\left(\underbrace{\frac{u_c(\hat{c}_{t+\tau}, \hat{m}_{t+\tau})}{u_c(\hat{c}_t, \hat{m}_t)}\frac{\hat{p}_t}{\hat{p}_{t+\tau}}}_{NominalRiskKernel}, \underbrace{\frac{dQ^{h^*(S_t)}_{t,t+\tau}}{dP_{t,t+\tau}}}_{AmbiguityKernel}\right)}_{AmbiguityPremium}$$

$$= \underbrace{cov_t^P\left(\underbrace{\frac{u_c(\hat{c}_{t+\tau}, \hat{m}_{t+\tau})}{u_c(\hat{c}_t, \hat{m}_t)}}_{IMRS}, \underbrace{\frac{dQ^{h^*(S_t)}_{t,t+\tau}}{dP_{t,t+\tau}}}_{AmbiguityKernel}\right)}_{=0} + cov_t^P\left(\underbrace{\frac{\hat{p}_t}{\hat{p}_{t+\tau}}}_{PriceDeflator}, \underbrace{\frac{dQ^{h^*(S_t)}_{t,t+\tau}}{dP_{t,t+\tau}}}_{AmbiguityKernel}\right). \tag{2.35}$$

In this paper, the source of ambiguity, $W^{\hat{M}}$, affects the price deflator and not the IMRS. I therefore, identify the inflation ambiguity premium as the covariance between the ambiguity kernel and the price deflator. The resulting interpretation is very clean. The inflation ambiguity premium equals the distance between the ambiguity adjusted expected inflation rate and the "true" expected inflation rate. I call this distance the inflation ambiguity premium $IAP_t(\tau)$ and summarize its analytical structure in the next proposition.

Proposition 9 (Identification of the Inflation Ambiguity Premium) *The term structure of the inflation ambiguity premium is given in analytical form as the covariance of the market price of uncertainty with inflation, i.e.*

$$IAP_t(\tau) := \underbrace{cov_t^P\left(\underbrace{\frac{\hat{p}_t}{\hat{p}_{t+\tau}}}_{PriceDeflator}, \underbrace{\frac{dQ^{h^*(S_t)}_{t,t+\tau}}{dP_{t,t+\tau}}}_{AmbiguityKernel} \right)}_{InflationAmbiguityPremium} = \underbrace{E_t^{Q^{h^*(S_t)}}\left[\frac{p_t}{p_{t+\tau}}\right] - E_t^P\left[\frac{p_t}{p_{t+\tau}}\right]}_{InflationAmbiguityPremium} = N_t(\tau) - \hat{N}_t(\tau). \tag{2.36}$$

My model provides a very easy way to extract the inflation ambiguity premium from the term structure of nominal bonds. First, the econometrician specifies the market price of inflation ambiguity. Second, the full model is estimated. Third, the estimated market prices of ambiguity are set to zero to get $\hat{N}_t(\tau)$. Fourth, the inflation ambiguity premium can be measured according to the last proposition.

Steps two to four are straight forward. In the next paragraph I show how I specify the ambiguity kernel for the econometric exercise.

2.4.1 Measuring Ambiguity in the Data

In order to determine the price of nominal bonds, one has to parameterize the upper bound of potential inflation misspecifications. I assume the following parameterization.

Assumption: [Parameterizing the Entropy Constraint] The parametric form for the upper bound of all potential inflation drift misspecifications is assumed to have the following law of motion

$$\eta(S_t) := \frac{1}{2}\left(q_{a_1}\sqrt{v_t} + \frac{q_{a_2}}{\sqrt{v_t}} + q_{a_3}\frac{z_t}{\sqrt{v_t}}\right)^2, \ \forall t \geq 0, \ q_{a_1}, q_{a_2}, q_{a_3} \in \mathbb{R}. \tag{2.37}$$

The corresponding market price of inflation ambiguity $h^*(S_t)$ takes the form

$$h^*(S_t) = \left(q_{a_1}\sqrt{v_t} + \frac{q_{a_2}}{\sqrt{v_t}} + q_{a_3}\frac{z_t}{\sqrt{v_t}}\right), \tag{2.38}$$

which generates an affine ambiguity adjustment in inflation

$$\kappa_M(t) = \frac{\sqrt{1-\rho^2}\sigma_M}{1-q_2}\left(q_{a_1}v_t + q_{a_2} + q_{a_3}z_t\right). \tag{2.39}$$

This affine structure has several features. First, the upper bound depends on the state variables that affect the conditional inflation volatility. This is sensible because it captures the fact that an increase in inflation volatility makes it more difficult to estimate the drift of inflation correctly. Second, the endogenous inflation ambiguity premium, which arises from (2.37) and (2.6) affects the bond price elasticity and changes the amount of priced risk that a nominal bond contains, even if the underlying macro risk remains the same. The intuition for that is that in times of *higher inflation volatility*, not only does the *inflation risk premium increase*[3], but it also becomes more difficult to estimate the drift of inflation precisely. This leads to an increasing set of potential inflation priors, from which nature will choose the worst one. Thus, the endogenously determined *inflation ambiguity premium increases*, because the *worst-case expected growth rate of inflation increases*.[4] Third, it generates a negative correlation between the nominal risk kernel and

[3]The inflation risk premium increases because the increasing inflation volatility increases the correlation between the intertemporal marginal rate of substitution and inflation. Said differently, the distance between the risky and the risky-neutral measure, i.e. $\frac{dP}{dQ}$, diverges.

[4]The inflation ambiguity premium increases because the estimation of the true inflation drift becomes more uncertain. Hence, the distance between the ambiguity adjusted measure and the risky measure increases, i.e. $\frac{dQ^{h^*(S_t)}}{dP}$.

the ambiguity kernel. This negative correlation helps to explain the empirically observed upward sloping Treasury yield curve. Moreover, it helps to explain why the nominal yield spread has been co-moving positively with inflation variance. Finally, the specification produces an affine term structure model.

The corresponding nominal bond price is given in the following proposition.

Proposition 10 (The Nominal Term Structure) [5] *The closed-form solution for the equilibrium price of a nominal zero-coupon bond $N_t(\tau)$ with time to maturity τ and ambiguity adjustment* (2.38) *is given by a log-linear function of the state variable vector S_t,*

$$N_t(\tau) = e^{-\rho\,\tau} E_t^{Q^{h^*(S_t)}} \left[\frac{u_c(c^*_{t+\tau}, m^*_{t+\tau})}{u_c(c^*_t, m^*_t)} \frac{p^*_t}{p^*_{t+\tau}} \right], \ \forall \tau \in \mathbb{R}^+ \tag{2.40}$$

$$N_t(\tau) = Z(\tau)\, e^{-b_z(\tau)\cdot z_t - b_\omega(\tau)\cdot \omega_t - b_v(\tau)\cdot v_t}, \ \forall \tau \in \mathbb{R}^+; \tag{2.41}$$

where $Z(\tau)$, $b_z(\tau)$, $b_\omega(\tau)$, $b_v(\tau)$ are deterministic functions of the structural parameters of the economy.

