Bond Liquidity Premia

Jean-Sébastien Fontaine
Université de Montréal and CIREQ
René Garcia
EDHEC Business School
First draft: January 2007. This draft: June 2009

Abstract

Recent asset pricing models of limits to arbitrage emphasize the role of funding conditions faced by financial intermediaries. In the US, the repo market is the key funding market. Then, the premium of on-the-run U.S. Treasury bonds should share a common component with risk premia in other markets. This observation leads to the following identification strategy. We measure the value of funding liquidity from the cross-section of on-the-run premia by adding a liquidity factor to an arbitrage-free term structure model. As predicted, we find that funding liquidity explains the cross-section of risk premia. An increase in the value of liquidity predicts lower risk premia for on-the-run *and* off-the-run bonds but higher risk premia on LIBOR loans, swap contracts and corporate bonds. Moreover, the impact is large and pervasive through crisis and normal times. We check the interpretation of the liquidity factor. It varies with transaction costs, S&P500 valuation ratios and aggregate uncertainty. More importantly, the liquidity factor varies with narrow measures of monetary aggregates and measures of bank reserves. Overall, the results suggest that different securities serve, in part, and to varying degrees, to fulfill investors' uncertain future needs for cash depending on the ability of intermediaries to provide immediacy.

JEL Classification: E43, H12.

We thank Greg Bauer, Antonio Diez, Darrell Duffie, Thierry Foucault, Francis Longstaff, Albert Menkveld, Monika Piazzesi, Robert Rasche, Jose Sheinkman, participants at the Econometric Society Summer Meeting (2007) and European Meeting (2007), Canadian Economic Association (2007), International Symposium on Financial Engineering and Risk Management (2007), the Risk Management Institute (2009), University of Lugano (2009), University of Piraeus (2009), University College Dublin (2009), WFA (2009) and EFA (2009). The first author gratefully acknowledges support from the IFM^2 and the Banque Laurentienne. The second author is a research fellow at CIRANO and CIREQ. He gratefully acknowledges support from FQRSC, SSHRC, MITACS, Hydro-Québec, and the Bank of Canada. *Correspondence:* `jsfontaine@bankofcanada.ca`.

"... a part of the interest paid, at least on long-term securities, is to be attributed to uncertainty of the future course of interest rates."
(p.163)

"... the imperfect 'moneyness' of those bills which are not money [...] causes the trouble of investing in them and [causes them] to stand at a discount."
(p.166)

"... In practice, there is no rate so short that it may not be affected by speculative elements; there is no rate so long that it may not be affected by the alternative use of funds in holding cash."
(p.166)

John R. Hicks, Value and Capital, 2nd edition, 1948.

Introduction

Bond traders know very well that liquidity affects asset prices. One prominent case is the on-the-run premium, whereby the most recently issued (on-the-run) bonds sell at a premium relative to seasoned (off-the-run) bonds with similar coupons and maturities. Moreover, systematic variations in liquidity sometimes drive interest rates across several markets. A case in point occurred around the Federal Open Market Committee [FOMC] decision, on October 15, 1998, to lower the Federal Reserve funds rate by 25 basis points. In the meeting's opening, Vice-Chairman McDonough, of the New York district bank, noted increases in the spread between the on-the-run and the most recent off-the-run 30-year Treasury bonds (0.05% to 0.27%), the spreads between the rate on the fixed leg of swaps and Treasury notes with two years and ten years to maturity (0.35% to 0.70%, and 0.50% to 0.95%, respectively), the spreads between Treasuries and investment-grade corporate securities (0.75% to 1.24%), and finally between Treasuries and mortgage-backed securities (1.10% to 1.70%). He concluded that we were seeing a run to quality and a serious drying up of liquidity[1]. These events attest to the sometimes dramatic impact of liquidity seizures[2].

A common explanation for that and the more recent market turmoil is based on a common wealth shock to capital-constrained intermediaries or speculators (Shleifer and Vishny (1997), Kyle and Xiong (2001), Gromb and Vayanos (2002)). Intuitively, lower wealth hinders the ability to pursue quasi-arbitrage opportunities across markets. In practice, the repo market is the key market where investment banks, hedge funds and other speculators obtain the marginal funds for their

[1] Minutes of the Federal Open Market Committee, October 15, 1998 conference call. See http://www.federalreserve.gov/Fomc/transcripts/1998/981015confcall.pdf.

[2] The liquidity crisis of 2007-2008 provides another example. Facing sharp increases of interest rate spreads in most markets, the Board approved reduction in discount rate, target Federal Funds rate as well as novel policy instruments to deal with the ongoing liquidity crisis.

activities and manage their leveraged exposure to risk (Adrian and Shin (2008)). Then, the risk premia for each market intermediated by a common set of intermediaries share a component measuring tightness in the funding market (Brunnermeier and Pedersen (2008), Krishnamurthy and He (2008)). This paper tests the implication that tightness of funding conditions in repo markets should be reflected in risk premia across financial markets.

We introduce liquidity as an additional factor in an otherwise standard term structure model. Indeed, the modern term structure literature has not recognized the importance of aggregate liquidity for government yields. We extend the no-arbitrage dynamic term structure model of Christensen et al. (2007) [CDR, hereafter] allowing for liquidity[3] and we extract a common factor driving on-the-run premia across maturities. Identification of the liquidity factor is obtained by estimating the model from a panel of pairs of U.S. Treasury securities where each pair has similar cash flows but different ages. This sidesteps credit risk issues and delivers direct estimates of funding liquidity value: it isolates price differences that can be attributed to liquidity. A recent empirical literature suggests that liquidity is priced on bond markets[4] but these empirical investigations are limited to a single market. Moreover, none consider the role of funding constraints.

Our main contribution is precisely to show that funding liquidity is an aggregate risk factor that drives a substantial share of risk premia across interest rate markets. In particular, we document large variations in the liquidity premium of U.S. Treasury bonds. By construction, an increase in the liquidity factor is associated with lower expected returns for on-the-run bonds. What we show is that the risk premium of any U.S. Treasury bonds also decreases substantially. On the other hand, tight funding conditions raise the risk premium implicit in LIBOR rates, swap rates and corporate bond yields. This pattern is consistent with accounts of flight-to-quality but the relationship is pervasive even in normal times. This adds considerably to the existing evidence pointing toward the importance of funding liquidity as an aggregate risk factor. Moreover, it suggests that different securities serve, in part and to varying degrees, to fulfill investors uncertain future needs for cash.

We estimate the model and obtain a measure of funding liquidity value from a sample of end-of-month bond prices running from December 1985 until the end of 2007. Hence, our results cannot be attributed to the extreme influence of 2008. In a concluding section, we repeat the estimation including 2008 and find, not unexpectedly, that importance of funding liquidity increases. Our empirical findings can be summarized as follows. Panel (a) of Figure 1 presents the measure of funding liquidity value. Clearly, it exhibits significant variations through normal and crisis periods. In particular, the stock crash of 1987, the Mexican Peso devaluation of December 1994, the LTCM failure of 1998 and the recent liquidity crisis are associated with peaks in investors' valuation of the funding liquidity of on-the-run bonds. The relationship with the risk premium of government

[3] This model captures parsimoniously the usual level, slope and curvature factors, while delivering good in-sample fit and forecasting power. Moreover, the smooth shape of Nelson-Siegel curves identifies small deviations, relative to an idealized curve, which may be caused by variations in market liquidity.

[4] See Longstaff (2000) for evidence that liquidity is priced for short-term U.S. Treasury security and Longstaff (2004) for U.S. Treasury bonds of longer maturities. See Collin-Dufresne et al. (2001), Longstaff et al. (2005), Ericsson and Renault (2006), Nashikkar and Subrahmanyam (2006) for corporate bonds.

bonds is illustrated in Figure 2. Panel (a) compares the funding liquidity factor with annual excess returns on a 2-year to maturity off-the-run bond. Clearly, an increase in the value of liquidity predicts lower expected excess returns and, thus, higher current bond prices. For that maturity, a one-standard deviation shock to liquidity predicts a decrease in excess returns of 85 basis points [bps] compared to an average excess returns of 69 bps. We obtain similar results using different maturities or investment horizons. Intuitively, while an off-the-run bond may be less liquid relative to an on-the-run bond with similar characteristics, it is still viewed as a liquid substitute. In particular, it can still be quickly converted into cash, at low costs, via the funding market.

Next, we consider the predictive power of funding liquidity for the risk premium on short-term Eurodollar loans. Panel (b) of Figure 2 shows that variations of LIBOR excess returns are positively linked to variations of funding liquidity. The relationship is significant, both statistically and economically. Consider excess returns from borrowing at the risk-free rate for 12 months and rolling a 3-month LIBOR loans. On average, returns from this strategy are not statistically different than zero since the higher term premium on the borrowing leg compensates for the 3-month LIBOR spread earned on the lending leg. However, following a one-standard deviation shock to the funding liquidity factor, rolling excess returns increase by 42 bps. We reach similar conclusions using LIBOR spreads as ex-ante measures of risk premium. The effect of funding liquidity also extends to swap markets. Panel (d) compares the liquidity factor with the spread, above the par Treasury yield, of a swap contract with 5 years to maturity. We find that a shock to funding liquidity predicts an increase of 6 bps the 5-year swap spread. This is economically significant given the higher sensitivity (i.e. duration) of this contract value to changes in yields. In each regression, we control for variations in the level and shape of the term structure of Treasury yields. The marginal contribution of liquidity to the predictive power is high.

Finally, we consider a sample of corporate bond spreads from the NAIC. We find that the impact of liquidity is significant and follows a flight-to-quality pattern across ratings. For bonds of the highest credit quality, spreads decrease, on average, following a shock to the funding liquidity factor. In contrast, spreads of bonds with lower ratings increase. We also compute excess returns on AAA, AA, A, BBB and High Yield Merrill Lynch corporate bond indices (see Figure 3) and reach similar conclusions. Bonds with high credit ratings were perceived to be liquid substitutes to government securities and offered lower risk premium following increases of the liquidity factor. This corresponds to an average effect through our sample, the recent events suggests that this is not always the case.

These results raise the all important issue of identifying macroeconomic drivers of the liquidity factor. Can we characterize the aggregate liquidity premium in terms of economic state variables? First, consistent with theory, our liquidity factor varies with measures of transaction costs on the bond market. Second, we find that funding liquidity is linked to stock market valuation ratios and option-implied volatility from S&P 500 index options. These results support empirically the link between conditions on the funding market, the ability of intermediaries to provide liquidity and the level and risk of aggregate wealth. Most importantly, we find that measures of changes in monetary aggregates and changes in bank reserves are key determinants of our liquidity measure. These findings support our interpretation of the liquidity factor as a measure of conditions on the funding market. This provides a third important empirical contribution.

Related Literature

A few empirical papers document the effects of intermediation constraints in specific markets[5] but we differ in significant ways from existing work. First, we measure the effect of intermediation constraints directly from observed prices rather than quantities. Prices aggregate information about and anticipations of intermediaries wealth, their portfolios and the margins they face. Second, we study a cross-section of money-market and fixed-income securities, providing evidence that funding constraints should be thought as an aggregate risk factor driving liquidity premia across markets.

We introduce a measure of funding liquidity *value* based on the higher valuation of on-the-run bonds relative to off-the-run bonds .[6] The on-the-run liquidity premium was first documented by Warga (1992). Amihud and Mendelson (1991) and, more recently, Goldreich et al. (2005) confirm the link between the premium and expected transaction costs. Duffie (1996) provides a theoretical channel between on-the-run premia and lower financing costs on the repo market. Vayanos and Weill (2006) extend this view and model search frictions in both the repo and the cash markets explicitly.[7] The key frictions differentiating bonds with identical cash flows lies in their segmented funding markets. The link between the repo market and the on-the-run premium has been confirmed empirically. (See Jordan and Jordan (1997), Krishnamurthy (2002), Buraschi and Menini (2002) and Cheria et al. (2004).)

We differ from the modern term structure literature in two significant ways. First, the latter focuses almost exclusively on bootstrapped zero-coupon yields[8]. This approach is convenient because a large family of models delivers zero-coupon yields which are linear in the state variables (see Dai and Singleton (2000)). However, we argue that pre-processing the data wipes out the most accessible evidence on liquidity, that is the on-the-run premium. Therefore, we use coupon bond prices directly. However, the state space is no longer linear and we handle non-linearities with the Unscented Kalman Filter [UKF], an extension of the Kalman Filter for non-linear state-space systems (Julier et al. (1995) and Julier and Uhlmann (1996)). We first estimate a model without liquidity and, notwithstanding differences in data and filtering methodologies, our results are consistent with CDR. However, pricing errors in this standard term structure model reveals systematic

[5]See Froot and O'Connell (2008) for catastrophe insurance, Gabaix et al. (2009) for mortgage-backed securities, Gârleanu et al. (2009) for index options and Adrian et al. (2009) for exchange rates.

[6]The U.S Treasury recognizes and takes advantages of this price differential: "In addition, although it is not a primary reason for conducting buy-backs, we may be able to reduce the government's interest expense by purchasing older, "off-the-run" debt and replacing it with lower-yield "on-the-run" debt." [Treasury Assistant Secretary for financial markets Lewis A. Sachs, Testimony before the House Committee on Ways and Means].

[7]Kiyotaki and Wright (1989) introduced search frictions in monetary theory and Shi (2005) extends this framework to include bonds. See Shi (2006) for a review. Search frictions can also rationalize the spreads between bid and ask prices offered by market intermediaries (Duffie et al. (2005)).

[8]The CRSP data set of zero-coupon yields is the most commonly used. It is based on the bootstrap method of Fama and Bliss (1987) [FB].

differences within pairs, correlated with ages. Estimation of the model with liquidity produces a persistent factor capturing differences between prices of recently issued bonds and prices of older bonds. The on-the-run premium increases with maturity but decays with the age of a bond. These new features complete our contributions to the modeling of the term structure of interest rates in presence of a liquidity factor.

We also differ from the recent literature using a reduced-form approach that model a convenience yield in interest rate markets (Duffie and Singleton (1997)). A one-factor model of the convenience yield cannot match the pattern of on-the-run premia across maturities. Moreover, the link between the premium and the age of a bond cannot be captured in a frictionless arbitrage-free model. Still, Grinblatt (2001) argues that the convenience yields of U.S. Treasury bills can explain the U.S. Dollar swap spread. Recently, Liu et al. (2006) and Fedlhütter and Lando (2007) evaluate the relative importance of credit and liquidity risks in swap spreads. Other empirical investigations are related to our work. Jump risk (Tauchen and Zhou (2006)) or the debt-gdp ratio (Krishnamurthy and Vissing-Jorgensen (2007)) have been proposed to explain the non-default component of corporate spreads. Finally, Pastor and Stambaugh (2003) and Amihud (2002) provide evidence of a liquidity risk factor in expected stock returns.

The link between interest rates and aggregate liquidity is supported elsewhere in the theoretical literature. Svensson (1985) uses a cash-in-advance constraint in a monetary economy. Bansal and Coleman (1996) allow government bonds to back checkable accounts and reduced transaction costs in a monetary economy. Luttmer (1996) investigates asset pricing in economies with frictions and shows that with transaction costs (bid-ask spreads) there is in general little evidence against the consumption-based power utility model with low risk-aversion parameters. Holmström and Tirole (1998) introduce a link between the liquidity demand of financially constrained firms and asset prices. Acharya and Pedersen (2004) propose a liquidity-adjusted CAPM model where transaction costs are time-varying. Alternatively, Vayanos (2004) takes transactions costs as fixed but introduces the risk of having to liquidate a portfolio. Lagos (2006) extends the search friction argument to multiple assets: in a decentralized exchange, agents with uncertain future hedging demand prefer assets with lower search costs.

The rest of the paper is organized as follows. The next section presents the model and its state-space representation. Section II describes the data and Section III introduces the estimation method based on the UKF. We report estimation results for models with and without liquidity in Section IV. Section V evaluates the information content of liquidity for excess returns and interest rate spreads while Section VI identifies economic determinants of liquidity. Section VIII concludes.

I A Term Structure Model With Liquidity

We base our model on the Arbitrage-Free Extended Nelson-Siegel [AFENS] model introduced in CDR. This model belongs to the affine family (Duffie and Kan (1996)). The latent state variables relevant for the evolution of interest rates are grouped within a vector F_t of dimension $k = 3$. Its dynamics under the risk-neutral measure Q is described by the stochastic differential equation

$$dF_t^Q = K^Q(\theta^Q - F_t) + \Sigma dW_t^Q, \tag{1}$$

where dW_t is a standard Brownian motion process. Combined with the assumption that the short rate is affine in all three factors, the model then leads to the usual affine solution for discount bond yields.

In this context, CDR show that if the short rate is defined as $r_t = F_{1,t} + F_{2,t}$ and if the mean-reversion matrix K^Q is restricted to

$$K^Q = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda & -\lambda \\ 0 & 0 & \lambda \end{pmatrix}, \tag{2}$$

then the absence of arbitrage opportunity implies the discount yield function,

$$y(F_t, m) = a(m) + F_{1,t}b_1(m) + F_{2,t}b_2(m) + F_{3,t}b_3(m), \tag{3}$$

with loadings given by

$$\begin{aligned} b_1(m) &= 1, \\ b_2(m) &= \left(\frac{1 - \exp(-m\lambda)}{m\lambda}\right), \\ b_3(m) &= \left(\frac{1 - \exp(-m\lambda)}{m\lambda} - \exp(-m\lambda)\right), \end{aligned} \tag{4}$$

where $m \geq 0$ is the length of time until maturity (see Appendix C for the $a(m)$ term).

These loadings are consistent with the static Nelson-Siegel representation of forward rates (Nelson and Siegel (1987), NS hereafter). Their shapes across maturities lead to the usual interpretations of factors in terms of level, slope and curvature. Moreover, the NS representation is parsimonious and imposes a smooth shape to the forward rate curve. Empirically, this approach is robust to over-fitting and delivers performance in line with, or better than, other methods for pricing out-of-sample bonds in the cross-section of maturities[9]. Conversely, its smooth shape is useful to identify deviations of observed yields from an idealized curve.

A dynamic extension of the NS model, the Extended Nelson-Siegel model [ENS], was first proposed by Diebold and Li (2006) and Diebold et al. (2006). Diebold and Li (2006) document large improvements in long-horizon interest rate forecasting. They argue that the ENS model performs better than the best essentially affine model of Duffee (2002) and point toward the model's parsimony to explain its successes. A persistent concern, though, was that the ENS model does not enforce the absence of arbitrage. This is precisely the contribution of CDR. They derive the class of

[9]See Bliss (1997) and Anderson et al. (1996) for an evaluation of yield curve estimation methods.

continuous-time arbitrage-free affine dynamic term structure models with loadings that correspond to the NS representation. Intuitively, an AFENS model corresponds to a canonical affine model in Dai and Singleton (2000) where the loading shapes have been restricted through over-identifying assumptions on the parameters governing the risk-neutral dynamics of latent factors. CDR compare the ENS and AFENS models and show that implementing these restrictions improves forecasting performances further.