The corresponding nominal yield curve is affine in the underlying state variables,

$$y_t(\tau) = -\frac{1}{\tau}\left(\ln Z(\tau) - b_z(\tau) z_t - b_\omega(\tau)\omega_t - b_v(\tau) v_t\right). \tag{2.42}$$

The nominal yield curve, $-\frac{1}{\tau}\ln(N_t(\tau))$ is linear in the three state variables. The assumed entropy constraint in equation (2.37) preserves the simple affine bond pricing structure that is known from models like Cox, Ingersoll, and Ross (1985b), Vasiček (1977), and others. As in Buraschi and Jiltsov (2005), the inflation target and output target of the central bank affects the intercept of the yield curve but not the slope with respect to the state variables. On the other hand, inflation ambiguity also affects the slope and curvature

[5] The characterization of the deterministic functions and the proof of the proposition can be found in the technical appendix of this paper.

of the yield curve. As in other multi-factor term structure models, such as Longstaff and Schwartz (1992), Constantinides (1992), and Buraschi and Jiltsov (2005), my model is able to capture different shapes of the yield curve.

2.4.2 Identifying the Ambiguity Premium in the Data

The corresponding inflation ambiguity premium in the yield curve, which I call $IAP_y(t,\tau)$, is defined by

$$IAP_y(t,\tau) := y_t(\tau) - \hat{y}_t(\tau) \tag{2.43}$$

$$= -\frac{1}{\tau}\left(\ln\frac{Z(\tau)}{\hat{Z}(\tau)} - (B(\tau) - \hat{B}(\tau))S_t\right), \tag{2.44}$$

where $B(\tau) = [b_z(\tau)\, b_v(\tau)\, b_\omega(\tau)]'$ and $\hat{B}(\tau) = [\hat{b}_z(\tau)\, \hat{b}_v(\tau)\, \hat{b}_\omega(\tau)]'$.

It can be shown that the entropy constraint in equation (2.37) combined with (2.6), (2.20) and (2.33), leads to the inflation ambiguity premium being proportional to the variance of inflation.

$$IAP_t(\tau) = a(\tau) \cdot var_t\left(\frac{p^*_{t+\tau}}{p^*_t}\right), a(\tau) \in \mathbb{R}. \tag{2.45}$$

3 Estimation

3.1 Methodology

I estimate the model by Quasi Maximum Likelihood (QML). The log-likelihood function $L_T(\Omega_P)$, where T represents the sample size and Ω_P summarizes the parameter space of the bond model, is fully characterized in the technical appendix of this paper. To conduct the estimation, I match a panel of six bond maturities (1, and 6 month, 1, 2, 5, and 7 years) as well as inflation, consumption- and money growth. I solve for the unobservable state vector S_t by inverting the affine yield relationship for the three-month, three-year, and ten-year zero-coupon bonds. This procedure is standard in empirical affine term structure modeling. It has been successfully applied by Chen and Scott (1993), Duffee (2002) and many others. Moreover, I assume that the measurement error shocks are conditionally joint normal distributed and orthogonal to the shocks to the unobservable states.

To reduce the parameter space, the following parameters are fixed ex-ante: I set the capital depreciation rate to 5 percent. I set the real output growth target (inflation target) $\hat{y}$ ($\hat{\pi}$) to the mean of consumption growth (inflation) in the data, i.e. $\hat{y} = 0.033$ ($\hat{\pi} = 0.047$). I set the subjective time discount factor to $\rho = 0.05$. Further, I set the transaction service of money to $\gamma = 0.05$. Using the terminology of Dai and Singleton (2000), my affine term structure model is an econometrically identified $A_3(3)$ model.

3.2 Data

The sample consists of smoothed continuously compounded Fama-Bliss yields and price, money supply and consumption data for the period January 1970 to December 2003. The interest rate data comprises nine different maturities (1, 3 and 6 month as well as 1, 2, 3, 5, 7 and 10 years).[6] I take inflation data from the Consumer Price Index (CPI) for all urban consumers. The money supply data is the M2 money stock from the official H.6 release of the Federal Reserve Board of Governors. The M2 money stock includes money market deposit accounts, which can be used to buy services and products. It is the most adequate representation of money in the model. In comparison, $M3$ contains instruments that pay significant interest and, therefore, it is too wide. As consumption growth data I

[6] I thank Rob Bliss for sharing his data and programs with me.

take price level deflated nondurable goods and services from the Personal Consumption Expenditures database. This database is provided by the Bureau of Economic Analysis.

4 Empirical Analysis

4.1 Parameter Estimates

I present the parameter estimates of the structural model in table 1. The three latent state variables are identified by the panel of nominal yields and by consumption growth, money growth and inflation. The estimated speed of mean-reversion shows that each factor has different effects on the nominal yield curve. The most persistent factor is ω_t which basically represents variations in expected inflation. Its estimated speed of mean-reversion of 0.0034 argues that shocks to expected inflation are the most persistent. Expected inflation is therefore the persistent nominal term structure factor. The second most persistent factor is the business cycle factor z_t with a speed of mean-reversion of 0.0693. This corresponds to a half-life of 4 years.[7] The least persistent factor is the inflation volatility factor v_t with a half-life of five months. A half-life of five months means that half of inflation volatility innovations die out after five months.

I present the cross-sectional in-sample yield fit of the model in table 2. The in-sample fit for the time period 1970 to 2003 is very good. The average pricing error across all maturities is 13 basis points. This is remarkable, given that the model fits also consumption growth, inflation and money growth. Table 3 reports the out-of-sample root-mean-square errors and its ratio compared to a random walk forecast. The forecast horizon is the period June 1998 to December 2003. I use the following estimation strategy: I estimate the model with bond and macro data from January 1970 to May 1998. Starting in May

[7] The half-life is defined as $\frac{\ln(2)}{\kappa}$

1998, for each consecutive month, I re-estimate the model using data up to and including that month and then forecast the next month's, next six months' and next year's nominal bond yield. The structural model forecasts the three-month nominal bond yield, which is often regarded as a good proxy for the Fed's policy instrument, well across all forecast horizons. It outperforms random walk forecasts by two percent at a monthly forecast horizon and by ten percent at an annual forecast horizon.

The good cross-sectional pricing performance suggests that my simple term structure model with inflation ambiguity describes the U.S. data sufficiently well. It is therefore interesting to analyze the different components of the model implied nominal yield curve, especially the inflation risk- and the inflation ambiguity premium.

4.2 Decomposing the Term Structure of Nominal Yields

Table 4 presents the estimated components of the Treasury yield curve for the period 1970 to 2003. The model-implied average one-year U.S. Government bond yield of 6.8 percent consists of a real yield of 2 percent, inflation expectations of 4.2 percent, an inflation ambiguity premium of -0.26 percent and an inflation risk premium of 0.83 percent. On the other hand, the model implied average ten-year nominal yield of 7.7 percent is decomposed into a ten-year real yield of 2.77 percent, an inflation risk premium of -0.19 percent, expected inflation of 4.35 percent and an inflation ambiguity premium of 0.79 percent.

The estimated nominal yield curve is upward sloping with a term spread of 1.2 percent. Three components contribute to the upward sloping term structure: First, the real yield curve generates a term spread of 1.7 percent. Second, the inflation ambiguity premium generates a term spread of 1.4 percent. Third, expected inflation generates a term spread of 0.3 percent. The estimated term spread of the inflation risk premium is -2.2 percent. The very different term spread of the inflation risk premium and the inflation ambiguity premium shows how different both premia behave.

The inflation risk premium on a six month bond is on average 1.98 percent, whereas the inflation ambiguity premium is -0.61 percent. For the five year nominal bond both premia are approximately zero and change their sign for longer maturities. A ten year nominal bond contains an inflation risk premium of -0.19 percent and an inflation ambiguity premium of 0.79 percent. It is known from equation (2.25) that the inflation risk premium is a measure for the negative future covariance between consumption and inflation. A high inflation risk premium at the six month horizon of 1.98 percent coincides with a highly negative conditional covariance of consumption with inflation over the next six months. This negative conditional covariance flattens towards zero for longer time horizons. This cross-sectional inflation risk premium arises from: First, the general equilibrium money market clearing condition $p_t := \frac{M_t^s}{m_t^d}$ which postulates a negative relation between changes in the price level and real activity. And Second it arises from the estimated cross-section of the business cycle bond yield factor loading, $b_z(\tau)$, that is shown in figure 1. This business cycle factor loading summarizes the cross-sectional impact that fluctuations in the business cycle factor z_t have on the nominal term structure. The impact is strongly positive for short-end yields and zero for long-term nominal yields. The negative relation between changes in real activity and changes in inflation which are driven by the business cycle factor z_t, together with the fast decaying business cycle factor loading $b_z(\tau)$ explains why the inflation risk premium is positive and downward sloping.

The underlying economic intuition for an upward sloping inflation ambiguity premium that is negative for short-maturity bonds and positive for long-maturity bonds is straight forward. In times of high inflation uncertainty, it is more difficult to predict the correct inflation model over the next ten years compared to forecasting the true inflation model over the next six months. Ambiguity averse investors therefore prefer to hold short-term bonds instead of long-term bonds, pushing up the price of short-term bonds and reducing the price of long-term bonds. The corresponding movements in the nominal yield curve are decreasing short-term yields and increasing long-term yields.

The term spread on the inflation risk premium remained rather constant during the monetary policy experimentation in $1979 - 1983$ and during the great moderation in $1984 - 2003$. The intuition for that finding is a stable equilibrium relation between inflation and real activity, as measured by the conditional negative covariance between consumption and inflation. In contrast, the inflation ambiguity premium became very steep during the monetary policy experimentation, producing a term spread of 4 percent. This can be seen in the left panel of figure 2. The economic intuition of the previous paragraph holds. In particular, the period of the monetary policy experimentation is characterized by high monetary policy uncertainty. The U.S. Fed experimented with different monetary policy regimes which made it difficult for bond investors to forecast the inflation model that governs inflation over the next ten years. Investors therefore preferred to hold short-term bonds instead of long-term bonds, which strongly pushed up long-term yields and strongly reduced yields on the short-end of the nominal yield curve. During the great moderation, inflation was easier to predict which led to a significant reduction in the inflation ambiguity premium. The right panel of figure 2 shows that during the great moderation, the term spread of the inflation ambiguity premium decreased to 1 percent.

4.3 Time-Variation of the Term Structure of Nominal Yields

Figure 3 presents the time-variation of the inflation risk premium and the inflation ambiguity premium over the business cycle. The left panel contrasts both premia for the six month bond, the middle panel depicts both premia for the two year bond and the right panel shows both premia for the ten year bond. These three panels show that the short-end inflation risk premium varies very smoothly around two percent, whereas the inflation ambiguity premium fluctuates very strongly around zero percent, with a negative peak of -5.8 percent at the end of the monetary policy experimentation and a positive peak of 2.4 percent in the mid 1970s and the early 2000. Nearly all fluctuations in the inflation premium are attributed to the inflation ambiguity premium. Empirically observed variations in the consumption factor z_t and in the volatile inflation variance factor v_t are the reason for this behavior. Model implied consumption growth and model implied consumption variance are a one-factor model of z_t. This consumption factor is identified by fitting U.S. consumption growth. Since consumption growth is rather smooth in the data, the extracted time series of z_t is smooth as well. The inflation risk premium picks up the negative covariance between consumption and inflation. All the time variation in the inflation risk premium is therefore attributed to variations in the smooth consumption factor z_t. In contrast, fluctuations of the inflation ambiguity premium depend, according to equation (2.38), on the consumption factor z_t and on the inflation variance factor v_t. Since z_t is very smooth, all the fluctuations in the inflation ambiguity premium go back to the inflation variance factor v_t. Empirically, inflation variance has been strongly fluctuating, it has been very high in the early 1980s and decreasing in the 1970s and in the period 1990 - 2003. This spiking behavior of inflation variance in the early 1980s and the mean reversion during the great moderation are picked up by the inflation ambiguity premium.