Interestingly, CDR show that we are free to choose the drift and variance term for the dynamics under the physical measure

$$dF_t^P = K^P(\theta^P - F_t) + \Sigma dW_t^P, \tag{5}$$

and we impose that Σ is lower triangular and that K^P is diagonal[10]. We can then cast the model within a discretized state-space representation. The state equation becomes

$$(F_t - \bar{F}) = \Phi(F_{t-1} - \bar{F}) + \Gamma\epsilon_t, \tag{6}$$

where the innovation ϵ_t is standard Gaussian, the autoregressive matrix Φ is

$$\Phi = \exp\left(-K\frac{1}{12}\right) \tag{7}$$

and the covariance matrix Γ can computed from

$$\Gamma = \int_0^{\frac{1}{12}} e^{-Ks}\Sigma\Sigma^T e^{-Ks} ds. \tag{8}$$

Finally, we define a new latent state variable, L_t, that will be driving the liquidity premium. Its transition equation is

$$(L_t - \bar{L}) = \phi^l(L_{t-1} - \bar{L}) + \sigma^l\epsilon_t^l, \tag{9}$$

where the innovation ϵ_t^l is standard Gaussian and uncorrelated with ϵ_t.

Typically, term structure models are not estimated from observed prices. Rather, coupon bond prices are converted to forward rates using the bootstrap method. This is convenient as affine term structure models deliver forward rates that are linear in state variables. Is is also thought to be innocuous because bootstrapped forward rates achieve near-exact pricing of the original sample of bonds. Unfortunately, this extreme fit means that a naive application of the bootstrap pushes any liquidity effects and other price idiosyncracies into forward rates. Fama and Bliss (1987) handle this sensitivity to over-fitting by excluding bonds with "large" price differences relative to their neighbors.[11] This approach is certainly justified for many of the questions addressed in the

[10]Formally, the assumption on Σ is required for identification purposes. In practice, the presence of the off-diagonal elements in the K^P matrix does not change our results. Moreover, CDR show that allowing for an unrestricted matrix K^P deteriorates out-of-sample performance.

[11]The CRSP data set of zero coupon yields is based on the approach proposed by Fama and Bliss. See also the

literature, butit removes any evidence of large liquidity effects. Moreover, the FB data set focuses on discount bond prices at annual maturity intervals. This smooths away evidence of small liquidity effects remaining in the data and passed through to forward rates. These effects would be apparent from reversals in the forward rate function at short maturity interval. Consider three quotes for bonds with successive maturities $M_1 < M_2 < M_3$. A relatively expensive quote at maturity M_2 induces a relatively small forward rate from M_1 to M_2. However, the following normal quote with maturity M_3 requires a relatively large forward rate from M_2 to M_3. This is needed to compensate the previous low rate and to achieve exact pricing as required by the bootstrap. However, the reversal cancels itself as we sum intra-period forward rates to compute annual rates.

Instead of using smoothed data, we proceed from observed coupon bonds with maturity, say, M and with coupons at maturities $m = m_1, \ldots, M$. The price, $D_t(m)$, of a discount bond with maturity m, used to price intermediate payoffs, is given by

$$D_t(m) = \exp\left(-m(a(m) + b(m)^T F_t)\right) \qquad m \geq 0,$$

which follows directly from equation (3) but where we use vector notation for factors F_t and factor loadings $b(m)$. In a frictionless economy, the absence of arbitrage implies that the price of a coupon bond equals the sum of discounted coupons and principal. That is, the frictionless price is

$$P^*(F_t, Z_t) = \sum_{m=m_1}^{M} D_t(m) \times C_t(m), \tag{10}$$

where Z_t includes (deterministic) characteristics relevant for pricing a bond. In this case, it includes the maturity M and the schedule of future coupons and principal payments, $C_t(m)$.

However, with a short-sale constraint on government bonds and a collateral constraint in the repo market, Luttmer (1996) shows that the set of stochastic discount factors consistent with the absence of arbitrage satisfies $P \geq P^*$. These constraints match the institutional features of the Treasury market. An investor cannot issue new bonds to establish a short position. Instead, she must borrow the bond on the repo market through a collateralized loan. Then, we model the price, $P(F_t, L_t, Z_t)$, of a coupon bond with characteristics Z_t as the sum of discounted coupons to which we add a liquidity term,

$$P(F_t, L_t, Z_{n,t}) = \sum_{m=1}^{M_n} D_t(m) \times C_{n,t}(m) + \zeta(L_t, Z_{n,t}).$$

CRSP documentation for a description of this procedure. Briefly, a first filter includes a quote if its yield to maturity falls within a range of 20 basis points from one of the moving averages on the 3 longer or the 3 shorter maturity instruments *or* if its yield to maturity falls between the two moving averages. When computing averages, precedence is given to bills when available and this is explicitly designed to exclude the impact of liquidity on notes and bonds with maturity of less than one year. Amihud and Mendelson (1991) document that yield differences between notes and adjacent bills is 43 basis point on average, a figure much larger than the 20 basis point cutoff. The second filter excludes observations that cause reversals of 20 basis points in the bootstrapped discount yield function. The impact of these filters has not been studied in the literature.

Here Z_t also includes the age of the bond. Note that the liquidity term should be positive to be consistent with a Luttmer (1996).

That the on-the-run premium is related to the short-sale constraint on government bonds and the collateral constraint in the repo market is justified by the results of Vayanos and Weill (2006) (see also Duffie (1996)). They show that the combination of these constraints with search frictions on the repo market induces differences in funding costs that favor recently issued bonds. Intuitively, the repo market provides the required heterogeneity between assets with identical payoffs. An investor cannot choose which bond to deliver to unwind a repo position; she must find and deliver the same security she had originally borrowed. Because of search frictions, then, investors are better off in the aggregate if they coordinate around one security to reduce search costs. In practice, the repo rate is lower for this special issue to provide an incentive for bond holders to bring their bonds to the repo market. Typically, recently issued bonds benefit from these lower financing costs, leading to the on-the-run premium. Moreover, these bonds offer lower transaction costs adding to the wedge between asset prices (Amihud and Mendelson (1986)). Empirically, both channels seem to be at work although the effect of lower transaction costs appears weaker than the effect of lower funding rates.[12]

Grouping observations together, and adding an error term, we obtain our measurement equation

$$P(F_t, L_t, Z_t) = C_t D_t + \zeta(L_t, Z_t) + \Omega \nu_t, \tag{11}$$

where C_t is the $(N \times M_{max})$ payoffs matrix obtained from stacking the N row vectors of individual bond payoffs and M_{max} is the longest maturity group in the sample. Shorter payoff vectors are completed with zeros. Similarly, $\zeta(L_t, Z_t)$ is a $N \times 1$ vector obtained by staking the individual liquidity premium. D_t is a $(M_{max} \times 1)$ vector of discount bond prices and the measurement error, ν_t, is a $(N \times 1)$ gaussian white noise uncorrelated with innovations in state variables. The matrix Ω is assumed diagonal and its elements are a linear function of maturity,

$$\omega_n = \omega_0 + \omega_1 M_n,$$

which reduce substantially the dimension of the estimation problem. However, leaving the diagonal elements of Ω unrestricted does not affect our results[13].

Our specification of the liquidity premium is based on a latent factor common to all bonds but with loadings that vary with maturity and age. The premium is given by

$$\zeta(L_t, Z_{n,t}) = L_t \times \beta_{M_n} \exp\left(-\frac{1}{\kappa}\ age_{n,t}\right) \tag{12}$$

[12] Amihud and Mendelson (1991) and Goldreich et al. (2005) consider transaction costs. Jordan and Jordan (1997), Krishnamurthy (2002) and Cheria et al. (2004) consider funding costs. See also, Buraschi and Menini (2002) for the German bonds market.

[13] This may be due to the fact that the level factor explains most of yields variability. Its impact on bond prices is linear in duration and duration is approximately linear in maturity, at least for maturities up to 10 years. Bid-ask spreads increase with maturity and may also contribute to an increase in measurement errors with maturity.

where age_t is the age, in years, of the bond at time t. The parameter β_M controls the average on-the-run premium at each fixed maturity M. Warga (1992) document the impact of age and maturity on the average premium. We estimate β for a fixed set of maturities and the shape of β is unrestricted between these maturities.[14] Next, the parameter κ controls the on-the-run premium's decay with age. The gradual decay of the premium with age has been documented by Goldreich et al. (2005). For instance, immediately following its issuance (i.e.: $age = 0$), the loading on the liquidity factor is $\beta_M \times 1$. Taking $\kappa = 0.5$, the loading decreases by half within any maturity group after a little more than 4 months following issuance : $\zeta(L_t, 4) \approx \frac{1}{2}\zeta(L_t, 0))$. While the specification above reflects our priors about the impact of age and maturity, the scale parameters are left unrestricted at estimation and we allow for a continuum of shapes for the decay of liquidity. However, we fix $\beta_{10} = 1$ to identify the level of the liquidity factor with the average premium of a just-issued 10-year bond relative to a very old bond with the same maturity and coupons.

Equation (11) shows that omitting the liquidity term will push the impact of liquidity into pricing errors, possibly leading to biased estimators and large filtering errors. Alternatively, adding a liquidity term amounts to filtering a latent factor present in pricing errors. However, Equation (11) shows that this factor captures that part of pricing errors correlated with bond ages. Our maintained hypothesis is that any such positive factor can be interpreted as a liquidity effect. Clearly, the impact of age on the price of a bond can hardly be rationalized in a frictionless economy.

Intuitively, our specification delivers a discount rate function consistent with off-the-run valuation but remains silent on the linkage with the equilibrium stochastic discount factor. A structural specification of the liquidity premium raises important challenges. The on-the-run premium is a real arbitrage opportunity unless we explicitly consider the costs of shorting the more expensive bond or, alternatively, the benefits accruing to the bondholder from a lower repo rate. These features are absent from the current crop of term structure models with the notable exception of Cheria et al. (2004) who allow for a convenience yield, due to lower repo rates accruing to holders of an on-the-run issue. Clearly, theory suggests that using repo rates may improve the identification of the premium. Unfortunately, this would restrict our analysis to a much shorter sample where repo data are readily available. In any case, a joint model of the term structure of repo rates and of government yields may still not be free of arbitrage unless we also model the convenience yield of holding short-term government securities. This follows from the observation that a Treasury bill typically offers a lower yield than a repo contract with the same maturity. Moreover, the stochastic properties of repo rates are not well known, as well as the form of their relationships with bond yields. This is beyond the scope of this paper. Our strategy bypasses these challenging considerations but still uncovers the key role funding liquidity. We now turn to a description of the data.

[14] Opportunities of arbitrage may arise if β follows a step process across maturities. We thank an anonymous referee for this remark.

II Data

We use end-of-month prices of U.S. Treasury securities from the CRSP data set. Our sample covers the period from January 1986 to December 2008. However, we estimate the model both with and without 2008 data. Before 1986, interest income had a favorable tax treatment compared to capital gains and investors favored high-coupon bonds. The resulting tax premium and the on-the-run premium cannot be disentangled in the earlier period. When interest rates are rising, recently issued bonds had relatively high coupons and were priced at a premium both for their liquidity and for their tax benefits. Green and Ødegaard (1997) document that the high-coupon tax premium mostly disappeared when the asymmetric treatment of interest income and capital gains was eliminated following the 1986 tax reform.

The CRSP data set[15] provides quotes on all outstanding U.S. Treasury securities. We filter unreliable observations and construct bins around maturities of 3, 6, 9, 12, 18, 24, 36, 48, 60, 84 and 120 months.[16] Then, at each date, and for each bin, we choose a pair of securities to identify the on-the-run premium. First, we want to pick the on-the-run security if any is available. Unfortunately, on-the-run bonds are not directly identified in the CRSP database. Instead, we use time since issuance as a proxy and pick the most recently issued security in each maturity bin. Second, we choose the security that most closely matches the bin's maturity (e.g. 3 months, 6 months,...). Note that pinning off-the-run securities at fixed maturities ensures a stable coverage of the term structure of interest rates. Also, by construction, securities within each pair have the same credit quality and very close times to maturity. We do not match coupon rates but coupon differences within pairs are low in practice.

The most important aspect of our sample is that whenever a security trades at premium relative to its pair companion, any large price difference cannot be rationalized from small coupon or maturity differences under the no-arbitrage restriction. On the other hand, price differences common across maturities and correlated with age will be attributed to liquidity. Note that the most recent issue for a given bin and date is not always an on-the-run security. This may be due to the absence of new issuance in some maturity bins throughout the whole sample (e.g. 18 months to maturity) or within some sub-periods (e.g. 84 months to maturity). Alternatively, the on-the-run bond may be a few months old, due to the quarterly issuance pattern observed in some maturity categories. In any case, this introduces variability in age differences which, in turn, identifies how the liquidity premium varies with age.

We now investigate some features of our sample of $265 \times 22 = 5830$ observations. The first two columns of Table I present means and standard deviations of age for each liquidity-maturity category. The average off-the-run security is always older than the corresponding on-the-run security. Typically, the off-the-run security has been in circulation for more than a year. In contrast, the on-the-run security is typically a few months old and only a few weeks old in the 6 and 24-month

[15] See Elton and Green (1998) and Piazzesi (2005) for discussion of the CRSP data set.

[16] See Apppendix A for more details on data filter.

categories. A relatively low average age for the recent issues indicates a regular issuance pattern. On the other hand, the relatively high standard deviations in the 36 and 84-month categories reflect the decision by the U.S. Treasury to stop the issuance cycles at these maturities.

[Table I about here.]

Next, Table I presents means and standard deviations of duration[17]. Average duration is almost linear in maturity. As expected, duration is similar within pairs implying that averages of cash flow maturities are very close. Finally, the last columns of Table I show that the term structure of coupons is upward sloping on average and the high standard deviations indicate important variations across the sample. This is in part due to the general decline of interest rates. Nonetheless, coupon rate *differences* within pairs are small on average. To summarize our strategy, differences in duration and coupon rates are kept small within each pair but differences of ages are highlighted so that we can identify any effect of liquidity on prices that is linked to age.

III Estimation Methodology

Equations (6), (9) and (11) can be summarized as a state-space system

$$\begin{aligned}(X_t - \bar{X}) &= \Phi_X (X_{t-1} - \bar{X}) + \Sigma_X \epsilon_t \\ P_t &= \Psi(X_t, C_t, Z_t) + \Omega \nu_t, \end{aligned} \tag{13}$$

where $X_t \equiv [F_t^T \; L_t]^T$ and Ψ is the (non-linear) mapping of cash flows C_t, bond characteristics, Z_t, and current states, X_t, into prices, P_t.

Estimation of this system is challenging because we do not know the joint density of factors and prices. Various strategies to deal with non-linear state-space systems have been proposed in the filtering literature: the Extended Kalman Filter (EKF), the Particle Filter (PF) and more recently the Unscented Kalman Filter[18] (UKF). The UKF is based on a method for calculating statistics of a random variable which undergoes a nonlinear transformation. It starts with a well-chosen set of points with given sample mean and covariance. The nonlinear function is then applied to each point and moments are computed from transformed points. This approach has a Monte Carlo flavor but the sample is drawn according to a specific deterministic algorithm. It delivers second-order accuracy with no increase in computing costs relative to the EKF. Moreover, analytical derivatives are not required. The UKF has been introduced in the term structure literature by Leippold and Wu (2003) and in the foreign exchange literature by Bakshi et al. (2005). Recently, Christoffersen et al. (2007) compared the EKF and the UKF for the estimation of term structure models. They conclude that the UKF improves filtering results and substantially reduces estimation bias.

[17]Duration is the relevant measure to compare maturities of bonds with different coupons.

[18]See Julier et al. (1995), Julier and Uhlmann (1996) and Wan and der Merwe (2001) for a textbook treatment. Another popular approach bypasses filtering altogether. It assumes that some prices are observed without errors and obtains factors by inverting the pricing equation. In our context, the choice of maturities and liquidity types that are not affected by measurement errors is not innocuous and impacts estimates of the liquidity factor.

To set up notation, we state the standard Kalman filter algorithm as applied to our model. We then explain how the unscented approximation helps overcome the challenge posed by a non-linear state-space system. First, consider the case where Ψ is linear in X and where state variables and bond prices are jointly Gaussian. In this case, the Kalman recursion provides optimal estimates of current state variables given past and current prices. The recursion works off estimates of state variables and their associated MSE from the previous step,

$$\begin{aligned}\hat{X}_{t+1|t} &\equiv E\left[X_{t+1}|\Im_t\right], \\ Q_{t+1|t} &\equiv E\left[(\hat{X}_{t+1|t} - X_{t+1})(\hat{X}_{t+1|t} - X_{t+1})^T\right], \end{aligned} \tag{14}$$

where $\Im_t$ belongs to the natural filtration generated by bond prices. The associated predicted bond prices, and MSE, are given by

$$\begin{aligned}\hat{P}_{t+1|t} &\equiv E\left[P_{t+1}|\Im_t\right] \\ &= \Psi(\hat{X}_{t+1|t}, C_{t+1}, Z_{t+1}), \qquad (15)\\ R_{t+1|t} &\equiv E\left[(\hat{P}_{t+1|t} - P_{t+1})(\hat{P}_{t+1|t} - P_{t+1})^T\right] \\ &= \Psi(\hat{X}_{t+1|t}, C_{t+1}, Z_{t+1})^T \hat{Q}_{t+1|t} \Psi(\hat{X}_{t+1|t}, C_{t+1}, Z_{t+1}) + \Omega, \qquad (16)\end{aligned}$$

using the linearity of Ψ. The next step compares predicted to observed bond prices and update state variables and their MSE,

$$\hat{X}_{t+1|t+1} = \hat{X}_{t+1|t} + K_{t+1}(P_{t+1} - \hat{P}_{t+1|t}), \tag{17}$$

$$Q_{t+1|t+1} = Q_{t+1|t} + K_{t+1}^T (R_{t+1|t})^{-1} K_{t+1}, \tag{18}$$

where

$$\begin{aligned}K_{t+1} &\equiv E\left[(\hat{X}_{t+1|t} - X_{t+1})(\hat{P}_{t+1|t} - P_{t+1})^T\right], \\ &= Q_{t+1|t}\Psi(\hat{X}_{t+1|t}, C_{t+1}, Z_{t+1}), \end{aligned} \tag{19}$$

measures co-movements between pricing and filtering errors. Finally, the transition equation gives us a conditional forecast of X_{t+2},

$$\hat{X}_{t+2|t+1} = \Phi_X \hat{X}_{t+1|t+1}, \tag{20}$$

$$Q_{t+2|t+1} = \Phi_X^T Q_{t+1|t+1} \Phi_X + \Sigma_X \Sigma_X^T. \tag{21}$$

The recursion delivers series $\hat{P}_{t|t-1}$ and $R_{t|t-1}$ for $t = 1, \cdots, T$. Treating $\hat{X}_{1|0}$ as a parameter, and setting $R_{1|0}$ equal to the unconditional variance of measurement errors, the sample log-likelihood is

$$L(\theta) = \sum_{t=1}^{T} l(P_t;\theta) = \sum_{t=1}^{T} \left[\log \phi(\hat{P}_{t+1|t}, R_{t+1|t})\right], \tag{22}$$

where $\phi(\cdot,\cdot)$ is the multivariate Gaussian density.