Table 5 presents a formal variance decomposition of the four nominal yield curve components. Equation (2.13) shows that the real yield curve is driven by the smooth consumption factor z_t only. The same holds for the inflation risk premium, as we have seen in the previous paragraph. Expected inflation is driven by the smooth consumption factor z_t and by the volatile nominal drift factor ω_t. Equation (2.38) shows that the inflation ambiguity premium is driven by the smooth consumption factor z_t and the volatile inflation variance factor v_t. The smoothness of the consumption factor z_t allows to view the variation of expected inflation as being driven by the nominal drift factor ω_t and the variation of the inflation ambiguity premium to be driven by v_t. Table 5 presents that the inflation risk premium accounts only for 0.01 percent of the variance of nominal yields. Since real yields are also driven by the same smooth consumption factor z_t, it is clear why real yields explain also only 0.01 percent of the variance of nominal yields. All the action in the variance of nominal yields arises from the inflation drift factor ω_t and from the inflation ambiguity factor v_t. More than 85 percent of the variance of the six month nominal yield is explained by expected inflation and more than 14 percent are explained by the inflation risk premium. Intuitively, this means that variations in the short-end of the nominal yield curve are primarily due to fluctuations in expected inflation. In contrast, 54 percent of the variance of the nominal ten year yield is driven by fluctuations in the inflation ambiguity premium and 45 percent are driven by expected inflation. Economically, this means that more than half of the variation in the long-end of the nominal yield curve are due to fluctuations in the inflation ambiguity premium.

Comparing the three panels in figure 3 shows that the time-series of the inflation ambiguity premium is very interesting and very distinct from the inflation risk premium. The ambiguity premium has different signs for different bond maturities. This result has already been indicated in the cross-section. There we have seen that a concern for inflation ambiguity reduces short-term bond yields at the cost of increasing long-term bond yields. The time-series draws the same picture. The inflation ambiguity premium for short-term bonds is nearly as volatile as the one for the ten-year bond. Both have often opposite signs. This finding suggests that in low inflation variance periods, investors feel confident about their estimate of the inflation model and prefer to buy long-term bonds over short-term bonds. This reduces long-term yields and increases short-term yields. In contrast, during periods of high inflation variance, the representative investor requires a lower yield for short-term bonds and a higher yield for long-term bonds. The time-variation of the inflation risk premium does not change for different nominal bonds, only the value of the inflation risk premium reduces the longer the maturity of the nominal bond. This is consistent with the cross-sectional behavior of the inflation risk premium that we have seen in the previous section.

Figure 4 contrasts the six month and ten year inflation ambiguity premium with the model implied variance of inflation. The model implied variance of inflation is obtained by integrating the quadratic norm of $\sigma_p(S_t)$ in equation (2.20). The correlation between the variance of inflation and the ten year inflation ambiguity premium is 0.7. Both time-series do clearly move very closely to each other. This supports the view of equation (2.45) that the inflation ambiguity premium is a measure for the inflation variance premium. The correlation between the six month inflation ambiguity premium and inflation variance is -0.6. Hence, a one percent increase in inflation variance leads ceteris paribus to a 60 basis points decline in the six-month bond yield, to a 70 basis points increase in the ten year bond yield and to a 1.3 percent increase in the nominal term spread.

4.4 Campbell-Shiller Coefficients

A popular way to measure conditional risk premia of nominal bonds is the method proposed by Campbell and Shiller (1991), CS afterwards. CS analyze whether the current slope of the nominal yield curve explains future yield changes. The expectation hypothesis holds if the CS coefficients are one. Many empirical papers reject this hypothesis. Dai and Singleton (2002) show an equivalence between matching the empirically observed CS coefficients and matching the empirical dynamic of the yield curve. The appendix to this paper presents the derivation of the model implied CS coefficients of my structural model. The next proposition summarizes the result

Proposition 11 (Population Campbell-Shiller Coefficients) *The population Campbell and Shiller (1991) coefficients of my structural model are given by*

$$\phi_n = \frac{cov(y_{t+m}(n-m) - y_t(n), y_t(n) - y_t(m))\frac{m}{n-m}}{(\frac{m}{n-m})^2 var(y_t(n) - y_t(m))} \tag{4.1}$$

$$= \frac{\frac{m}{n-m}\sum_{i\in\{z,v,\omega\}}(\frac{b_i(n-m)}{n-m}b_i^0(n,m)cov(i(t+m),(i(t))) - \frac{b_i(n)}{n}b_i^0(n,m)var(i(t)))}{\left(\frac{m}{n-m}\right)^2\sum_{i\in\{z,v,\omega\}}(b_i^0(n,m))^2 var(i(t))} \tag{4.2}$$

with

$$var(i(t)) = \theta\frac{\sigma_i^2}{2\kappa}, \ \forall i \in \{z, v, \omega\} \tag{4.3}$$

$$cov(i(t), i(t+m)) = e^{-\kappa m}(\frac{\sigma_i^2\theta}{2\kappa}), \ \forall i \in \{z, v, \omega\} \tag{4.4}$$

$$b_i^0(n,m) = \frac{b_i(n)}{n} - \frac{b_i(m)}{m}, \ \forall i \in \{z, v, \omega\}. \tag{4.5}$$

The factor loadings $b_i(\tau)$, $\forall i \in \{z, v, \omega\}$ are derived in proposition (10). These loadings contain the market prices of risk, equation (7), and the market prices of inflation ambiguity, equation (2.38). In order to derive the CS coefficients that only contain the risk premium one applies the last proposition with the factor loadings $\hat{b}_i(\tau)$, $\forall i \in \{z, v, \omega\}$ that are derived in proposition (2.27). The resulting CS coefficients coincide with the risk premium component of the model implied population CS coefficients. I call them $\hat{\phi}_n$. The ambiguity premium component within the CS coefficients coincides with the difference $\phi_n - \hat{\phi}_n$. The model therefore allows the decomposition of the population CS coefficients into its risk premium component $\hat{\phi}_n$ and into its ambiguity premium component $\phi_n - \hat{\phi}_n$. Figure 5 presents the empirically observed CS coefficients and model implied risk and ambiguity components. The overall fit to the empirically observed CS coefficients is very good. The inflation ambiguity premium is the main driver for the CS coefficients for short- and medium-term bonds. The ambiguity premium explains the non-linear shape of the empirically observed CS coefficients for short- and medium-term bonds very well. In contrast, the CS coefficients for long-term bond are mostly affected by time-varying output and inflation risk premia.

5 Literature Overview

This paper is the first paper that incorporates possible inflation misspecification into a term structure model. Recent empirical models of the term structure find that business cycle factors and inflation factors describe changes in nominal yields or nominal bond risk premia (Ang and Piazzesi (2003), Ang, Piazzesi, and Wei (2006), Duffee (2007), among others). My model extends this line of research by taking into account that investors do not know the true statistical distribution of future inflation. My model confirms that nominal yields and nominal bond premia are driven by business cycle and inflation factors. My estimated structural model argues that changes in inflation expectations and changes in the inflation ambiguity premium create most of the time-variation that we observe in the nominal yield curve. I confirm the results of Gürkaynak, Sack, and Swanson (2005) and Ang, Bekaert, and Wei (2008) who find that it is not the inflation risk premium which drives most of the time-variation of nominal bond yields.