However, because $\Psi(\cdot)$ is not linear, equations (15) and (16) do not correspond to the conditional expectation of prices and the associated MSE. Also, (19) does not correspond to the conditional covariance between pricing and filtering errors. Still, the updating equations (17) and (18) remain justified as optimal linear projections. Then, we can recover the Kalman recursion provided we obtain approximations of the relevant conditional moments. This is precisely what the unscented transformation achieves, using a small deterministic sample from the conditional distribution of factors while maintaining a higher order approximation than linearization[19]. We can then use the likelihood given in (22), but in a QML context. Using standard results, we have $\hat{\theta} \approx N(\theta_0, T^{-1}\Omega)$ where $\hat{\theta}$ is the QML estimator of θ_0 and the covariance matrix is

$$\Omega = E\left[\left(\zeta_H \zeta_{OP}^{-1} \zeta_H\right)^{-1}\right], \tag{23}$$

where ζ_H and ζ_{OP} are the alternative representations of the information matrix, in the Gaussian case. These can be consistently estimated via their sample counterparts. We have

$$\hat{\zeta_H} = -T^{-1}\left[\frac{\partial^2 L(\hat{\theta})}{\partial\theta\partial\theta'}\right] \tag{24}$$

and

$$\hat{\zeta_{OP}} = T^{-1}\sum_{t=1}^{T}\left[\left(\frac{\partial l(t,\hat{\theta})}{\partial\theta}\right)\left(\frac{\partial l(t,\hat{\theta})}{\partial\theta}\right)^{T}\right]. \tag{25}$$

Finally, the model implies some restrictions on the parameter space. In particular, ϕ_l and diagonal elements of Φ must lie in $(-1,1)$ while κ and λ must remain positive. In practice, large values of κ or λ lead to numerical difficulties and are excluded. Finally, we maintain the second covariance contour of state variables inside the parameter space associated with positive interest rates. The filtering algorithm often fails outside this parameter space. None of these constraints binds around the optimum and estimates remain unchanged when the constraints are relaxed.

IV Estimation Results

We first estimate a restricted version of our model, excluding liquidity. Filtered factors and parameter estimates are consistent with results obtained by CDR from zero-coupon bonds. More interestingly, the on-the-run premium reveals itself in the residuals from the benchmark model. This provides a direct justification for linking the premium with the age and maturity of each

[19] See Appendix B.

bond. We then estimate the unrestricted liquidity model. The null of no liquidity is easily rejected and the liquidity factor captures systematic differences between on-the-run and off-the-run bonds. Finally, estimates imply that the on-the-run premium increases with maturity but decreases with the age of a bond.

A Results For The Benchmark Model Without Liquidity

Estimation[20] of the benchmark model put the curvature parameter at $\hat{\lambda} = 0.6786$ when time periods are measured in years. The standard error is 0.0305 and 0.0044 when using the QMLE and MLE covariance matrix, respectively. This estimate pins the maximum curvature loading at a maturity close to 30 months.

[Table II about here.]

Figure 1 displays the time series of the liquidity (Panel (a)) and the term structure (Panel (b)) factors. Estimates for the transition equation are given in Table IIa. The results imply average short and long term discount rates of 3.73% and 5.45%, respectively. The level factor is very persistent, perhaps a unit root. This standard result in part reflects the gradual decline of interest rates in our sample. The slope factor is slightly less persistent and exhibits the usual association with business cycles. Its sign changes before the recessions of 1990 and 2001. The slope of the term structure is also inverted starting in 2006, during the so-called "conundrum" episode. Finally, the curvature factor is closely related to the slope factor.

Standard deviations of pricing errors are given by

$$\begin{array}{rcccc} \sigma(M_n) = & 0.0229 & + & 0.0284 \times M_n, \\ & (0.017,\ 0.0012) & & (0.021,\ 0.0006) \end{array}$$

with QMLE and MLE standard errors for each parameter. This implies standard deviations of %0.05 and \$0.31 dollars for maturities of 1 and 10 years, respectively. Using durations of 1 and 7 years, this translates into yield errors of 5.1 and 4.4 bps. Table IIIa gives more information on the fit of the benchmark model. Root Mean Squared Errors (RMSE) increase from \$0.047 and \$0.046 for 3-month on-the-run and off-the-run securities, respectively, to \$0.35 and \$0.39 at 10-year maturity. As discussed above, the monotonous increase of RMSE with maturity reflects the higher sensitivity of longer maturity bonds to interest rates. It may also be due to higher uncertainty surrounding the true prices, as signaled by wider bid-ask spreads. In addition, for most maturities, the RMSE is larger for on-the-run bonds. For the entire sample, the RMSE is \$0.188.

Notwithstanding differences between estimation approaches, our results are consistent with CDR. Estimating using coupon bonds or using bootstrapped data provides similar pictures of the

[20]Estimation is implemented in MATLAB via the *fmincon* routine with the medium-scale (active-set) algorithm. Different starting values were used. For standard errors computations, we obtain the final Hessian update (BFGS formula) and each observation gradient is obtained through a centered finite difference approximation evaluated at the optimum.

underlying term structure of interest rates. Also, the approximation introduced when dealing with nonlinearities is innocuous. However, preliminary estimation of forward rate curves smooths away any effect of liquidity. In contrast, our sample comprises on-the-run and off-the-run bonds. Any systematic price differences not due to cash flow differences will be revealed in the pricing errors.

[Table III about here.]

Table IIIa confirms that Mean Pricing Errors (MPE) are systematically higher for on-the-run securities. On-the-run residuals are systematically higher than off-the-run residuals. For a recent 12-month T-Bill, the average difference is close to \$0.08, controlling for cash flow differences. Similarly, a recently issued 5-year bond is \$0.25 more expensive on average than a similar but older issue.[21] To get a clearer picture of the link between age and price differences, consider Figure 4. The top panels plot residual differences within the 12-month and 48-month categories. The bottom panels plot the ages of each bond in these categories. Panel (c) shows that the U.S. Treasury stopped regular issuance of the 12-month Notes in 2000. The liquidity premium was generally positive until then but stopped when issuance ceased. Afterwards, each pair is made of old 2-year Notes, and evidence of a premium disappears from the residuals. Panel (d) shows that there has been regular issuance of 4-year bonds early in the sample. As expected, the difference between residuals is generally positive whenever there is a significant age difference between the two issues. Moreover, in each case, on-the-run (i.e. low age) bonds appear overpriced compared to off-the-run (i.e. high age) bonds. This correspondence between issuance patterns and systematic pricing errors can be observed in each maturity category. The premium increases with maturity but decreases with age.

Bonds with 24 months to maturity seem to carry a smaller liquidity premium than what would be expected given the regular monthly issuance for this category. Note that a formal test rejects the null hypothesis of zero-mean residual differences. Interestingly, Jordan and Jordan (1997) could not find evidence of a liquidity or specialness effect at that maturity[22]. A smaller price premium for 2-year Notes is intriguing and we can only conjecture as to its causes. Recall that the magnitude of the premium depends on the benefits of higher liquidity, both in terms of lower transaction costs and lower repo rates. However, it also depends on the expected length of time a bond will offer these benefits. Results in Jordan and Jordan (1997) suggests that 2-year Notes remain "special" for shorter periods of time (see Table I, p.2057). Similarly, Goldreich et al. (2005) find that the on-the-run premium on 2-year Notes goes to zero faster than other maturities, on average. This is consistent with its short issuance cycle. Alternatively, holders of long-term bonds may re-allocate funds from their now short maturity bonds into newly issued longer term securities. If the two-year mark serves as a focus point for buyers and sellers, this may cause a larger volume of transactions around this key maturity, increasing the liquidity value of surrounding assets.

[21] Note that the price impact of liquidity increases with maturity. This is consistent with the results of Amihud and Mendelson (1991).

[22] See Jordan and Jordan (1997) p. 2061: "With the exception of the 2-year notes [...], the average price differences in Table II are noticeably larger when the issue examined is on special."

B Results For The Liquidity Model

Estimation of the unrestricted model leads to a substantial increase of the log-likelihood. The benchmark model is nested with 15 parameter restrictions and the improvement in likelihood is such that the LR test-statistic leads to a p-value that is essentially zero[23]. The estimate for the curvature parameter is now $\hat{\lambda} = 0.7304$ with QMLE and MLE standard errors of 0.0857 and 0.0043. Results for the transition equations are given in Table IIb. These imply average short and long term discount rates of 4.09% and 5.76% respectively. Interestingly, the yield curve level is higher once we account for the liquidity premium. Intuitively, the off-the-run yield curve is higher than an otherwise unadjusted estimate would suggest. The standard deviations of measurement errors are given by

$$\begin{array}{rcc} \sigma(M_n)^2 = & 0.0227+ & 0.0251 \times M_n, \\ & (0.016, 0.001) & (0.0021, 0.0006) \end{array}$$

with QMLE and MLE standard errors for each parameter in parenthesis. Then, standard deviations are \$0.048 and \$0.274 for bonds with one and ten years to maturity, respectively. Using durations of 1 and 7, this translates into standard deviations of 4.8 and 3.9 bps when measured in yields. Overall, parameter estimates and latent factors are relatively unchanged compared to the benchmark model.

We estimate the decay parameter at $\hat{\kappa} = 1.89$ with QMLE and MLE standard errors of 1.23 and 0.45 respectively. Estimates of β are given in Table IV. Note that the level of the liquidity premium increases with maturity.[24] The pattern accords with the observations made from residuals of the model without liquidity. Moreover, Table IIIa shows that the model eliminates most of the systematic differences between on-the-run and off-the-run bonds. There is still some evidence of a systematic difference in the 10-year category where the average error decreases from \$0.31 to \$0.26. We conclude that part of the variations in the 10-year on-the-run premium is not common with variations in other maturity groups. Finally, Table IIIb shows RMSE improvements for almost all maturities while the overall sample RMSE decreases from \$0.188 to \$0.151.

[Table IV about here.]

Figure 5 draw the residual differences within the 12-month and 60-month category, respectively. This is another way to see that the model removes systematic differences between residuals. Overall, the evidence points toward a large common factor driving the liquidity premium of on-the-run U.S. Treasury securities. We interpret this liquidity factor as a measure of the value of funding liquidity to investors. The results below show that its variations also explain a substantial share of the risk premia observed in different interest rate markets.

[23] The benchmark model reached a maximum at 1998.6 while the liquidity model reached a maximum at 3482.6.

[24] The estimated average level is lower in the 10-year group relative to the 5-year and 7-year group. This is due to the lower average age of bonds in this groups.

V Liquidity And Bond Risk Premia

In this section, we present evidence that variations in the value of funding liquidity, as measured from a cross-section of on-the-run premia, share a common components with variations of risk premia in other interest rate markets. In other words, conditions prevailing on the funding market induce an aggregate risk factor that affects each of these markets. Of course, an increase in the liquidity factor necessarily leads to lower excess returns for on-the-run bonds. We show here that it also leads to lower risk premia for off-the-run bonds as well as higher risk premia on LIBOR loans, swap contracts and corporate bonds. Thus, although the payoffs of these assets are not directly related to the higher liquidity of on-the-run securities, their risk premium and, hence, their price, is affected by a common liquidity factor. To summarize, exposure to liquidity risk in the U.S. Treasury funding market carries a substantial price of risk in the cross-section of bond returns. The impact across assets is similar to the often cited "flight-to-liquidity" phenomenon but remains pervasive in normal market conditions. This commonality across liquidity premia accords with a substantial theoretical literature supporting the existence of an economy-wide liquidity premium (Svensson (1985), Bansal and Coleman (1996), Holmtröm and Tirole (1998, 2001), Acharya and Pedersen (2004), Vayanos (2004), Lagos (2006), Brunnermeier and Pedersen (2008), Krishnamurthy and He (2008).). The following section presents our results.[25]

A Off-The-Run U.S. Treasury Bonds

We first document the negative relationship between liquidity and expected excess returns on off-the-run bonds. This is the return, over a given investment horizon, from holding a long maturity bond, in excess of the risk-free rate for that horizon. Figure 3a displays annual excess returns on a 2-year off-the-run bond along with the liquidity factor. The negative relationship is visually apparent throughout the sample but note the sharp variations around the crash of October 1987, the Mexican Peso crisis late in 1994, around the LTCM crisis in August 1998 and until the end of the millennium. At first, this tight link between on-the-run premia and returns from off-the-run Treasury bonds may be surprising. Recall that on-the-run bonds trade at a premium due to their anticipated transaction costs and funding advantages on the cash and repo markets. However, off-the-run bonds can be readily converted into cash via the repo market. This is especially true relative to other asset classes. In that sense, seasoned bonds are close substitutes to on-the-run bonds. Then, the risk premium of all Treasury bonds decreases in periods of high demand for the relative funding liquidity of on-the-run bonds. Longstaff (2004) documents price differences

[25] All the results below are robust to choice of the off-the-run yield curve used to compute excess returns or spreads. Unless otherwise stated we use off-the-run yields from the Svensson, Nelson and Siegel method (Gurkaynak et al. (2006)) available at (http://www.federalreserve.gov/pubs/feds/2007). Using model-implied zero-coupon yields does not affect the results. Also, for ease of interpretation, we standardize each regressor by subtracting its mean and dividing by its standard deviation. For each risk premium regression, the constant corresponds to an estimate of the average risk premium and the coefficient on the liquidity factor measures the impact on expected returns, in basis points, of a one-standard deviation shock to liquidity.

between off-the-run U.S Treasury bonds and Refcorp bonds[26] with similar cash flows. He argues that discounts on Refcorp bond are due to "...the liquidity of Treasury bonds, especially in unsettled markets.".

[Table V about here.]

We test this hypothesis through predictive regressions of off-the-run bond excess returns on the liquidity factor. We use the off-the-run curve from the model to compute excess returns and include term structure factors to control for the information content of forward rates (Fama and Bliss (1987), Campbell and Shiller (1991), Cochrane and Piazzesi (2005a)). The term structure factors spans forward rates but do not suffer from their near-collinearity. Table V presents the results. We consider (annualized) excess returns from holding off-the-run bonds with maturities of 2, 3, 4, 5, 7 and 10 years and for investment horizons of 1, 3, 6, 12, and 24 months. First, Panel (a) presents average risk premia. These range from 153 to 471 bps at one-month horizon and from 69 to 358 bps at annual horizon. These large excess returns are consistent with an average positive term structure slope and with a period of declining interest rates. Panel (b) presents estimates of the liquidity coefficients. The results are conclusive. Estimates are negative and significant at all horizons and maturities. Moreover, the impact of liquidity on excess returns is economically significant. At a one-month horizon a one-standard deviation shock to our measure of funding liquidity lowers expected excess returns obtained from off-the-run bonds by 187 and 571 bps for maturities of two and ten years respectively. At this horizon, R^2 statistics range from 7.34% to 4.23% (see Panel (c)). Regressions based on excess returns at an annual horizon correspond to the cased studied by Cochrane and Piazzesi (2005a) who document the substantial predictability of US Treasury excess returns from forward rates. The impact of funding liquidity is substantial. A one-standard deviation shock decreases expected excess returns by 103 basis points at 2-year maturity and by as much as 358 basis points at 10-year maturity. At this horizon, R^2 are substantially higher, ranging from 43% and 50%. Of course, these coefficients of variation pertain to the joint explanatory power of all regressors. Panel (c) also presents, in bracket, the R^2 of the same regressions but excluding the liquidity factor. The liquidity factor accounts for more or less half of the predictive power of the regressions.

The regressions above used excess returns and term structure factors computed from the term structure model. One concern is that model misspecification leads to estimates of term structure factors that do not correctly capture the information content of forward rates or that it induces spurious correlations between excess returns and liquidity. As a robustness check against both possibilities, we re-examine the predictability regressions but using excess returns and forward rates available from the CRSP zero-coupon yield data set. From this alternative data set, we compute annual excess returns on zero-coupon bonds with maturity from 2 to 5 years. As regressors, we include annual forward rates from CRSP at horizon from 1 to 5 years along with the liquidity factor from the model. Table VIa presents the results. Estimates of the liquidity coefficients are very close

[26]Refcorp is an agency of the U.S. government. Its liabilities have their principals backed with U.S. Treasury bonds and coupons explicitly guaranteed by the U.S. Treasury.

to our previous results (see Table Vb) and highly significant. We conclude that the predictability power of the liquidity factor is robust to how we compute excess returns and forward rates.

[Table VI about here.]

Furthermore, this alternative set of returns allows to check whether the AFENS model captures important aspects of observed excess returns. Table VIb provides results for the regressions of CRSP excess returns on CRSP forward rates, excluding the liquidity factor. This is a replication of the unconstrained regressions in Cochrane and Piazzesi (2005a) but for our shorter sample period. This exercise confirms their stylized predictability results in this sample. That is, the predictive power of forward rates is substantial and we recover a tent-shaped pattern of coefficients across maturities. Next, Table VIc provides results of a similar regressions with CRSP forward rates but using excess returns computed from the *model*. Comparing the last two panels, we see that average excess returns, forward rate coefficients, as well as R^2s are similar across data sets. This is striking given that excess returns were recovered using very different approaches. The AFENS model captures the stylized facts of bond risk premia, which is an important measure of success for term structure models.[27]

The evidence shows that variations of funding liquidity value induce variations in the liquidity premium of Treasury bonds. Empirically, off-the-run US Treasury bonds are viewed as liquid substitutes to their recently issued counterparts and provide a hedge against fluctuations in funding liquidity. Note that this link between conditions on the funding market and the risk premium on a Treasury bond can hardly be attributed to traditional explanations of bond risk premia such as inflation risk or interest rate risk. Instead, we argue that frictions in the financial intermediation sector affect the Treasury market. The following section considers the impact of funding liquidity on LIBOR rates.

B LIBOR Loans

In this section, we link variations of the liquidity factor with variations in the risk compensation from money market loans. We consider the returns obtained from rolling over a lending position in the London inter-bank market at the LIBOR rate and funding this position at a fixed rate. This measures the reward of providing liquidity in the inter-bank market. In contrast with the government bond market, higher valuation of funding liquidity predicts higher excess returns. Figure 3b highlights the positive correlation between liquidity and rolling excess returns. Again, note the spikes in 1987, 1994, in 1998 and around the end of the millennium.