There is a small but growing literature on model uncertainty and asset pricing. Early contributions to that research are Gilboa and Schmeidler (1989), Epstein and Wang (1994), Epstein and Schneider (2003), Anderson, Hansen, and Sargent (2003), Chen and Epstein (2002), Epstein and Miao (2003). The commonality among this research is that market prices of risk and market prices of ambiguity are endogenously determined as a solution of an agent‘s optimization problem. The analog in my paper are proposition 1 and proposition 7. The biggest part of this research focuses on equity pricing and portfolio allocation.[8]

[8]Garlappi, Uppal, and Wang (2007) analyze a portfolio selection problem where an investor accounts for uncertainty about the estimated expected returns. Uppal and Wang (2003) study an intertemporal portfolio choice problem where an investor faces an ambiguous return distribution of his stock portfolio. They find that ambiguity can lead to underdiversified portfolios relative to the mean-variance portfolio. Liu, Pan, and Wang (2005) introduce ambiguity aversion about the jump probability in the return of

Recently, some papers focus on ambiguity and the term structure of real bonds. Kleshchelski and Vincent (2007) study how model uncertainty with regard to the consumption model affects the real term structure. Compared to my model framework this means that the business cycle innovation W^z in equation (2.1) is risky and uncertain at the same time. My paper takes a different approach. It argues that the investor is not ambiguity averse with regard to the risky business cycle and inflation shocks, but faces model uncertainty with regard to a central bank shock $W^{\hat{M}}$ that is orthogonal to the risky state variable shocks W^v, W^z, W^ω of the economy. Risk and ambiguity shocks are therefore separated in my model. Further, my model uses a nominal framework and addresses the question of how inflation ambiguity affects inflation expectations, the inflation risk premium and the inflation ambiguity premium. Gagliardini, Porchia, and Trojani (2008) construct a real production economy where the representative investor faces model uncertainty with regard to the variance process of the production technology. The authors work in a real framework and analyze the impact on real bonds. Compared to my model framework which introduces model uncertainty with regard to the central bank shock $W^{\hat{M}}$, this means that the stochastic volatility process of the business cycle factor $z(t)$ in equation (2.2) is risky and uncertain at the same time. Their resulting market price of uncertainty has the same structure as mine in proposition 1. In contrast to their assumption of a constant upper boundary $\eta(S_t)$, I allow the upper boundary to be linear in the norm of inflation volatility, i.e. (2.37), which intuitively allows the set of inflation priors

stocks and show that his helps to explain the implied volatility smirk of stock options. Sbuelz and Trojani (2002), Maenhout (2004), Leippold, Trojani, and Vanini (2008) apply ambiguity aversion to the equity premium puzzle and/or excess volatility puzzle. Dow and Werlang (1992), Trojani and Vanini (2004) and Cao, Wang, and Zhang (2005) study the impact of ambiguity aversion on the limited stock market participation. Miao and Wang (2006) study the impact of ambiguity aversion on the optimal exercise decision. Easley and O'Hara (2008) study the link between ambiguity aversion and stock market participation.

to vary with the volatility of inflation. In addition, it follows from equation (2.37) that the inflation ambiguity premium coincides with the inflation variance premium. Allowing for inflation ambiguity is a much more reasonable approach for the modeling of the nominal yield curve given past work who finds that inflation plays the largest role in understanding nominal bond prices and nominal bond price fluctuations (Gürkaynak, Sack, and Swanson (2005), Ang, Bekaert, and Wei (2008), Stock (2001), Cogley and Sargent (2002), and others).

6 Conclusion

This paper specifies and estimates a three-factor model for the nominal term structure which accounts for two sources of inflation premia. The first premium is determined by the product of risk aversion and the covariance between inflation and consumption. The second premium is determined by the product of model uncertainty aversion and the volatility of inflation.

The term structure model contains an inflation risk premium and an inflation ambiguity premium. The inflation ambiguity premium in Treasury bond prices coincides with the negative covariance of inflation and the market price of inflation ambiguity. I determine the market price of inflation ambiguity via the solution of an agent's max-min problem. The term structure of real bonds, inflation expectations and nominal bonds is determined in closed-form. When pricing nominal bonds, the ambiguity averse agent protects himself against unfavorable inflation misspecifications by using a robust inflation forecast. The difference between the robust inflation forecast and the inflation forecast if the model was perfectly known determines the inflation ambiguity premium.

I test the model with U.S. data. I show that investors command an inflation ambiguity premium. I estimate the model using a data set comprising nominal Government bond yields and a panel of inflation, consumption growth and money growth, at a monthly frequency from 1970 to 2003.

I find that the term structures of the inflation ambiguity premium is upward sloping. The inflation ambiguity premium is negative for short-maturity bonds and positive for long-maturity bonds. The underlying economic intuition for an upward sloping inflation ambiguity premium that is negative for short-maturity bonds and positive for long-maturity bonds is straight forward. In times of high inflation uncertainty, it is more difficult to predict the correct inflation model over the next ten years compared to forecasting the true inflation model over the next six months. Ambiguity averse investors therefore prefer to hold short-term bonds instead of long-term bonds, pushing up the price of short-term bonds and reducing the price of long-term bonds. The corresponding movements in the nominal yield curve are decreasing short-term yields and increasing long-term yields. During monetary policy experimentation of the $1979 - 1983$, investors expected high inflation to mean-revert to lower levels and therefore priced nominal bonds with a lower expected inflation. At the same time, investors charged a steep inflation ambiguity premium, because they were highly concerned that their inflation model might not be the correct one.

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Table 1: **Parameter Estimates of the Bond Model based on Quasi Maximum Likelihood Estimation**

δ	θ_z	γ	q_{a_1}	$\hat{y}$
0.05	0.2571 (0.0212)	0.05	1.0316(< 0.0001)	0.033
κ_ω	κ_v	κ_z	μ_A	q_1
0.0034 (0.0002)	0.7193 (< 0.0001)	0.0693 (0.0057)	3.0531 (0.0023)	1.015 (< 0.0001)
q_2	$\hat{\pi}$	ρ	ρ_{Mv}	σ_M
0.7621 (< 0.0001)	0.047	0.05	0.6956 (< 0.0001)	0.6776 (< 0.0001)
σ_ω	σ_v	σ_z	σ_A	θ_ω
0.1284 (< 0.0001)	0.6840 (< 0.0001)	0.2189 (0.0001)	1.8595 (0.0235)	45.2123 (3.1708)
ν_A	θ_v	q_{a_2}	q_{a_3}	
0.2039 (0.0157)	0.0062 (0.0004)	0.3471 (0.0012)	40.3509 (5.9402)	

This table presents the Quasi Maximum Likelihood parameter estimates of the structural model and their standard errors. Estimates without standard errors were fixed prior to the estimation.

Table 2: **In-Sample Mean Pricing Error**

Bond Maturity	Mean Pricing Error
1 mth	7.5
6 mth	0.8
1 year	2.5
2 year	2.2
5 year	38.0
7 year	32.8

This table presents the mean cross-sectional pricing errors (in-sample) for the estimated structural model (QML) with ambiguity. The pricing error is given in basis points.

Table 3: **Bond Yield Out-Of-Sample Forecasting**

Mat	Full (Relative Ratio)	Random Walk (Relative Ratio)
1 Month Ahead Forecast Errors in Basis Points		
3	20.48 (0.98)	20.76 (1)
36	29.75 (1.01)	29.45 (1)
6 Month Ahead Forecast Errors in Basis Points		
3	70.87 (0.89)	79.50 (1)
36	78.57 (0.99)	79.33 (1)
12 Month Ahead Forecast Errors in Basis Points		
3	129.23 (0.92)	139.91 (1)
36	104.99 (0.89)	117.09 (1)

This table presents the RMSE for the out-of-sample forecasting exercise and its "Ratio" compared to a random walk forecast. The latter is shown in brackets. It contains one-, six, and twelve month ahead forecasts of the 3 month and 3 year nominal bond. The model is estimated from January 1970 to May 1995. The period June 1995 to December 2003 is the out-of-sample forecast period.

Table 4: **Nominal Yield Curve - Decomposition**

1970 - 2003						
Mat	y^{real}	$E^P[\pi]$	IRP	IAP	$y^{\$}$	$y^{\$,Data}$
6 mth	1.05 (0.03)	4.11 (3.54)	1.98 (0.025)	-0.61 (1.45)	6.53	6.534
1 year	2.0 1(0.03)	4.21 (3.34)	0.83 (0.029)	-0.26 (1.0)	6.79	6.764
2 years	2.47 (0.03)	4.53 (3.34)	0.25 (0.024)	-0.17 (0.87)	7.08	7.052
5 years	2.72 (0.02)	4.55 (2.25)	-0.1 (0.004)	-0.07(0.65)	7.10	7.472
7 years	2.75 (0.19)	4.46 (1.37)	-0.16 (<0.002)	0.22 (0.07)	7.267	7.606
10 years	2.77 (0.01)	4.35 (1.21)	-0.19 (0.003)	0.79 (1.31)	7.7	7.7

This table presents the components of the nominal yield curve as estimated over the entire sample January 1970 to December 2003. The corresponding standard deviations are given in parentheses. The estimation is performed with a panel of smoothed Fama-Bliss yields (1, 3, 6 month and 1, 2, 3, 5, 7, 10 years), CPI inflation, M2 money growth and consumption growth data. The sample period is January 1970 to December 2003.

Table 5: **Nominal Yield Curve - Variance Decomposition**

1970 - 2003				
Mat	y^{real}	$E^P[\pi]$	IRP	IAP
6 mth	0.01%	85.53%	<0.01%	14.45%
1 year	0.01%	91.72%	0.01%	8.26%
2 years	0.01%	93.57%	0.01%	6.42%
5 years	0.01%	92.21%	<0.01%	7.78%
7 years	0.01%	99.81%	<0.01%	0.18%
10 years	0.01%	45.9%	<0.01%	54.09%

This table presents the variance decomposition for components of the nominal yield curve.

Figure 1: **Bond Yield Factor Loadings, 1970.1 - 2003.12**

This figure presents the factor loadings of the structural model. The estimation is performed with a panel of smoothed Fama-Bliss yields (1, 3, 6 month and 1, 2, 3, 5, 7, 10 years), CPI inflation, M2 money growth and consumption growth data. The sample period is January 1970 to December 2003.