Thus, interbank loans are poor substitutes to U.S. Treasury securities in time of funding stress. The reward for providing funds in the inter-bank market is higher when the relative value of on-

[27] Fama (1984b) originally identified this modeling challenge but see also Dai and Singleton (2002). Other stylized facts are documented in Fama (1976), (1984a), and(1984b), as well as Startz (1982) for maturities below 1 year. See also Shiller (1979), Fama and Bliss (1987), Campbell and Shiller (1991). Our conclusions hold if we use Campbell and Shiller (1991) as a benchmark. We also conclude that the empirical facts highlighted by Cochrane and Piazzesi (2005a) are not an artefact of the bootstrap method. See the discussion in Dai et al. (2004) and Cochrane and Piazzesi (2005b).

the-run bonds increases. Thus, the spread of a LIBOR rate above the Treasury yield reflects the opportunity costs, in terms of future liquidity, of an interbank loan compared to the liquidity of a Treasury bonds on the repo or the cash markets. Indeed, in order to convert a loan back to cash, a bank must enter into a new bilateral contract to borrow money. The search costs of this transaction depend on the number of willing counterparties in the market and it may be difficult at critical times to convert a LIBOR position back to cash.[28]

As in the previous section, we test this hypothesis formally through predictive regressions of excess rolling returns on the liquidity factor. Again, we use term structure factors to control for the information content of forward rates. We consider investment horizons of 1, 3, 6, 12 and 24 months and rolling investments in LIBOR loans with 1, 3, 6 and 12 months to maturity. The LIBOR data is available from the web site of the BBA and we use a sample from January 1987 to December 2007. Table VII presents the results. For each loan maturity, the average excess returns is around 25 bps for the shortest horizon. Returns then decrease with longer horizon and become negative at the longest horizons. This reflects the average positive slope of the term structure. In practice, funding rolling short-term investments at a fixed rate does not produce positive returns on average. Still, the impact of liquidity is unambiguously positive for all horizons and maturities with t-statistics above 5 in most cases.

Interestingly, the impact of the liquidity *increases* with the horizon. A one-standard deviation shock to the value of liquidity increases returns on a rolling investment in one-month LIBOR loans by 16 and 90 bps at horizons 3 and 24 months, respectively. Results are similar for other maturities. In fact, the impact is sufficiently large that returns are positive on average, and the risk premium is higher than the slope of the term structure. This reflects the persistence of the liquidity premium. The R^2 from these regressions range from 30% to 50%. Moreover, the contribution of the liquidity factor to the predictability of LIBOR returns is substantial, generally doubling the R^2, or more. In the case of annual excess rolling returns from 3-month loans, the predictive power increases from 10.8% to 43.2% when we include the liquidity factor.

An alternative indicator of ex-ante returns from investment in the inter-bank market is the simple spread of LIBOR rates above risk-free zero-coupon yields. As an alternative test, we compute LIBOR spreads on loans with maturities of 1, 3, 6 and 12 months and consider regressions of these spreads on the liquidity and term structure factors. Panel (c) shows the positive relationship between liquidity and the 12-month LIBOR spread. Table VIIIa presents results from the regressions. A one-standard deviation shock to liquidity is associated with concurrent increases of 16, 12, 8 and 6 bps for loans with maturity of 1, 3, 6 and 12 months, respectively.

C Swap Spreads

The impact of funding liquidity extends to the swap market. This section documents the link between the liquidity factor and the spread of swap rates above the off-the-run curve. To the extent

[28]Note that this does not preclude that part of the LIBOR spread is due to the higher default risk of the average issuer compared to the U.S. government.

that swap rates are determined by anticipations of future LIBOR rates, results from the previous section suggest that swap spreads increase with the liquidity factor. Moreover, variations in funding liquidity may affect the swap market directly since the same intermediaries operate in the Treasury and the swap markets. We do not distinguish between these alternative channels here.

[Table VIII about here.]

We obtain a sample of swap rates from DataStream, starting in April 1987 and up to December 2007. We focus on swaps with maturities of 2, 5, 7 and 10 years and compute their spreads above the yield to maturity of the corresponding off-the-run par coupon bond. Figure 3d compares the liquidity factor with the 5-year swap spread. The positive relationship is apparent. Table VIIIb shows the results from regressions of swap spreads on the liquidity and term structure factors. First, the average spread rises with maturity, from 44 to 53 bps, and extends the pattern of LIBOR risk premia. Next, estimates of the liquidity coefficients imply that, controlling for term structure factors, a one-standard deviation shock to liquidity raises swap spreads from 5 to 7 basis points across maturities. The estimates are significant, both statically and economically, given the higher price sensitivities of swap to change in yields. For a 5-year swap with duration of 4.5, say, the price impact of a 6 basis point change is \$0.27 while the price impact of the 6.3 bps rate change for a 1-year LIBOR loand is \$0.063.[29] Finally, the explanatory power of liquidity is high and increases with maturity.

Interestingly, funding liquidity affects swap spreads and LIBOR spreads similarly. This suggests that anticipations of liquidity compensation in the interbank loan market, rather than liquidity risk, is the main driver behind the aggregate liquidity component of swap risk premium. This supports previous literature (Grinblatt (2001), Duffie and Singleton (1997), Liu et al. (2006) and Fedlhütter and Lando (2007)) pointing toward LIBOR liquidity premium as an important driver of swap spreads. However, we show that the liquidity risk underlying a substantial part of that premium is not specific to the LIBOR market but reflects risks faced by intermediaries in funding markets.

D Corporate Spreads

The impact of funding liquidity extends to the corporate bond market. This section measures the impact of the liquidity factor on the risk premium offered by corporate bonds. Empirically, we find that the impact of liquidity has a "flight-to-quality" pattern across credit ratings. Following an increase of the liquidity factor, excess returns decrease for the higher ratings but increase for the lower ratings. Our results are consistent with the evidence that default risk cannot rationalize corporate spreads. Collin-Dufresne et al. (2001) find that most of the variations of non-default corporate spreads are driven by a single latent factor. We formally link this factor with funding risk. Our evidence is also consistent with the differential impact of liquidity across ratings found by

[29] We do not use returns on swap investment to measure expected returns. Swap investment requires zero initial investment. Determining the proper capital-at-risk to use in returns computation is somewhat arbitrary. It should be clear from Figure 3d that receiving fixed, and being exposed to short-term LIBOR fluctuations, will provide greater compensation when the liquidity premium is elevated.

Ericsson and Renault (2006). However, while they relate bond spreads to bond-specific measures of liquidity, we document the impact of an aggregate factor in the compensation for illiquidity.

Our analysis begins with Merrill Lynch corporate bond indices. We consider end-of-month data from December 1988 to December 2007 on 5 indices with credit ratings of AAA, AA, A, BBB and High Yield [HY] ratings (i.e. HY Master II index), respectively. In a complementary exercise, below, we use a sample of NAIC transaction data.[30] As in earlier sections, we measure the impact of liquidity on corporate bonds through predictive excess returns regressions. For each index, and each month, we compute returns in excess of the off-the-run zero coupon yield for investment horizons of 1, 3, 6, 12 and 24 months. We then project returns on the liquidity and term structure factors. Again, term structure factors are included to control for the information content of the yield curve. The first Panel of Table IX presents the results.

First, as expected, average excess returns are higher for lower ratings. Next, estimates of the liquidity coefficients show that the impact of a rising liquidity factor is negative for the higher ratings and becomes positive for lower ratings. A one-standard deviation shock to the liquidity factor leads to decreases in excess returns for AAA, AA and A ratings but to increases in excess returns for BBB and HY ratings. Excess returns decrease by 2.27% for AAA index but increase by 2.38% for the HY index. For comparison, the impact on Treasury bonds with 7 and 10 years to maturity was -4.52% and -5.42%. Thus, on average, high quality bonds were considered substitutes, albeit imperfect, to U.S. Treasuries as a hedge against variations in funding conditions. On the other hand, lower-rated bonds were exposed to funding market shocks.

The differential impact of liquidity on excess returns across ratings suggests a flight-to-liquidity pattern. We consider an alternative sample, based on individual bond transaction data from the NAIC. While this sample covers a shorter period, from February 1996 until December 2001, the sample comprises actual transaction data and provides a better coverage of the rating spectrum. Once restricted to end-of-month observations, the sample includes 2,171 transactions over 71 months. To preserve parsimony, we group ratings in five categories.[31] We consider regression of NAIC corporate spreads on the liquidity and term structure factors but we also include the control variables used by Ericsson and Renault (2006). These are the VIX index, the returns on the S&P500 index, a measure of market-wide default risk premium and an on-the-run dummy signalling whether that particular bond was on-the-run at the time of the transaction. Control variables also include the level and the slope of the term structure of interest rates.[32].

The panel regressions of credit spreads for bond i at date t are given by

$$sprd_{i,t} = \alpha + \beta_1 L_t I(G_i = 1) + \cdots + \beta_5 L_t I(G_i = 5) + \gamma_h^T X_t + \epsilon_{i,t} \tag{26}$$

[30] We thank Jan Ericsson for providing the NAIC transaction data and control variables. See Ericsson and Renault (2006) for a discussion of this data set.

[31] Group 1 includes ratings from AAA to A+, group 2 includes ratings A and A-, group 3 includes ratings BBB+, BBB and BBB-, group 4 includes ratings CCC+, CCC and CCC- while group 5 includes the remaining ratings down to C-

[32] We do not include individual bond fixed-effects as our sample is small relative to the number (998) of securities.

where L_t is the liquidity factor and $I(G_i = j)$ is an indicator function equal to one if the credit rating of bond i belongs in group $j = 1, \ldots, 5$. Control variables are grouped in the vector X_{t+h}. Table IXb presents the results. The flight-to-quality pattern clearly emerges from the results. For the highest rating category, an increase in liquidity value of one standard deviation decreases spreads by 31 and 20 basis points in groups 1 and 2 respectively. The effect is smaller and statistically undistinguishable from zero for group 3. Coefficients then become positive implying increases in spreads of 25 and 26 basis points for groups 4 and 5, respectively. This is an average effect through time and across ratings within each group.[33]

The results obtained from spreads computed from Merrill Lynch indices and spreads computed from NAIC transactions differ. While results from Merrill Lynch were inconclusive, estimates of liquidity coefficients obtained from NAIC data confirm that a shock to funding liquidity leads to lower corporate spreads in the highest rating groups but higher corporate spreads in the lowest rating groups. Two important differences between samples may explain the results. First, the composition of the index is different from the composition of NAIC transaction data. The impact of liquidity on corporate spreads may not be homogenous across issues. For example, the maturity or the age of a bond, the industry of the issuer and security-specific option features may introduce heterogeneity. Second, Merrill Lynch indices cover a much longer time span. The pattern of liquidity premia across the quality spectrum may be time-varying.

E Discussion

Focusing on the common component of on-the-run premia filters out local or idiosyncratic demand and supply effects on Treasury bond prices. The results above show that this measure of funding liquidity is an aggregate risk factor affecting money market instruments and fixed-income securities. These assets carry a significant, time-varying and common liquidity premium. That is, when the value of the most-easily funded collateral rises relative to other securities, we observe variations in risk premia for off-the-run U.S. government bonds, eurodollar loans, swap contracts, and corporate bonds. Empirically, the impact of aggregate liquidity on asset pricing appears strongly during crisis and the pattern is suggestive of a flight-to-quality behavior. Nevertheless, its impact is pervasive even in normal times.

Note that these regressions assumed a stable relationship between risk premium and funding liquidity. One important alternative is that the sign and the size of the impact of funding conditions itself depend on the intensity of the funding shock, as suggested by the recent experience. In particular, while corporate bonds with high ratings may be substitutes to Treasury bonds in good times, they experience large risk premium increases in funding crisis. We leave this for further research but note that this may explain the weak statistical evidence above in the case of corporate bonds. In any case, the main result of this section is that a substantial fraction of the risk premia is linked to variations in funding liquidity.

[33] We do not report other coefficients. Briefly, the coefficient on the level factor is negative and significant. All other coefficients are insignificant but these results are are not directly comparable with Ericsson and Renault (2006) due to differences of models and sample frequencies.

Jointly, the evidence is hard to reconcile with theories based on variations of default probability, inflation or interest rates and their associated risk premia. Instead, we link risk premium variations with conditions in the funding markets. This supports the theoretical literature that emphasizes the role of borrowing constraints faced by financial intermediaries (Gromb and Vayanos (2002), He and Krishnamurthy (2007)) and, in particular, that highlights the role of funding markets in financial intermediation (Brunnermeier and Pedersen (2008)). Different securities serve, in part, and to varying degrees, to fulfill investors' uncertain future needs for cash and their risk premium depend on the ability of intermediaries to provide immediacy in each market. In this context, it is interesting that the liquidity premium of government bonds appears to *decrease* when funding liquidity become scarce. This confers a special status to government obligations, and possibly to high-quality corporate bonds, as a hedge against variations in funding liquidity. We leave for further research the cause of this special attribute of government bonds. The next section identifies candidate determinants of liquidity valuation and characterizes aggregate liquidity in terms of known economic indicators.

VI Determinants Of Liquidity Value

The liquidity factor aggregates very diverse economic information. The value of liquidity services on the funding market depends on investors' demand for immediacy on markets where intermediaries are active. Next, funding costs will also vary with the capital position and the access to capital (present and future) of financial intermediaries that obtain leverage through secured loans. Finally, conditions on the funding market are affected by the availability of funds and, thus, by the relative tightness of monetary policy. In this section, we find that the value of funding liquidity, measured by the on-the-run factor, varies with changes in monetary aggregates and in bank reserves. Also, the value of funding liquidity increases with aggregate wealth and aggregate uncertainty as measured by valuation ratios and option-implied volatility of the SP500 stock index. Finally, the on-the-run premium rises when recently issued bonds offers relatively lower bid-ask spreads[34].

A Macroeconomic Variables

Ludvigson and Ng (2009) [LN hereafter] summarize 132 US macroeconomic series into 8 principal components. They then explore parsimoniously the predictive content of this large information set for bond returns. Their main result is that that a "real" and an "inflation" factor[35] have substantial predictive power for bond excess returns beyond the information content of forward rates. They also find that a "financial" factor is significant but that much of its information content is subsumed in the Cochrane-Piazzesi measure of bond risk premium.

Table X displays results from a regression of liquidity on macroeconomic factors (Regression A)

[34] We also considered the Pastor-Stambaugh measure of aggregate stock market liquidity and found no relationship.

[35] Ludvigson and Ng (2009) use univariate regressions of individual series on each principal component to characterize its information content. For example, the "real" factor was labeled as such because it has high explanatory power for real quantities (e.g. Industrial Production).

from LN.[36] This shows that the funding liquidity factor shares tight linkages with the macroeconomy. Macroeconomic factors with significant coefficients are $F1$, $F2$ and $F4$, the "real", "financial" and "inflation" factors of LN that also predict bond risk premium. In addition, factors $F5$, $F6$, and $F7$ are also significant. As we discuss below, $F6$, and $F7$ can be interpreted as "monetary conditions" factors and $F5$ is a "housing activity" factor. Finally, the R^2 is 58% and individual coefficients have similar magnitude.

[Table X about here.]

The "financial" factor relates to different interest rate spreads, which is consistent with the evidence above that the liquidity factor is related to risk premia across markets.[37] The $F6$ and $F7$ factors share a similar and extremely interesting interpretation: these are "monetary conditions" factors. Both have highest explanatory power for the rate of change in reserves and non-borrowed reserves of depository institutions. Next, factor $F6$ has most information for the rate of change of the monetary base and the M1 measure of money stock and some information from the PCE indices. Beyond bank reserves, factor $F7$ is most informative for the spreads of commercial paper and three-month Treasury bills above the Federal Reserve funds rate. This suggests an important channel between monetary policy and the intermediation mechanism and, ultimately, with variations in the valuation of marketwide liquidity. These results are consistent with Longstaff (2004), who establishes a link between variations of RefCorp spreads and measures of flows into money market mutual funds, Longstaff et al. (2005), who document a similar link for the non-default component of corporate spreads and, finally, Chordia et al. (2005), who document that money flows and monetary surprises affect measures of bond market liquidity.

We find that the liquidity factor is also related to the "real", "inflation" factors, indicating that some of the predictability of macro factors for bond risk premium could be measured in funding markets. This may also result from the impact of the Fed's actions on funding markets. The $F5$ is a "housing activity" factor and is also significant. It contains information on housing starts and new building permits. Nonetheless, its significance appears to be limited to the early part of the sample and it is not robust to the inclusion of bid-ask spreads information (see below). Finally, the "real" and "inflation" factors are not robust to the inclusion of stock index implied volatility.

B Transaction Costs Variables

Coupon bond quotes from the CRSP data set include bid and ask prices. At each point in time, we consider the entire cross-section of bonds and compute the difference between the median and the minimum bid-ask spreads. This measures the difference in transaction costs between the most liquid bond and a typical bond. Table X presents the results from a regression of liquidity on this

[36] A significant link between liquidity and one of the principal components of LN does not necessarily require that this component predicts bond excess returns. The liquidity factor is endogenous and its loadings on the underlying macroeconomic variables is unlikely to be linear nor constant through time.

[37] LN found that the information content of the "financial" factor for excess returns is subsumed in the CP factor. Recall from Section A that the information content of the funding liquidity factor is not subsumed by the Cochrane-Piazzesi factor.

measure of relative transaction costs. The coefficient is positive and significant. The liquidity factor increases when the median bid-ask spread moves further away from the minimum spread. That is, on-the-run bonds become more expensive when they offer relatively lower transaction costs. The explanatory power of bid-ask information is substantial, as measured by an R^2 of 37.7%. However, there is a sharp structural break in this relationship. Most of the explanatory power and all of the statistical evidence is driven by observations preceding 1990 as made clear by Figure 7a. The first break in this process coincides with the advent of the GovPX platform while the second break, around 1999, matches the introduction of the eSpeed electronic trading platform. Although transaction costs contribute to the on-the-run premium, the lack of variability since these breaks implies a lesser role in the variations of liquidity on Treasury markets.

C Aggregate Uncertainty

The valuation of liquidity should increase with higher aggregate uncertainty. We use implied volatility from options on the S&P 500 stock index as proxy for aggregate uncertainty. The S&P500 index comprises a large share of aggregate wealth and its implied volatility can be interpreted as a forward looking indicator of wealth volatility. The sample comprises monthly observations of the CBOE VOX index from January 1986 until the end of 2007. Table X presents results from a regression of liquidity on aggregate uncertainty (Regression C). The R^2 is 7.9% and the coefficients is positive but the evidence is statistically weak. Figure 7b shows the measures of volatility and funding liquidity until the end of 2008. Clearly, peaks in volatility are often associated with rises in liquidity valuation. The weak statistical evidence is due to the period around 2002 where very low funding liquidity value was not matched with a proportional decrease of implied volatility. In any case, the coefficient estimate suggests that a one-standard deviation shock to implied volatility raises the liquidity factor by 0.052.

D Combining Regressors

Finally, Table X reports the results from a regression combining all the economic information considered above (Regression D). The coefficient on the relative bid-ask spread decreases but remains significant. On the other hand, the information from the VIX measure is subsumed in other regressors. Its coefficient changes sign and becomes insignificant. In particular, the VIX measure is positively correlated with the stock market factor and this factor's coefficient doubles. Next, the inflation, real and housing activity factor become insignificant. However, the "monetary conditions" factors also remain significant when conditioning on transaction costs and aggregate uncertainty information.