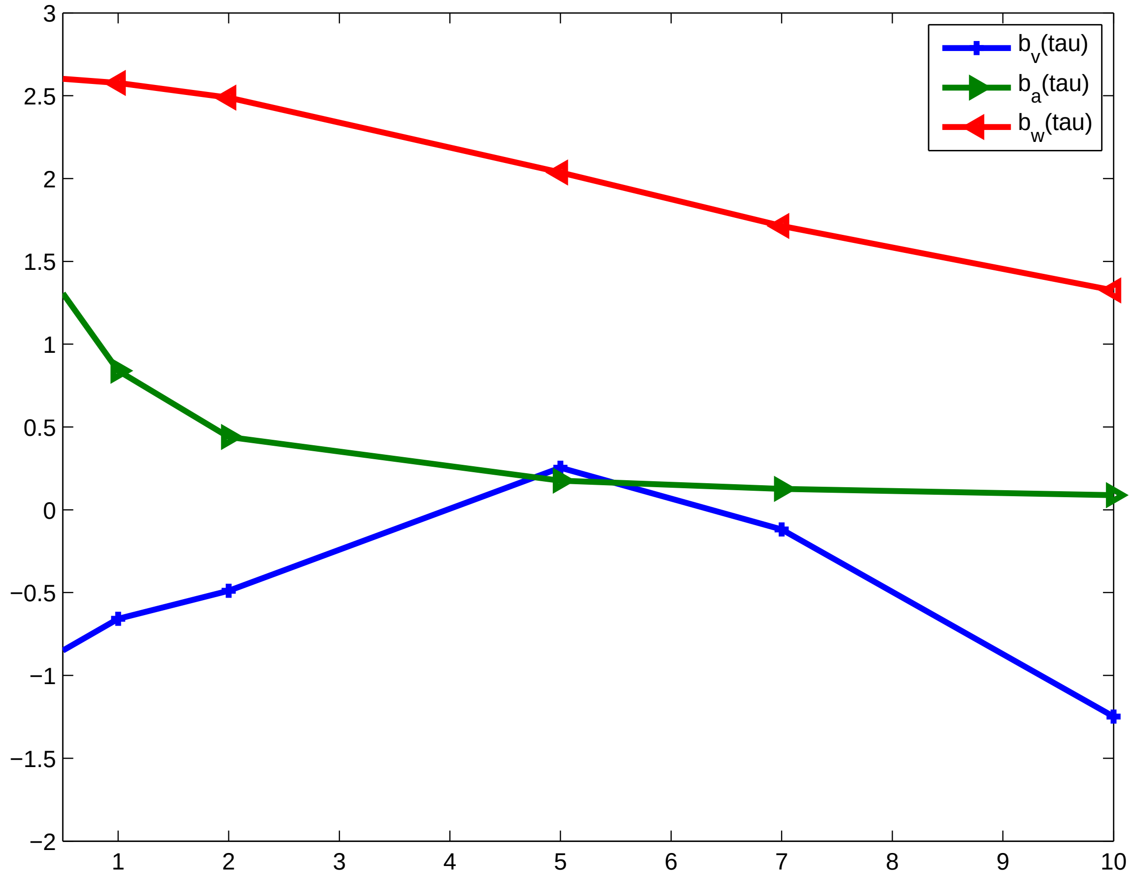

Figure 2: **Components of Nominal Yields during the Monetary Policy Experimentation** 1979 – 1983 **and during the Great Moderation** 1984 – 2003

This figure presents the term structure of the components of the nominal yield curve. The components are real yields, expected inflation, inflation risk premium and inflation ambiguity premium. The left panel presents the components for the time period of the monetary policy experimentation 1979 – 1983. The right panel presents the components for the time period of the great moderation 1984 – 2003. The estimation is performed with a panel of smoothed Fama-Bliss yields (1, 3, 6 month and 1, 2, 3, 5, 7, 10 years), CPI inflation, M2 money growth and consumption growth data. The estimation period is January 1970 to December 2003.

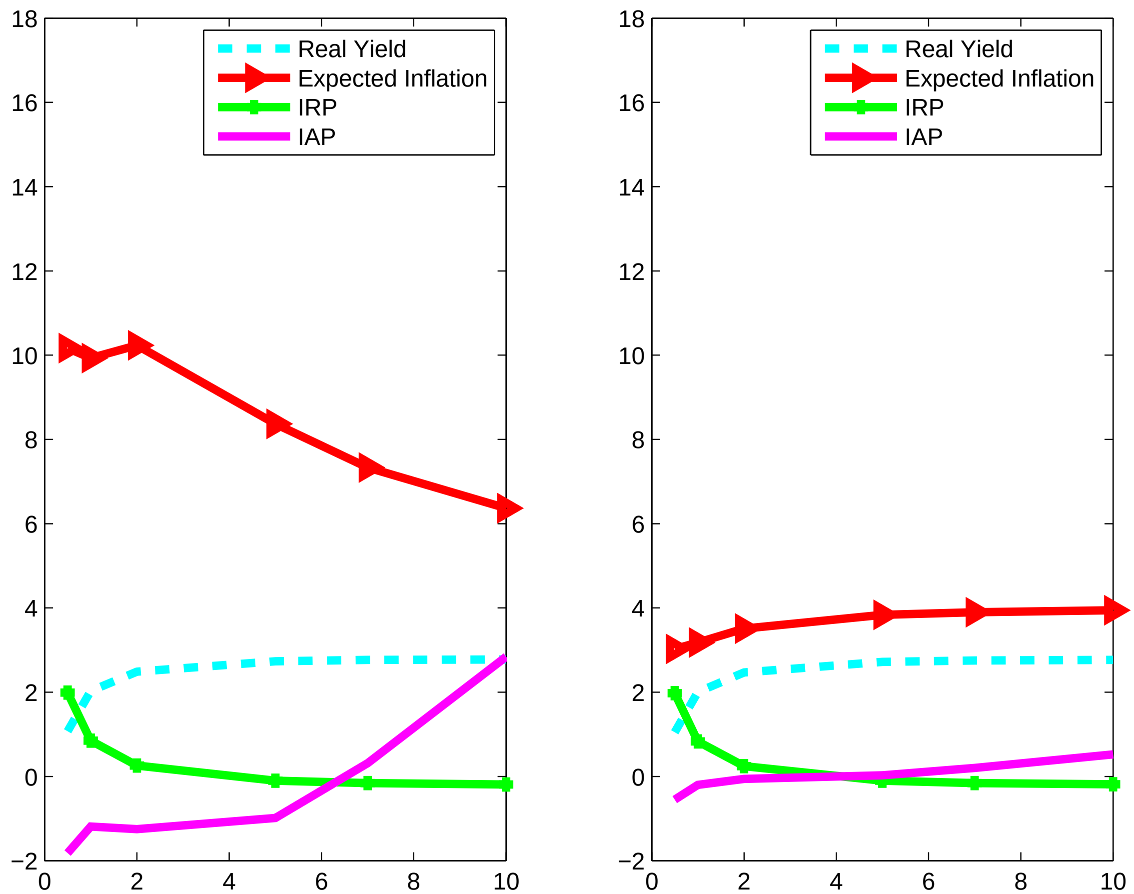

Figure 3: **Time Series of Inflation Ambiguity Premium and Inflation Risk Premium, 1970.1 - 2003.12**

This figure presents the model implied annualized inflation ambiguity premium and inflation risk premium. The left panel shows the premia for the six month nominal yield, the panel in the middle shows the premia for the two year nominal yield and the panel on the right shows the premia for the ten year nominal yield. The estimation is performed with a panel of smoothed Fama-Bliss yields (1, 3, 6 month and 1, 2, 3, 5, 7, 10 years), CPI inflation, M2 money growth and consumption growth data. The sample period is January 1970 to December 2003. The black line presents the overall term premium.

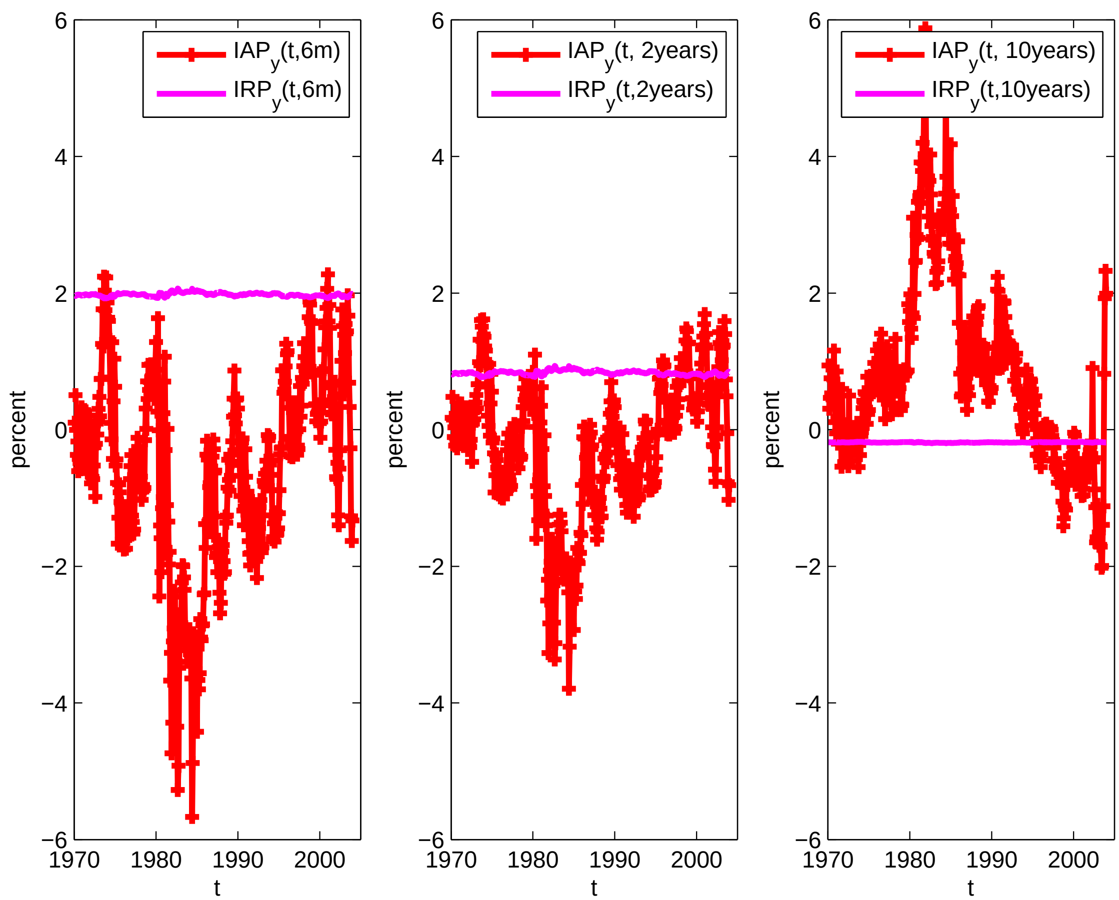

Figure 4: **Variance of Inflation and Inflation Ambiguity Premium**

This figure contrasts the estimated inflation ambiguity premium with the model implied monthly variance of inflation (annualized).
The estimation is performed with a panel of smoothed Fama-Bliss yields (1, 3, 6 month and 1, 2, 3, 5, 7, 10 years), CPI inflation, M2 money growth and consumption growth data. The sample period is January 1970 to December 2003.

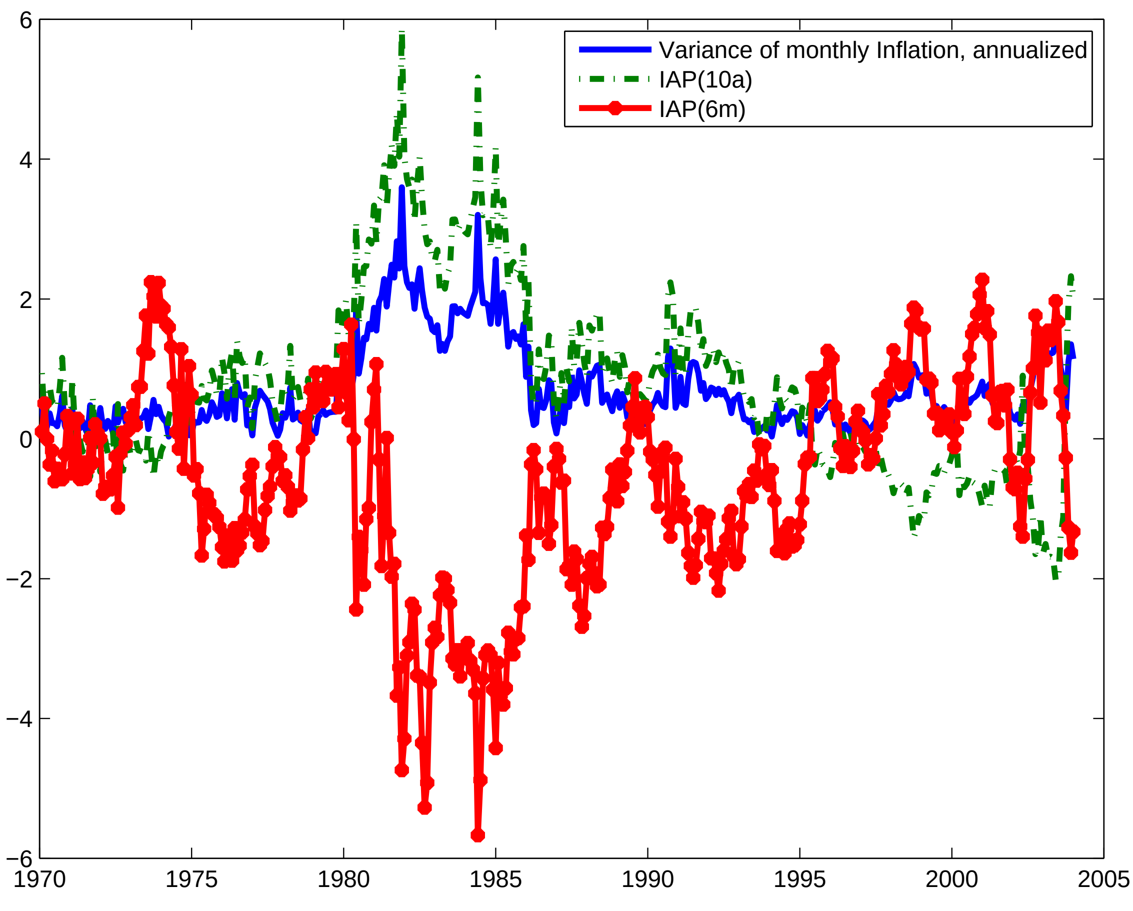

Figure 5: **Campbell-Shiller Coefficients, 1970.I - 2003.IV**

This figure decomposes the model implied population Campbell-Shiller (CS) coefficients into its risk and ambiguity premium. The estimation is performed with a panel of smoothed Fama-Bliss yields (1, 3, 6 month and 1, 2, 3, 5, 7, 10 years), CPI inflation, M2 money growth and consumption growth data. The sample period is January 1970 to December 2003.