Overall the evidence points toward two broad channels in the determination of the value of funding liquidity. First, similar to the model of Krishnamurthy and He (2008), aggregate uncertainty and aggregate wealth affect the intermediaries' ability to provide liquidity. Second, the Fed implements its monetary policy primarily through the funding market. To some extent, it also support to the stability of the financial system through that channel. Then, these policies, through their impact on funding conditions directly impact risk premium in other markets.

VII The Events Of 2008

We repeat the estimation of the model including data from 2008. Figure 7 presents the liquidity (Panel 8a) and the term structure (Panel 8b)factors. The latter shows a sharp increase in the cross-section of on-the-run premium. In fact, this large shock increases the volatility of the liquidity factor substantially. Looking at Figure 7b and 7a we see that this spike was associated with a large increase in the SP500 implied volatility but, interestingly, the spread between the minimum and median bid-ask spread remained stable. This supports our interpretation that the liquidity factor finds its roots in the funding market.

Adding 2008 only increases the measured impact of the common funding liquidity factor on bond risk premia. Each of the regression above leads to higher estimate for the liquidity coefficient. An interesting case, though, is the behavior of corporate bond spreads. Clearly corporate bond spreads increased sharply over that period, indicating an increase in expected returns. What is interesting is that this was the case for any ratings. Figure 8 compares the liquidity factor with the spread of the AAA and BBB Merrill Lynch index. In the sample excluding 2008, the estimated average impact a shock to funding liquidity was negative for AAA bonds and positive for BBB. The large and positively correlated shock in 2008 reverses this conclusion for AAA bonds. But note that AAA spreads and the liquidity factor were also positively correlated in 1998. This confirms our conjecture that the behavior of high-rating bonds is not stable and depends on the nature or the size of the shock to funding liquidity. Note that this does not affect our conclusion that corporate bond liquidity premium shares a component with other risk premium due to funding risk. Instead, it suggests that the relationship exhibits regimes through time.

VIII Conclusion

We augment the Arbitrage Free Extended Nelson-Siegel term structure model of Christensen et al. (2007) by allowing for a liquidity factor driving the on-the-run premium. Estimation of the model proceeds directly from coupon bond prices using a non-linear filter. We identify from a panel of Treasury bonds a common liquidity factor driving on-the-run premia at different maturities. Its effect increases with maturity and decreases with the age of a bond.

This liquidity factors measures the value of the lower funding and transaction costs of on-the-run bonds. It predicts a substantial share of the risk premium on off-the-run bonds. It also predicts LIBOR spreads, swap spreads and corporate bond spreads. The pattern across interest rate markets and credit ratings is consistent with accounts of flight-to-liquidity events. However, the effect is pervasive in normal times. The evidence points toward the importance of funding liquidity for the intermediation mechanism and, hence, for asset pricing. Our results are robust to changes in data set and to the inclusion of term structure information.

The liquidity factor varies with transaction costs on the secondary bond market. More importantly, we find that the value of liquidity is related to narrow measures monetary aggregates and measures of bank reserves. It also varies with measures of stock market valuations and aggregate uncertainty. The ability of intermediaries to meet the demand for immediacy depends, in part, on funding conditions and induces a large common liquidity premium in key interest rate markets. In particular, our results suggest that the behavior of the Fed is a key determinants of the liquidity premium. It remains to be seen if the impact of aggregate liquidity extends to the risk premium for stocks. In this context, the measure of funding liquidity proposed here can be used as real-time measure of liquidity premia.

IX Appendix

A Data

We use end-of-month prices of U.S. Treasury securities from the CRSP data set. We exclude callable bonds, flower bonds and other bonds with tax privileges, issues with no publicly outstanding securities, bonds and bills with less than 2 months to maturity and observations with either bid or ask prices missing. Our sample covers the period from January 1986 to December 2008. We also exclude the following suspicious quotes.

CRSP ID	Date
#19920815.107250	August 31^{st} 1987
#19950331.203870	December 30^{th}
#19980528.400000	May 30^{th} 1998
#20011130.205870	October 31^{th}
#20041031.202120	November 29^{th} 2002
#20070731.203870	May 31^{st} 2006
#20080531.204870	November 30^{th} 2007

CRSP ID #20040304.400000 has a maturity date preceding its issuance date, as dated by the U.S. Treasury. Finally, CRSP ID #20130815.204250 is never special and is excluded.

B Unscented Kalman Filter

The UKF is based on an approximation to any non-linear transformation of a probability distribution. It has been introduced in Julier et al. (1995) and Julier and Uhlmann (1996) (see Wan and der Merwe (2001) for textbook treatment) and was first imported in finance by Leippold and Wu (2003).

Given $\hat{X}_{t+1|t}$ a time-t forecast of state variable for period $t+1$, and its associated MSE $\hat{Q}_{t+1|t}$ the unscented filter selects a set of Sigma points in the distribution of $X_{t+1|t}$ such that

$$\bar{\mathbf{x}} = \sum_i w^{(i)} x^{(i)} = \hat{X}_{t+1|t}$$
$$\mathbf{Q}_x = \sum_i w^{(i)} (x^{(i)} - \bar{\mathbf{x}})(x^{(i)} - \bar{\mathbf{x}})' = \hat{Q}_{t+1|t}.$$

Julier et al. (1995) proposed the following set of Sigma points,

$$x^{(i)} = \begin{cases} \bar{\mathbf{x}} & i = 0 \\ \bar{\mathbf{x}} + \left(\sqrt{\frac{N_x}{1-w^{(0)}} \sum_x}\right)_{(i)} & i = 1, \ldots, K \\ \bar{\mathbf{x}} - \left(\sqrt{\frac{N_x}{1-w^{(0)}} \sum_x}\right)_{(i-K)} & i = K+1, \ldots, 2K \end{cases}$$

with weights

$$w^{(i)} = \begin{cases} w^{(0)} & i = 0 \\ \frac{1-w^{(0)}}{2K} & i = 1, \ldots, K \\ \frac{1-w^{(0)}}{2K} & i = K+1, \ldots, 2K \end{cases}$$

where $\left(\sqrt{\frac{N_x}{1-w^{(0)}}\sum_x}\right)_{(i)}$ is the i-th row or column of the matrix square root. Julier and Uhlmann (1996) use a Taylor expansion to evaluate the approximation's accuracy. The expansion of $y = g(x)$ around $\bar{x}$ is

$$\begin{aligned}\bar{y} &= E\left[g(\bar{x}+\Delta x)\right] \\ &= g(\bar{x}) + E\left[D_{\Delta x}(g) + \frac{D^2_{\Delta x}(g)}{2!} + \frac{D^3_{\Delta x}(g)}{3!} + \cdots\right]\end{aligned}$$

where the $D^i_{\Delta x}(g)$ operator evaluates the total differential of $g(\cdot)$ when perturbed by Δx, and evaluated at $\bar{x}$. A useful representation of this operator in our context is

$$\frac{D^i_{\Delta x}(g)}{i!} = \frac{1}{i!}\left(\sum_{j=1}^{n}\Delta x_j \frac{\partial}{\partial x_j}\right)^i g(x)\Bigg|_{x=\bar{x}}$$

Different approximation strategies for $\bar{y}$ will differ by either the number of terms used in the expansion or the set of perturbations Δx. If the distribution of Δx is symmetric, all odd-ordered terms are zero. Moreover, we can re-write the second terms as a function of the covariance matrix P_{xx} of Δx,

$$\bar{y} = g(\bar{x}) + \left(\nabla^T P_{xx}\nabla\right) g(\bar{x}) + E\left[\frac{D^4_{\Delta x}(g)}{4!} + \cdots\right]$$

Linearisation leads to the approximation $\hat{\bar{y}}_{lin} = g(\bar{x})$ while the unscented approximation is exact up to the third-order term and the σ-points have the correct covariance matrix by construction. In the Gaussian case, Julier and Ulhmann (1996) show that same-variable fourth moments agree as well and that all other moments are lower than the true moments of Δx. Then, approximation errors of higher order terms are necessarily smaller for the UKF than for the EKF. Using a similar argument, but for approximation of the MSE, Julier and Uhlmann (1996) show that linearisation and the unscented transformation agree with the Taylor expansion up to the second-order term and that approximation errors in higher-order terms are smaller for the UKF.

C Arbitrage-Free Yield Adjustment Term

Christensen et al. (2007) show that the constant, $a(m)$ is given by

$$\begin{aligned}a(m) = &-\sigma_{11}^2\frac{m^2}{6} - (\sigma_{21}^2+\sigma_{22}^2)\left[\frac{1}{2\lambda^2} - \frac{1-e^{-m\lambda}}{m\lambda^3} + \frac{1-e^{-2m\lambda}}{4m\lambda^3}\right] \\ &- (\sigma_{31}^2+\sigma_{32}^2+\sigma_{33}^2) \\ &\quad\times\left[\frac{1}{2\lambda^2} + \frac{e^{-m\lambda}}{\lambda^2} - \frac{me^{-2m\lambda}}{4\lambda} - \frac{3e^{-2m\lambda}}{4\lambda^2} - \frac{2(1-e^{-m\lambda})}{m\lambda^3} + \frac{5(1-e^{-2m\lambda})}{8m\lambda^3}\right] \\ &- (\sigma_{11}\sigma_{21})\left[\frac{m}{2\lambda} + \frac{e^{-m\lambda}}{\lambda^2} - \frac{1-e^{-m\lambda}}{m\lambda^3}\right] \\ &- (\sigma_{11}\sigma_{31})\left[\frac{3e^{-m\lambda}}{\lambda^2} + \frac{m}{-2\lambda} + \frac{me^{-m\lambda}}{\lambda}\right] \\ &- (\sigma_{21}\sigma_{31}+\sigma_{22}\sigma_{32}) \\ &\quad\times\left[\frac{1}{\lambda^2} + \frac{e^{-m\lambda}}{\lambda^2} - \frac{e^{-2m\lambda}}{\lambda^2} - \frac{3(1-e^{-m\lambda})}{m\lambda^3} + \frac{3(1-e^{-2m\lambda})}{4m\lambda^3}\right].\end{aligned}$$

References

Acharya, V. V. and L. Pedersen (2004). Asset pricing with liquidity risk. *NBER* (W10814).

Adrian, T., E. Etula, and H. Shin (2009). Risk appetite and exchange rates. Working Paper.

Adrian, T. and H. Shin (2008). Liquidity and leverage. *Journal of Financial Intermediation*, forthcoming.

Amihud, Y. (2002). Illiquidity and stock returns: cross-section and time-series effects. *Journal of Financial Markets 5*, 31–56.

Amihud, Y. and H. Mendelson (1986). Asset pricing and the bid-ask spreads. *Journal of Financial Economics 17*, 223–249.

Amihud, Y. and H. Mendelson (1991). Liquidity, maturity and the yields on U.S. treasury securities. *The Journal of Finance 46*, 1411–1425.

Anderson, N., F. Breedon, M. Deacon, A. Derry, and G. Murphy (1996). *Estimating and interpreting the yield curve.* Series in Financial Economics and Quantitative Analysis. John Wiley & Sons.

Bakshi, G., P. Carr, and L. Wu (2005). Stochastic risk premiums, stochastic skewness in currency options, and stochastic discount factors in international economies. *Journal of Financial Economics*. forthcoming.

Bansal, R. and W. Coleman (1996). A monetary explanation of the term premium, equity premium and risk-free rate puzzle. *The Journal of Political Economy 104*, 396–409.

Bliss, R. (1997). Testing term structure estimation methods. *Advances in Futures and Options Research 9*, 197–231.

Brunnermeier, M. K. and L. Pedersen (2008). Market liquidity and funding liquidity. *Review of Financial Studies*, forthcoming.

Buraschi, A. and D. Menini (2002). Liquidity risk and specialness. *Journal of Financial Economics 64*, 243–284.

Campbell, J. and R. Shiller (1991). Yield spreads and interest rate movements: A bird's eye view. *Review of Economic Studies 58*, 495–514.

Cheria, J., E. Jacquier, and R. Jarrow (2004). A model of the convenience yields in on-the-run treasuries. *Review of Derivatives Research 7*, 79–97.

Chordia, T., A. Sarkar, and A. Subrahmanyam (2005). An empirical analysis of stock and bond market liquidity. *Review of Financial Studies 18*, 85–129.

Christensen, J., F. Diebold, and G. Rudebusch (2007). The affine arbitrage-free class of nelson-siegel term structure models. Technical Report 20, FRBSF.

Christoffersen, P., K. Jacobs, L. Karoui, and K. Mimouni (2007). Estimating term structure modesl using swap rates. McGill University.

Cochrane, J. and M. Piazzesi (2005a). Bond risk premia. *American Economic Review 95*, 138–160.

Cochrane, J. and M. Piazzesi (2005b). Reply to Dai, Singleton and Yang.

Collin-Dufresne, P., R. Goldstein, and J. Spencer Martin (2001). The determinants of credit spread changes. *The Journal of Finance 56*, 2177–2207.

Dai, Q. and K. Singleton (2000). Specification analysis of affine term structure models. *The Journal of Finance 55*, 1943–1978.

Dai, Q. and K. Singleton (2002). Expectation puzzles, time-varying risk premia and affine model of the term structure. *Journal of Financial Economics 63*, 415–441.

Dai, Q., K. Singleton, and W. Yang (2004). Predictability of bond risk premia and affine term structure models. University of Chicago.

Diebold, F. and C. Li (2006). Forecasting the term structure of government bond yields. *Journal of Econometrics 130*, 337–364.

Diebold, F., G. Rudebusch, and S. Aruoba (2006). The macroeconomy and the yield curve : A dynamic latent factor approach. *Journal of Econometrics 131*, 309–338.

Duffee, G. (2002). Term premia and interest rate forecast in affine model. *The Journal of Finance 57*, 405–443.

Duffie, D. (1996). Special repo rates. *The Journal of Finance 51*, 493–526.

Duffie, D., N. Gârleanu, and L. Pedersen (2005). Over-the-counter markets. *Econometrica 73*, 1815–1847.

Duffie, D. and R. Kan (1996). A yield-factor model of interest rates. *Mathematical Finance 6*, 379–406.

Duffie, D. and K. Singleton (1997). An econometric model of the term structure of interest-rate swap yields. *The Journal of Finance 52*, 1287–1321.

Elton, E. and T. Green (1998). Tax and liquidity effects in pricing governement bonds. *The Journal of Finance 53*, 1533–1562.

Ericsson, J. and O. Renault (2006). Liquidity and credit risk. *The Journal of Finance 61*, 2219–2250.

Fama, E. (1976). Forward rates as predictors of future spot rates. *Journal of Financial Economics 3*, 361–377.

Fama, E. (1984a). The information in the term structure. *Journal of Financial Economics 13*, 509–528.

Fama, E. (1984b). Term premium in bond returns. *Journal of Financial Economics 13*, 529–546.

Fama, E. and R. Bliss (1987). The information in long-maturity forward rates. *American Economic Review 77*, 680–692.

Fedlhütter, P. and D. Lando (2007). Decomposing swap spreads. Copenhagen Business School.

Froot, K. and P. O'Connell (2008). On the pricing of intermediated risks: Theory and application to catastrophe reinsurance. *Journal of Banking and Finance 32*, 69–85.

Gabaix, X., A. Krishnamurthy, and O. Vigneron (2009). Limits of arbitrage: Theory and evidence from the mortgage-backed securities market. *The Journal of Finance*. Forthcoming.

Gârleanu, N., L. Pedersen, and A. Pothesman (2009). Demand-based option pricing. *Review of Financial Studies*. Forthcoming.

Goldreich, D., B. Hanke, and P. Nath (2005). The price of future liquidity : Time-varying liquidity in the U.S. treasury market. *Review of Finance 9*, 1–32.

Green, R. and B. Ødegaard (1997). Are there tax effects in the relative pricing of the U.S. government bonds? *The Journal of Finance 52*, 609–633.

Grinblatt, M. (2001). An analytic solution for interest rate swap spreads. *International Review of Finance 2*, 113–149.

Gromb, D. and D. Vayanos (2002). Equilibrium and welfare in markets with financially constrained arbitrageurs. *Journal of Financial Economics 66*, 361–407.

Gurkaynak, R., B. Sack, and J. Wright (2006). The u.s. treasury curve: 1961 to present. Technical Report 2006-28.

He, Z. and A. Krishnamurthy (2007). Intermediated asset prices. Technical report.

Holmström, B. and J. Tirole (1998). Private and public supply of liquidity. *The Journal of Political Economy 106*, 1–40.

Jordan, B. and S. Jordan (1997). Special repo rates: An empirical analysis. *The Journal of Finance 52*, 2051–2072.

Julier, S. and J. Uhlmann (1996). A general method for approximating nonlinear transformations of probability distributions. Robotics Research Group, University of Oxford.

Julier, S., J. Uhlmann, and H. Durrant-Whyte (1995). A new approach for filtering nonlinear systems. *The Proceedings of the American Control Conference 3*, 1628–1632. Seattle.

Kiyotaki, N. and R. Wright (1989). On money as a medium of exchange. *The Journal of Political Economy 97*, 927–954.

Krishnamurthy, A. (2002). The bond/old-bond spread. *Journal of Financial Economics 66*, 463–506.

Krishnamurthy, A. and Z. He (2008). Intermediary asset prices. W14517.

Krishnamurthy, A. and J. Vissing-Jorgensen (2007). The demand for treasury debt. W12881.

Kyle, A. and W. Xiong (2001). Contagion as a wealth effect. *The Journal of Finance 56*, 1401–1440.

Lagos, R. (2006). Asset prices and liquidity in an exchange economy. Technical Report 373, Federal Reserve Bank of Minneapolis.

Leippold, M. and L. Wu (2003). Design and estimation of multi-currency quadratic models. *European Finance Review 7*, 47–73.

Liu, J., F. Longstaff, and R. Mandell (2006). The market price of risk in interest rate swaps: The roles of default and liquidity risks. *Journal of Business 79*, 2337–2359.

Longstaff, F. (2000). The term structure of very short-term rates : New evidence for the expectation hypothesis. *Journal of Financial Economics 58*, 397–415.

Longstaff, F. (2004). The flight-to-liquidity premium in U.S. treasury bond prices. *The Journal of Business 77*, 511–526.

Longstaff, F., S. Mithal, and E. Neis (2005). Corporate yield spreads: Default risk or liquidity? new evidence from the credit-default swap market. *The Journal of Finance 60*, 2213–2253.

Ludvigson, S. and S. Ng (2009). Macro factors in bond risk premia. *Review of Financial Studies*. forthcoming.

Luttmer, E. (1996). Asset pricing in economies with frictions. *Econometrica 64*, 1439–1467.

Nashikkar, A. and M. Subrahmanyam (2006). Latent liquidity and corporate bond yield spreads. Stern School of Business.

Nelson, C. and A. Siegel (1987). Parsimonious modeling of yield curves. *The Journal of Business 60*, 473–489.

Pastor, L. and R. Stambaugh (2003). Liquidity risk and expected stock returns. *The Journal of Political Economy 111*, 642–685.

Piazzesi, M. (2005). *Affine term structure models.* in Handbook of Financial Econometrics. Elsevier.

Shi, S. (2005). Nominal bonds and interest rates. *International Economic Review 46*, 579–612.

Shi, S. (2006). Viewpoint: a microfoundation of monetary economics. *Canadian Journal of Economics 39*, 643–688.

Shiller, R. (1979). The volatility of long-term interest rates and expectation models of the term structure. *The Journal of Political Economy 87*, 1190–1219.

Shleifer, A. and R. Vishny (1997). The limits of arbitrage. *The Journal of Finance 52*, 35–55.

Startz, R. (1982). Do forecast errors of term premia really make a difference between long and short rates. *Journal of Financial Economics 10*, 323–329.

Svensson, L. (1985). Money and asset prices in a cash-in-advance economy. *The Journal of Political Economy 93*, 919–944.

Tauchen, G. and H. Zhou (2006). Realized jumps on financial markets and predicting credit spreads. Technical Report 2006-35, Federal Reserve Board.

Vayanos, D. (2004). Flight to quality, flight to liquidity, and the pricing of risk. *NBER* (W10327).

Vayanos, D. and P.-O. Weill (2006). A search-based theory of the on-the-run phenomenon. *The Journal of Finance*. forthcoming.

Wan, E. and R. V. der Merwe (2001). *Kalman filtering and neural network*, Chapter The unscented Kalman filter. Wiley and Sons.

Warga, A. (1992). Bond returns, liquidity and missing data. *The Journal of Financial and Quantitative Analysis 27*, 605–617.

Table I: Summary statistics of bond characteristics

We present summary statistics of age (in months), duration (in months) and coupon (in %) for each maturity and liquidity category. *New* refers to the on-the-run security and *Old* refers to the off-the-run security (see text for details). In each case, the first column gives the sample mean and the second column gives the sample standard deviations. Coupon statistics are not reported for maturity categories of 12 months and less as T-bills do not pay coupons. End-of-month data from CRSP (1985:12-2007:12).

	Age				Duration				Coupon			
	Old		New		Old		New		Old		New	
Maturity												
3	12.01	9.31	1.64	0.09	3.01	0.03	4.38	0.09				
6	16.93	6.27	0.12	0.11	6.00	0.10	5.90	0.11				
9	14.45	6.05	4.42	4.88	8.89	0.11	10.00	0.40				
12	13.11	5.78	2.51	3.90	111.77	0.23	12.14	1.08				
18	28.29	11.92	6.74	0.62	17.14	0.50	16.81	0.59	7.12	2.81	6.84	3.06
24	22.90	13.45	0.33	0.52	22.56	0.59	22.68	0.72	7.11	2.82	6.74	3.13
36	24.64	10.17	4.61	6.74	32.56	1.42	32.75	2.63	7.49	2.96	7.10	2.82
48	18.42	9.57	4.42	3.00	41.95	2.30	44.17	4.40	7.38	3.03	7.25	2.86
60	29.06	21.58	2.29	3.85	50.41	3.09	51.36	3.02	7.72	2.80	7.13	2.90
84	34.41	8.61	12.51	11.82	65.85	5.04	68.71	8.45	7.74	2.56	7.55	2.63
120	14.91	18.59	4.02	7.56	84.43	8.34	85.55	9.16	7.15	2.24	7.44	2.71

Table II: Parameter estimates of transition equations.

Panel (a) presents estimation results for the AFENS model without liquidity. Panel (b) presents estimation results for the AFENS model with liquidity. For each parameter, the first standard error (in parenthesis) is computed from the QMLE covariance matrix (see Equation 23) while the second is computed from the outer product of scores (see Equation 25). End-of-month data from CRSP (1985:12-2007:12).

(a)

	$\bar{F}$	K	Σ $(\times 10^2)$		
	0.0545	0.169	0.68		
Level	(0.0136)	(0.177)	(0.42)		
	(0.0093)	(0.069)	(0.03)		
	-0.0172	0.182	0.76	0.84	
Slope	(0.0277)	(0.088)	(0.75)	(0.46)	
	(0.013)	(0.071)	(0.06)	(0.04)	
	-0.0128	0.891	-0.14	0.41	2.31
Curvature	(0.0061)	(0.860)	(1.86)	(1.64)	(0.66)
	(0.0045)	(0.283)	(0.15)	(0.17)	(0.13)

(b)

	$\bar{F}$	K	Σ $(\times 10^2)$		
	0.0576	0.198	0.85		
Level	(0.0165)	(0.165)	(0.86)		
	(0.0154)	(0.098)	(0.02)		
	-0.0167	0.222	-0.81	0.85	
Slope	(0.0092)	(0.293)	(0.85)	(0.44)	
	(0.0165)	(0.145)	(0.06)	(0.05)	
	-0.0189	0.887	0.57	0.25	2.27
Curvature	(0.0057)	(1.414)	(0.82)	(1.91)	(1.66)
	(0.0088)	(0.325)	(0.13)	(0.20)	(0.12)

	$\bar{L}$	ϕ_l	σ_l
	0.32	0.955	0.06
Liquidity	(0.42)	(0.034)	(0.066)
	(0.09)	(0.021)	(0.011)

Table III: Mean Pricing Errors and Root Mean Squared Pricing Errors

Panel (a) presents MPE and Panel (b) presents RMSPE from AFENS models with and without liquidity. The columns correspond to liquidity category where New refers to on-the-run issues and Old refers to off-the-run issues. End-of-month data from CRSP (1985:12-2007:12).

(a) Mean Pricing Errors

Mean Pricing Errors				
	Benchmark Model		Liquidity Model	
Maturity	Old	New	Old	New
3	0.009	0.032	-0.010	0.001
6	-0.003	0.022	0.009	-0.011
9	-0.035	0.024	-0.008	0.016
12	-0.043	0.035	-0.015	0.029
18	-0.057	-0.054	-0.010	-0.004
24	-0.022	0.000	0.007	$4e^{-5}$
36	0.001	0.068	-0.020	0.013
48	-0.002	0.082	-0.060	0.018
60	-0.010	0.177	0.021	0.034
84	-0.080	0.014	0.024	-0.021
120	-0.402	0.249	-0.075	0.104
All	-0.058	0.059	-0.011	0.016

(b) Root Mean Squared Errors

Root Mean Squared Pricing Errors				
	Benchmark Model		Liquidity Model	
Maturity	Old	New	Old	New
3	0.048	0.060	0.037	0.031
6	0.036	0.046	0.033	0.030
9	0.055	0.060	0.039	0.054
12	0.076	0.081	0.052	0.073
18	0.091	0.088	0.048	0.048
24	0.069	0.094	0.053	0.082
36	0.105	0.138	0.103	0.109
48	0.199	0.200	0.184	0.134
60	0.234	0.271	0.231	0.179
84	0.363	0.301	0.276	0.238
120	0.710	0.500	0.290	0.413
All	0.264	0.216	0.157	0.167

Table IV: On-the-run Premium

Each line corresponds to a maturity category (months). The first two columns provide the average of residual differences in each category for the AFENS model with and without maturity, respectively. The last three columns display estimates of the liquidity level, $\hat{\beta}$, followed by standard errors (in parenthesis). The first standard error is computed from the QMLE covariance matrix (see Equation 23) while the second is computed from the outer product of scores. (see Equation 24). End-of-month data from CRSP (1985:12-2008:12).

Maturity	Redidual Differences		$\hat{\beta}$	Standard Error	
	Benchmark	Liquidity		QMLE	MLE
3	0.0111	-0.0053	0.2642	0.030411409	0.023289472
6	0.0221	-0.0295	0.2837	0.032610756	0.027397712
9	0.0566	0.0202	0.3158	0.03709391	0.033157944
12	0.0783	0.0396	0.3026	0.036220666	0.033527915
18	0.0025	-0.0036	0.0428	0.024812485	0.035277936
24	0.028	-0.0117	0.2005	0.032073934	0.035007491
36	0.0644	-0.026	0.5325	0.073912661	0.084293764
48	0.0892	0.0165	0.7446	0.094527023	0.088060245
60	0.2477	0.0102	1.227	0.136949189	0.119759631
84	0.125	-0.0509	1.2174	0.102685574	0.097803022
120	0.3106	0.264	1	-	-

Table V: Results from off-the-run excess returns regressions

Results from predictive regression,

$$xr_{t+h}^{(m)} = \alpha_h^{(m)} + \delta_h^{(m)} L_t + \beta_h^{(m)T} F_t + \epsilon_{(t+h)}^{(m)},$$

the liquidity, L_t, and term structure factors, F_t, from the AFENS model where $xr_{t+h}^{(m)}$ is the excess returns at horizon h (months) on a bond of maturity m (years). Regressors are demeaned and divided by its standard deviation. Panel (a) contains estimates of α and Panel (b) contains estimates of δ with t-statistics based on Newey-West standard errors (h+3 lags) in parenthesis. Panel (c) presents R^2 of including or excluding [in brackets] the liquidity factor. End-of-month data from CRSP (1985:12-2007:12).

(a) Average risk premia

	Bond Maturity											
Horizon	2		3		4		5		7		10	
1	1.53	(7.07)	2.09	(11.17)	2.59	(15.00)	3.03	(18.53)	3.80	(24.86)	4.71	(33.49)
3	1.36	(4.17)	1.90	(6.64)	2.39	(8.89)	2.83	(10.89)	3.57	(14.36)	4.44	(18.90)
6	1.10	(2.67)	1.63	(4.38)	2.10	(5.89)	2.51	(7.22)	3.21	(9.53)	3.99	(12.53)
12	0.69	(1.37)	1.21	(2.59)	1.66	(3.62)	2.07	(4.50)	2.78	(6.03)	3.58	(8.07)
24	0.00	(0.00)	0.61	(0.96)	1.11	(1.67)	1.56	(2.20)	2.34	(2.94)	3.26	(3.78)

(b) Liquidity Coefficients

	Bond Maturity											
Horizon	2		3		4		5		7		10	
1	-1.39	(-2.49)	-2.27	(-2.53)	-3.01	(-2.47)	-3.61	(-2.39)	-4.52	(-2.27)	-5.42	(-2.07)
3	-1.35	(-3.28)	-2.12	(-3.14)	-2.74	(-2.97)	-3.23	(-2.84)	-3.98	(-2.64)	-4.70	(-2.34)
6	-1.25	(-4.67)	-2.00	(-4.51)	-2.59	(-4.29)	-3.07	(-4.09)	-3.84	(-3.75)	-4.69	(-3.26)
12	-0.85	(-5.47)	-1.63	(-5.63)	-2.24	(-5.63)	-2.73	(-5.57)	-3.44	(-5.18)	-4.08	(-4.15)
24	0.00	(0.00)	-0.53	(-3.24)	-0.91	(-3.23)	-1.17	(-3.27)	-1.51	(-3.29)	-1.75	(-2.91)

(c) R^2

	Bond Maturity											
Horizon	2		3		4		5		7		10	
1	4.74	[2.28]	4.65	[2.02]	4.51	[1.95]	4.34	[1.93]	4.03	[1.92]	3.55	[1.89]
3	13.56	[6.84]	13.33	[6.83]	13.07	[7.03]	12.78	[7.18]	12.07	[7.17]	10.52	[6.57]
6	24.23	[10.34]	24.50	[11.21]	24.57	[12.26]	24.61	[13.11]	24.44	[14.10]	22.92	[14.02]
12	35.36	[11.23]	37.71	[12.66]	39.24	[14.96]	40.32	[17.20]	41.46	[21.00]	40.54	[24.42]
24	0.00	[0.00]	35.53	[16.92]	31.91	[13.91]	29.46	[11.92]	26.56	[10.32]	25.82	[12.69]

Table VI: Off-the-run excess returns and funding liquidity

Results from the regressions,

$$xr^{(m)}_{t+12} = \alpha^{(m)} + \delta^{(m)} L_t + \beta^{(m)T} f_t + \epsilon^{(m)}_{(t+12)},$$

where $xr^{(m)}_{t+h}$ is the annual excess returns on a bond with maturity m (years), L_t is the liquidity factor and f_t is a vector of annual forward rates $f^{(h)}_t$ from 1 to 5 years. Regressors are demeaned and divided by their standard deviations. Panel (a) presents results using returns and forward rates directly from CRSP data but with the liquidity factor from the model. Panel (b) excludes the liquidity factor. Panel (c) excludes the liquidity factor and uses excess returns from the model. Newey-West t-statistics (in parenthesis) with 15 lags. End-of-month data from CRSP (1985:12-2007:12).

(a) Excess returns and forward rates from Fama-Bliss data with the liquidity factor

Maturity	cst	$f^{(1)}_t$	$f^{(2)}_t$	$f^{(3)}_t$	$f^{(4)}_t$	$f^{(5)}_t$	L_t	R^2
2	0.72	0.29	-1.31	1.88	0.93	-0.95	-0.78	41.65
	(3.49)	(0.49)	(-1.18)	(1.50)	(1.04)	(-1.60)	(-5.97)	
3	1.31	0.15	-2.26	4.32	0.76	-1.49	-1.55	41.66
	(3.41)	(0.14)	(-1.13)	(1.89)	(0.48)	(-1.27)	(-5.93)	
4	1.79	-0.51	-1.74	4.58	1.53	-1.85	-2.18	42.82
	(3.53)	(-0.35)	(-0.66)	(1.51)	(0.75)	(-1.13)	(-6.07)	
5	1.98	-1.51	-0.24	4.57	0.36	-0.81	-2.66	40.87
	(3.23)	(-0.84)	(-0.07)	(1.24)	(0.15)	(-0.39)	(-5.83)	

(b) Excess returns and forward rates from Fama-Bliss data

Maturity	cst	$f^{(1)}_t$	$f^{(2)}_t$	$f^{(3)}_t$	$f^{(4)}_t$	$f^{(5)}_t$	L_t	R^2
2	0.72	-0.43	-1.34	2.66	0.99	-1.53		21.04
	(2.95)	(-0.57)	(-1.06)	(1.50)	(0.95)	(-2.13)		
3	1.31	-1.27	-2.33	5.86	0.88	-2.64		19.29
	(2.87)	(-0.87)	(-1.04)	(1.77)	(0.46)	(-1.86)		
4	1.79	-2.52	-1.83	6.74	1.70	-3.46		19.86
	(2.95)	(-1.26)	(-0.62)	(1.51)	(0.67)	(-1.76)		
5	1.98	-3.96	-0.35	7.20	0.56	-2.79		18.27
	(2.71)	(-1.65)	(-0.10)	(1.35)	(0.19)	(-1.14)		

(c) Excess returns from the model and forward rates from Fama-Bliss data

Maturity	cst	$f^{(1)}_t$	$f^{(2)}_t$	$f^{(3)}_t$	$f^{(4)}_t$	$f^{(5)}_t$	L_t	R^2
2	0.66	-0.13	-1.91	2.97	0.93	-1.51		21.10
	(2.71)	(-0.17)	(-1.53)	(1.69)	(0.91)	(-2.09)		
3	1.27	-1.15	-2.04	4.97	1.19	-2.43		18.19
	(2.82)	(-0.79)	(-0.90)	(1.50)	(0.63)	(-1.73)		
4	1.74	-2.46	-1.26	6.09	1.18	-2.92		17.22
	(2.83)	(-1.24)	(-0.41)	(1.34)	(0.46)	(-1.47)		
5	2.09	-3.86	0.00	6.62	1.06	-3.12		17.15
	(2.80)	(-1.61)	(0.00)	(1.20)	(0.34)	(-1.26)		

Table VII: LIBOR rolling excess returns and funding liquidity

Results from the regressions,

$$xr_{t+h}^{(m)} = \alpha_h^{(m)} + \delta_h^{(m)} L_t + \beta_h^{(m)T} F_t + \epsilon_{(t+h)}^{(m)},$$

where $xr_{t+h}^{(m)}$ is the returns at time $t+h$ (months) on rolling investment in loans with maturity m (months), L_t is the liquidity factor and F_t is the vector of term structure factor. Each regressor is demeaned and divided by its standard deviation for interpretation. Panel (a) contains estimates of average returns. Panel (b) contains estimates of $\delta_h^{(m)}$. Newey-West t-statistics (h+3 lags) in parenthesis. Panel (c) presents R^2 from the regressions including and excluding [in brackets] the liquidity factor. End-of-month data from CRSP (1985:12-2007:12).

(a) Average Excess Returns

	Loan Maturity							
Horizon	1		3		6		12	
1	0.277	(0.347)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)
3	0.183	(0.248)	0.265	(0.245)	0.000	(0.000)	0.000	(0.000)
6	0.062	(0.322)	0.144	(0.264)	0.239	(0.165)	0.000	(0.000)
12	-0.153	(0.615)	-0.070	(0.560)	0.029	(0.439)	0.253	(0.151)
24	-0.537	(1.120)	-0.453	(1.079)	-0.351	(0.985)	-0.120	(0.743)

(b) Liquidity Coefficients

	Loan Maturity							
Horizon	1		3		6		12	
1	0.184	(7.837)	0.000	(0.000)	0.000	(0.000)	0.000	(0.000)
3	0.162	(7.853)	0.149	(6.364)	0.000	(0.000)	0.000	(0.000)
6	0.193	(6.139)	0.173	(6.985)	0.101	(5.699)	0.000	(0.000)
12	0.360	(5.700)	0.340	(6.364)	0.277	(7.329)	0.076	(3.695)
24	0.732	(5.578)	0.715	(5.909)	0.664	(6.395)	0.526	(7.366)

(c) R^2

	Loan Maturity							
Horizon	1		3		6		12	
1	46.4	[28.0]	0.0	[0.0]	0.0	[0.0]	0.0	[0.0]
3	44.7	[16.8]	50.6	[26.5]	0.0	[0.0]	0.0	[0.0]
6	24.7	[1.4]	30.7	[2.9]	44.8	[20.4]	0.0	[0.0]
12	29.2	[7.1]	30.3	[6.6]	32.3	[6.7]	35.2	[18.6]
24	38.8	[12.3]	38.9	[11.7]	39.4	[11.2]	41.2	[10.1]

Table VIII: LIBOR and swap spreads and funding liquidity

Results from regressions,

$$sprd_t^{(m)} = \alpha^{(m)} + \delta^{(m)} L_t + \beta^{(m)T} F_t + \epsilon_{(t)}^{(m)},$$

where $sprd_t^{(m)}$ is the spread at time t and for maturity m (months), L_t is the liquidity factor and F_t is the vector of term structure factor. Spreads are computed above the off-the-run U.S. Treasury yield curve and we use par yields to compute swap spreads. Each regressors is demeaned and divided by its standard deviation. Panel (a) presents results for LIBOR spreads. Panel (b) presents results for swap spreads. Newey-West t-statistics (3 lags) in parenthesis. Finally, R^2 from regressions including and excluding [in brackets] the liquidity factor. End-of-month data from CRSP (1985:12-2007:12).

(a) LIBOR Spreads

	1 month		3 months		6 months		12 months	
Avg Spread	0.423	(0.027)	0.422	(0.023)	0.406	(0.019)	0.429	(0.019)
$\delta_m^{(h)}$	0.183	(6.463)	0.153	(5.939)	0.106	(5.166)	0.080	(4.410)
R^2	58.4	[44.9]	59.4	[47.8]	53.2	[42.2]	53.9	[37.7]

(b) Swap Spreads

	24		60		84		120	
Avg. Spread	0.384	(0.016)	0.483	(0.018)	0.477	(0.019)	0.432	(0.020)
$\delta_m^{(h)}$	0.094	(4.556)	0.104	(4.525)	0.107	(4.395)	0.095	(3.917)
R^2	37.8	[35.4]	38.0	[34.2]	45.5	[38.6]	51.7	[38.5]

Table IX: Corporate bond excess returns and funding liquidity

Results from the regressions

$$y_t = \alpha_h^{(r)} + \delta_h^{(r)} L_t + \beta_h^{(r)T} F_t + \epsilon_{(t+h)}^{(r)},$$

where y_t is either a spread, $sprd_t^r$, observed a time t for rating r or an excess returns, $xr_{t+h}^{(r)}$ over an horizon h (months) on an investment the Corporate index with rating r, L_t is the liquidity factor and F_t is the vector of term structure factor. See Equation 26 for the spread panel specification. Panel (b) presents results for corporate spreads. Panel (a)) presents results for excess returns. Individual corporate bond yields are obtained from NAIC. Corporate bond returns are computed using Merrill Lynch indices obtained from Bloomberg. Spreads and excess returns are computed above the Treasury off-the-run yield curve. Each regressor is demeaned and divided by its standard deviation for interpretation. Newey-West t-statistics in parenthesis and R^2 from regressions including and excluding [in brackets] the liquidity factor. Results from Merrill Lynch indices cover the entire sample. Results from NAIC corporate bond yields is monthly from February 1996 until December 2001.

(a) Merrill Lynch Indices Excess Returns

	AAA		AA		A		BBB		HY	
Avg. Spread	3.162	(15.502)	3.130	(15.291)	3.162	(15.618)	3.204	(16.196)	3.785	(23.400)
$\delta_m^{(G)}$	-1.775	(-1.396)	-1.626	(-1.341)	-1.154	(-0.913)	0.073	(0.057)	3.117	(1.461)
R^2	4.5	[3.7]	4.9	[4.2]	4.4	[4.1]	3.4	[3.4]	6.3	[5.2]

(b) NAIC Corporate Spreads

	G1		G2		G3		G4		G5	
Avg.	1.51	(0.19)	1.65	(0.21)	2.25	(0.30)	3.38	(0.59)	3.70	(0.54)
$\delta_m^{(G)}$	-0.31	(-2.98)	-0.20	(-1.96)	-0.04	(-0.34)	0.25	(2.29)	0.26	(2.47)
R^2	3.9	[2.0]	5.7	[2.0]	6.5	[2.0]	7.0	[2.0]	7.5	[2.0]

(c) Merrill Lynch Spread Indices

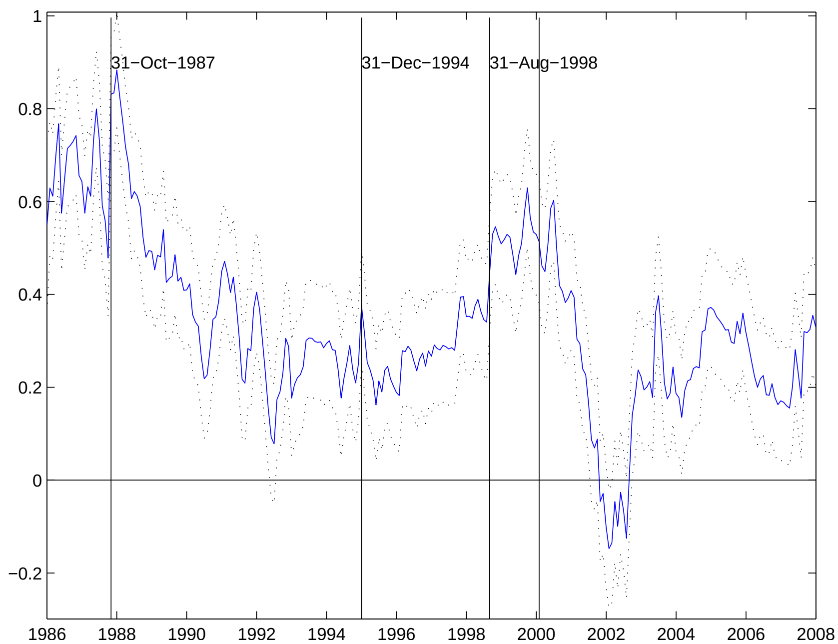
	Aaa		Aa		A		Baa		HY	
Avg. Spread	0.930	(0.036)	0.976	(0.049)	1.227	(0.046)	1.856	(0.077)	5.385	(0.270)
$\delta_m^{(h)}$	0.065	(2.294)	0.060	(1.188)	0.073	(1.268)	0.119	(1.379)	0.334	(1.168)
R^2	59.5	[55.5]	31.4	[29.6]	39.6	[34.9]	49.7	[42.7]	39.2	[29.9]

Table X: Macroeconomic determinants

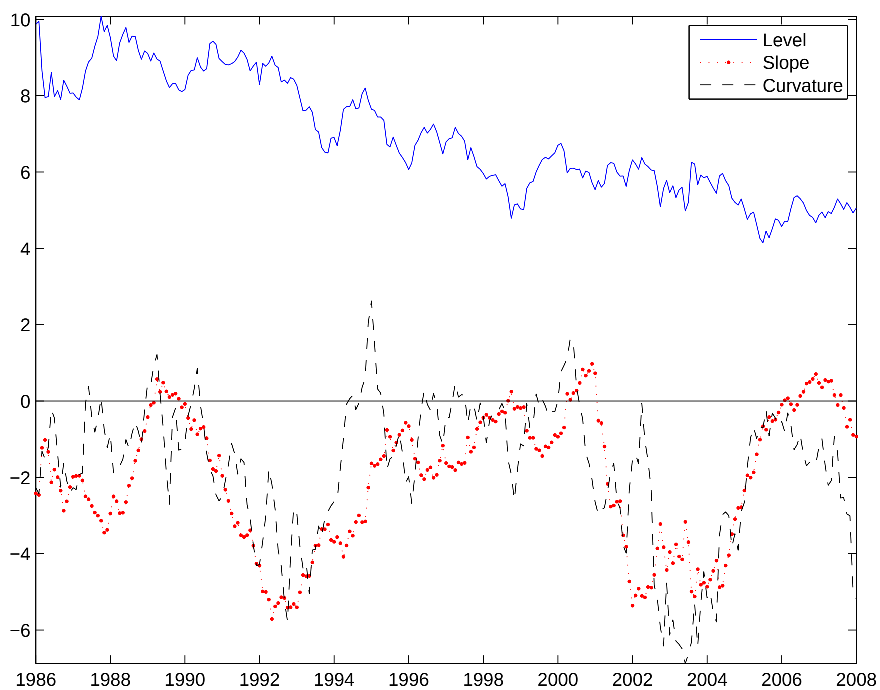
Results from regressions of the liquidity factor on selected economic variables. BA is the difference between the minimum and the median bid-ask spreads across bonds on any given date. VXO is the implied volatility from S&P500 call options. $F1$ to $F8$ are principal components of macroeconomic series from Ludvigson and Ng (2009). Newey-West standard errors (3 lags) are included in parenthesis. End-of-month data (1986:01-2004:12).

	Regressors											
Model	cst	BA	VXO	$F1$	$F2$	$F3$	$F4$	$F5$	$F6$	$F7$	$F8$	R^2
A	0.36			0.046	0.091	-0.001	0.051	0.050	-0.035	0.037	-0.030	45.0
	(16.7)			(2.13)	(5.73)	(-0.06)	(2.51)	(3.14)	(-2.34)	(2.28)	(-1.84)	
B	0.34	0.114										37.7
	(17.5)	(5.35)										
C	0.34		0.052									7.9
	(13.4)		(1.91)									
D	0.36	0.076	-0.087	0.218	0.075	0.004	0.023	0.021	-0.030	0.031	-0.059	55.5
	(19.9)	(4.13)	(0.43)	(1.40)	(4.50)	(0.37)	(1.45)	(1.35)	(-2.10)	(2.29)	(4.90)	

Figure 1: Liquidity and Term structure factors
Factors from the AFENS model with liquidity. Panel (a) displays the liquidity factor. The scale is in dollar. Panel (b) displays the term structure factors. The scale is in percentage. End-of-month data from CRSP (1985:12-2007:12).



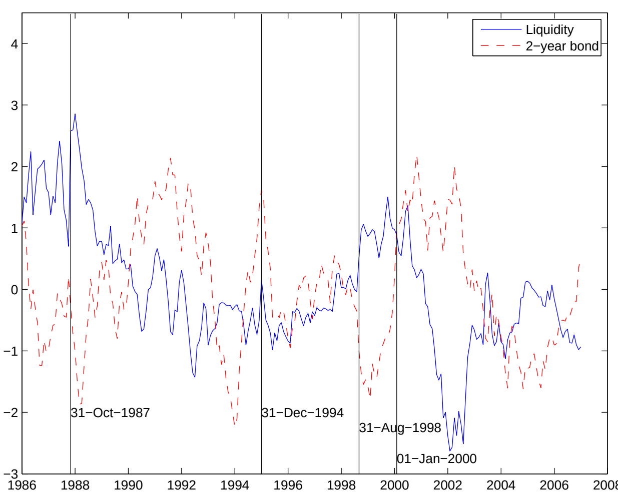
(a) Liquidity Factor



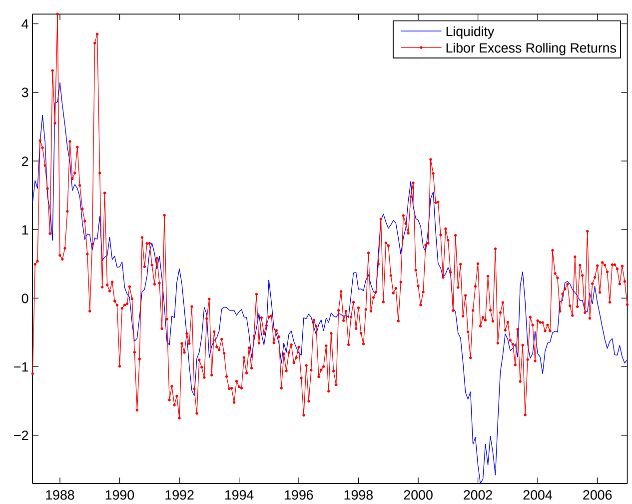
(b) Term Structure Factors

Figure 2: Excess returns and funding liquidity

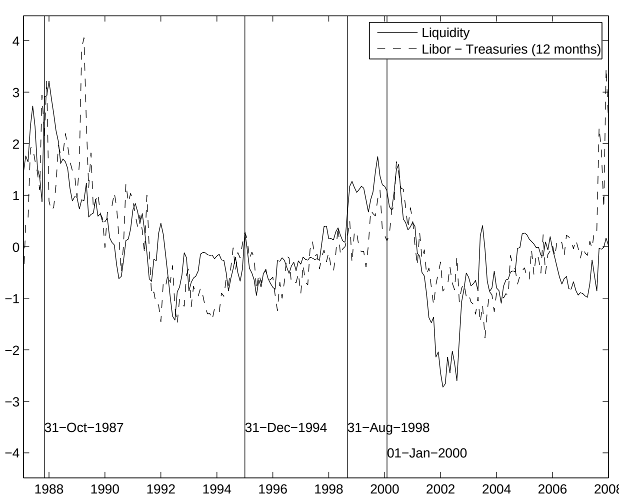
The liquidity factor and the risk premium in different markets. Panel (a) displays annual excess returns on 2-year off-the-run U.S. Treasury bonds. Panel (b) displays annual excess rolling returns on a 12-month LIBOR loan. Panel (d) displays the spread of the 1-year LIBOR rate above the off-the-run 1-year zero yield. Panel (d) displays the spread of the 5-year swap rate. Excess returns are computed above the off-the-run Treasury risk-free rate. End-of-month data from CRSP (1985:12-2007:12).



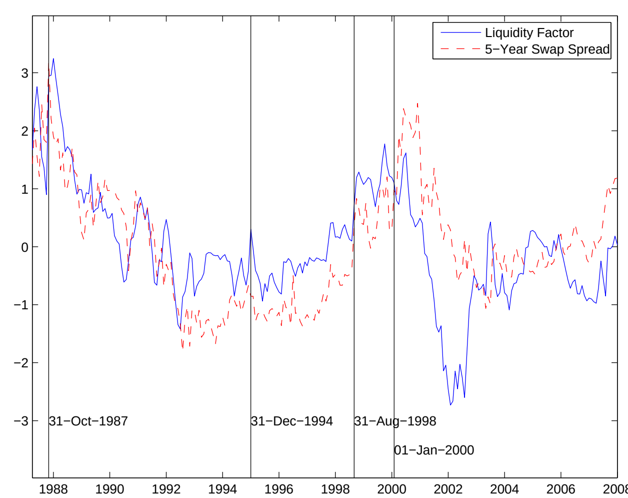
(a) Off-the-run excess returns



(b) LIBOR excess rolling returns



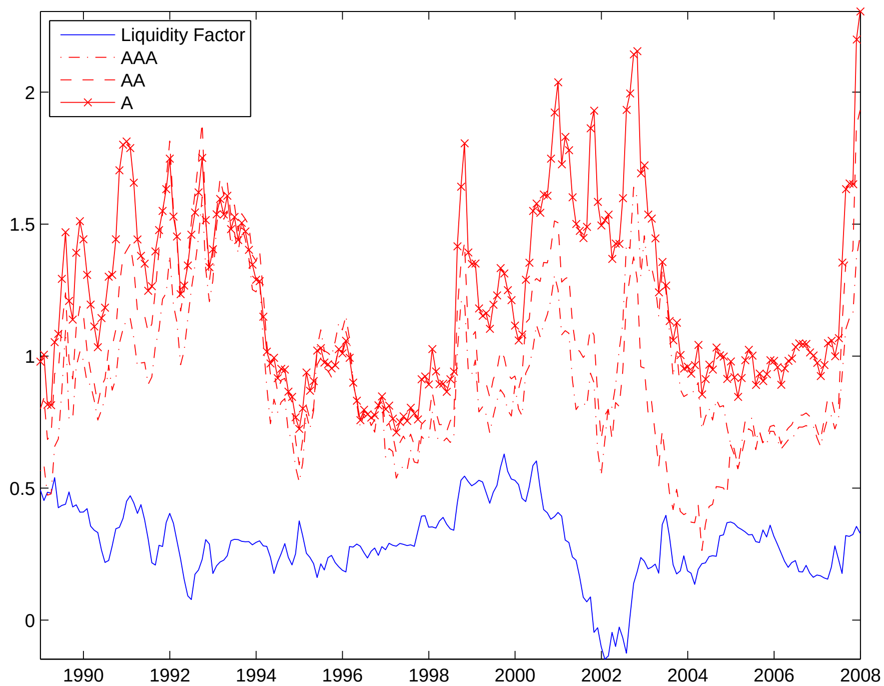
(c) 1-year LIBOR spread



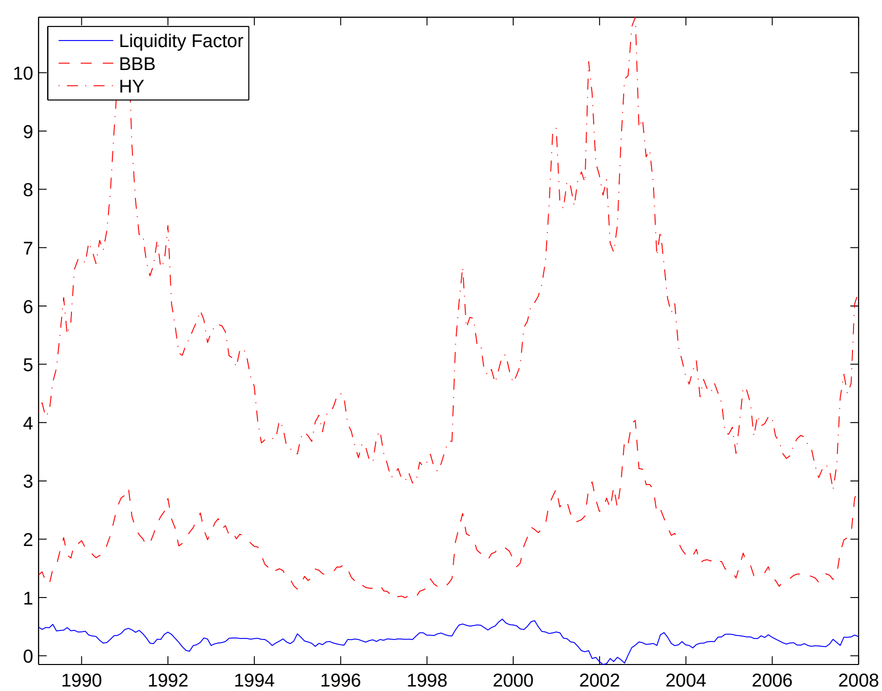
(d) 5-year swap spread

Figure 3: Corporate spread and funding liquidity

The liquidity factor with corporate bond spreads for different ratings. Panel (a) compares with the spreads of Merrill Lynch indices for high quality bonds: AAA, AA and A ratings. Panel (b) compares with the spread of Merrill Lynch BBB and High Yield corporate bond indices. Spreads are computed above the off-the-run 10-year Treasury par yield.



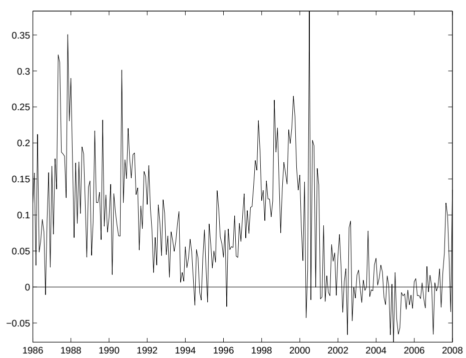
(a) Liquidity and Merrill Lynch AAA, AA, and A indices



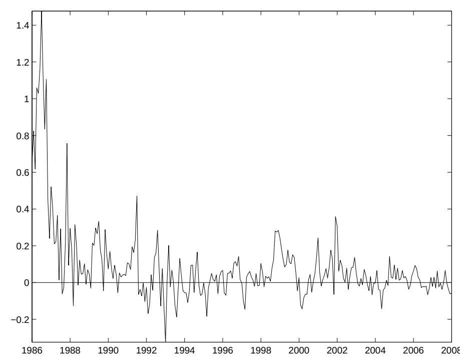
(b) Liquidity and Merrill Lynch BBB and High Yield indices

Figure 4: Residual Differences - Benchmark Model

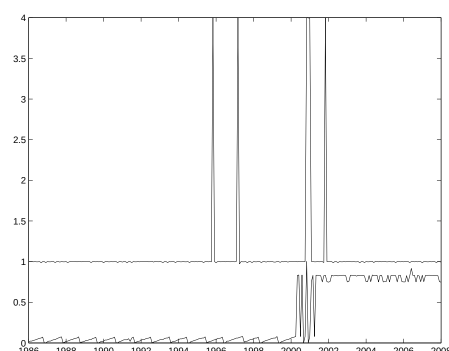
Comparison of residual differences and ages for the benchmark AFENS model without liquidity. Panel (a) presents differences between the residuals (dollars) of the on-the-run and off-the-run bonds in the 12-month category. Panel (b) presents the residuals 48-month category. Panel (c) and (d) displays years from issuance for the more recent and the seasoned bonds in the 12-month and the 48-month category, respectively. End-of-month data from CRSP (1985:12-2007:12).



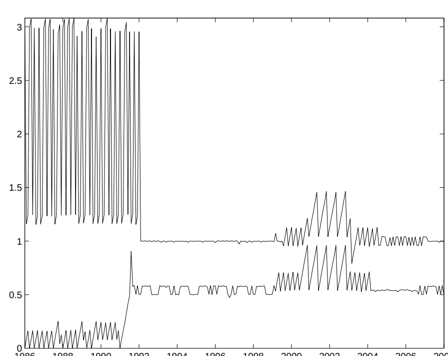
(a) 12-month residual difference



(b) 48-month residual difference



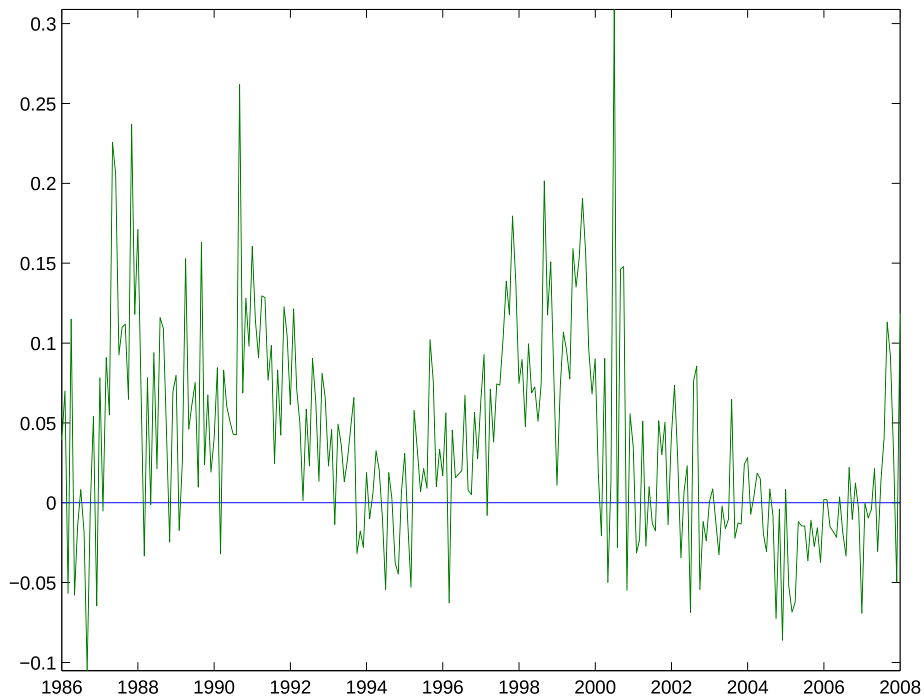
(c) 12-month ages



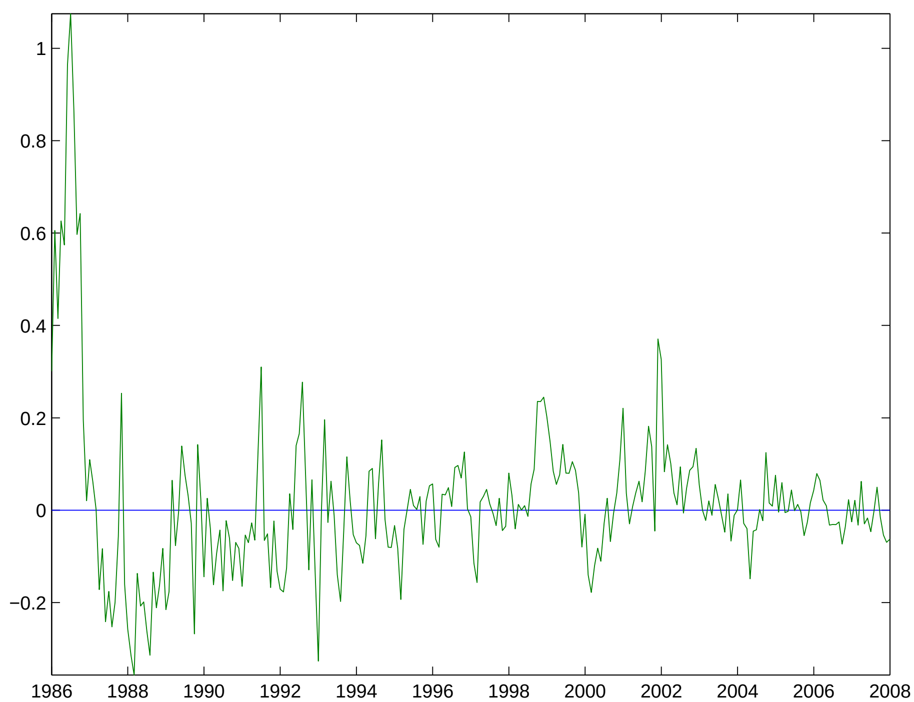
(d) 48-month ages

Figure 5: Residual differences - Liquidity Model

Comparison of residual differences for the AFENS model with liquidity. Panel (a) present differences between residuals (dollars) of on-the-run and off-the-run bonds in the 12-month category. Panel (b) presents differences between residuals (dollars) in the 48-month category. End-of-month data from CRSP (1985:12-2007:12).



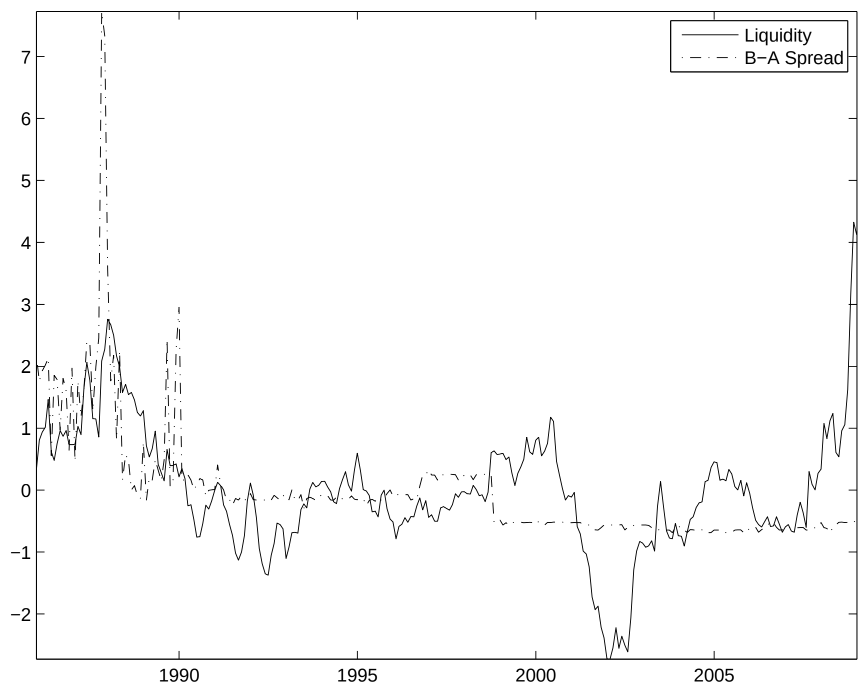
(a) 12-month residual differences



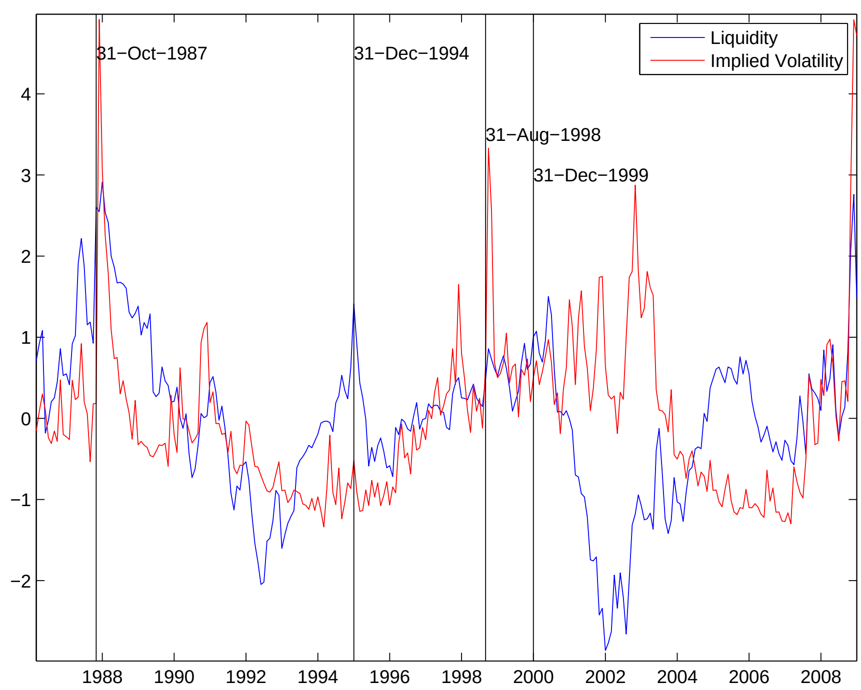
(b) 48-month residual differences

Figure 6: Determinants of Liquidity

Panel (a) traces the liquidity factor and the difference between the median and the minimum bid-ask spread at each observation date. Panel (b) traces the liquidity factor and implied volatility from S&P 500 call options. The liquidity factor is obtained from the AFENS model with liquidity. End-of-month data from CRSP (1985:12-2008:12)

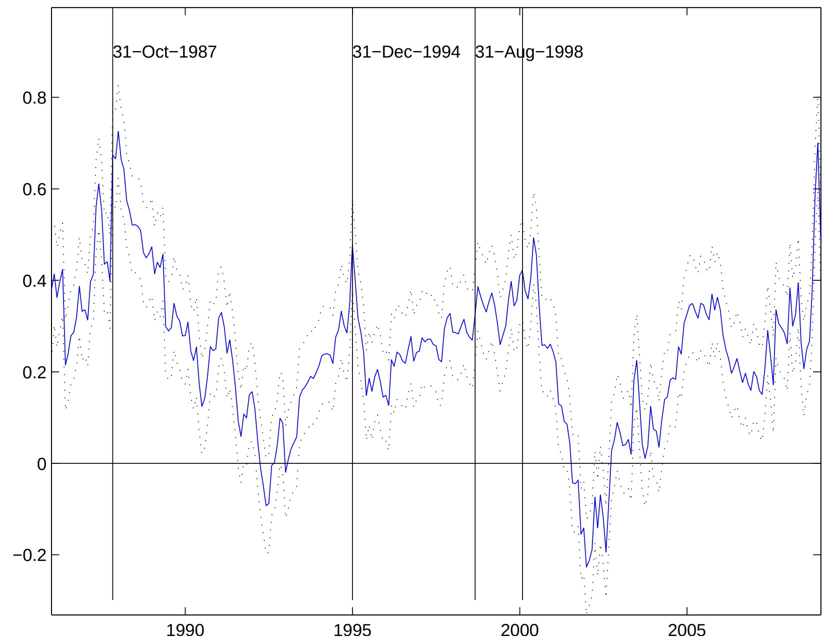


(a) Bid-Ask Spread and Liquidity

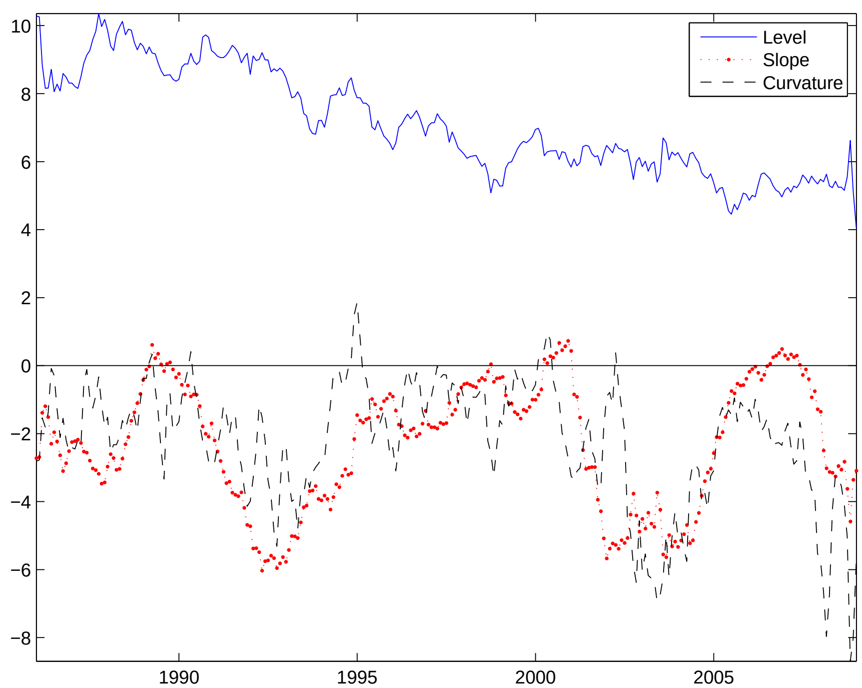


(b) Volatility and Liquidity

Figure 7: Liquidity and Term structure factors including 2008 Data
Factors from the AFENS model with liquidity. Panel (a) displays the liquidity factor. The scale is in dollar. Panel (b) displays the term structure factors. The scale is in percentage. End-of-month data from CRSP (1985:12-2008:12).



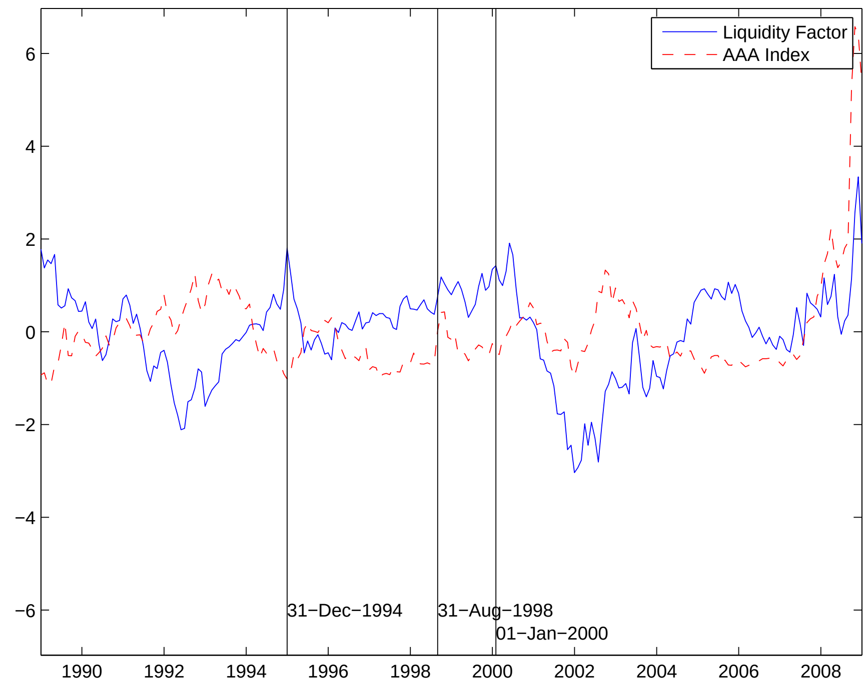
(a) Liquidity Factor



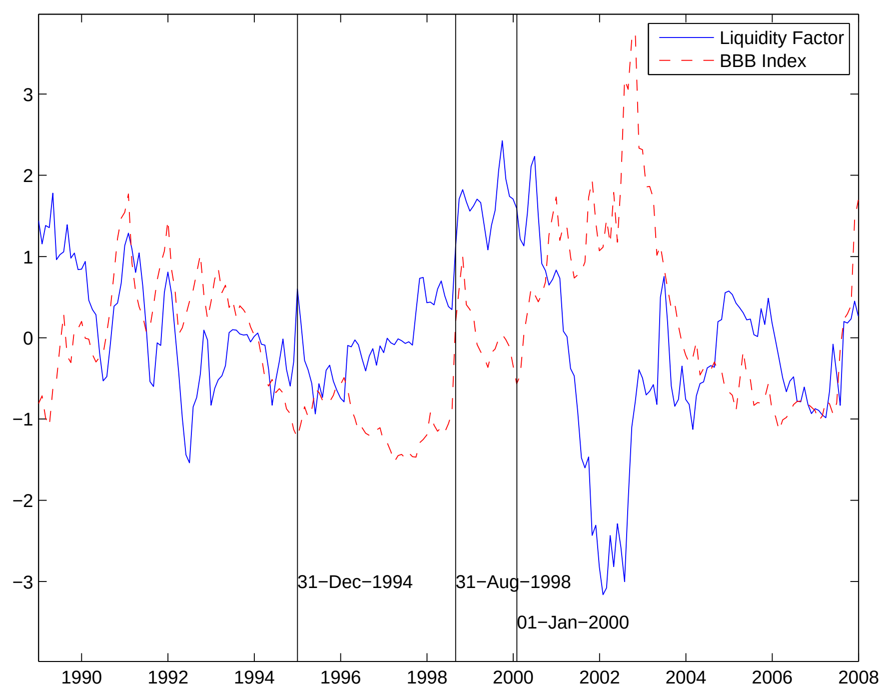
(b) Term Structure Factors

Figure 8: Corporate spread and funding liquidity including 2008 data

The liquidity factor with corporate bond spreads for different ratings. Panel (a) compares with the spreads of Merrill Lynch index for AAA bonds. Panel (b) compares with the spread of Merrill BBB corporate bond index. Spreads are computed above the off-the-run 10-year Treasury par yield. End-of-month data from CRSP and Merrill Lynch (1988:12-2008:12).



(a) Liquidity and Merrill Lynch AAA indiex



(b) Liquidity and Merrill Lynch BBB index